

*Challenging GR with future detectors: the case of **3G** ground based interferometers*

@ Connecting the dots

Albert Einstein Institute, 16th June 2023



Andrea Maselli

3G

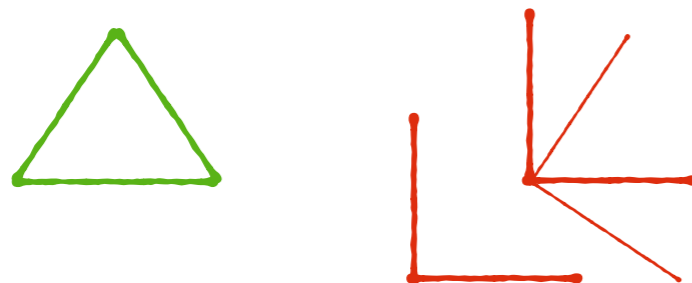
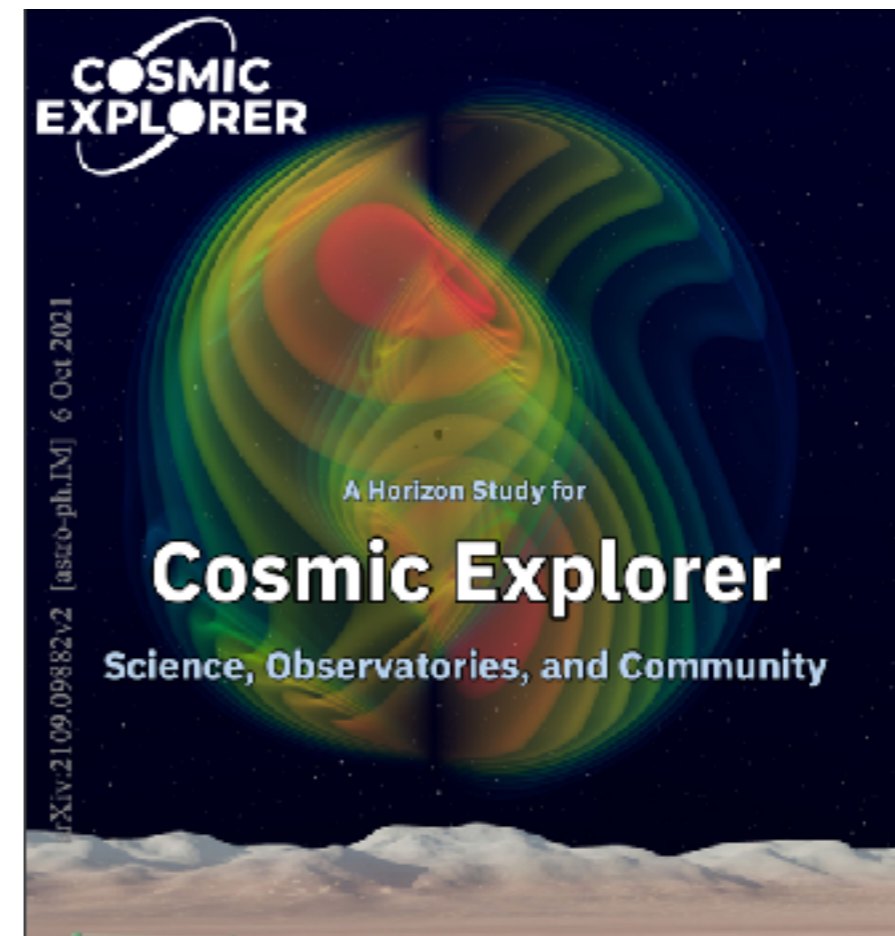
Latest on the 3G design performances

M. Branchesi + arXiv gr-qc: 2303.15923

Science with the Einstein Telescope: a comparison of different designs

Marica Branchesi,^{1,2} Michele Maggiore,^{3,4} David Alonso,⁵ Charles Badger,⁶ Biswajit Banerjee,^{1,2} Freija Beirnaert,⁷ Swetha Bhagwat,^{8,9} Guillaume Boileau,^{10,11} Ssohrab Borhanian,¹² Daniel David Brown,¹³ Man Leong Chan,¹⁴ Giulia Cusin,^{15,3,4} Stefan L. Danilishin,^{16,17} Jerome Degallaix,¹⁸ Valerio De Luca,¹⁹ Arnab Dhani,²⁰ Tim Dietrich,^{21,22} Ulyana Dupletsa,^{1,2} Stefano Foffa,^{3,4} Gabriele Franciolini,⁸ Andreas Freise,^{23,26} Gianluca Gemme,²⁴ Boris Goncharov,^{1,2} Archisman Ghosh,⁷ Francesca Gulminelli,²⁵ Ish Gupta,²⁹ Pawan Kumar Gupta,^{16,26} Jan Harms,^{1,2} Nandini Hazra,^{1,2,27} Stefan Hild,^{16,17} Tanja Hinderer,²⁸ Ik Siong Heng,²⁹ Francesco Iacovelli,^{3,4} Justin Janquart,^{16,28} Kamiel Janssens,^{10,11} Alexander C. Jenkins,³⁰ Chinmay Kalaghatgi,^{16,26,31} Xhesika Koroveshi,^{32,33} Tjonnie G. F. Li,^{34,35} Yufeng Li,³⁶ Eleonora Loffredo,^{1,2} Elisa Maggio,²² Michele Mancarella,^{3,4,37,38} Michela Mapelli,^{39,40,41} Katarina Martinovic,⁶ Andrea Maselli,^{1,2} Patrick Meyers,⁴² Andrew L. Miller,^{43,16,26} Chiranjib Mondal,²⁵ Niccolò Muttoni,^{3,4} Harsh Narola,^{16,26} Micaela Oertel,⁴⁴ Gor Oganessian,^{1,2} Costantino Pacilio,^{8,37,38} Cristiano Palomba,⁴⁵ Paolo Pani,⁸ Antonio Pasqualetti,⁴⁶ Albino Perego,^{47,48} Carole Pérois,^{39,40,41} Mauro Pieroni,^{49,50} Ornella Juliana Piccinni,⁵¹ Anna Puecher,^{16,26} Paola Puppo,⁴⁵ Angelo Ricciardone,^{52,39,40} Antonio Riotto,^{3,4} Samuele Ronchini,^{1,2} Mairi Sakellariadou,⁹ Anuradha Samajdar,²¹ Filippo Santoliquido,^{39,40,41} B.S. Sathyaprakash,^{20,53,54} Jessica Steinlechner,^{16,17} Sebastian Steinlechner,^{16,17} Andrei Utina,^{16,17} Chris Van Den Broeck,^{16,26} and Teng Zhang^{9,17}

M. Evans + arXiv gr-qc: 2109.09882



The Inspiral

Inspiral tests of gravity are the most easy/common/safe tool used so far

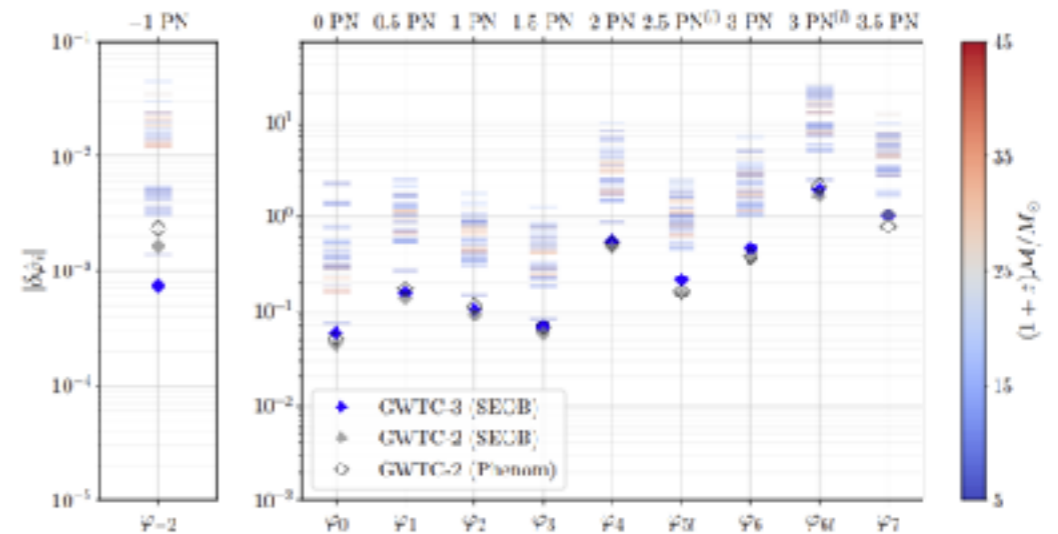
- easiest implementation of the two flavours



- changes into binary's binding energy/flux

→ GW signal modification

- accumulate with many cycles
- perturbatively included within pN



Abbott +, PRL 2112.06861 (2021)

$$\tilde{h}(f) = \mathcal{A}(f)e^{i\phi(f)}$$

$$\phi(f) \longrightarrow \phi(f) + \beta(\mathcal{M}\pi f)^{(2\gamma-5)/3}$$

amplitude ← type →

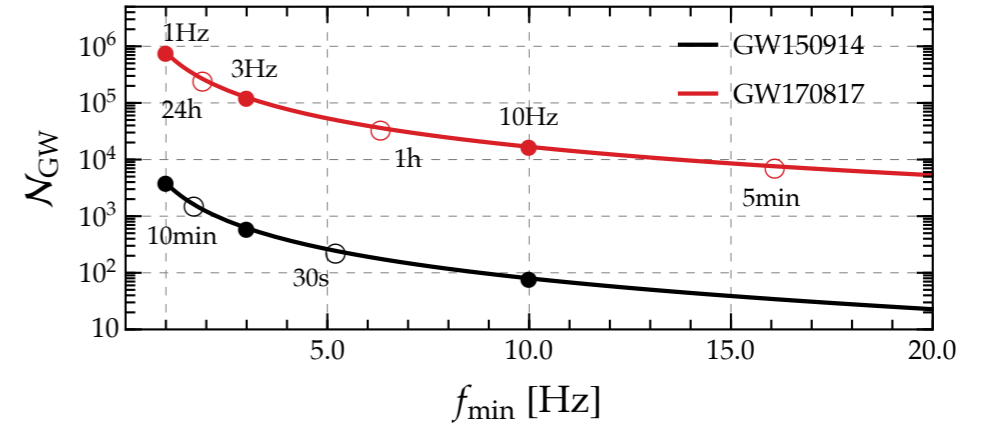
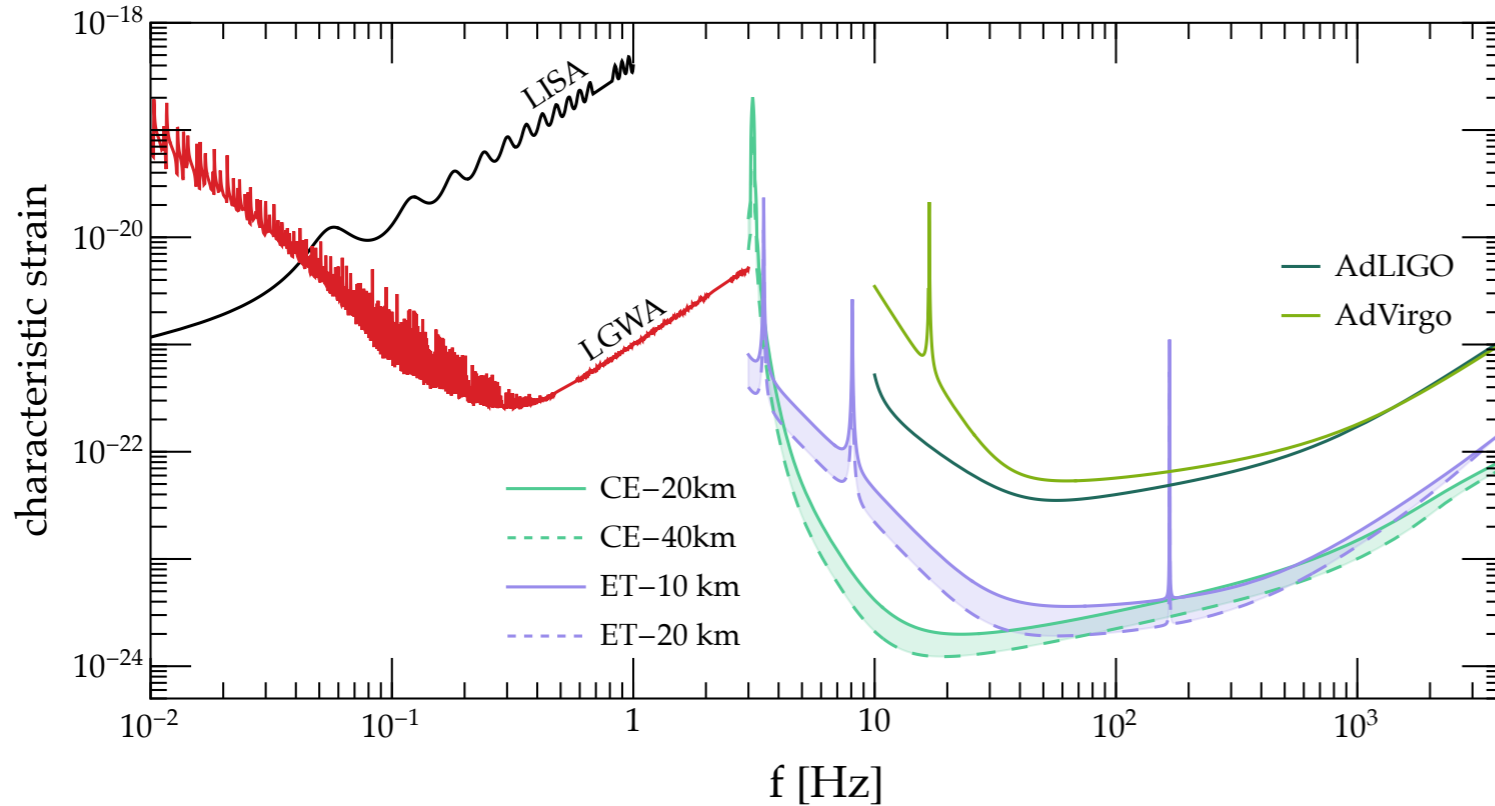
(γ, β)

can be potentially mapped to "specific" theories

The Inspiral

Deeper and wider in frequencies

J. Harms +, Astrophys. J. 910, 1 (2021)

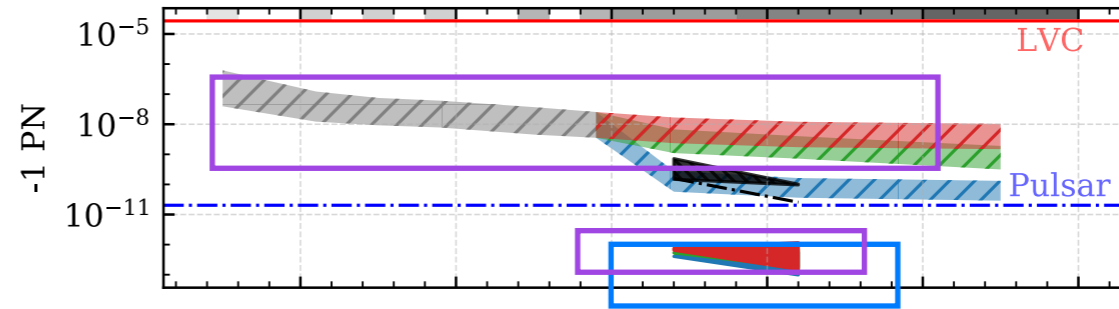
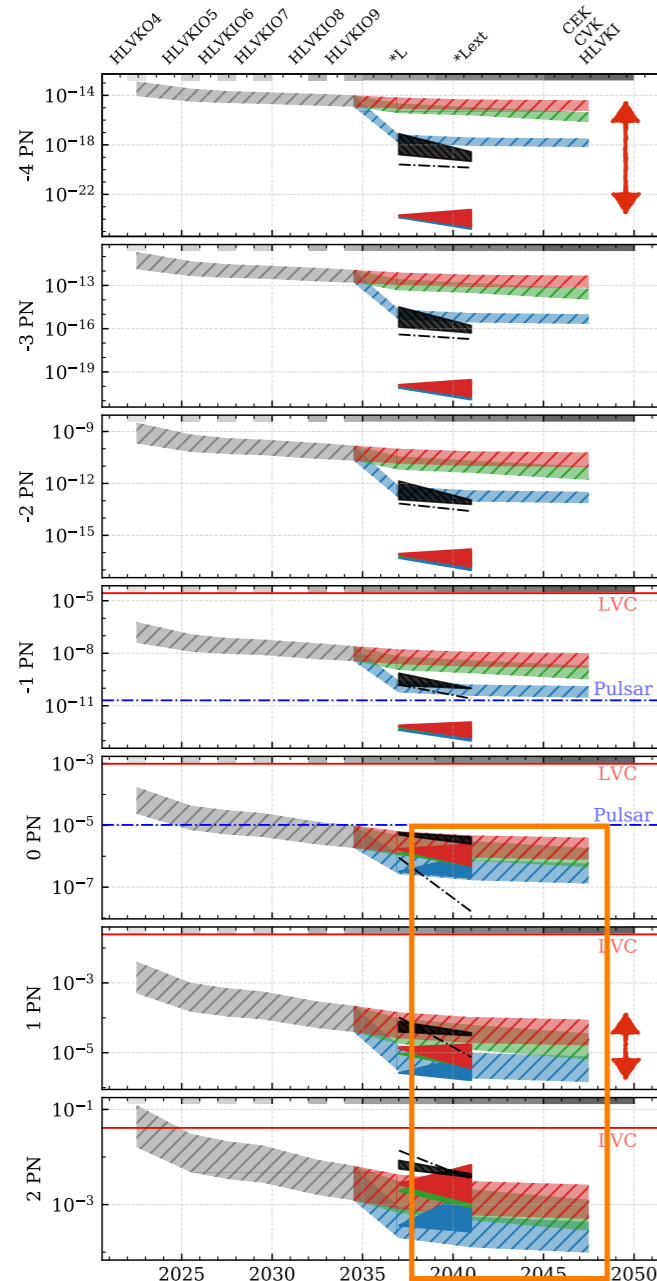


- opportunity for pre-Newtonian effects, working at low frequencies (e.g. dipole, but also environmental)
- multi-band observations (IMRI potentially surfing all frequency bands)
- More asymmetric binaries

The Inspiral

Constraints from 3G detectors

S. Perkins+, PRD 103, 044024 (2021)



- sources in multi-band do the best at negative pN
- stellar mass sources for 3G worse than massive for LISA at negative pN
- stellar-mass sources for 3G potentially better than massive for LISA at positive pN
- ground based upgrades (don't) play a role for (positive) negative pN



what do we learn?

The Inspiral

Map to specific theories of gravity

$$\phi(f) \longrightarrow \phi(f) + \beta(\mathcal{M}\pi f)^{(2\gamma-5)/3}$$

Theory or physical process	Physical modification	G/P	PN order	β	Theory parameter	$2\gamma - 5$
Generic dipole radiation	Dipole radiation	G	-1	(B2)	$\delta\dot{E}$	-7
Einstein-dilaton Gauss-Bonnet	Dipole radiation	G	-1	(B3)	$\sqrt{\alpha_{\text{EdGB}}}$	-7
Black Hole Evaporation	Extra dimensions	G	-4	(B6)	\dot{M}	-13
Time varying G	LPI	G	-4	(B7)	\dot{G}	-13
Massive Graviton	Nonzero graviton mass	P	1	(B11)	m_g	-3
dynamical Chern-Simons	Parity violation	G	2	(B8)	$\sqrt{\alpha_{\text{dCS}}}$	-1
Noncommutative gravity	Lorentz violation	G	2	(B10)	$\sqrt{\Lambda}$	-1

- same type may correspond to different physical effect when mapped to a given theory beyond GR
 - coupling's theory can depend on the source's properties
 - actual constraints can change due to scalings and/or correlations

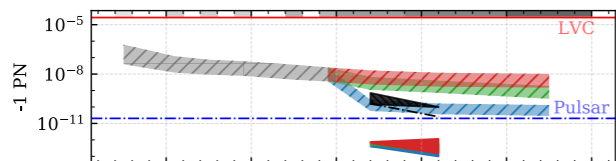
The Inspiral

Gauss Bonnet

$$\beta_{\text{EdGB}} = -\frac{5}{7168} \frac{\zeta_{\text{EdGB}} (m_1^2 s_2^{\text{EdGB}} - m_2^2 s_1^{\text{EdGB}})^2}{\eta^{18/5} m^4}$$

$$s_i^{\text{EdGB}} = \frac{2 \left[(1 - \chi_i^2)^{1/2} - 1 + \chi_i^2 \right]}{\chi_i^2},$$

$$\zeta_{\text{EdGB}} \sim \alpha_{\text{EdGB}}^2 / m^4$$



Chern Simons

$$\beta_{\text{dCS}} = -\frac{5}{8192} \frac{\zeta_{\text{dCS}} (m_1 s_2^{\text{dCS}} - m_2 s_1^{\text{dCS}})^2}{\eta^{14/5} m^2}$$

$$+ \frac{15075}{114688} \frac{\zeta_{\text{dCS}} (m_2^2 \chi_1^2 - \frac{350}{201} m_1 m_2 \chi_1 \chi_2 + m_1^2 \chi_2^2)}{m^2}$$

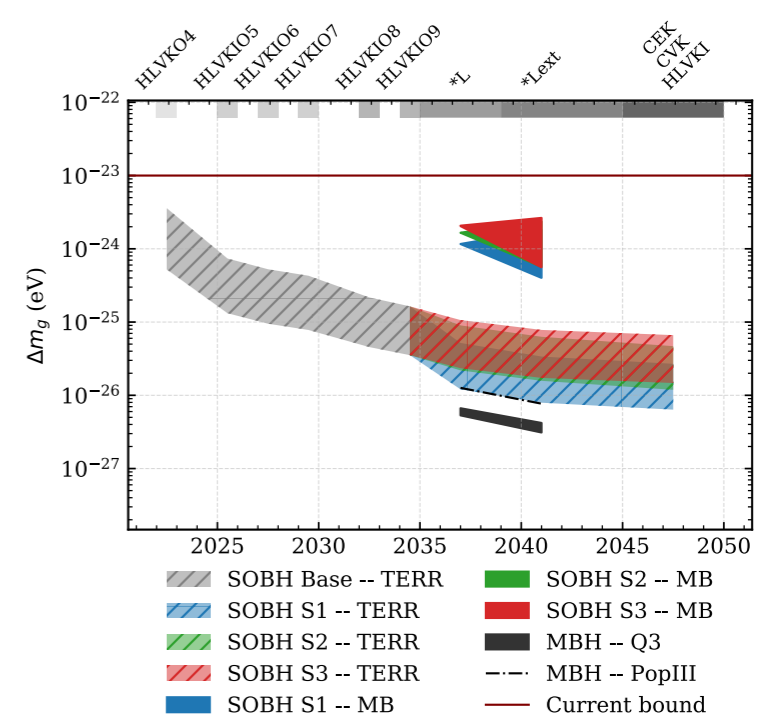
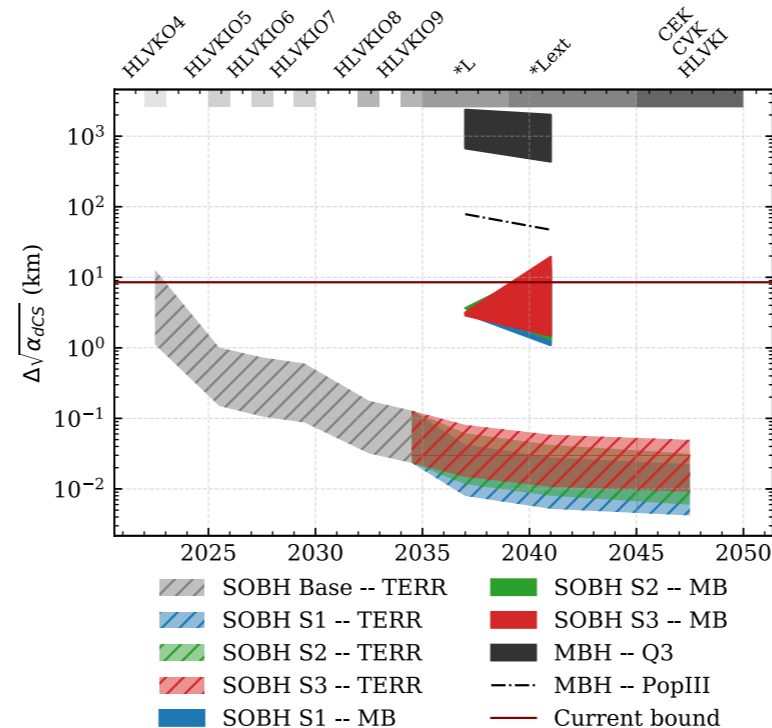
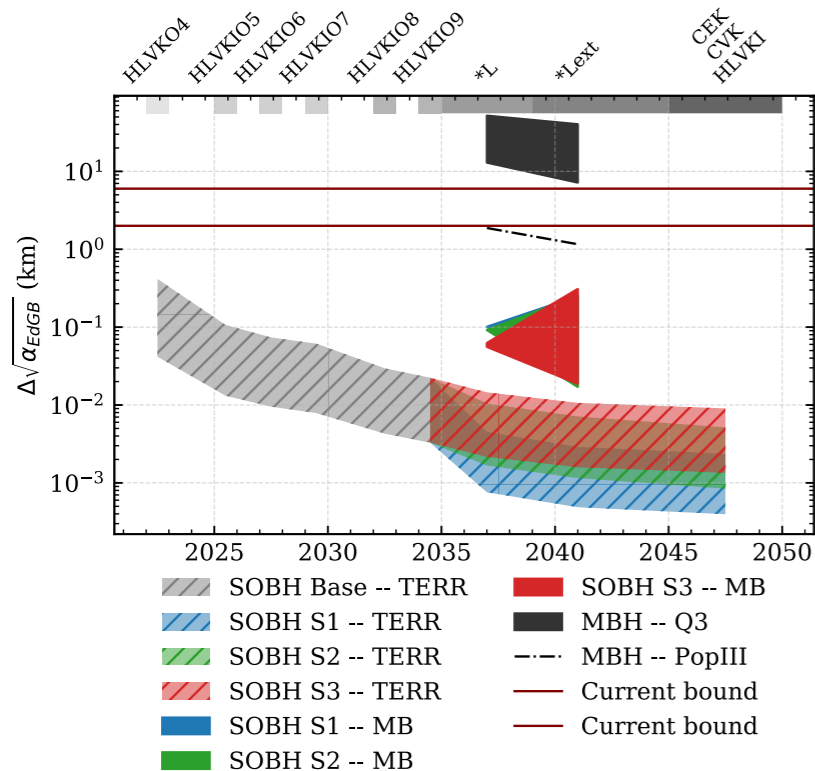
$$s_i^{\text{dCS}} = \frac{2 + 2\chi_i^4 - 2(1 - \chi_i^2)^{1/2} - \chi_i^2 [3 - 2(1 - \chi_i^2)^{1/2}]}{2\chi_i^3},$$

$$\zeta_{\text{dCS}} \sim \alpha_{\text{dCS}}^2 / m^4$$

Massive graviton

$$\beta_{\text{MG}} = \pi^2 \frac{D_0}{1+z} \frac{M_z}{\lambda_{\text{MG}}^2}$$

$$D_0 \equiv (1+z) \int_0^z \frac{1}{H(z') (1+z')^2} dz'$$



The Inspiral

More on agnostic-to-specific deviations

- generic deviations from the Kerr multipolar structure $M_\ell^{\text{kerr}} + iS_\ell^{\text{kerr}} = M^{\ell+1} (i\chi)^\ell$

$$M_{20} = -\kappa M^3 \chi^2 \quad \longrightarrow \quad \kappa = \kappa(\text{couplings}, M, \chi)$$

$$M_{20} = -M^3 \chi^2 [(1 + 0.061618 \zeta_{\text{EdGB}}^2 + \dots) - \chi^2 \zeta_{\text{EdGB}}^2 (0.0308519 + \dots)]$$

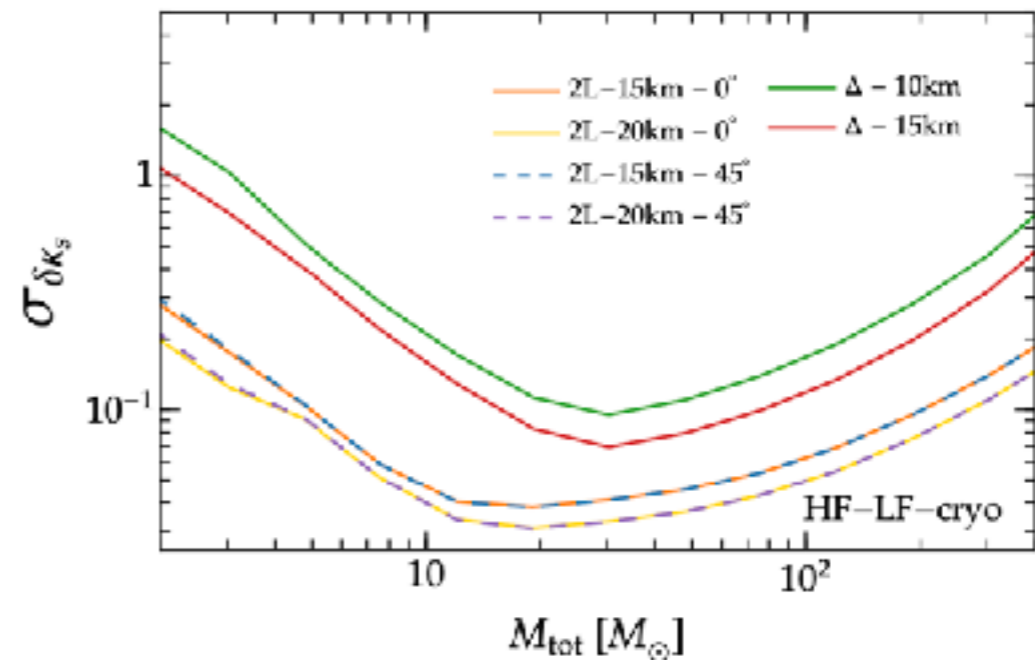
A. M. + PRD 92, 083014 (2015)

Quadrupole entering the waveform at 2PN

$$\kappa_s = \frac{\kappa_1 + \kappa_2}{2}$$

$$\kappa_a = \frac{\kappa_1 - \kappa_2}{2}$$

$$\kappa_s = 1 + \delta\kappa_s \quad \longrightarrow$$



M. Branchesi + arXiv gr-qc: 2303.15923

- More sophisticated models with broken equatorial symmetry and non-axisymmetric

N. Loutrel+, PRD 105, 124050 (2022)

The Inspiral

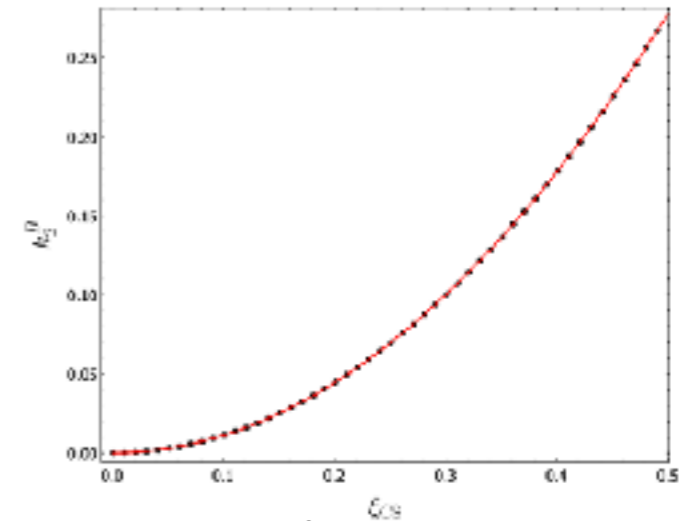
- Tidal Love numbers

- tidal parameters for Kerr BHs vanish

- scalar tidal love numbers

L. Bernard + PRD 101, 021501 (2020)

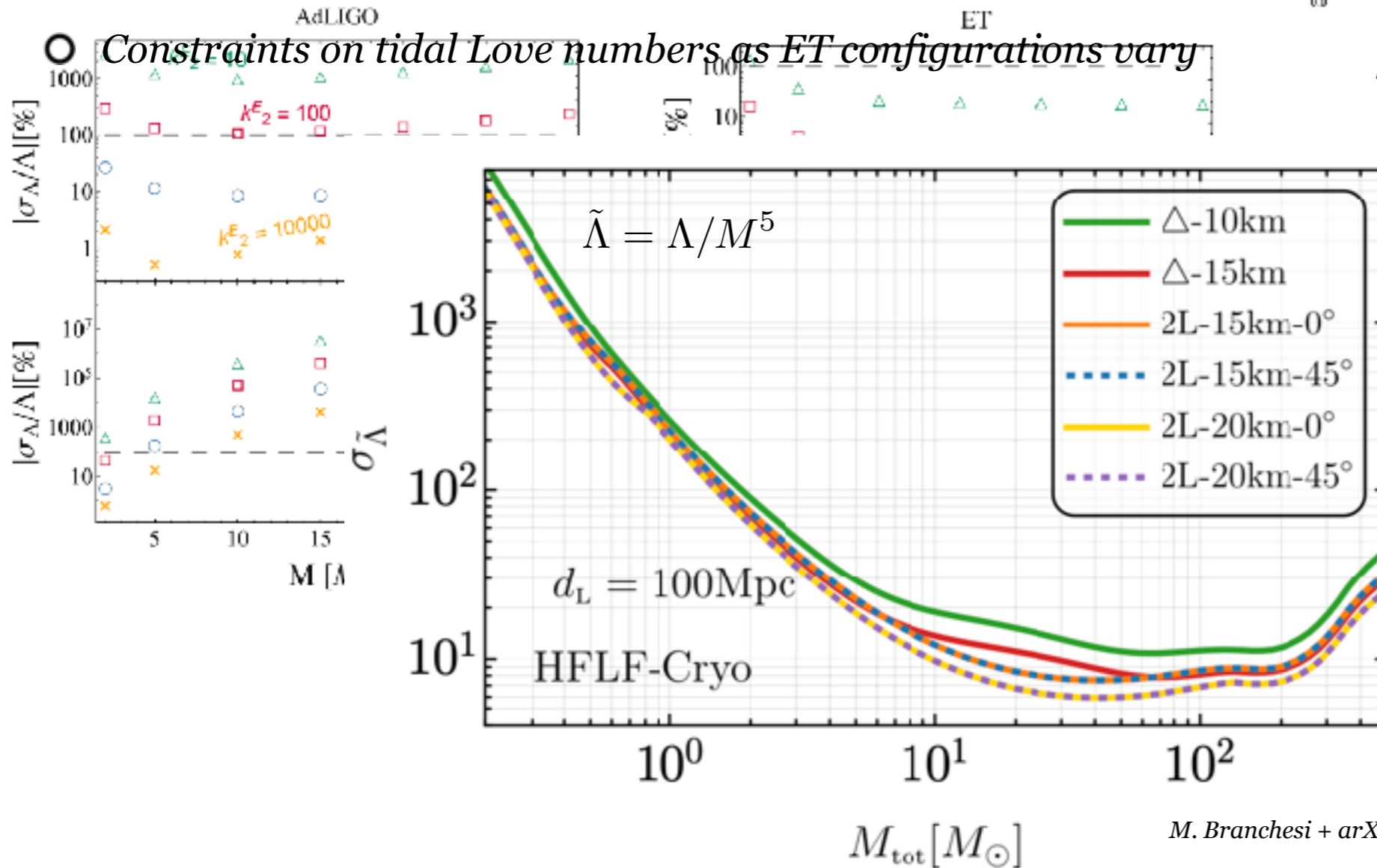
agnostic constraints on $\Lambda \sim k_2 M^5$



$$k_2^B = \frac{9}{5} [8\zeta(3) - 9] \zeta_{dCS}^2$$

V. Cardoso + PRD 95, 084014 (2017)

- Constraints on tidal Love numbers as ET configurations vary

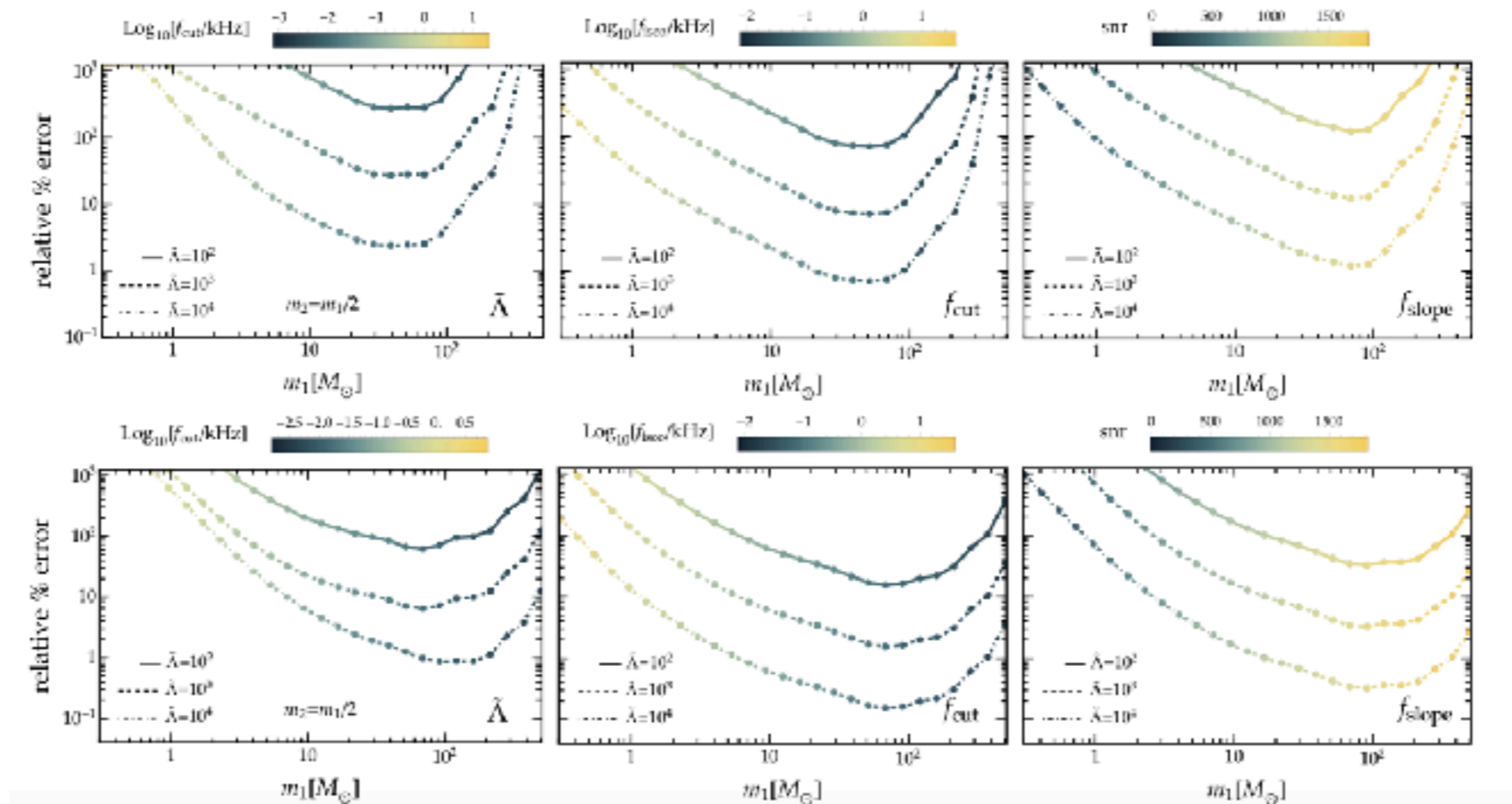


M. Branchesi + arXiv gr-qc: 2303.15923

The Inspiral

- Frequency dependent Love numbers of dressed BHs

$$\tilde{\Lambda} \rightarrow \mathcal{S}(f) \cdot \tilde{\Lambda} = \left[\frac{1 + e^{-f_{\text{cut}}/f_{\text{slope}}}}{1 + e^{(f-f_{\text{cut}})/f_{\text{slope}}}} \right] \cdot \tilde{\Lambda}$$



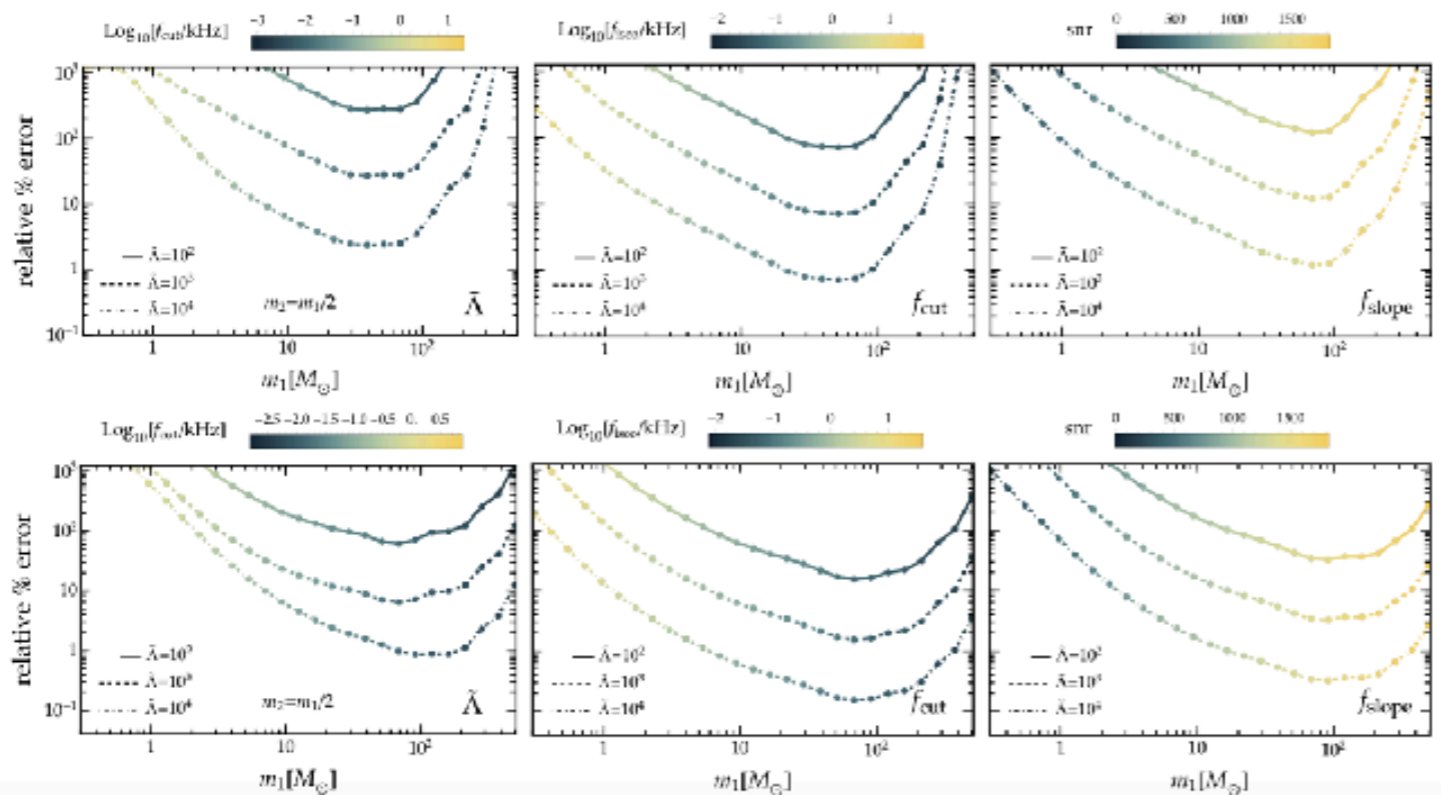
V. De Luca, A.M., P. Pani, PRD 107, 044058 (2023)

- paved for multi-band analysis LISA-ET exploiting different frequency regimes

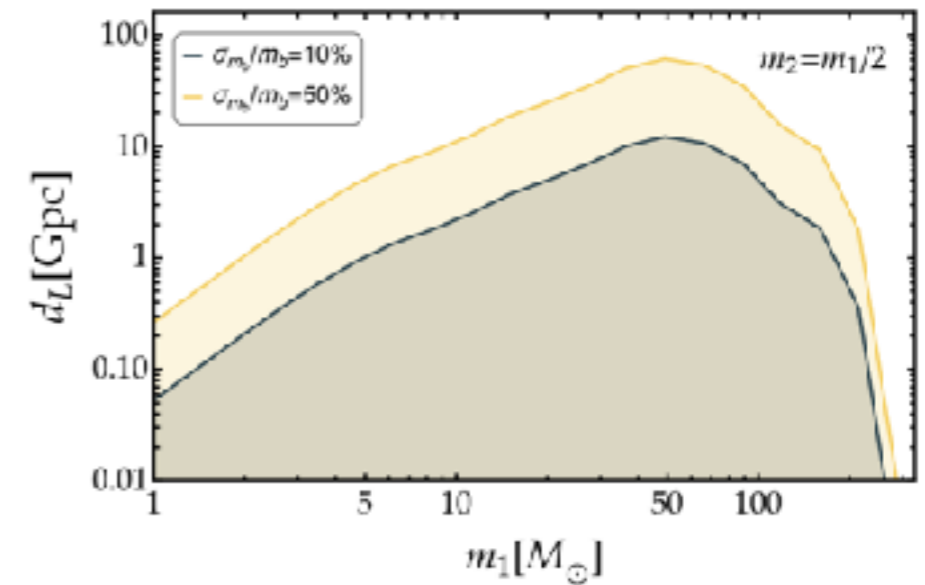
The Inspiral

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maximum luminosity. distance for ultralight boson cloud



V. De Luca, A.M., P. Pani, PRD 107, 044058 (2023)

- suited for multi-band analysis LISA-ET

The Inspiral

- Varying multiple parameters each time

- improvement from multi-band observations

A. Gupta +, PRL 125, 201101 (2020)
Perkins & Yunes, PRD 105, 124047 (2022)

- Precessing ppE formalism

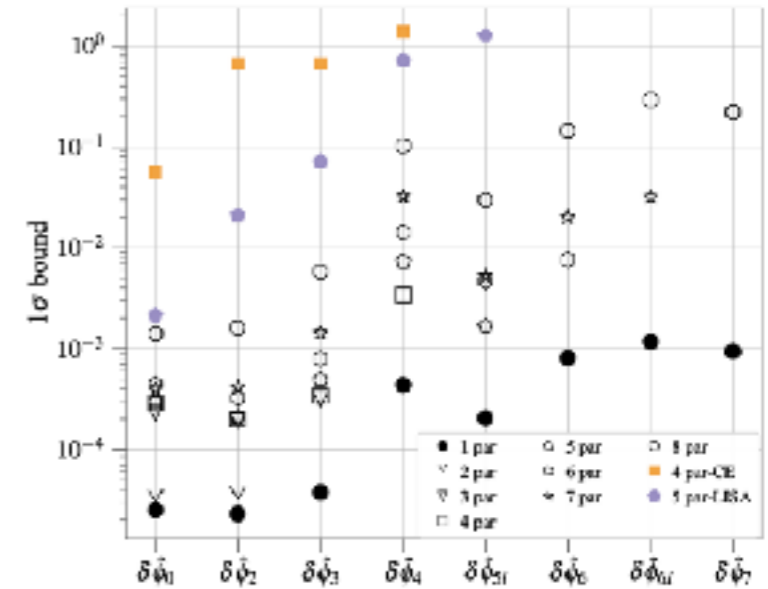
N. Loutrel +, PRD 107, 044046 (2023)

- Dynamical scalarization

M. Khalil +, PRD 100, 124013 (2019)

- Inspiral Merger Ringdown tests

Z. Carson & K. Yagi +, PRD 101, 084050 (2020)



Coherent waveform models encoding multiple effects/coefficients

- Less number of parameters to constrain, possibly alleviate degeneracies

- lesson learnt from other sources (and phenomena)

M. Vaglio +, gr-qc: 2302.13954 (2023)
C. Pacilio +, PRL 122, 101101 (2022)

QNMs beyond Kerr

Modelling frequencies and damping times beyond Kerr

- Within GR extension QNMs can be written in terms of Kerr shifts

$$\omega = \omega^{\text{Kerr}} + \delta\omega$$
$$\tau = \tau^{\text{Kerr}} + \delta\tau$$
$$(M, \chi)$$

generic deviation

$$\delta\omega \ll \omega^{\text{Kerr}}$$
$$\delta\tau \ll \tau^{\text{Kerr}}$$

- In GR the full QNM spectrum is specified by the mass and the spin of Kerr BHs

- Measuring one mode allows to determine mass and spin

- Multiple modes provide independent null-hypothesis tests of GR

C. Pacilio, S. Bhagwat PRD 107, 083021, (2021)

- Deviations proportional to the fundamental coupling constants of the theory

- Are QNMs competitive with other diagnostics?

Parametrised Ringdown Spin Expansion Coefficients

A. M., P. Pani, L. Gualtieri, E. Berti, PRD 101, 024043, (2020)

G. Carullo, PRD 103, 124043, (2021)

ParSpec

Assume $i = 1 \dots N$ ringdown observations for which q modes are detected

- Parametrise QNM in terms of a double expansion of spin χ & GR deviation

$$M_i \omega_i^{\ell m} = \sum_{k_1=0}^{n_1} \chi_i^{k_1} \bar{\omega}_{\ell m}^{(k_1)} + \sum_{k_2=0}^{n_2} \chi_i^{k_2} \bar{\omega}_{\ell m}^{(k_2)} \gamma_i \delta \omega_{\ell m}^{(k_2)}$$

$$\tau_i^{\ell m} / M_i = \sum_{k_1=0}^{n_1} \chi_i^{k_1} \bar{\tau}_{\ell m}^{(k_1)} + \sum_{k_2=0}^{n_2} \chi_i^{k_2} \bar{\tau}_{\ell m}^{(k_2)} \gamma_i \delta \tau_{\ell m}^{(k_2)}$$

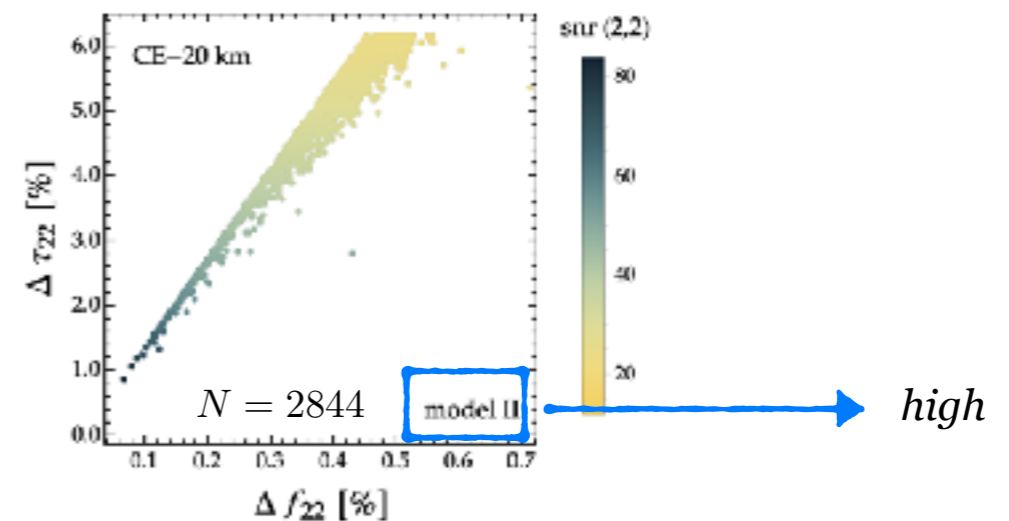
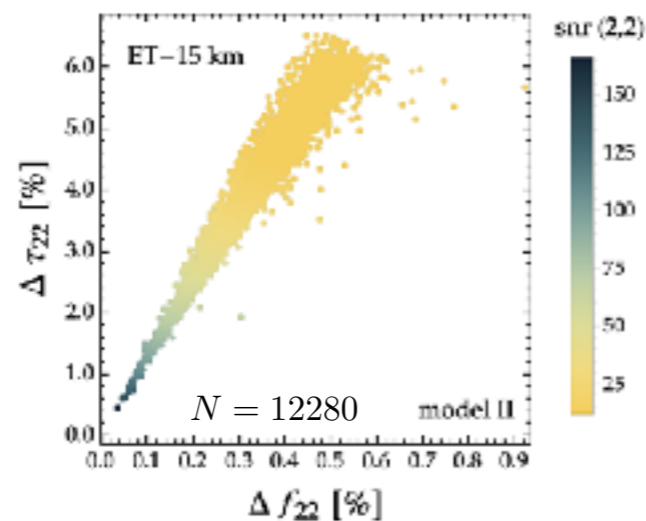
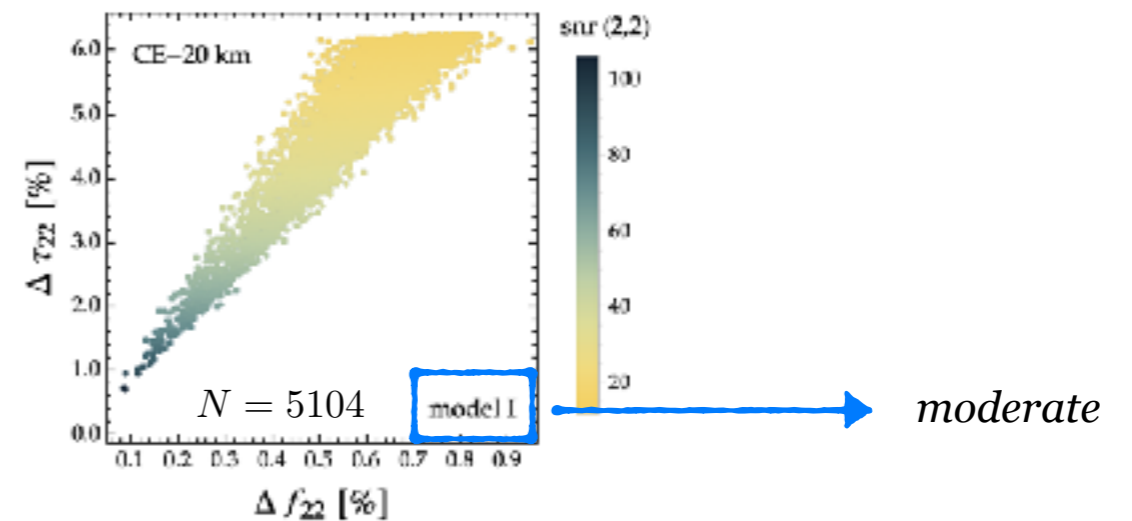
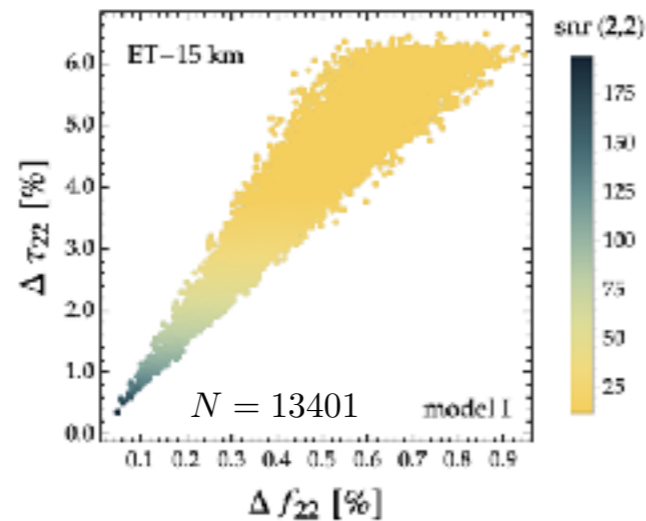
- coefficients of the expansion for Kerr QNMs in GR

- beyond-Kerr corrections to the QNMs: **universal** & $\gamma_i[\delta \tau_{\ell m}^{(k_2)}, \delta \omega_{\ell m}^{(k_2)}] < 1$

- dimensionless coupling constants, possibly depending on the source

QNM uncertainties

Errors on frequencies and damping times of the fundamental mode for 2 populations of BBH



- errors on $f_{22} \sim 1\%$ for both ET and CE
- errors on $\tau_{22} \sim 1\%$ only for $\rho \gtrsim 100$
- for the secondary, 33 measured in general with better accuracy than 21

Specific models: GB gravity

Gauss Bonnet gravity with exponential coupling

L. Pierini & L. Gualtieri, PRD 106, 104009 (2023)

J. Salcedo +, PRD 94, 104024 (2016)

$$S_{\text{GB}} = \int d^4x \sqrt{-g} \left(R - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{\alpha_{\text{GB}}}{4} e^\varphi \mathcal{R}_{\text{GB}}^2 \right)$$

- QNM at the second order in the spin $\Omega_{\ell m} = \omega_{\ell m} - i/\tau_{\ell m}$ $\beta_{\text{sGB}} = \alpha_{\text{GB}}/M^2$

$$M\Omega_{\ell m} = (\Omega_0^A + \beta_{\text{sGB}}^2 \Omega_0^B) + m\chi (\Omega_1^A + \beta_{\text{sGB}}^2 \Omega_1^B) + \chi^2 [(\Omega_{2a}^A + \beta_{\text{sGB}}^2 \Omega_{2a}^B) + m^2 (\Omega_{2b}^A + \beta_{\text{sGB}}^2 \Omega_{2b}^B)]$$

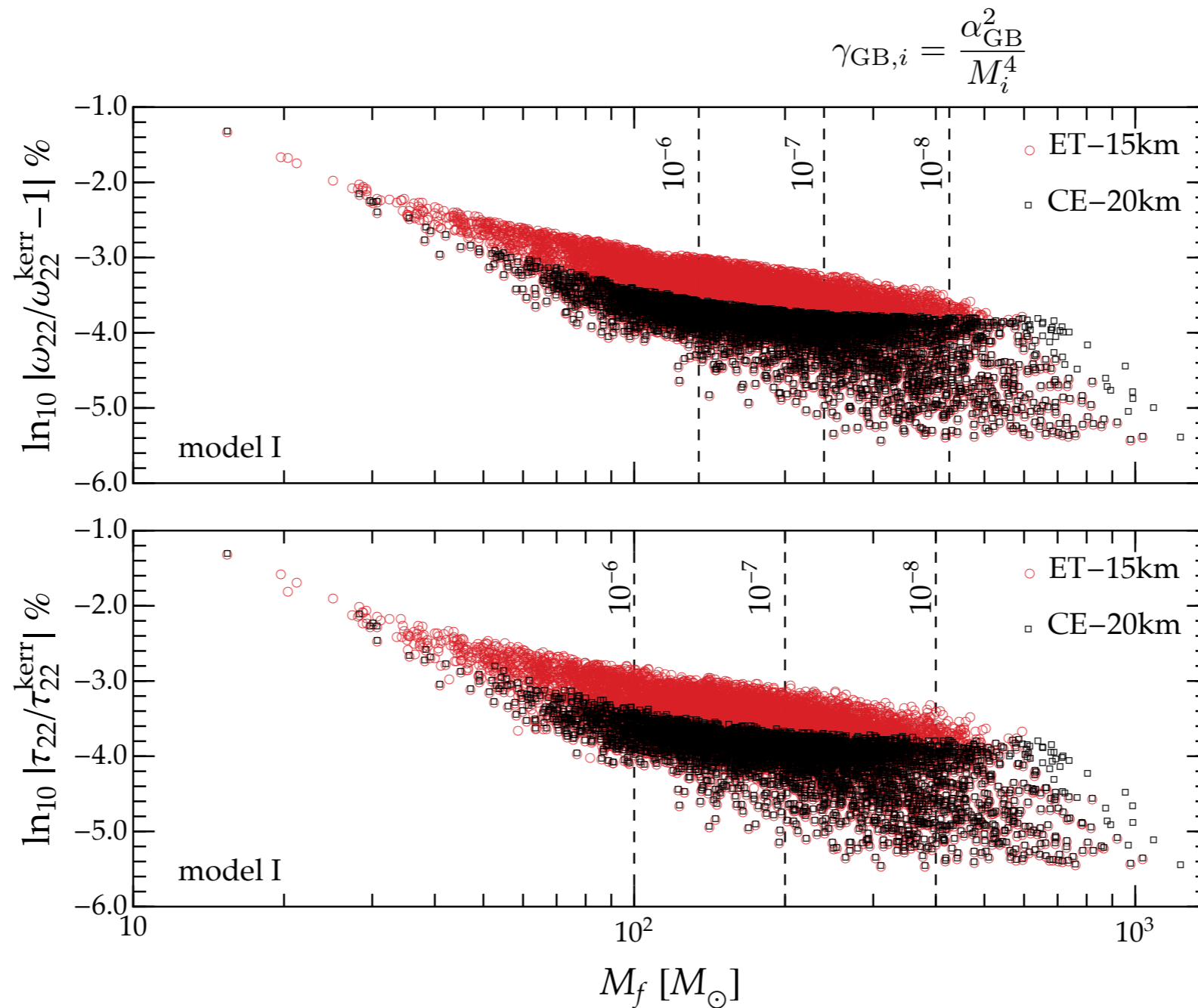
ParSpec map

$$\begin{aligned} \omega_{\ell m} = & \frac{1}{M} \left[\bar{\omega}_{\ell m}^{(0)} \left(1 + \beta_{\text{GB}}^2 \delta\omega_{\ell m}^{(0)} \right) + \chi \bar{\omega}_{\ell m}^{(1)} \left(1 + \beta_{\text{GB}}^2 \delta\omega_{\ell m}^{(1)} \right) + \chi^2 \bar{\omega}_{\ell m}^{(2)} \left(1 + \beta_{\text{GB}}^2 \delta\omega_{\ell m}^{(2)} \right) \right] + \\ & + \frac{1}{M} \sum_{k_1=3}^6 \bar{\omega}_{\ell m}^{(k_1)} \\ \tau_{\ell m} = & M \left[\bar{\tau}_{\ell m}^{(0)} \left(1 + \beta_{\text{GB}}^2 \delta\tau_{\ell m}^{(0)} \right) + \chi \bar{\tau}_{\ell m}^{(1)} \left(1 + \beta_{\text{GB}}^2 \delta\tau_{\ell m}^{(1)} \right) + \chi^2 \bar{\tau}_{\ell m}^{(2)} \left(1 + \beta_{\text{GB}}^2 \delta\tau_{\ell m}^{(2)} \right) \right] + \\ & + M \sum_{k_1=3}^6 \bar{\tau}_{\ell m}^{(k_1)} \end{aligned}$$

k_1	$\bar{\omega}_{22}^{(k_1)}$	$\bar{\tau}_{22}^{(k_1)}$	$\bar{\omega}_{33}^{(k_1)}$	$\bar{\tau}_{33}^{(k_1)}$	$\bar{\omega}_{21}^{(k_1)}$	$\bar{\tau}_{21}^{(k_1)}$
0	0.37367	11.2407	0.59944	10.7871	0.37367	11.2407
1	0.12578	0.2522	0.20205	0.2276	0.06288	0.1281
2	0.07178	0.6650	0.10717	0.8238	0.04487	0.7701
3	0.04800	0.5840	0.06890	0.7345	0.02183	0.4026
4	0.03601	0.6420	0.04841	0.6744	0.01642	0.3563
5	0.01389	0.0733	0.05295	0.5408	0.00590	1.2600
6	0.13021	3.9385	-0.11339	1.3153	0.05889	0
k_2	$\delta\omega_{22}^{(k_2)}$	$\delta\tau_{22}^{(k_2)}$	$\delta\omega_{33}^{(k_2)}$	$\delta\tau_{33}^{(k_2)}$	$\delta\omega_{21}^{(k_2)}$	$\delta\tau_{21}^{(k_2)}$
0	-0.03773	-0.0528	-0.09112	-0.0778	-0.03773	-0.0528
1	-0.16679	-0.0802	-0.32149	2.4495	-0.16679	-0.0176
2	-0.27788	3.9141	-0.56101	2.6150	0.11256	1.4773

How much change

Relative change in frequencies and damping times

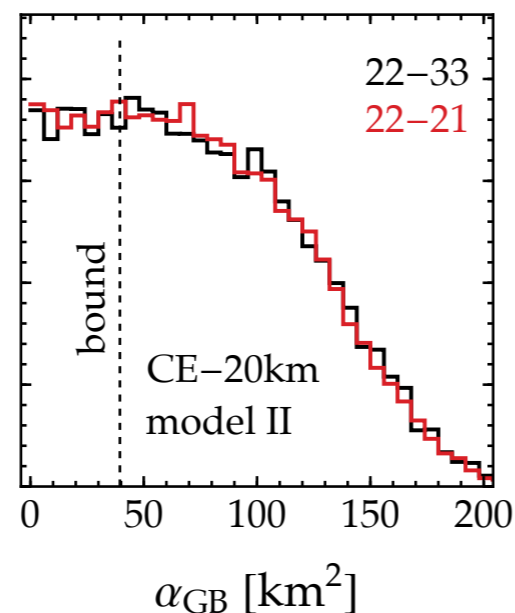
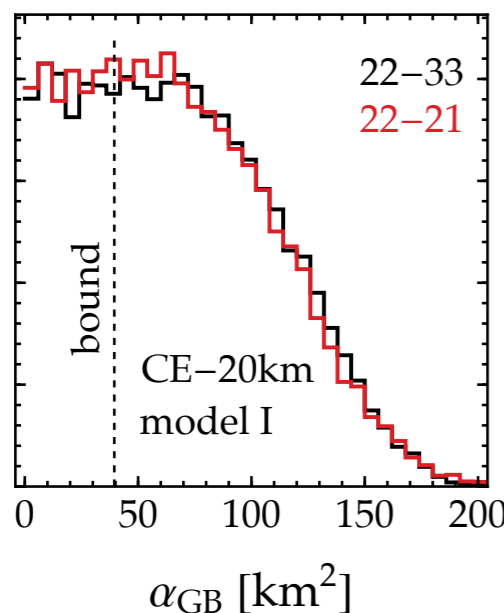
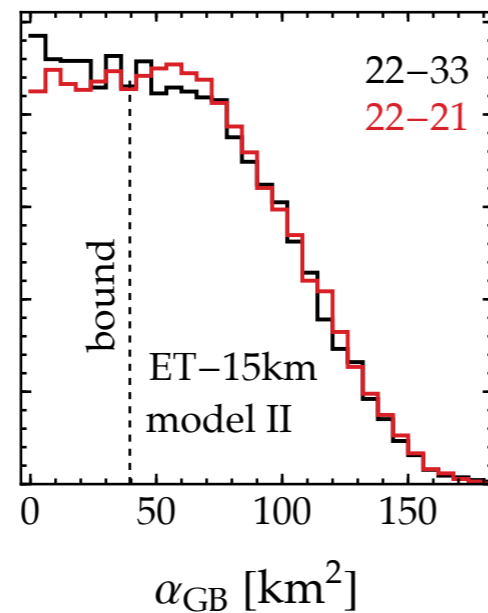
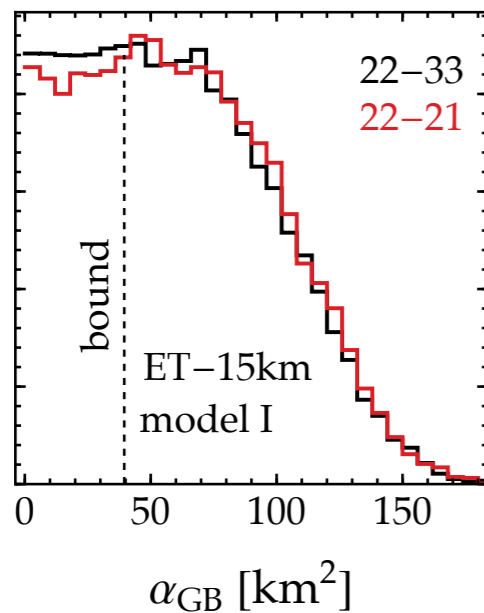


○ Current bound $\alpha_{\text{GB}}^{\text{bound}} \simeq 40 \text{ km}^2$

sGB + ParSpec: bounds

Injected events in GB gravity with $\alpha_{\text{GB}}^{\text{bound}}$, reconstructed with ParSpec

○ *sampling on α_{GB}*



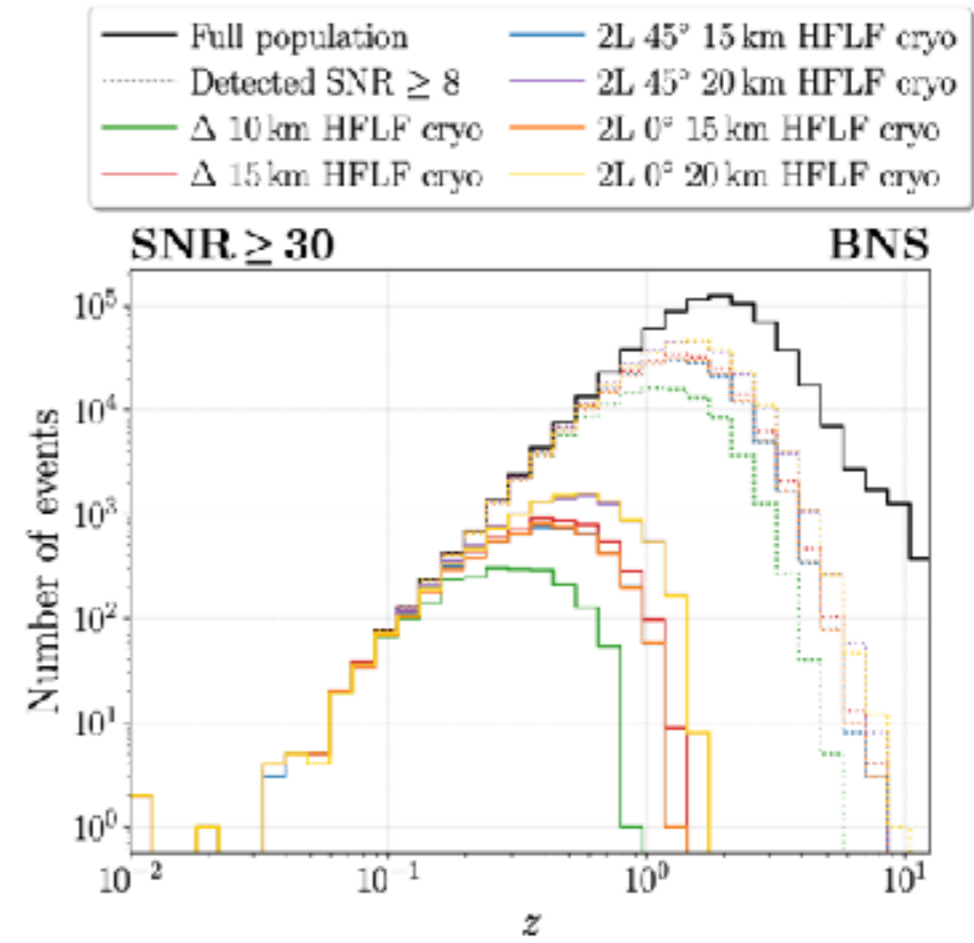
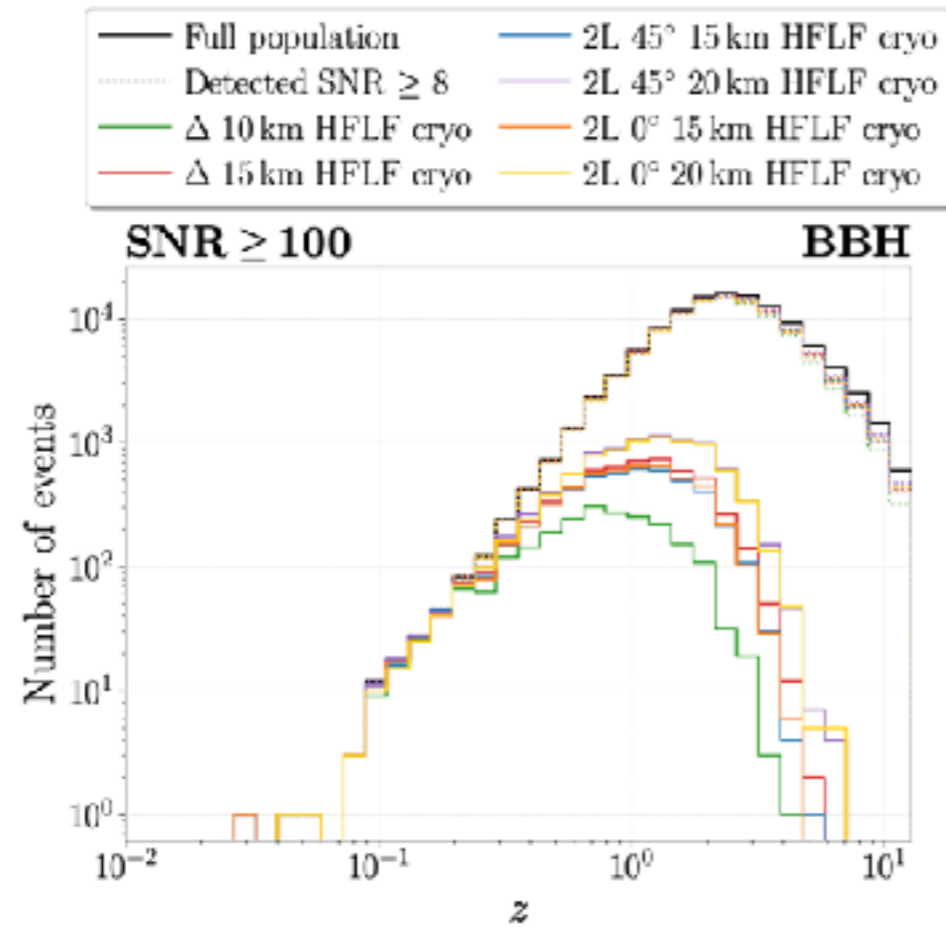
○ *Spin corrections seem to play a minor role*

○ *Bounds dominated by light BHs, with $M_f \lesssim 60M_{\odot}$*

○ *Results weaker than best constraint to date*

Summary

3G interferometers expectations



M. Branchesi + arXiv gr-qc: 2303.15923

Back up

Science Cases

Accommodate multiple scenarios

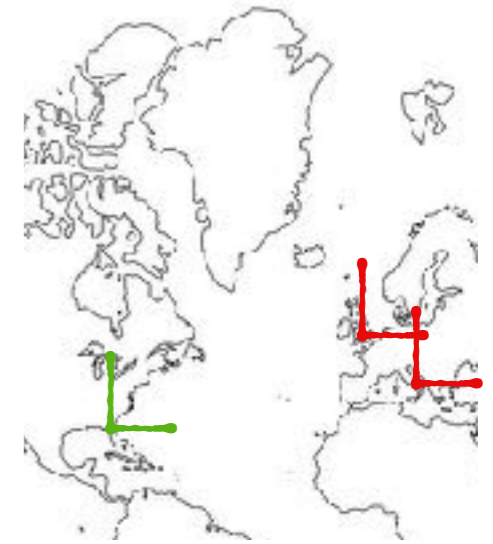
agnostic tests the Kerr QNM spectrum

scale free couplings $\gamma_i = \text{const}$

coupling can be re-absorbed in the shifts $\gamma_i \delta\omega_{\ell m}^{(k_2)} \rightarrow \delta\omega_{\ell m}^{(k_2)}$

dimensional coupling constant $[\alpha] = [\text{mass}^p]$ $\gamma_i = \frac{\alpha^p}{M_i^p}$ $p = 4$

coupling can also be re-absorbed $\alpha \delta\omega_{\ell m}^{(k_2)} \rightarrow \delta\omega_{\ell m}^{(k_2)}$



QNM spectrum in specific beyond GR theories

Gauss Bonnet and Chern Simons gravity $\gamma_i = \frac{\alpha_{\text{GB}}^2}{M_i^4}$ $\gamma_i = \frac{\alpha_{\text{CS}}^2}{M_i^4}$

map frequencies/damping times to the ParSpec shifts $\delta\omega_{\ell m}^{(k_2)}, \delta\tau_{\ell m}^{(k_2)}$

sampling is on the couplings $\alpha_{\text{GB,CS}}$

Population models

N sources drawn from binary black hole catalogues

- *POWER LAW + PEAK* phenomenological model (GWTC3 inspired)

R. Abbott + arXiv PRX 13, 011048 (2023)

$$P(m_1) \propto [(1 - \lambda)P_{\text{law}}(m_1|\gamma_1, m_{\text{max}}) + \lambda G(m_1|\mu_m, \sigma_m)] S(m_1|m_{\text{min}}, \delta_m)$$

$$P(q) \propto q^{\gamma_q} S(m_1 q|m_{\text{min}}, \delta_m)$$

- *BBH* sampled up to $z=10$, with z -distribution following a *Madau-Dickinson* curve for the *SFR*

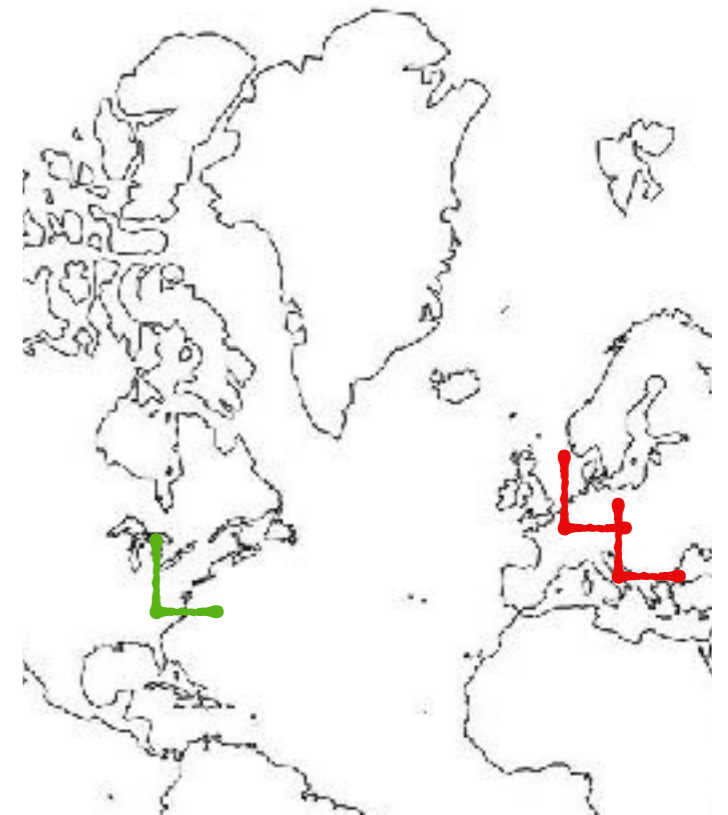
- 2 spin assumptions

- **model I** $\chi_{1,2} \in \beta$ -distribution $(\alpha, \beta) = (2, 5)$

- **model II** $\chi_{1,2} \in \text{uniform}[-1,1]$

- 3G detectors: 2 x **E**instein **T**elescope & **C**osmic **E**xplorer

M. Branchesi + arXiv gr-qc: 2303.15923 (2023)

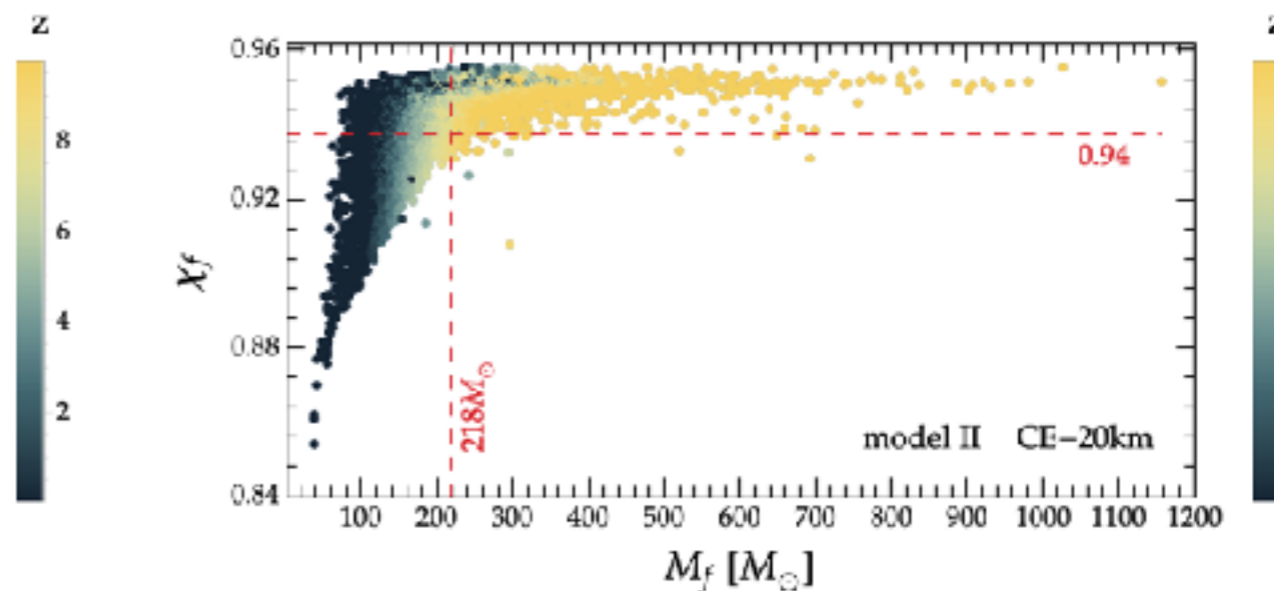
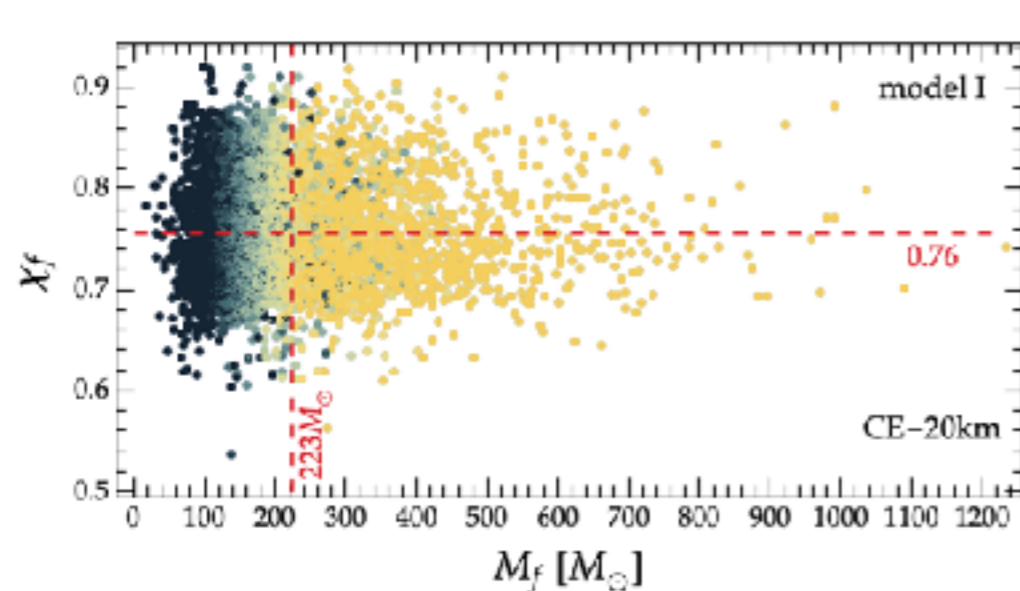
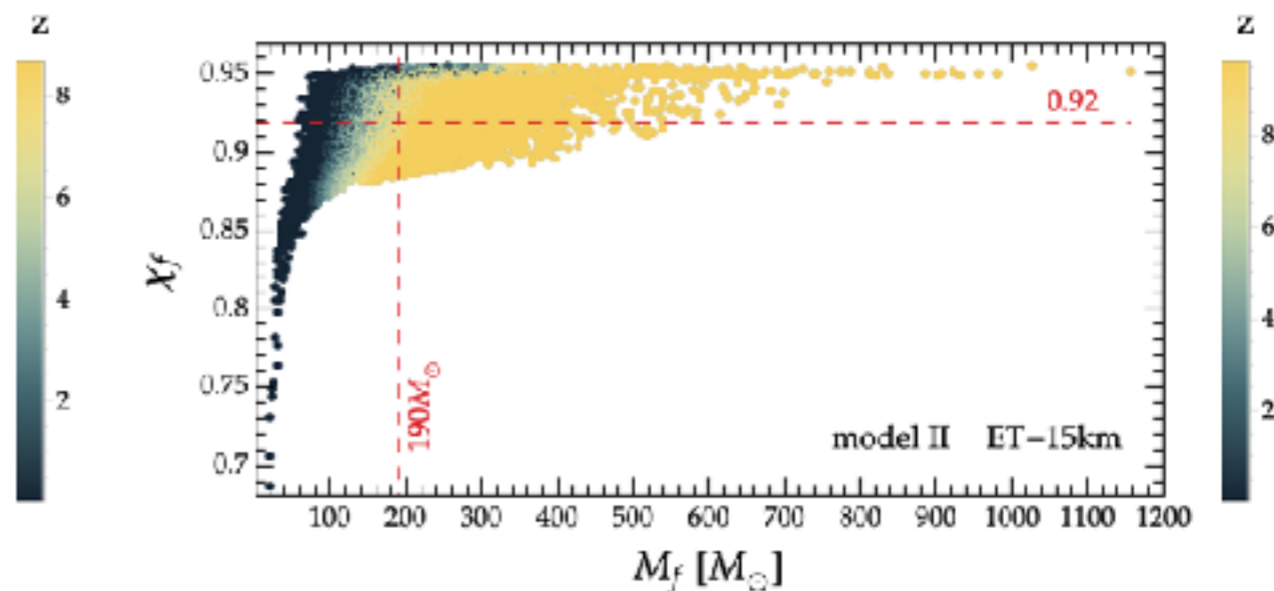
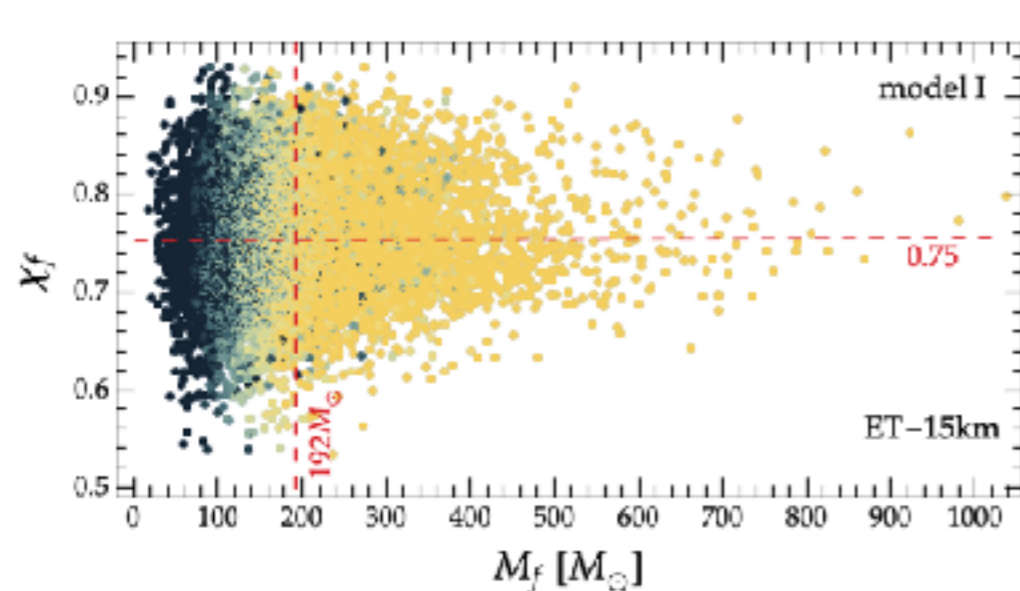


(sub)Population models

Mass-spin distributions for the selected BH remnants

β -distribution

uniform [-1,1]



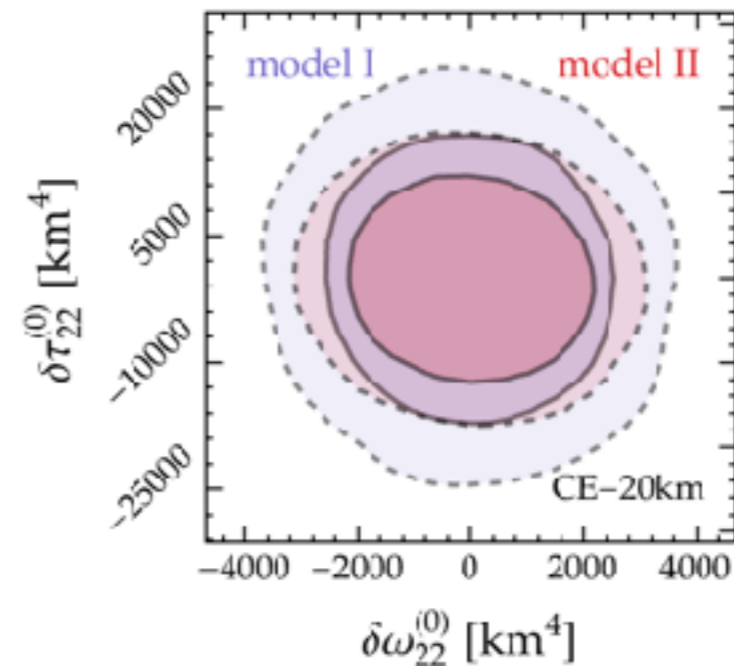
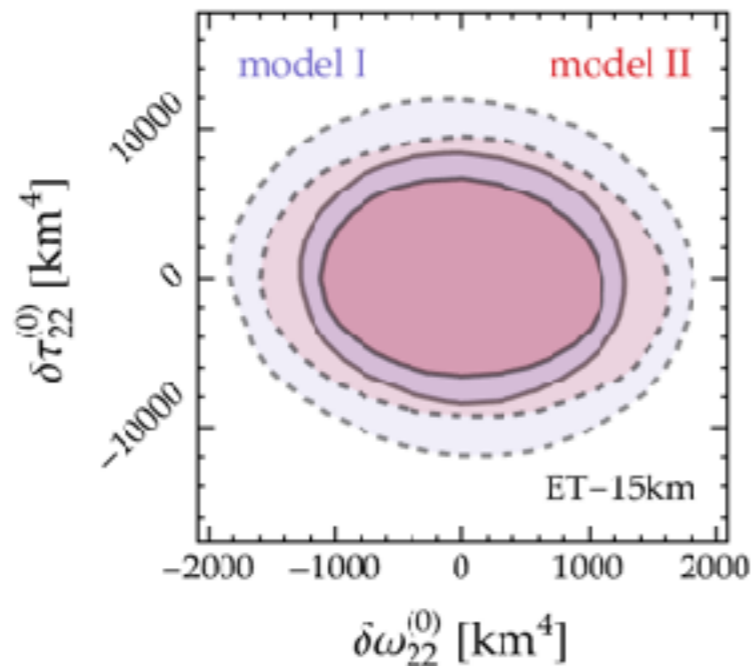
Spin-less corrections (again)

$$M_i \omega_i^{\ell m} = \sum_{k_1=0}^6 \chi_i^{k_1} \bar{\omega}_{\ell m}^{(k_1)} + \bar{\omega}_{\ell m}^{(0)} \gamma_i \delta \omega_{\ell m}^{(0)}$$

$$\tau_i^{\ell m} / M_i = \sum_{k_1=0}^6 \chi_i^{k_1} \bar{\tau}_{\ell m}^{(k_1)} + \bar{\tau}_{\ell m}^{(0)} \gamma_i \delta \tau_{\ell m}^{(0)}$$

$$\gamma_i = \frac{\alpha^4}{M_i^4}$$

2D posteriors stacking all set of events



- bounds insensitive to spin models, strongly dependent on BH mass
 - posteriors dominated by $\mathcal{O}(100)$ events with low masses
- projected bound on the coupling $\alpha \sim [\delta \omega_{22}^{(0)}]^{1/4} \sim \mathcal{O}(10\text{km})$
- Adding spin terms does makes the posteriors uninformative