Analytical modeling of dynamical scalarization in an effective-action approach

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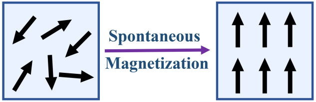
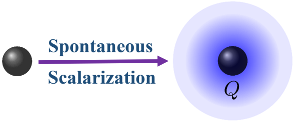
Based on
MK, R. Mendes, N. Ortiz, J. Steinhoff, Phys. Rev. D (2022)
MK, N. Sennett, J. Steinhoff, A. Buonanno, Phys. Rev. D (2019)

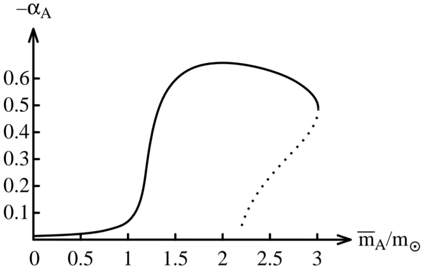
Connecting the dots workshop
AEI Potsdam
June 15, 2023

Spontaneous scalarization

- Spontaneous scalarization: a phase transition for a compact object in the strong-field due to a symmetry breaking of the scalar field. [Damour, Esposito-Farèse '93]







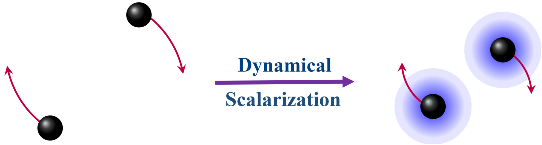
plot from [Damour, Esposito-Farèse arXiv:gr-qc/9602056]

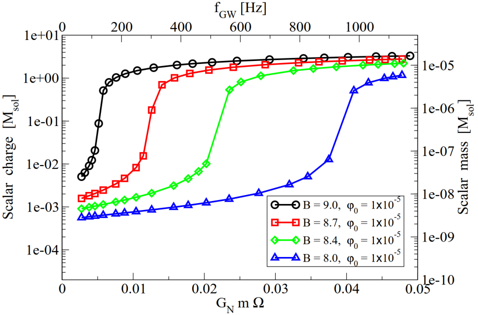
Theories that exhibit spontaneous scalarization

- Spontaneous scalarization can occur in scalar-tensor (ST) theories for NSs above a certain compactness
  - massless [Damour, Esposito-Farèse '93]
  - massive [Chen, Suyama, Yokoyama 1508.01384]
    [Ramazanoğlu, Pretorius 1601.07475]
  - Rotating NSs [Doneva, Yazadjiev, Stergioulas, Kokkotas 1309.0605]
- It can also occur in scalar Gauss-Bonnet theories for BHs and NSs
  - curvature induced [Doneva, Yazadjiev 1711.01187]
    [Silva, Sakstein, Gualtieri, Sotiriou, Berti 1711.02080]
  - spin induced [Dima, Barausse, Franchini, Sotiriou 2006.03095]
    [Herdeiro, Radu, Silva, Sotiriou, Yunes 2009.03904]
- Generalized ST theories (Horndeski action)
  [Andreou, Franchini, Ventagli, Sotiriou 1904.06365]
- Theories with vector, spinor, or tensor fields can exhibit vectorization, spinorization, or tensorization [Ramazanoğlu 1706.01056, 1804.00594, 1901.10009]

Dynamical scalarization

- Dynamical scalarization occurs during the coalescence of a binary system, above a certain compactness or spin. [Barausse, Palenzuela, Ponce, Lehner 1212.5053] [Shibata, Taniguchi, Okawa, Buonanno 1310.0627], [Elley, Silva, Witek, Yunes 2205.06240]





plot from [Taniguchi, Shibata, Buonanno 1410.0738]

Linear mode instability and nonlinear saturation

- Action for a ST theory

$$S = \int \mathrm{d}^4x \, \frac{\sqrt{-g}}{16\pi} \left(R - 2\partial_\mu \varphi \partial^\mu \varphi - f[g, \varphi^2, \ldots] \right)$$

- Scalar field equation

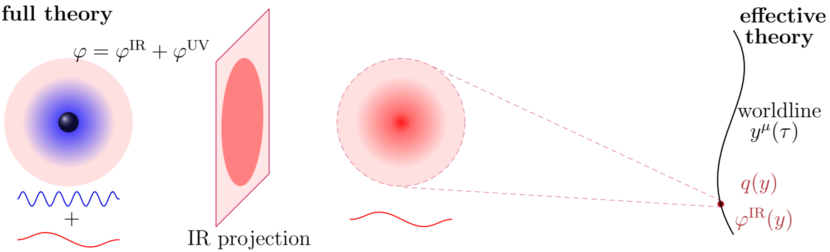
$$\Box \varphi = m_{\text{eff}}^2[g, \varphi^2, \ldots]\, \varphi, \qquad m_{\text{eff}}^2 \equiv \frac{1}{2} \frac{\partial f}{\partial \varphi^2}$$

- Dispersion relation $\omega^2 = \boldsymbol{k}^2 + m_{\text{eff}}^2$; if m_{eff}^2 is sufficiently negative, then ω^2 is also negative, leading to a tachyonic instability.

- Scalarized equilibrium configurations exist if the mode instability saturates in the nonlinear regime. [Ramazanoğlu 1710.00863]

Effective action for the scalarization phase transition

[**MK**, Sennett, Steinhoff, Buonanno 1906.08161]



- Split the fields of the full theory into small (UV) and long (IR) wavelength regimes, e.g. $\varphi = \varphi^{\mathrm{IR}} + \varphi^{\mathrm{UV}}$, and integrate out the UV parts.
- Dynamical variable $q(y)$ represents the monopolar scalar oscillation mode that becomes unstable at scalarization.
- Make an ansatz for an effective action respecting certain symmetries, in particular scalar-inversion symmetry ($\varphi^{\mathrm{IR}} \to -\varphi^{\mathrm{IR}}$, $q \to -q$), and including terms up to a given power in the cutoff between IR and UV scales, $\mathcal{O}(R/r)$.

Effective action close to the critical point

- Oscillator equation for the mode q driven by the IR field φ^{IR}

$$c_{\dot{q}^2}\,\ddot{q} + V'(q) = \varphi^{\mathsf{IR}} + \varphi_0$$

$$V(q) = \frac{c_{(2)}}{2!}q^2 + \frac{c_{(4)}}{4!}q^4 + \dots$$

φ_0 is a possible cosmological value of the scalar field.

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- Effective action close to the critical point

$$S^{\mathsf{eff}}_{\mathsf{CO}} = \int \mathrm{d}\tau \left[\frac{c_{\dot{q}^2}}{2}\dot{q}^2 + \varphi^{\mathsf{IR}} q - m(q) + \dots\right]$$
$$m(q) \equiv m_{(0)} + \varphi_0 q + \underbrace{\frac{c_{(2)}}{2!}q^2 + \frac{c_{(4)}}{4!}q^4 + \dots}_{V(q)}$$

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- Effective action close to the critical point

$$S_{\text{CO}}^{\text{eff}} = \int \mathrm{d}\tau \left[\frac{c_{\dot{q}^2}}{2}\dot{q}^2 + \varphi^{\text{IR}} q - m(q) + \dots\right]$$
$$m(q) \equiv m_{(0)} + \varphi_0 q + \underbrace{\frac{c_{(2)}}{2!}q^2 + \frac{c_{(4)}}{4!}q^4 + \dots}_{V(q)}$$

- Matching the asymptotic behavior of the IR and UV fields leads to

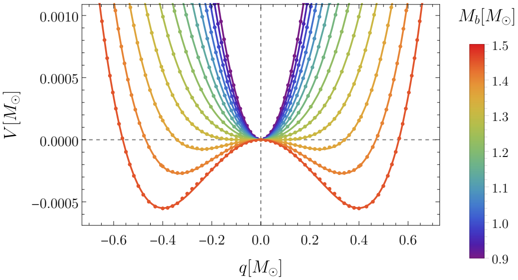
$$S_{\text{CO}} = -\int \mathrm{d}\tau\, m_E(\varphi)$$
$$m_E(\varphi^{\text{IR}}) = m(q) - \varphi^{\text{IR}} q \quad \longrightarrow \quad m'(q) = \varphi^{\text{IR}}$$

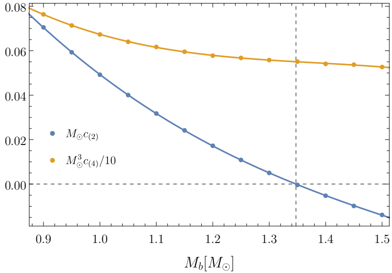
Scalarized solutions in a specific scalar-tensor theory

- Action for massless scalar-tensor theory [Damour, Esposito-Farése '93]

$$S_{\mathsf{ST}} = \int \mathrm{d}^4 x \, \frac{\sqrt{-g}}{16\pi} \left(R - 2\partial_\mu \varphi \partial^\mu \varphi\right) + S_m[\mathcal{A}^2(\varphi) g_{\mu\nu}], \qquad \mathcal{A}(\varphi) = \exp(\beta\varphi^2/2)$$

- Numerically solve the structure equations for an isolated object, computing a sequence of solutions with different values for φ_∞ at fixed baryon mass M_b.
- Determine $(c_{(2)}, c_{(4)})$ by fitting a polynomial to $m(q) = m_E(\varphi_\infty) + \varphi_\infty q$.





$V(q)$, $c_{(2)}$ and $c_{(4)}$ for DEF theory with $\beta = -5$.

[**MK**, Mendes, Ortiz, Steinhoff 2206.13233]

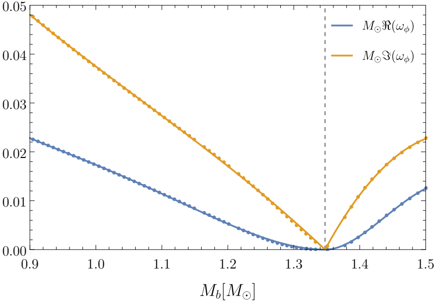
Matching the $c_{\dot{q}^2}$ coefficient

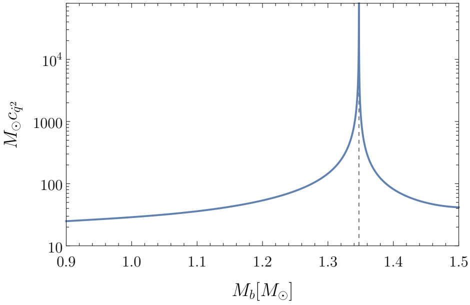
- Oscillator equation around the nonscalarized equilibrium $q = 0$, and taking $\varphi^{\mathrm{IR}} + \varphi_0 \simeq 0$,

$$\ddot{q} \simeq -\frac{c_{(2)}}{c_{\dot{q}^2}} q \equiv -\omega_0^2 q$$

- Identify ω_0^2 as the scalar quasi-normal mode frequency $|\omega_\varphi|^2$ [Mendes, Ortiz 1802.07847]

$$c_{\dot{q}^2} = \frac{c_{(2)}}{|\omega_\varphi|^2}$$





Hamiltonian and equations of motion for a binary

- Hamiltonian for a binary at leading order in the center-of-mass frame

$$H = m_A + m_B + \frac{\boldsymbol{p}^2}{2\mu} + \frac{p_{q,A}^2}{2c_{\dot{q}^2,A}} + \frac{p_{q,B}^2}{2c_{\dot{q}^2,B}} - \frac{m_A m_B}{r} - \frac{q_A q_B}{r}$$

$p_{q,A/B}$ are canonical conjugate variables to $q_{A/B}$.

- Approximate stable solution for q, assuming identical bodies and $\dot{q} \approx 0$

$$q = \begin{cases} 0 & \text{for} \quad r > 1/c_{(2)} \\ \pm\sqrt{\frac{6}{c_{(4)}}}\sqrt{\frac{1}{r} - c_{(2)}} & \text{for} \quad r < 1/c_{(2)} \end{cases}$$

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- Equations of motion with leading order RR force

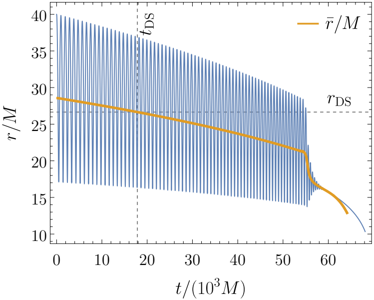
$$\dot{r} = \frac{\partial H}{\partial p_r}, \qquad \dot{p}_r = -\frac{\partial H}{\partial r} + \mathcal{F}_r^{\text{quad}} + \mathcal{F}_r^{\text{dip}},$$

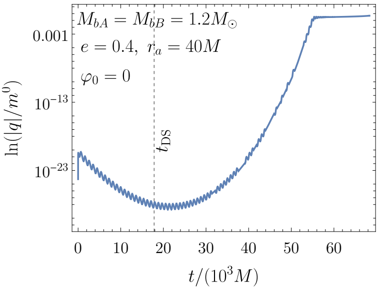
$$\dot{\phi} = \frac{\partial H}{\partial L}, \qquad \dot{L} = -\frac{\partial H}{\partial \phi} + \mathcal{F}_\varphi^{\text{quad}} + \mathcal{F}_\varphi^{\text{dip}},$$

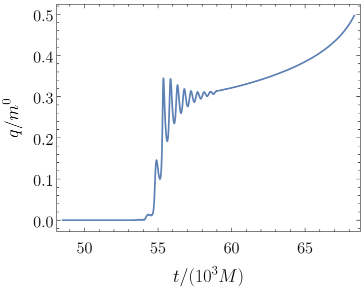
$$\dot{q}_A = \frac{\partial H}{\partial p_{q,A}}, \qquad \dot{p}_{q,A} = -\frac{\partial H}{\partial q_A} - \dot{q}_A - \dot{q}_B,$$

Dynamical scalarization of a binary in eccentric orbit

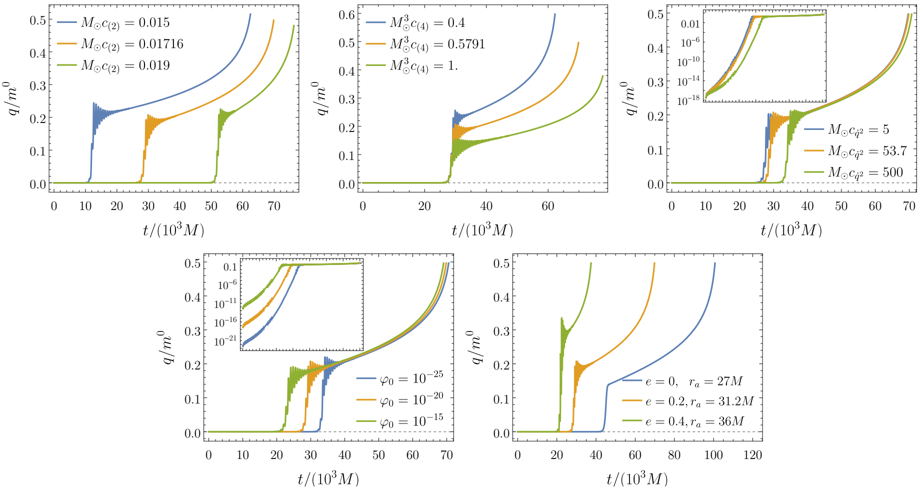
- Maximum separation allowing for dynamical scalarization $r_{\rm DS} \equiv 1/c_{(2)}$
- Scalar charge grows exponentially after $\bar{r}$ crosses $r_{\rm DS}$
- q grows as $1/\sqrt{r}$ after saturation





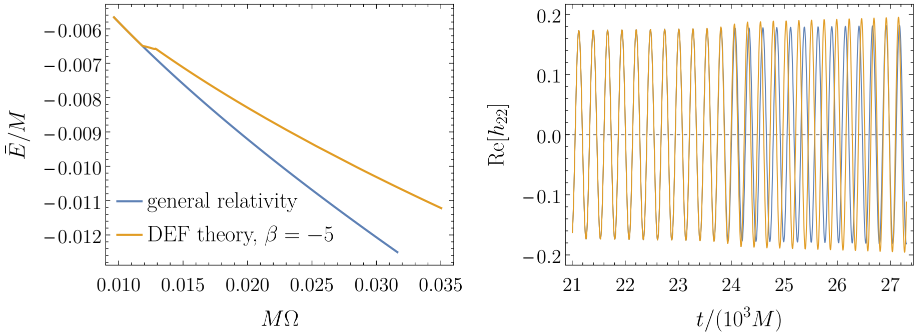


Effect of the effective-action parameters on the scalar charge



configurations with baryon mass $M_{bA} = M_{bB} = 1.2 M_\odot$,
one parameter is varied in each panel

Binding energy and waveform around scalarization

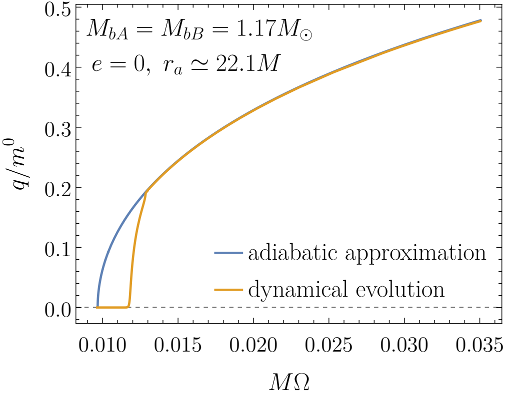


configuration with baryon mass $M_{bA} = M_{bB} = 1.17 M_{\odot}$,
in quasicircular inspiral with initial separation $\sim 22M$

Conclusions

- Developed a simple model for spontaneous and dynamical scalarization, based on an effective-action approach.
- The model is theory agnostic and captures the dynamical evolution around the scalarization phase transition.
- This approach can be used in theory-specific or theory-agnostic tests, and used to develop waveform models for dynamical scalarization.

Quasi-stationary versus dynamical evolution



Binary in a highly-eccentric orbit