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Effective field theory of black hole quasinormal modes

based on
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Testing general relativity with gravitational waves

- One main goal of gravitational-wave astronomy is to test General Relativity.
- Different ways to test GR:

(1) Null test: one assumes by default that GR is correct and then tests whether the data reject or fail to reject such hypothesis.

(2) Concrete models help guide our thinking on the form of possible deviations and the associated model testing.

(3) Parametrize the deviations from general relativistic expectations in a model-independent way.

Testing general relativity with gravitational waves

(3) Parametrize the deviations from general relativistic expectations in a model-independent way.

This can take the form of:

i) "phenomenological parametrization"

[Loutrel, Yunes and Pretorius '14], [Cardoso et al. '19], [McManus et al. '19], [Glampedakis and Silva '19], [Maselli et at. '19]…

$\Longrightarrow$ has the advantage of easier connection to the data.

ii) parametrization at the level of an Effective Field Theory (EFT).

[Endlich et al. '17], [Cardoso et al '18], [Franciolini, Hui, Penco, LS and Trincherini '18], [Cano et al '20], [Hui, Podo, LS and Trincherini '21]…

$\Longrightarrow$ better control on the theory side: fundamental principles, symmetries…

Testing general relativity with gravitational waves

- Usually hard in more phenomenological parametrizations to see which variations of the parameters correspond to theories that respect physical principles like locality, Lorentz invariance, the equivalence principle, etc.

- A parametrization at the level of an EFT makes instead more transparent which types of deformations correspond to theories that respect physical principles, and makes it easier to connect UV theories with observations through a systematic matching procedure.

- The complexity of black hole merger events suggests that the theory space should first be reduced as much as possible before comparing with data.
  EFTs are more `amenable' to being constrained by positivity, causality, unitarity, … arguments.
  [Camanho, Edelstein, Maldacena and Zhiboedov '14], [Caron-Huot, Mazac, Rastelli and Simmons-Duffin '21], [Caron-Huot, Li, Parra-Martinez and Simmons-Duffin '22], [Serra, Serra, Trincherini and Trombetta '22]…

EFT for BH perturbations

Strategy:

- Fix the low-energy degrees of freedom:
  (1) if only h_+ and $h_\times$, include powers of the Riemann tensor;
  [Endlich et al. '17], [Cardoso, Kimura, Maselli, Senatore '18], [Cano et al. '20]…

  (2) additional (scalar, vector, ...) modes.
  [Franciolini, Hui, Penco, LS and Trincherini '18], [Hui, Podo, LS and Trincherini '21]…

- Include all possible operators, up to a certain order in derivatives, compatible with the symmetries.

EFT for BH perturbations

- I will focus on case (2):

  [Franciolini, Hui, Penco, LS and Trincherini '18], [Hui, Podo, LS and Trincherini '21]

$$S_{\rm EFT} = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}(\partial\Phi)^2 + \alpha\, M_{\rm Pl}\Phi\mathscr{G}_{\rm GB} + c\,\frac{(\partial\Phi)^2\Box\Phi}{\Lambda^3} + c_{n,m}\frac{\nabla^n\Phi^m}{\Lambda^{n+m-4}} + \dots\right]$$

- Absence of dipolar radiation: $\alpha \lesssim (1.7\,\mathrm{km})^2$ [Witek et al. '18], [Perkins et al. '19, '21]
- $\Lambda \sim (M_{\rm Pl}/\alpha)^{1/3}$ for all the leading interactions involving the scalar Φ to become strong at the same scale.
- Under these conditions, $(\partial\Phi)^2\Box\Phi$ can induce *order unity* changes to the observables compared to $\Phi\mathscr{G}_{\rm GB}$ (that is, compared e.g. to [Sotiriou and Zhou '14], [Blázquez-Salcedo et al '16]).
- An EFT is a convenient model-independent framework that allows to capture all of them in a single shot.

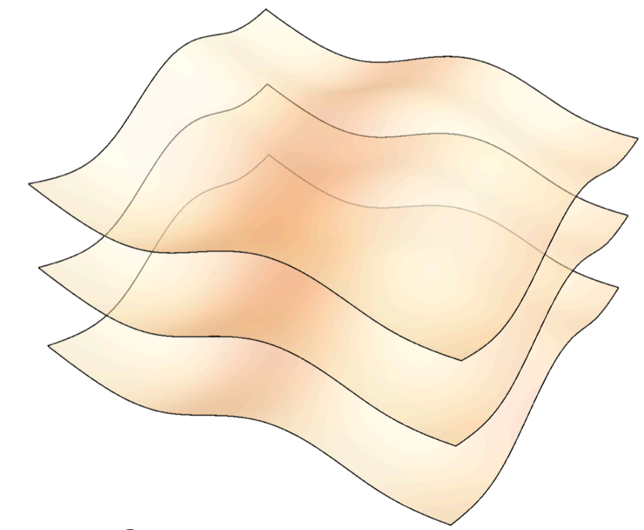
EFT for BH perturbations

- I will work under the assumptions that the theory:
  (i) is testable with gravitational-wave observations;
  (ii) is consistent with short-distance tests of GR;
  (iii) satisfies the principles of locality, causality and unitarity.

- A cautionary note: (i)-(iii) not always straightforward to satisfy from a purely bottom-up perspective. [Caron-Huot et al '22], [Serra et al '22]

EFT for BH QNMs in scalar-tensor theories

[Franciolini, Hui, Penco, LS and Trincherini '18]
[Hui, Podo, LS and Trincherini '21]



- The black hole is equipped with an extra scalar d.o.f. (hair)

$$\Phi(x) = \bar{\Phi}(r) + \pi(x) \, .$$

- In the unitary gauge ($\pi = 0$), the most general action for GR + scalar field is

$$S = \int d^4x \sqrt{-g} \, \mathscr{L}\left(g_{\mu\nu}, \epsilon^{\mu\nu\rho\sigma}, R_{\mu\nu\rho\sigma}, g^{rr}, K_{\mu\nu}, \nabla_\mu ; r \right) .$$

π is the goldstone boson that nonlinearly realizes spatial translations.

$$\begin{aligned} S = \int d^4x \sqrt{-g} \Bigg[& \frac{M_{Pl}^2}{2} R - \Lambda(r) - f(r) g^{rr} - \alpha(r) \bar{K}_{\mu\nu} K^{\mu\nu} \\ & + M_2^4(r)(\delta g^{rr})^2 + M_3^3(r) \delta g^{rr} \delta K + M_4^2(r) \bar{K}_{\mu\nu} \delta g^{rr} \delta K^{\mu\nu} + M_5^2(r)(\partial_r \delta g^{rr})^2 + M_6^2(r)(\partial_r \delta g^{rr}) \delta K \\ & + M_7(r) \bar{K}_{\mu\nu} (\partial_r \delta g^{rr}) \delta K^{\mu\nu} + M_8^2(r)(\partial_a \delta g^{rr})^2 + M_9^2(r)(\delta K)^2 + M_{10}^2(r) \delta K_{\mu\nu} \delta K^{\mu\nu} + M_{11}(r) \bar{K}_{\mu\nu} \delta K \delta K^{\mu\nu} \\ & + M_{12}(r) \bar{K}_{\mu\nu} \delta K^{\mu\rho} \delta K^{\nu}{}_{\rho} + \lambda(r) \bar{K}_{\mu\rho} \bar{K}^{\rho}{}_{\nu} \delta K \delta K^{\mu\nu} + M_{13}^2(r) \delta g^{rr} \delta \hat{R} + M_{14}(r) \bar{K}_{\mu\nu} \delta g^{rr} \delta \hat{R}^{\mu\nu} + \ldots \Bigg] . \end{aligned}$$

EFT for BH QNMs in scalar-tensor theories

[Franciolini, Hui, Penco, LS and Trincherini '18]
[Hui, Podo, LS and Trincherini '21]

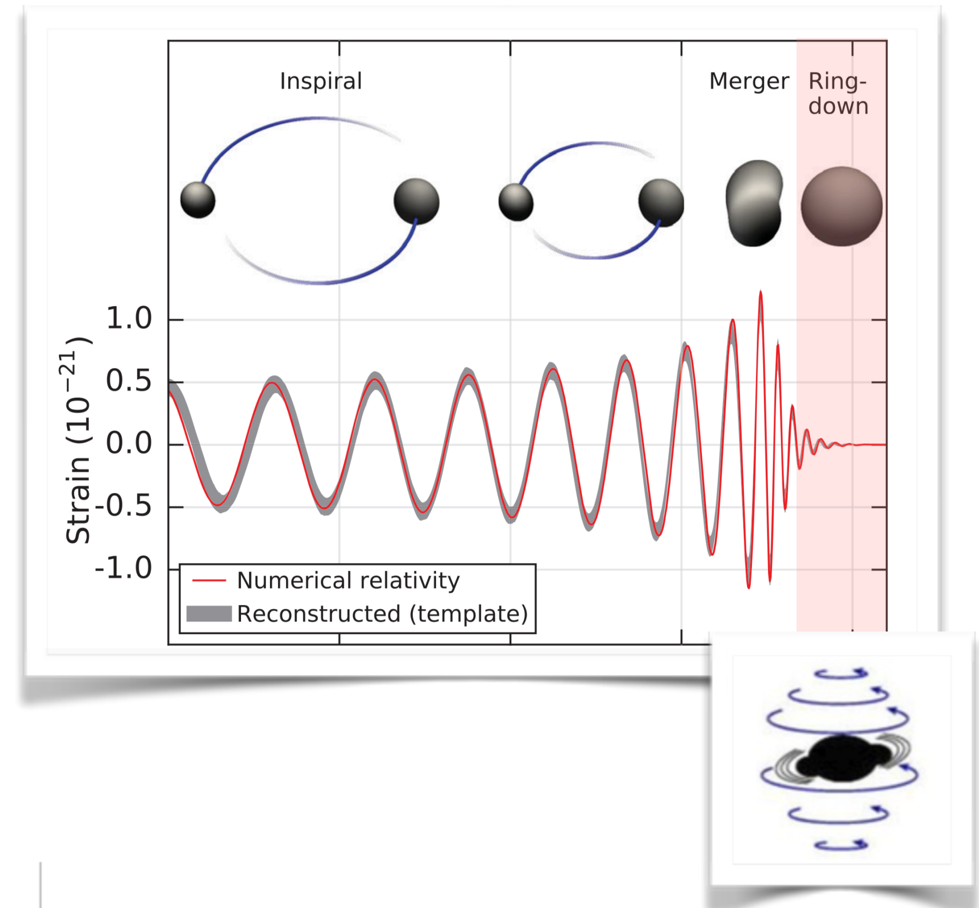
$$
\begin{aligned}
S = \int d^4x \sqrt{-g}\Bigg[& \frac{M_{Pl}^2}{2}R - \Lambda(r) - f(r)g^{rr} - \alpha(r)\bar{K}_{\mu\nu}K^{\mu\nu} \\
& + M_2^4(r)(\delta g^{rr})^2 + M_3^3(r)\delta g^{rr}\delta K + M_4^2(r)\bar{K}_{\mu\nu}\delta g^{rr}\delta K^{\mu\nu} + M_5^2(r)(\partial_r\delta g^{rr})^2 + M_6^2(r)(\partial_r\delta g^{rr})\delta K \\
& + M_7(r)\bar{K}_{\mu\nu}(\partial_r\delta g^{rr})\delta K^{\mu\nu} + M_8^2(r)(\partial_a\delta g^{rr})^2 + M_9^2(r)(\delta K)^2 + M_{10}^2(r)\delta K_{\mu\nu}\delta K^{\mu\nu} + M_{11}(r)\bar{K}_{\mu\nu}\delta K\delta K^{\mu\nu} \\
& + M_{12}(r)\bar{K}_{\mu\nu}\delta K^{\mu\rho}\delta K^{\nu}{}_{\rho} + \lambda(r)\bar{K}_{\mu\rho}\bar{K}^{\rho}{}_{\nu}\delta K\delta K^{\mu\nu} + M_{13}^2(r)\delta g^{rr}\delta\hat{R} + M_{14}(r)\bar{K}_{\mu\nu}\delta g^{rr}\delta\hat{R}^{\mu\nu} + \dots \Bigg].
\end{aligned}
$$

- The coefficients $M_i(r)$ encode all possible deviations from GR for BH with scalar hair.
- Generalized to slowly-rotating black holes in [Hui, Podo, LS and Trincherini '21].

- How to connect the EFT with the observables?

Black hole ringdown

- The final stage of the merger (ringdown) is described in terms of damped sinusoids with discrete quasinormal frequencies $\omega_{n\ell m}$.



- In GR, perturbing the Einstein equations leads to

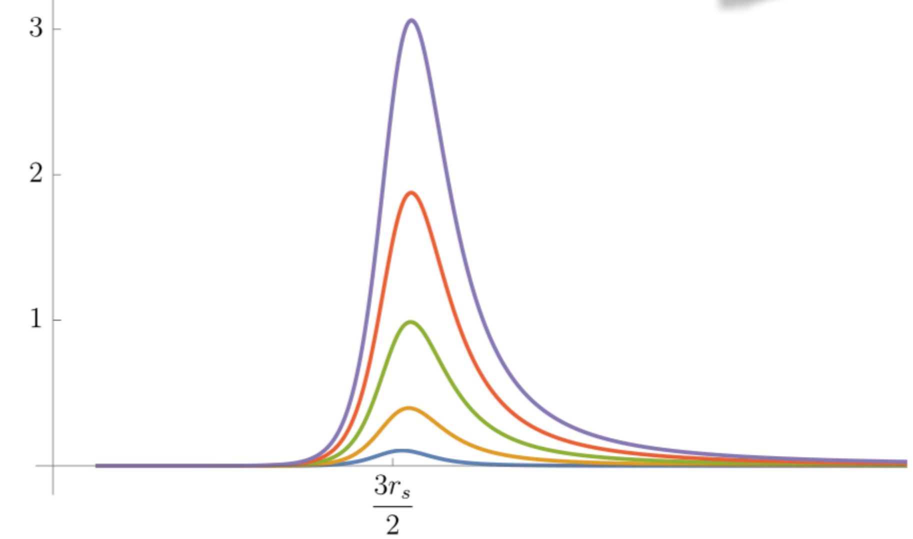
$$\frac{d^2h}{dr_\star^2} + \left(\omega^2 - V\right) h = 0\,, \qquad \frac{dr}{dr_\star} = 1 - \frac{r_s}{r}\,.$$

- Different methods to compute the QNMs (direct integration, continued-fraction method…).

- An analytic method to estimate the QNMs for single equations was developed by [Schutz and Will '85]. The result is an approximate "quantization rule":

$$\frac{\omega_n^2 - V(\tilde{r})}{\sqrt{-2V''(\tilde{r})}} = i\left(n + \frac{1}{2}\right) \qquad n = 0,1,2...$$

where and $\tilde{r}$ denotes the maximum of the potential, $V'(\tilde{r}) = 0$.



QNMs of coupled linear systems

- In theories beyond GR that involve extra degrees of freedom, the equations for the perturbations are often coupled:

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}r_\star^2} + \omega^2 - \mathbb{V}(r)\right) \cdot \vec{h} = 0\,.$$

- It is in general possible to put the system in Schrödinger-like form, when an action formulation is available. [Hui, Podo, LS and Trincherini '22]

- QNMs for coupled equations have been computed with different approaches in the context of:
  _ phenomenological parametrization of ringdown [Cardoso et al '19];
  _ eikonal (i.e., large-ℓ) limit [Glampedakis and Silva '19], [Bryant, Silva, Yagi and Glampedakis '21];
  _ theory-agnostic potential reconstruction [Völkel, Franchini and Barausse '22];
  _ first-order formulation [Langlois, Noui and Roussille '21, '22].

- An analytic approach that generalizes Schutz & Will's method to coupled equations was unknown.

An analytic approach to QNMs for coupled linear systems

- In [Hui, Podo, LS and Trincherini ’22] we introduced an analytic method to compute QNMs that generalizes Schutz & Will’s approach to coupled linear systems.

An analytic approach to QNMs for coupled linear systems

[Hui, Podo, LS and Trincherini '22]

- The system

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}r_\star^2}+\omega^2-\mathbb{V}(r)\right)\cdot\vec{h}=0$$

admits an Hamiltonian formulation ($r_\star \to \tau$ and $h_i \to q_i$):

$$H(q,p,\tau)=\tfrac{1}{2}p_ip_i+\tfrac{1}{2}q_i\left(\omega^2\delta_{ij}-V_{ij}(\tau)\right)q_j\,,\qquad \frac{\mathrm{d}}{\mathrm{d}\tau}\begin{pmatrix}q_i\\p_i\end{pmatrix}=\begin{pmatrix}0 & \delta_{ij}\\ -\omega^2\delta_{ij}+V_{ij}(\tau) & 0\end{pmatrix}\begin{pmatrix}q_j\\p_j\end{pmatrix}.$$

Performing a canonical transformation that diagonalizes the time evolution in the asymptotic region:

$$\xi_i^+=\frac{1}{\sqrt{2}}\left(\sqrt{\omega}\,q_i+\mathrm{i}\frac{p_i}{\sqrt{\omega}}\right),\qquad \xi_i^-=\frac{1}{\sqrt{2}}\left(\sqrt{\omega}\,q_i-\mathrm{i}\frac{p_i}{\sqrt{\omega}}\right),$$

the evolution equation becomes

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\tau}\begin{pmatrix}\xi_i^+\\ \xi_i^-\end{pmatrix}=\left[\begin{pmatrix}+\omega & 0\\ 0 & -\omega\end{pmatrix}\delta_{ij}-\frac{V_{ij}(\tau)}{2\omega}\begin{pmatrix}+1 & +1\\ -1 & -1\end{pmatrix}\right]\begin{pmatrix}\xi_j^+\\ \xi_j^-\end{pmatrix},$$

with boundary conditions $\begin{cases}\xi_i^+\to 0,\quad \xi_i^-\sim e^{\mathrm{i}\omega\tau}, & \text{for}\quad \tau\to+\infty\\ \xi_i^+\sim e^{-\mathrm{i}\omega\tau},\quad \xi_i^-\to 0, & \text{for}\quad \tau\to-\infty\,.\end{cases}$

An analytic approach to QNMs for coupled linear systems

[Hui, Podo, LS and Trincherini '22]

- The boundary conditions

$$\begin{cases} \xi_i^+ \to 0, \quad \xi_i^- \sim e^{\mathrm{i}\omega\tau}, & \text{for} \quad \tau \to +\infty \\ \xi_i^+ \sim e^{-\mathrm{i}\omega\tau}, \quad \xi_i^- \to 0, & \text{for} \quad \tau \to -\infty \end{cases}$$

correspond to the requirement that the system undergoes a transition from an eigenstate with eigenvalue $+\omega$ at early times to one with eigenvalue $-\omega$ at late times.

- For a slowly-varying Hermitian evolution operator, the adiabatic theorem of quantum mechanics implies that the transition occurs efficiently only when two eigenvalues $\lambda_{i,\pm} = \pm\sqrt{\omega^2 - v_i}$ become degenerate:

$$\omega^2 - v_i(\tilde{\tau}_i) = 0 \,, \qquad \text{with} \;\; v_i'(\tilde{\tau}_i) = 0 \;\; \text{(single crossing).}$$

- For a single equation, this recovers Schutz & Will's condition that ω^2 must equal the maximum of the potential.

An analytic approach to QNMs for coupled linear systems

[Hui, Podo, LS and Trincherini '22]

- The recipe:

(1) find the eigenvalues $v_i(r)$ of $\mathbb{V}(r)$ and identify the maxima of $v_i(r)$ (each maximum will be associated with a tower of QNMs);

(2) Taylor expand around each maximum and write

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = -\omega^2 \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} + \begin{pmatrix} a_0 - a_2 x^2 + \ldots & \varepsilon b_1 x + \ldots \\ \varepsilon c_1 x + \ldots & d_0 + \ldots \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}, \qquad x \equiv \tau - \tilde{\tau}_i\,;$$

(3) the leading-order quantization condition is

$$\omega_n^2 = a_0 - \mathrm{i}\sqrt{4a_2}\left(n + \frac{1}{2}\right)\,;$$

(4) mixing/anharmonic corrections can be computed in standard perturbation theory.

- An example:

$$\mathbb{V}(r) = \left(1 - \frac{r_s}{r}\right)\begin{pmatrix} -\frac{8r_s}{r^3} + \frac{\ell^2+\ell+4}{r^2} & \frac{\varepsilon(\ell^2+\ell-2)(2r-3r_s)}{r^3} \\ \frac{2\varepsilon}{r^2} & \frac{r_s}{r^3} + \frac{\ell^2+\ell-2}{r^2} \end{pmatrix}$$

	$\mathrm{Re}(\omega r_s)$			$-\mathrm{Im}(\omega r_s)$		
ε	Analytic	Numerical	%	Analytic	Numerical	%
0.1	0.847	0.849	0.2	0.183	0.176	4.0
0.2	0.852	0.854	0.2	0.184	0.176	4.5
0.3	0.861	0.861	< 0.1	0.185	0.175	5.7
0.4	0.871	0.868	0.3	0.186	0.175	6.3
0.5	0.882	0.876	0.7	0.188	0.175	7.4
0.6	0.893	0.883	1.1	0.190	0.176	8.0
0.7	0.905	0.891	1.6	0.192	0.178	7.9
0.8	0.917	0.898	2.1	0.194	0.181	7.2
0.9	0.929	0.906	2.5	0.195	0.185	5.4
1.0	0.941	0.915	2.8	0.196	0.190	3.2

An analytic approach to QNMs for coupled linear systems

[Hui, Podo, LS and Trincherini '22]

- The approximation scheme does not depend on the exact shape of the potential away from the maxima of the eigenvalues.

- This makes it particularly suitable when combined with an EFT approach: one can use the approximation scheme to connect deviations from the GR QNM spectrum to the EFT couplings M_i (and derivatives thereof) computed at the maxima of the potential eigenvalues (*light-ring expansion*). [Franciolini, Hui, Penco, LS and Trincherini '18]

- Clear signatures of new physics at the level of the QNM spectrum include:
  _ the presence of extra modes in addition to the GW polarizations (if the scalar-matter coupling is gravitational, or bigger);
  _ modified spectrum of QNMs with possible breaking of isospectrality (if the scalar-matter coupling is absent or very weak).

Conclusions

Challenges and future directions

- An effective approach is useful to capture possible deviations in the ringdown signal in a model-independent way, while having a handle on fundamental principles and assumptions.

- It is complementary to more phenomenological approaches.

- Future tests of GR will crucially benefit from:
  _ better understanding of the connection between EFTs and parametrized approaches;
  _ more systematic characterization of how different operators in the EFT affect the observables and the spectrum of QNMs;
  _ generalization of the EFT to rotating black holes (not only slow spin);
  _ finding a more convenient organization of the perturbative expansion (e.g. nonlinearities and *merger*);
  _ connection to the *inspiral* phase;
  _ understanding symmetry structure of gravitational EFTs, which can provide robust and model-independent ways of discriminating among large classes of theories.
  (Ex: symmetry of vanishing Love numbers [Hui, Joyce, Penco, LS and Solomon '21, '22], [Charalambous, Dubovsky and Ivanov '21, '22])