Status of PN and EOB calculations in modified theories of gravity

Laura Bernard

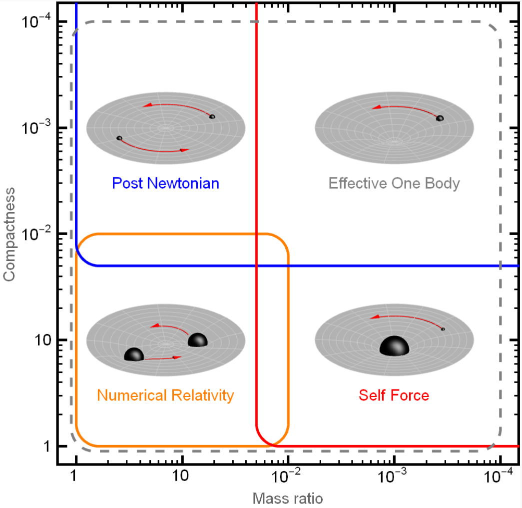
"Connecting the dots" workshop

AEI, Potsdam - June 14-16 2023

LUTH l'Observatoire de Paris | PSL Université Paris Cité cnrs

Starting point: GR

Gravitational wave modelling



Credits: M. van de Meent

- ▷ Synergies with *scattering amplitudes*

Starting point: general relativity

$$R^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu}\, R = 8\pi\, T^{\mu\nu}$$

- Defining $h^{\mu\nu} = \sqrt{-g}\, g^{\mu\nu} - \eta^{\mu\nu}$, we rewrite it as

$$\boxed{\Box_\eta\, h^{\mu\nu} = 16\pi\, |g|\, T^{\mu\nu} + \underbrace{\Lambda^{\mu\nu}}_{\propto\, h\partial^2 h + \partial h \partial h + \ldots} \;\equiv\; \tau^{\mu\nu}}$$

Matter field equations

- harmonic gauge $\partial_\nu\, h^{\mu\nu} = 0$ $\Leftrightarrow$ geodesic equation $\boxed{\nabla_\nu T^{\mu\nu} = 0}$

The basics of the PN formalism

$$h^{\mu\nu}(\mathbf{x},t) = \frac{16\pi G}{c^4}\left(\Box_{\text{ret}}^{-1}\tau^{\mu\nu}\right)(\mathbf{x},t)$$

Flat-space retarded propagator

$$\left(\Box_{\text{ret}}^{-1}\tau\right)(\mathbf{x},t) \equiv -\frac{1}{4\pi}\int_{\mathbb{R}^3}\frac{\mathrm{d}^3\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|}\,\tau\left(\mathbf{x}', t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}\right)$$

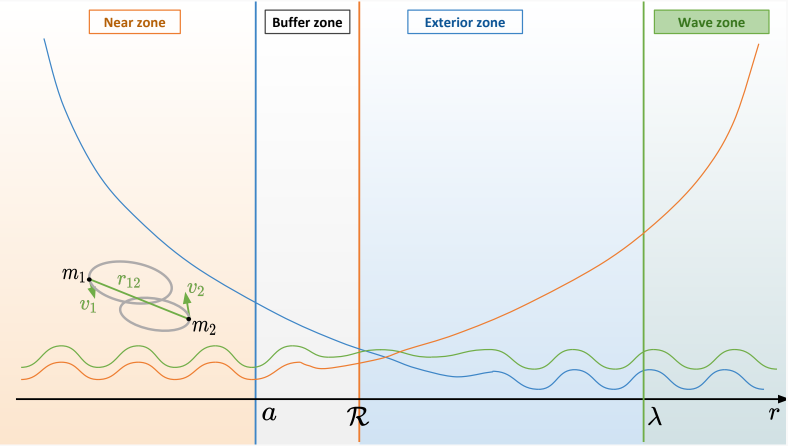
- near zone: ill-behaved when $r \gg \lambda_{\text{GW}}$:

$$\bar{h}^{\mu\nu} = \sum_{m=2}^{\infty}\frac{1}{c^m}\bar{h}^{\mu\nu}_m, \text{ with } \begin{cases} \Box\bar{h}^{\mu\nu}_m = 16\pi G\,\bar{\tau}^{\mu\nu}_m \text{ and } \tau^{\mu\nu} = \underbrace{T^{\mu\nu}}_{\text{source}} + \underbrace{\Lambda^{\mu\nu}}_{\text{nonlinearities}} \\ \partial_\nu \bar{h}^{\mu\nu}_m = 0 \end{cases}$$

- wave zone: ill-behaved when $r \to 0$:

$$h^{\alpha\beta}_{\text{ext}} = \sum_{n=1}^{\infty} G^n h^{\alpha\beta}_{(n)}, \text{ with } \begin{cases} \Box h^{\alpha\beta}_{(n)} = \Lambda^{\alpha\beta}_n\left[h_{(1)},\ldots,h_{(n-1)}\right] \\ \partial_\beta h^{\alpha\beta}_{(n)} = 0 \end{cases}$$

The multipolar PM - PN algorithm



Motion

Propagation

Generation

Reaction

State-of-the-art in GR - dynamics

	Dynamics				
PN order	non-spinning	spinning			tides
		SO	SS	higher spins	
0	$\surd$	-	-	-	-
1	$\surd$	-	-	-	-
1.5	-	$\surd$	-	-	-
2	$\surd$	-	$\surd$	-	-
2.5	$\surd$	$\surd$	-	-	-
3	$\surd$	-	$\surd$	-	-
3.5	$\surd$	$\surd$	-	$\surd$ (S^3)	-
4	$\surd$	-	$\surd$	$\surd$ (S^4)	-
4.5	$*$	$\surd$	-	$\surd$ (S^3)	-
5	$*$	-	$\surd$	$\surd$ (S^4)	$\surd$
5.5	$*$			$\surd$ (S^5)	-
6				$\surd$ (S^6)	$\surd$ (7PN)

State-of-the-art in GR - flux and GW modes

	Dissipative flux			
PN order	non-spinning	spinning		
		SO	SS	higher spins
2.5	$\surd$	-	-	-
3	-	-	-	-
3.5	$\surd$	-	-	-
4	$\surd$	$\surd$	-	-
4.5	$\surd$	-	$\surd$	-
5	$\surd$	$\surd$	-	-
5.5	$\surd$	$\surd$	$\surd$	-
6	$\surd$	$\surd$	$\surd$	$\surd$ (S^3)
6.5	$*$	$\surd$	$\surd$	
7	$\surd$			

- Tidal effects at 2.5PN beyond leading order
- Small eccentricity at 4PN

Going beyond GR

Adding one scalar degree of freedom

Generalised Brans-Dicke theory

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\mathcal{G}(\phi)\, R - \frac{1}{2} \nabla_a \varphi \nabla^a \varphi - \mathcal{V}(\varphi) \right] + S_m[\psi_\mathrm{m};\, g]$$

Simplifying: minimally coupled

$$S_{\mathrm{ST}} = \frac{c^3}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left[R - \frac{1}{2}g^{\alpha\beta}\partial_\alpha\varphi\,\partial_\beta\varphi\right] + S_m\left[\mathfrak{m}, A(\varphi)\,g_{\alpha\beta}\right]$$

Field equations in Einstein frame

$$\begin{cases} \Box h^{\mu\nu} = 16\pi G\,\bar{\tau}_m^{\mu\nu} \text{ with } \tau^{\mu\nu} = \underbrace{T^{\mu\nu}}_{\text{source}} + \underbrace{\Lambda^{\mu\nu}}_{\nabla h\cdot\nabla h} + \underbrace{\Lambda_{\mathrm{ST}}^{\mu\nu}}_{\sim\nabla\varphi\cdot\nabla\varphi} \\ \Box\varphi = 16\pi G\,\bar{\tau}_{\mathrm{s}} \text{ with } \tau_{\mathrm{s}} = \underbrace{T_{\mathrm{S}}}_{\text{source}} + \underbrace{\Sigma_{\mathrm{S}}}_{\sim h\partial^2\varphi+\partial h\partial\varphi+\nabla\varphi\cdot\nabla\varphi} \end{cases}$$

- PN expansion for $\varphi \sim h^{00}$
- no hair theorem but **scalarized neutron stars**
- a good starting point for more complicated theories
  - Einstein-scalar-Gauss-Bonnet, Einstein-Maxwell-dilaton

Coupling to matter

Skeletonization [Eardley '75]

- Incorporate the internal structure of compact, self-gravitating bodies

$$S_{\rm m} = -c \sum_A \int \mathrm{d}\tau_A \, m_A(\varphi)$$

- Sensitivities: $s_A = \left.\frac{\mathrm{d} \ln m_A(\varphi)}{\mathrm{d} \ln \varphi}\right|_0$ + higher orders
  - related to the **scalar charge** $\alpha_A \propto 1 - 2 s_A$
  - no hair theorem for BHs: $s_A = 0.5$ ($\Longleftrightarrow \alpha_A = 0$)

Coupling to matter

Skeletonization [Eardley '75]

- Incorporate the internal structure of compact, self-gravitating bodies

$$S_{\mathrm{m}} = -c \sum_A \int \mathrm{d}\tau_A \, m_A(\varphi)$$

- Sensitivities: $s_A = \left.\frac{\mathrm{d}\ln m_A(\varphi)}{\mathrm{d}\ln \varphi}\right|_0$ + higher orders
  - related to the **scalar charge** $\alpha_A \propto 1 - 2s_A$
  - no hair theorem for BHs: $s_A = 0.5$ ($\iff \alpha_A = 0$)
- Can be calcuated by matching to a known solution
  - NSs: depends on the equation of state ($s_A \sim 0.2$)
  - scalarized black holes in other theories (EMD, EsGB) [Julié 2017, Julié-Berti 2019]

The equations of motion

$$\frac{\mathrm{d}\mathbf{v_1}}{\mathrm{d}t} = \underbrace{-\frac{G_{\text{eff}}\, m_2}{r_{12}^2}\mathbf{n_{12}} + \frac{\mathbf{A}_{1\text{PN}}}{c^2}}_{\text{conservative terms}} + \underbrace{\frac{\mathbf{A}_{1.5\text{PN}}}{c^3}}_{\text{rad. reac.}} + \underbrace{\frac{\mathbf{A}_{2\text{PN}}}{c^4}}_{\text{cons.}} + \underbrace{\frac{\mathbf{A}_{2.5\text{PN}}}{c^5}}_{\text{rad. reac.}}$$

$$+ \underbrace{\frac{\mathbf{A}^{\text{inst}}_{3\text{PN}}}{c^6}}_{\text{cons, local}}$$

Differences w.r.t. GR

- Dissipative effects start at 1.5PN (*v.s.* 2.5PN in GR)

[Mirshekari & Will 2013; LB 2018, 2019]

The equations of motion

$$\frac{\mathrm{d}\mathbf{v_1}}{\mathrm{d}t} = \underbrace{-\frac{G_{\mathrm{eff}}\, m_2}{r_{12}^2}\mathbf{n_{12}} + \frac{\mathbf{A}_{1\mathrm{PN}}}{c^2}}_{\text{conservative terms}} + \underbrace{\frac{\mathbf{A}_{1.5\mathrm{PN}}}{c^3}}_{\text{rad. reac.}} + \underbrace{\frac{\mathbf{A}_{2\mathrm{PN}}}{c^4}}_{\text{cons.}} + \underbrace{\frac{\mathbf{A}_{2.5\mathrm{PN}}}{c^5}}_{\text{rad. reac.}}$$

$$+ \underbrace{\frac{\mathbf{A}^{\mathrm{inst}}_{3\mathrm{PN}}}{c^6}}_{\text{cons, local}} + \underbrace{\frac{\mathbf{A}^{\mathrm{tail}}_{3\mathrm{PN}}}{c^6}}_{\text{cons, nonloc.}}$$

Differences w.r.t. GR

- Dissipative effects start at 1.5PN (*v.s.* 2.5PN in GR)
- A conservative scalar tail term at 3PN : $\mathbf{A}^{\mathrm{tail}}_{3\mathrm{PN}} \propto \int_{-\infty}^{+\infty} \frac{\mathrm{d}t'}{|t-t'|}\, I_{s\,i}^{(4)}(t')$

[Mirshekari & Will 2013; LB 2018, 2019]

The equations of motion

$$\frac{\mathrm{d}\mathbf{v_1}}{\mathrm{d}t} = \underbrace{-\frac{G_{\text{eff}}\, m_2}{r_{12}^2}\mathbf{n_{12}} + \frac{\mathbf{A}_{1\text{PN}}}{c^2}}_{\text{conservative terms}} + \underbrace{\frac{\mathbf{A}_{1.5\text{PN}}}{c^3}}_{\text{rad. reac.}} + \underbrace{\frac{\mathbf{A}_{2\text{PN}}}{c^4}}_{\text{cons.}} + \underbrace{\frac{\mathbf{A}_{2.5\text{PN}}}{c^5}}_{\text{rad. reac.}}$$

$$+ \underbrace{\frac{\mathbf{A}^{\text{inst}}_{3\text{PN}}}{c^6}}_{\text{cons, local}} + \underbrace{\frac{\mathbf{A}^{\text{tail}}_{3\text{PN}}}{c^6}}_{\text{cons, nonloc.}} + \underbrace{\frac{\mathbf{A}^{\text{tidal}}_{3\text{PN}}}{c^6}}_{\text{cons, local}}$$

Differences w.r.t. GR

- Dissipative effects start at 1.5PN (*v.s.* 2.5PN in GR)
- A conservative scalar tail term at 3PN : $\mathbf{A}^{\text{tail}}_{3\text{PN}} \propto \int_{-\infty}^{+\infty} \frac{\mathrm{d}t'}{|t-t'|} I_{s\,i}^{(4)}(t')$
- Tidal effects start at 3PN (*v.s.* 5PN in GR)

[Mirshekari & Will 2013; LB 2018, 2019]

The scalar and gravitational fluxes

$$\mathcal{F} = \frac{32c^5\nu^2x^5}{5G_{\text{eff}}}\left[1 + \frac{\mathcal{F}^{\text{grav}}_{\text{1PN}}}{c^2} + \frac{\mathcal{F}^{\text{grav}}_{\text{1.5PN}}}{c^3} + \frac{\mathcal{F}^{\text{grav}}_{\text{2PN}}}{c^4}\right] + \frac{4c^5\nu^2x^4}{3G_{\text{eff}}}\,\zeta S_-^2 \cdot \frac{\mathcal{F}^{\text{scal, tidal}}_{\text{2PN}}}{c^4}$$

$$+ \frac{4c^5\nu^2x^5}{3G_{\text{eff}}}\,\zeta S_-^2\left[x^{-1} + \frac{\mathcal{F}^{\text{scal}}_{\text{0PN}}}{c^0} + \frac{\mathcal{F}^{\text{scal}}_{\text{0.5PN}}}{c^1} + \frac{\mathcal{F}^{\text{scal}}_{\text{1PN}}}{c^2} + \frac{\mathcal{F}^{\text{scal}}_{\text{1.5PN}}}{c^3}\right]$$

$$x \equiv \left(\frac{G_{\text{eff}}\,m\omega}{c^3}\right)^{2/3},\ \nu \equiv \frac{m_1 m_2}{m^2}$$

Differences w.r.t. GR

- Scalar flux starts at -1PN, known at 1.5PN: **2PN in progress**

[Lang 2013, LB, Blanchet, Trestini 2022, LB, Trestini in prep.]

The scalar and gravitational fluxes

$$\mathcal{F} = \frac{32c^5\nu^2x^5}{5G_{\text{eff}}}\left[1 + \frac{\mathcal{F}^{\text{grav}}_{\text{1PN}}}{c^2} + \frac{\mathcal{F}^{\text{grav}}_{\text{1.5PN}}}{c^3} + \frac{\mathcal{F}^{\text{grav}}_{\text{2PN}}}{c^4}\right] + \frac{4c^5\nu^2x^4}{3G_{\text{eff}}}\zeta S_-^2 \cdot \frac{\mathcal{F}^{\text{scal, tidal}}_{\text{2PN}}}{c^4}$$

$$+ \frac{4c^5\nu^2x^5}{3G_{\text{eff}}}\zeta S_-^2\left[x^{-1} + \frac{\mathcal{F}^{\text{scal}}_{\text{0PN}}}{c^0} + \frac{\mathcal{F}^{\text{scal}}_{\text{0.5PN}}}{c^1} + \frac{\mathcal{F}^{\text{scal}}_{\text{1PN}}}{c^2} + \frac{\mathcal{F}^{\text{scal}}_{\text{1.5PN}}}{c^3} + \frac{\mathcal{F}^{\text{scal}}_{\text{2PN}}}{c^4}\right]$$

$$x \equiv \left(\frac{G_{\text{eff}}\, m\omega}{c^3}\right)^{2/3},\ \nu \equiv \frac{m_1 m_2}{m^2}$$

Differences w.r.t. GR

- Scalar flux starts at -1PN, known at 1.5PN: **2PN in progress**
- Scalar tail term at 0.5PN: $\delta U_i^s \propto \frac{M}{c^3}\int \mathrm{d}\tau\, \ln\left(\frac{\tau}{\tau_0}\right) I_{s\,i}^{(3)}(t-\tau)$
- Scalar memory term at 1.5PN: $\delta U_{ij} \propto \frac{1}{c^3}\int \frac{\mathrm{d}t'}{|t-t'|} I_{s\,i}^{(2)}(t') I_{s\,j}^{(2)}(t')$

[Lang 2013, LB, Blanchet, Trestini 2022, LB, Trestini in prep.]

The scalar and gravitational fluxes

$$\mathcal{F} = \frac{32c^5\nu^2x^5}{5G_{\mathrm{eff}}}\left[1 + \frac{\mathcal{F}^{\mathrm{grav}}_{\mathrm{1PN}}}{c^2} + \frac{\mathcal{F}^{\mathrm{grav}}_{\mathrm{1.5PN}}}{c^3} + \frac{\mathcal{F}^{\mathrm{grav}}_{\mathrm{2PN}}}{c^4}\right] + \frac{4c^5\nu^2x^4}{3G_{\mathrm{eff}}}\,\zeta S_-^2 \cdot \frac{\mathcal{F}^{\mathrm{scal,\ tidal}}_{\mathrm{2PN}}}{c^4}$$

$$+ \frac{4c^5\nu^2x^5}{3G_{\mathrm{eff}}}\,\zeta S_-^2\left[x^{-1} + \frac{\mathcal{F}^{\mathrm{scal}}_{\mathrm{0PN}}}{c^0} + \frac{\mathcal{F}^{\mathrm{scal}}_{\mathrm{0.5PN}}}{c^1} + \frac{\mathcal{F}^{\mathrm{scal}}_{\mathrm{1PN}}}{c^2} + \frac{\mathcal{F}^{\mathrm{scal}}_{\mathrm{1.5PN}}}{c^3} + \frac{\mathcal{F}^{\mathrm{scal}}_{\mathrm{2PN}}}{c^4}\right]$$

$$x \equiv \left(\frac{G_{\mathrm{eff}}\,m\omega}{c^3}\right)^{2/3},\ \nu \equiv \frac{m_1 m_2}{m^2}$$

Differences w.r.t. GR

- Scalar flux starts at -1PN, known at 1.5PN: **2PN in progress**
- Scalar tail term at 0.5PN: $\delta U_i^s \propto \frac{M}{c^3}\int \mathrm{d}\tau\ \ln\left(\frac{\tau}{\tau_0}\right)\ I_{s}{}_i^{(3)}(t-\tau)$
- Scalar memory term at 1.5PN: $\delta U_{ij} \propto \frac{1}{c^3}\int \frac{\mathrm{d}t'}{|t-t'|}\,I_{s}{}_i^{(2)}(t')I_{s}{}_j^{(2)}(t')$
- Scalar tidal contribution at 2PN

[Lang 2013, LB, Blanchet, Trestini 2022, LB, Trestini in prep.]

Simple generalization

Massive scalar field

$$S = \int \mathrm{d}^4x\,\sqrt{-g}\,\left[R - \frac{1}{2}\nabla_a\varphi\nabla^a\varphi - V(\varphi)\right] \;+\; S_m[\psi_\mathrm{m};\, g]$$

- Yukawa decay $V_\varphi = [\dots]\,\mathrm{e}^{-m_s\,r}$; flux $P_s^{(\mathrm{dip})} \propto r^2\Omega^4\,\left(1 - \frac{m_s^2}{\Omega^2}\right)^{3/2}$
- 1PN results [Huang et al. 2018]

Tensor mutli-scalar gravity

$$S = \int \mathrm{d}^4x\,\sqrt{-g}\,\left[R - \frac{1}{2}f_{ab}\,\nabla_\mu\varphi^a\nabla^\mu\varphi^b - V(\phi_c)\right] \;+\; S_m[\psi_\mathrm{m};\, g]$$

- 2PN dynamics and 1PN flux [Damour & Esposito-Farèse 1992, 1996]
- 2.5PN* dynamics (* no matter description) [Schön & Doneva 2022]

Complexifying: beyond BD-like ST theories

Adding a Maxwell field

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[R - \frac{1}{2}\nabla_a\varphi\nabla^a\varphi - \mathrm{e}^{-2a\,\varphi}\,F^{ab}F_{ab}\right] + S_m[\psi_\mathrm{m};\, g]$$

Skeletonization

$$S_\mathrm{m} = -c\sum_A \int \mathrm{d}\tau_A\, m_A(\varphi) + \sum_A q_A \int A_\mu\, \mathrm{d}x_A^\mu$$

- matching to known BH solutions *e.g.* $m_A(\varphi) = \sqrt{\mu_A^2 + q_A^2\frac{\mathrm{e}^{2\varphi}}{2}}$
- Effective gravitational cst. to include electric charge $e_A \equiv \frac{q_A}{m_A}\mathrm{e}^{a\varphi_0}$
  - 1PN dynamics [Julié 2017]
  - 1PN flux [Julié 2018; Khalil et al. 2018]

Small coupling curvature corrections

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[R - \frac{1}{2}\nabla_a\varphi\nabla^a\varphi + \alpha\, f(\varphi)\, \mathcal{G} \right] + S_m[\psi_{\rm m};\, g]$$

Some examples with hairy BH solutions

- Einstein-scalar-Gauss-Bonnet

$$\mathcal{G} = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2$$

- Chern-Simmons gravity (parity-violating)

$$\mathcal{G} = \frac{1}{2}\,\epsilon^{\rho\sigma\alpha\beta}\, R_{\nu\mu\rho\sigma}\, R^{\mu\nu}{}_{\alpha\beta}$$

Small coupling curvature corrections (EsGB)

$$\frac{\mathrm{d}\mathbf{v_1}}{\mathrm{d}t} = \underbrace{\frac{\mathbf{A}_{3\mathrm{PN}}}{c^6}}_{\mathrm{ST}} + \underbrace{\alpha\, f'(\varphi)\frac{\mathbf{a}^{(\mathrm{LO})}}{c^2}}_{\text{new th.}}$$

- ▷ change in effective parameters, *i.e.* G_{eff}, *etc.*
- ▷ only leading order(LO) correction in α needed to get the full 3PN
  - ◦ dimension-full coupling constant $[\alpha] = [\mathrm{length}]^2$

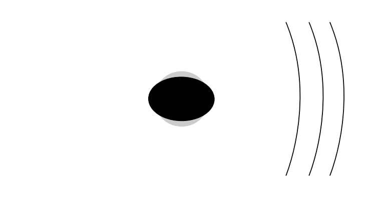
$$\zeta = \frac{\alpha}{M^2}$$

  - ◦ small α ($\sim \frac{1}{c^4}$) approximation $\longrightarrow$ 3PN correction
- ▷ LO dynamics and flux [Julié 2018, Julié & Berti 2019]

Adding specific effects

Scalar tidal effects

Response to an external scalar dipolar field

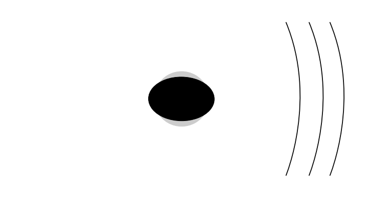


$$\mathcal{E}_i^{(s)} \sim \partial_i \varphi$$

$$U = \frac{M}{R} - \mathcal{E}_i^{(s)} x^i + \frac{\mathcal{Q}_i x^i}{r^3}$$

Scalar tidal effects

Response to an external scalar dipolar field



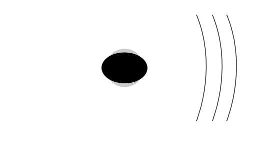
$$\mathcal{E}_i^{(s)} \sim \partial_i \varphi$$

$$U = \frac{M}{R} - \left(1 + \frac{\lambda_s}{r^3}\right) \mathcal{E}_i^{(s)} x^i$$

- adiabatic approximation: $\mathcal{Q}_\mu^{(s)} = -\lambda_{(s)} \mathcal{E}_\mu^{(s)}$
- dimensionfull scalar tidal deformability: $\left[\frac{G\,\lambda_s}{c^2}\right] = [\text{length}]^3$
- formally 3PN order correction with small ST parameters (GR: 5PN)

Scalar tidal effects

In the action



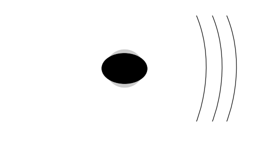
$$S_{\rm m} = S_{\rm pp} - \frac{1}{2}\sum_A \lambda_A^{(s)}(\phi)\int \mathrm{d}\tau_A \,\left(g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi\right)_A + \text{high. orders}$$

Consequence on the dynamics

$$\Delta\mathbf{a}_{(fs)} \propto \mathbf{a}_{(\mathrm{N})}\cdot\left[\tfrac{m_2}{m_1}\,\bar{\delta}_2\,\lambda_1^{(s)} + \tfrac{m_1}{m_2}\,\bar{\delta}_1\,\lambda_2^{(s)}\right]\frac{Gm}{c^2}\,\frac{1}{r^3}$$

Scalar tidal effects

In the action



$$S_{\mathrm{m}} = S_{\mathrm{pp}} - \frac{1}{2} \sum_A \lambda_A^{(s)}(\phi) \int \mathrm{d}\tau_A \left(g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi\right)_A + \text{high. orders}$$

Consequence on the dynamics

$$\Delta \mathbf{a}_{(fs)} \propto \mathbf{a}_{(\mathrm{N})} \cdot \left[\frac{m_2}{m_1} \bar{\delta}_2 \, k_1^{(s)} + \frac{m_1}{m_2} \bar{\delta}_1 \, k_2^{(s)} \right] \frac{R^3}{r^3}$$

- dimensionless scalar Love number: $k_s \equiv \frac{G \lambda_s}{c^2 R^3} \longrightarrow$ 3PN
  - LO dynamics and flux in ST theories [LB 2018] (NNLO [Dones, Mougiakakos & LB in prep.])
  - LO dynamics and flux in EsGB [van Gemeren et al. 2023]

Spins

Effective field theory description (as for GR)

$$S_{\mathrm{mat}} = \int \mathrm{d}\tau_A \left[p_\mu u^\mu + \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{m^2}\mathcal{C}_\varphi{}^* S_\nu^{\ \alpha} u^\nu \nabla_\mu \varphi \underbrace{-\frac{1}{6} J^{\gamma\nu\sigma\alpha} R_{\gamma\nu\sigma\alpha}}_{\text{quadrupole corr.}} \right]_A$$

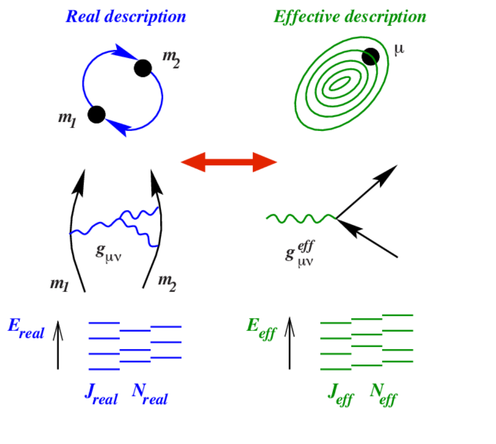
- obtained by doing $p_\mu \longrightarrow \mathcal{P}_\mu \left(u^\mu, \Omega^{\mu\nu}, g_{\mu\nu}, R_{\mu\nu\rho\sigma}, \varphi, \partial\varphi, \ldots\right)$
  - modified spin supplementary condition $S^{\mu\nu}\mathcal{P}_\nu = 0$
- LO (2PN) dynamics: SS, monopole-quadrupole, scalar-dipole [Loutrel et al. 2018]
- Corrections to the phase at 1.5PN (SO, SS) and 2PN (scalar dipole) [Loutrel et al. 2022]

Scalarizations

- PN results can apply to already (adiabatically) scalarized objects
  - scalar charges computed by matching to known solutions
- dynamical scalarization
  - effective description: see Mohammed Khalil's talk

Towards full IMR waveforms: EOB

The basics of EOB

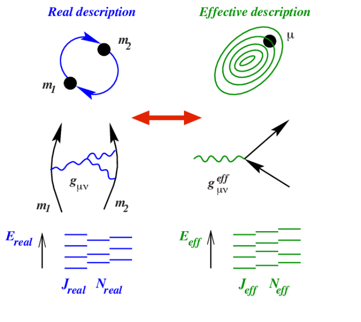


Credits: Buonanno & Sathyaprakash

- Canonical transformation: $(r, \phi, p_r, p_\phi) \longrightarrow (R, \Phi, P_R, P_\Phi)$

$$H_{2\text{body}} \longrightarrow H_{\text{eff}} \longrightarrow H_{EOB} = M\sqrt{1 + 2\nu\left(H_{\text{eff}} - 1\right)}$$

Current status



EOB Hamiltonian

- 2PN in simple ST theories [Julié 2017]
- 3PN [Julié et al. 2022; Jain et al. 2022]
  - includes non-local in time tail terms
  - extended to include LO contribution in EsGB

Conclusion

Final thoughts

- Many recent progresses in the simplest theories
- Synergies with numerical relativity results: full IMR
- Important for the future: non-perturbative effects (scalarization), spins, tides
- Missing theories or too preliminary: Lorentz-violating, DHOST, massive gravity, *etc.*
- A natural question: is it really usefull even for next generation detectors?
  - should we look only for striking signatures?