

Numerical Relativity beyond General Relativity

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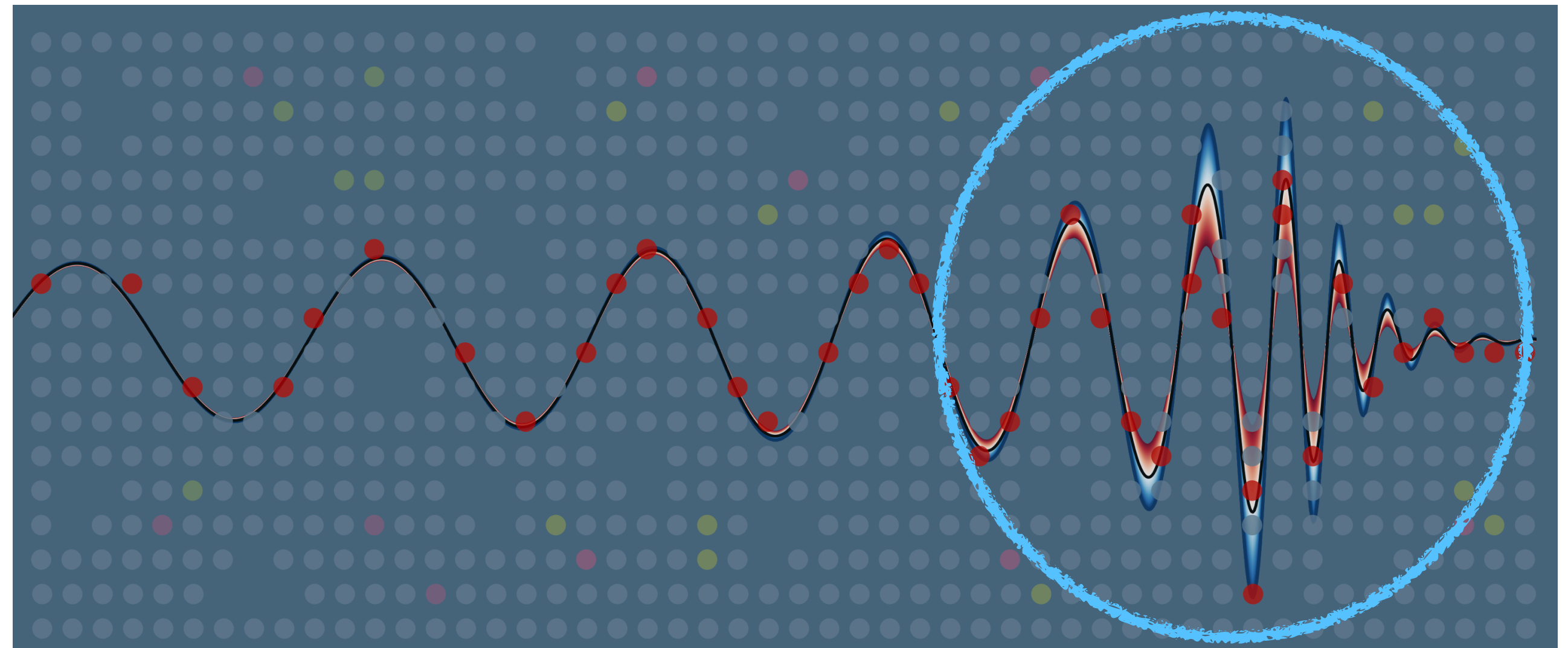
University of Illinois Urbana-Champaign

Workshop “Connecting the dots”

15 June 2023, AEI Potsdam



Connecting theory and observations



Credit: H. O. Silva

- Waveform **templates** crucial for **signal characterisation**
- Formation and (stability) **properties** of **black holes** and compact objects
- Construct consistent inspiral-**merger**-ringdown waveform models **in** and **beyond General Relativity**
- Combine theory-agnostic waveforms (e.g. Maggio, Silva, Buonanno, Ghosh '22) and **theory-specific waveforms**

See *Laura and Mohammed's talks for PN calculation; Geraint's and Nathan's talks on parameter estimation.*

Landscape of Numerical Relativity beyond GR

In **four spacetime dimensions** the only divergence-free symmetric rank-2 tensor constructed **solely from the metric and its derivatives up to second order**, and preserving **diffeomorphism invariance**, is the Einstein tensor plus a cosmological constant. [Lovelock '69-'72]

Extra fields

- **Scalar-tensor theory** (Healy+'11, Barausse+ '12, Shibata+ '13, Berti et al '13, Palenzuela+ '15, Gerosa+ '16, Sperhake+ '17, Sagunski+'17, Rosca-Mead+ '19, '20, Mendes+'21)
- **Einstein-Maxwell-Dilaton** (Hirschmann+ '17, Liebling '19)
- **Horndeski gravity** (Ripley+ '19, Bernard+'19, Kovacs+ '20, Figueras+ '20, '21, Dima+ '21, Bezares+ '20, '21, Lara+ '22; Review: Ripley '22)
- **Boson stars** (Liebling & Palenzuela '12, Palenzuela+ '17, Helfer+ '18, '19, '21, Clough+'18, Bezares+ '18-'22, Alcubierre+ '19, Di Giovanni+'21, Jaramillo+ '22, ...)
- **Ultralight scalars** (Bamber+ '22, Ficarra+ in prep.)

Higher curvature

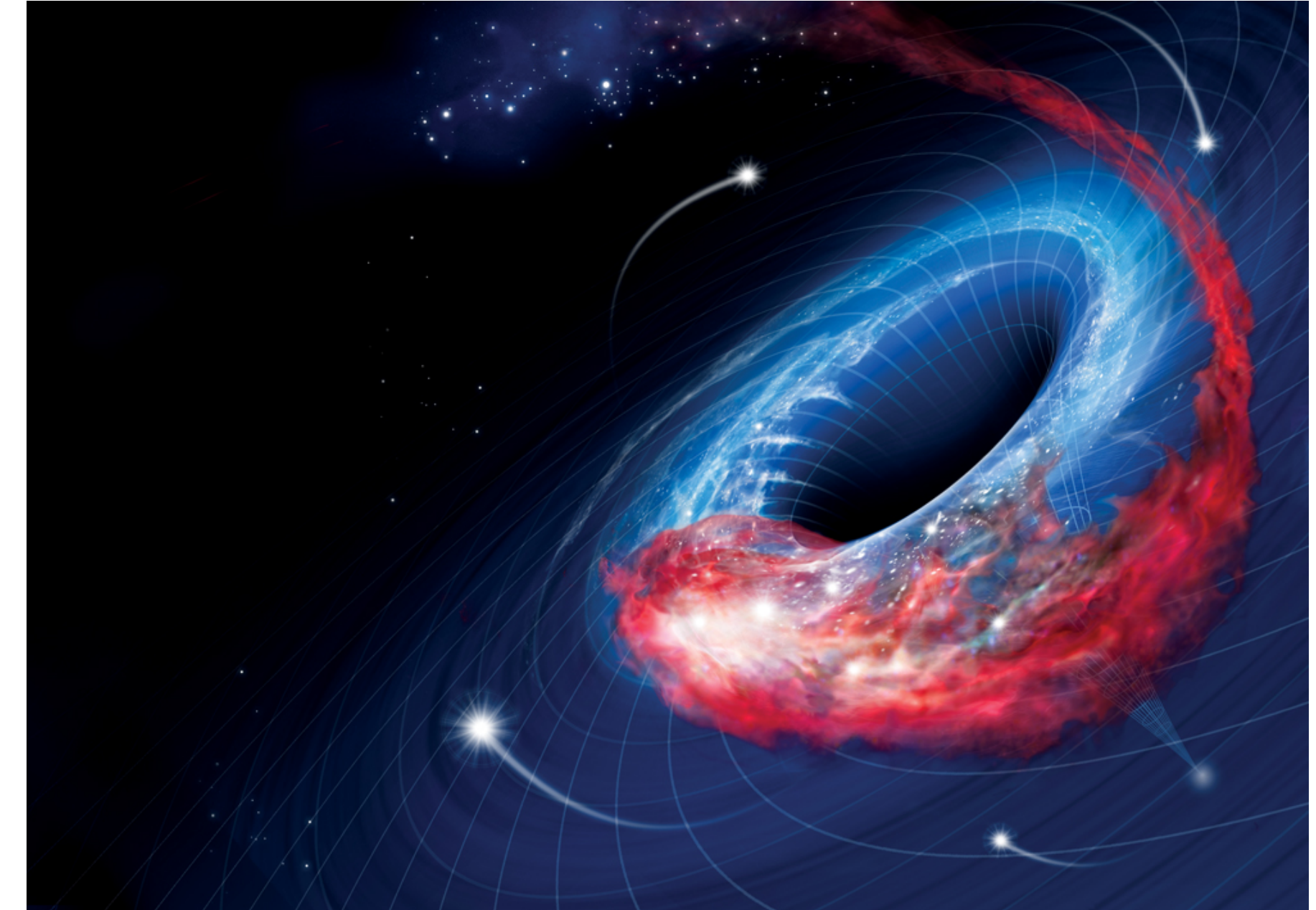
- **Effective field theory of gravity** (Held+ '21, Held+ '23, Cayuso+ '23)
- **Semi-classical gravity** (Benitez+ '20)
- **Black holes with near-horizon fluctuations** (Liebling+ '17)
- **Scalar Gauss–Bonnet gravity** (Benkel+ '16; Ripley+ '19, '20, Ramazanoglu '19, Dima+ '20, Hegade+ '22; HW+ '18, Okounkova '20, East+ '21, Silva+'21; Elley+ '22, Franchini+ '22, Salo+ '22, Corman+ '22, Thaalba+ '23)
- **Dynamical Chern-Simons gravity** (Okounkova+ '17 - '19, Doneva+ '20, '21, Richards+ '23)

Lorentz violations

- **Einstein-æther theory** (Garfinkle+ '07; Barausse, Sarbach et al'19)
- **Bimetric gravity** (Torsello+ '19, Kocic+ '20)

Focus today: higher derivative gravity

- Higher curvature corrections relevant in strong-curvature regime
- Bottom-up: series expansion in curvature
- Top-down: inspired by low-energy limit of quantum gravity candidates, e.g., string theories
(Gross & Sloan '87, Kanti et al '95, Moura & Schiappa 06, Cano et al '21)



$$\mathcal{L} = R - \frac{1}{2}(\nabla\Phi)^2 + \frac{\alpha_{GB}}{4}f(\Phi)\mathcal{G} - \frac{1}{2}(\nabla\theta)^2 + \frac{\alpha_{CS}}{4}\theta R^*R + F_2(\mathcal{R}^2) + F_3(\mathcal{R}^3) + \dots$$

Scalar Gauss-Bonnet Dynamical Chern-Simons Effective field theory of gravity

$$\mathcal{G} = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2$$

How to formulate higher derivative gravity theories?

- Field equations may have (i) mixed hyperbolic-elliptic character, (ii) higher time derivatives,...
- Causal and **well-posed** initial value formulation **necessary** for numerical stability

Order-by-order expansion

- Expand metric and scalars in coupling parameter

$$G_{\mu\nu}^{(0)}[g^{(n)}] \sim T_{\mu\nu}^{eff}(g^{(n-1)}, \dots, \Phi^{(n-1)}, \dots)$$

$$\square^{(0)} \Phi^{(n)} \sim F(\mathcal{R}^{(n-1)}, \dots, \Phi^{(n-1)}, \dots)$$

- Easy to implement
- BUT: secular effects
- Consider RG flow [Gherzi & Stein '21]

Fixing a la Israel-Stewart

- Inspired by relativistic hydrodynamics (Cayuso, Ortiz, Lehner '17; Allwright, Lehner '18,...)

- Toy model: $\square \Phi \sim \lambda \partial_x^4 \Phi$

- Superluminal modes

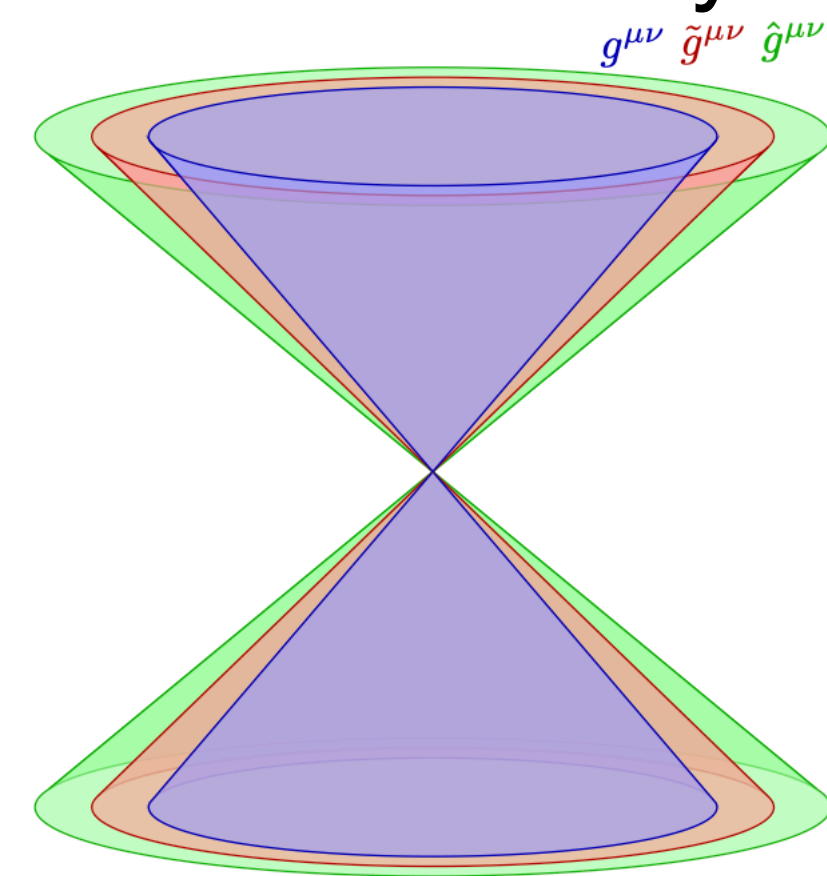
- Fix:

$$\Pi = \partial_x^2 \Phi \rightarrow \square \Phi \sim \lambda \partial_x^2 \Pi$$

$$\sigma \square \Pi \sim \partial_x^2 \Phi - \tau \partial_t \Pi - \Pi$$

Modified gauge conditions

- Modified generalized harmonic gauge (Kovacs, Reall '20)
- Gauge dofs propagate along light cone of auxiliary metric



Building a (hairy) black hole

Ultralight scalars:

Black holes anchor long-lived scalar clouds

$$\square \theta - \mu^2 \theta = 0$$

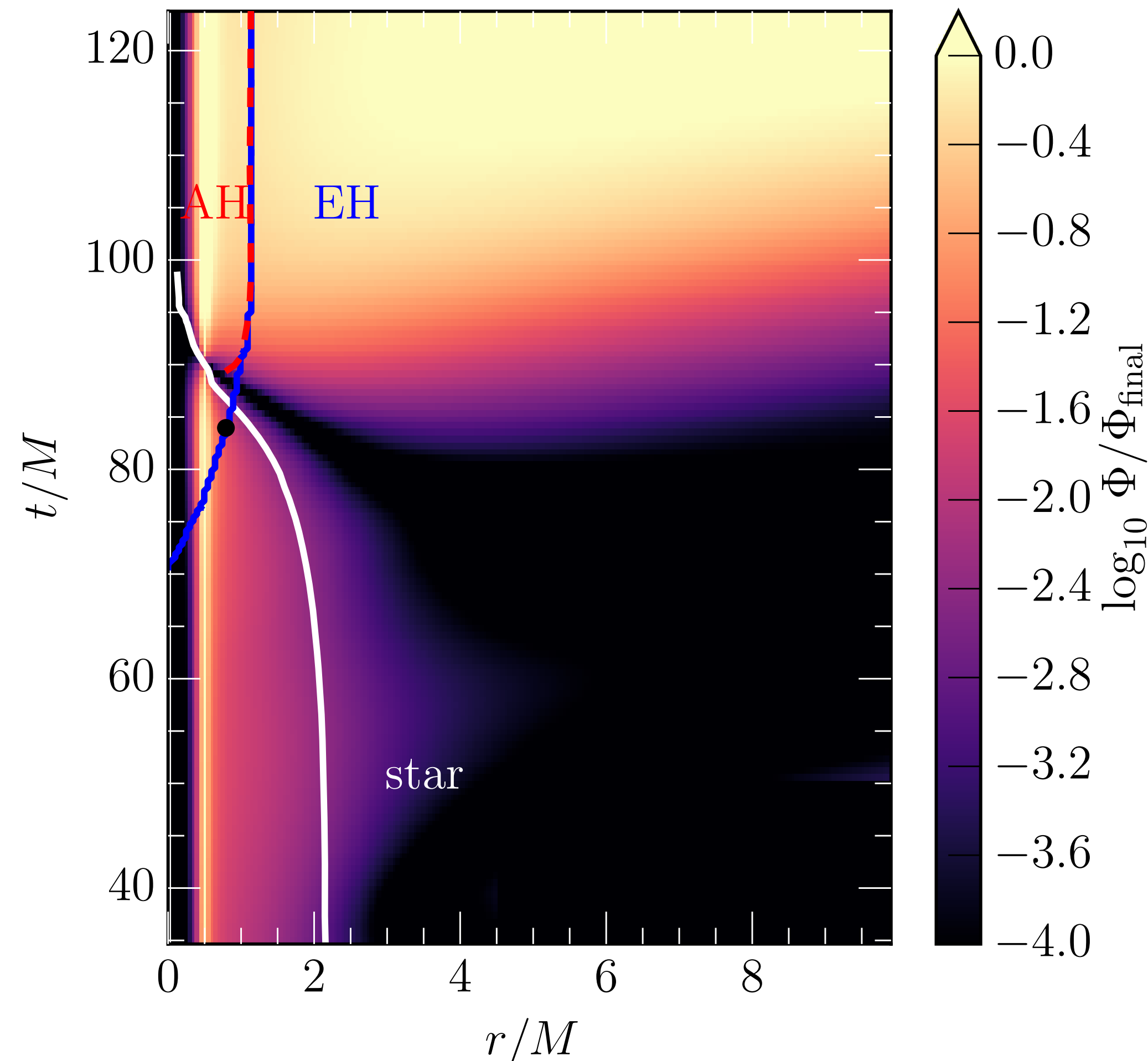
E.g.: in quadratic gravity:

Black holes have scalar charge, but neutron stars do not!

$$\square \Phi + \frac{\alpha_{GB}}{4} \mathcal{G} = 0 \quad \square \theta + \frac{\alpha_{CS}}{4} R^* R = 0$$

Growing monopole and dipole hair in quadratic gravity

(A. Hegade, E.R. Most, J. Noronha, HW, N. Yunes '21, '22)



How does the black hole hair form?

- simulated stellar collapse
- scalar field growth triggered by formation of event horizon

(BH formation: Benkel, Sotiriou, **HW** '16, **HW** et al '19, Ripley & Pretorius '19, Doneva et al '21;

BHs: Kanti et al '95, Torii et al '96, Pani et al '09, '11, Yunes & Stein '11, Sotiriou & Zhou '14, Ayzenberg et al '14, Maselli et al '15, Kunz et al '11, '14

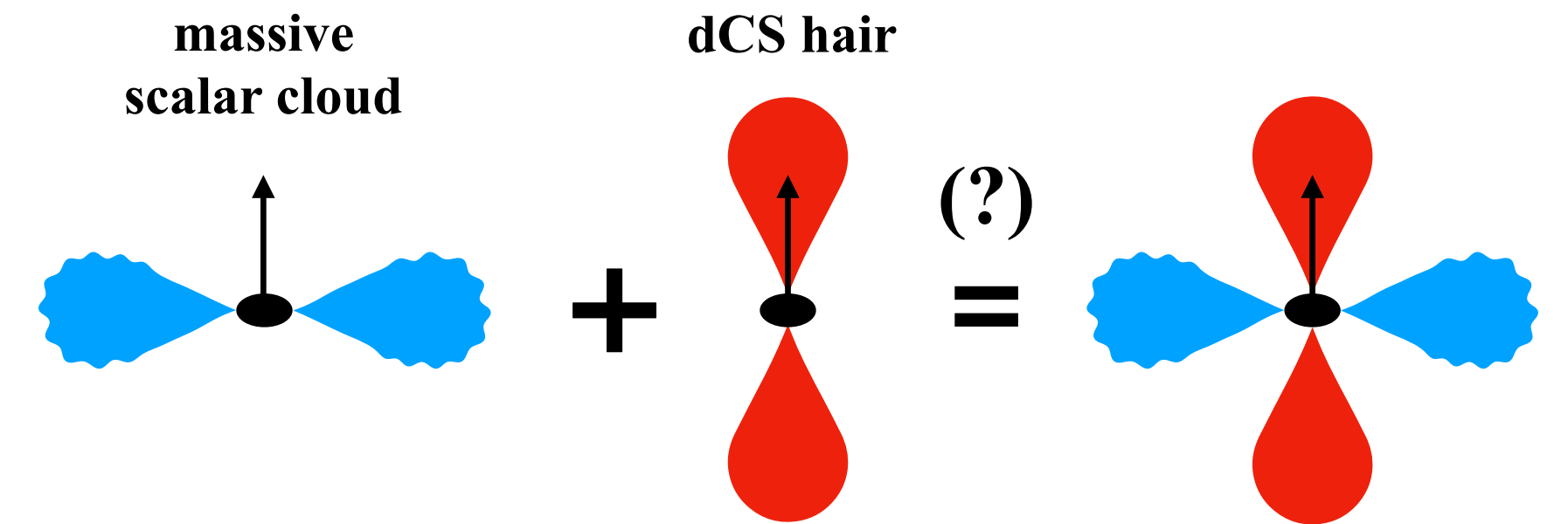
Scalarization: Silva et al '17, Doneva et al '17, Antoniou et al '17, Macedo et al '19, Ripley & Pretorius '20, Silva et al '19, Blazquez-Salcedo et al '18, '20, Dima et al '20, Hod '20, Doneva et al '20, Herdeiro et al '20, Berti et al '20; see review by Doneva, Silva, Sotiriou et al '22, ...)

Scalar clouds non minimally coupled to gravity

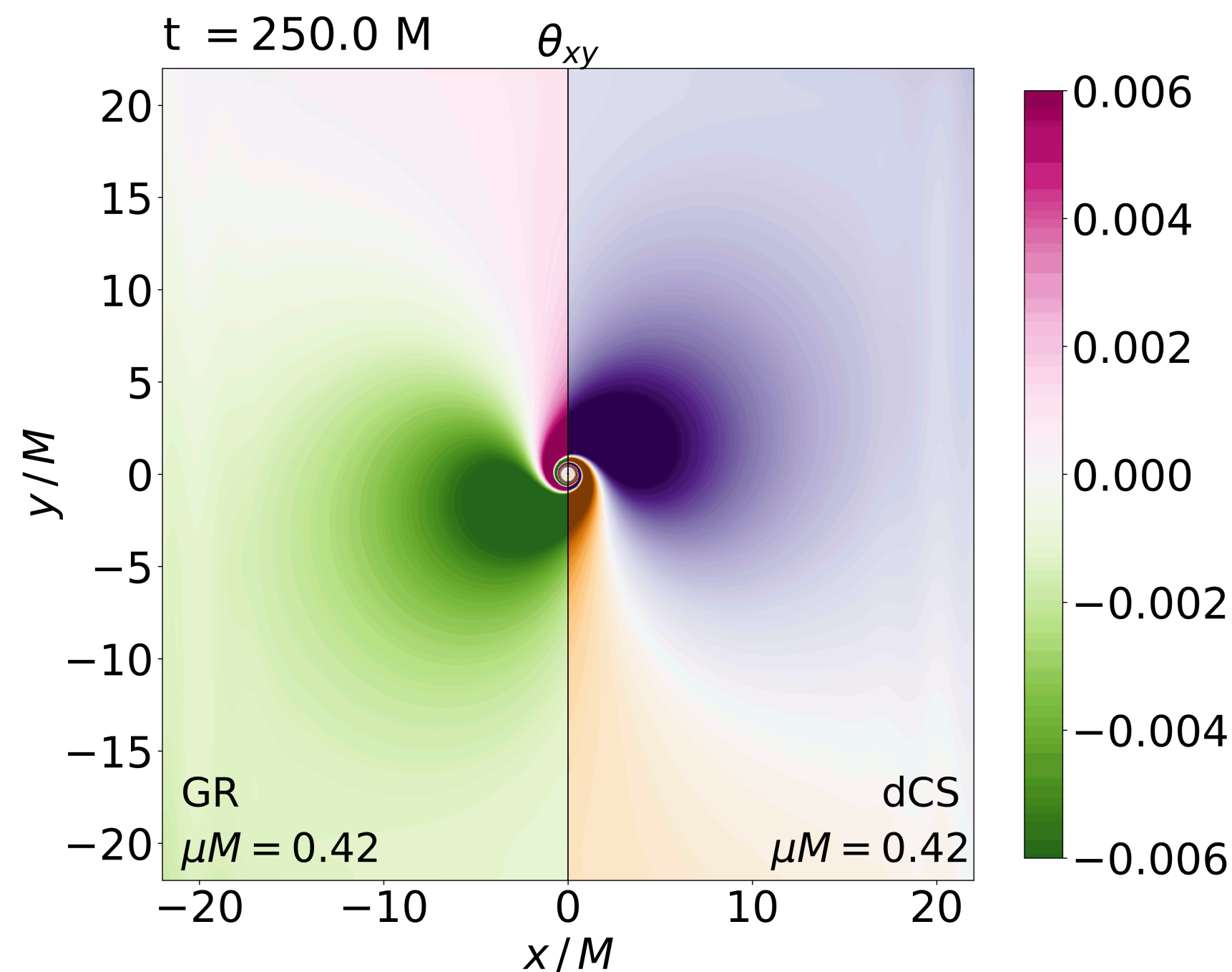
(C. Richards, A. Dima, HW '23)

- Evolving massive scalar cloud around single black hole
- Non minimally coupled to gravity via Pontryagin density

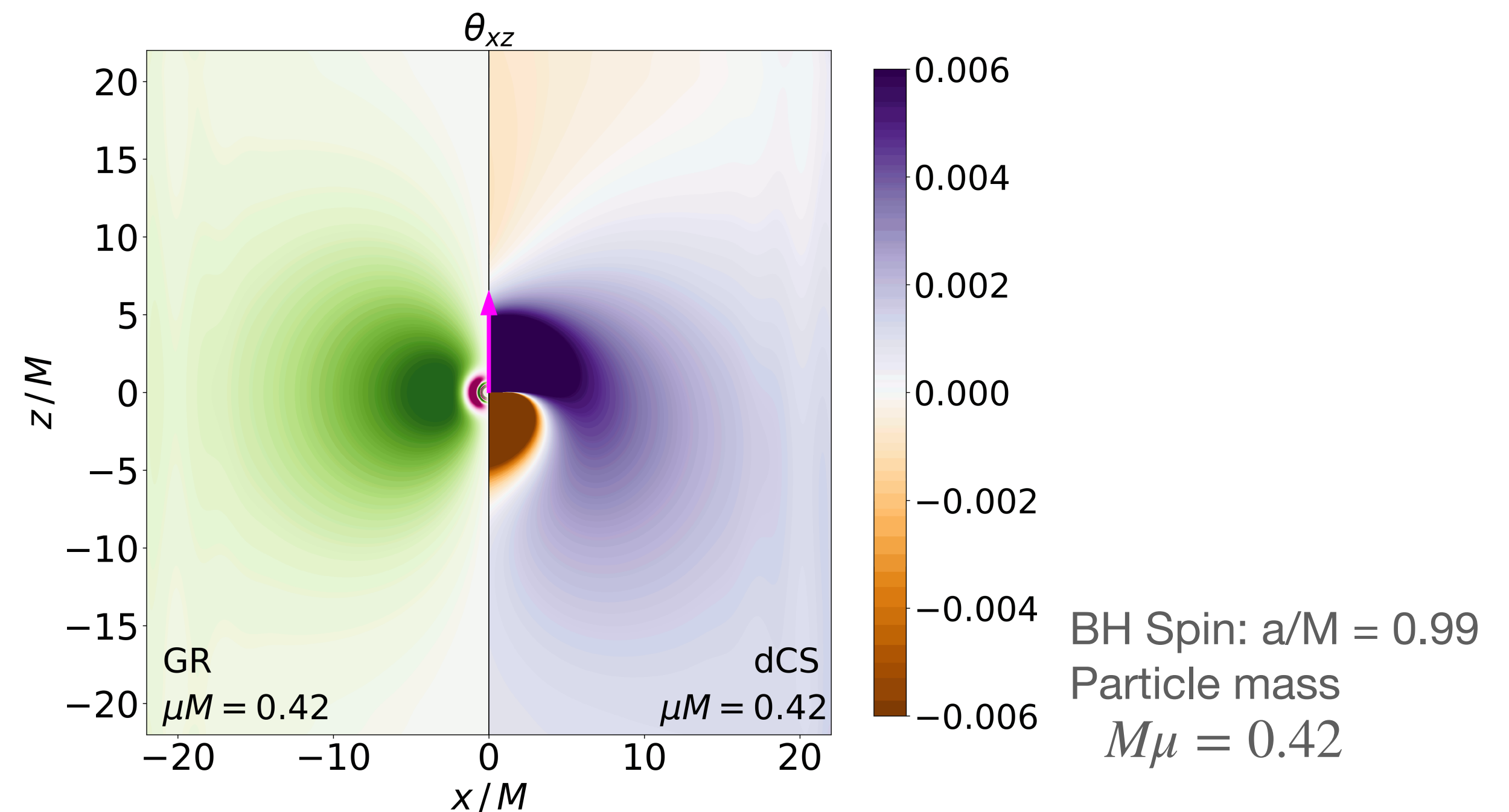
$$\square \theta + \frac{\alpha_{CS}}{4} R^* R - \mu^2 \theta = 0$$



Massive scalar cloud



Oscillating Chern-Simons hair

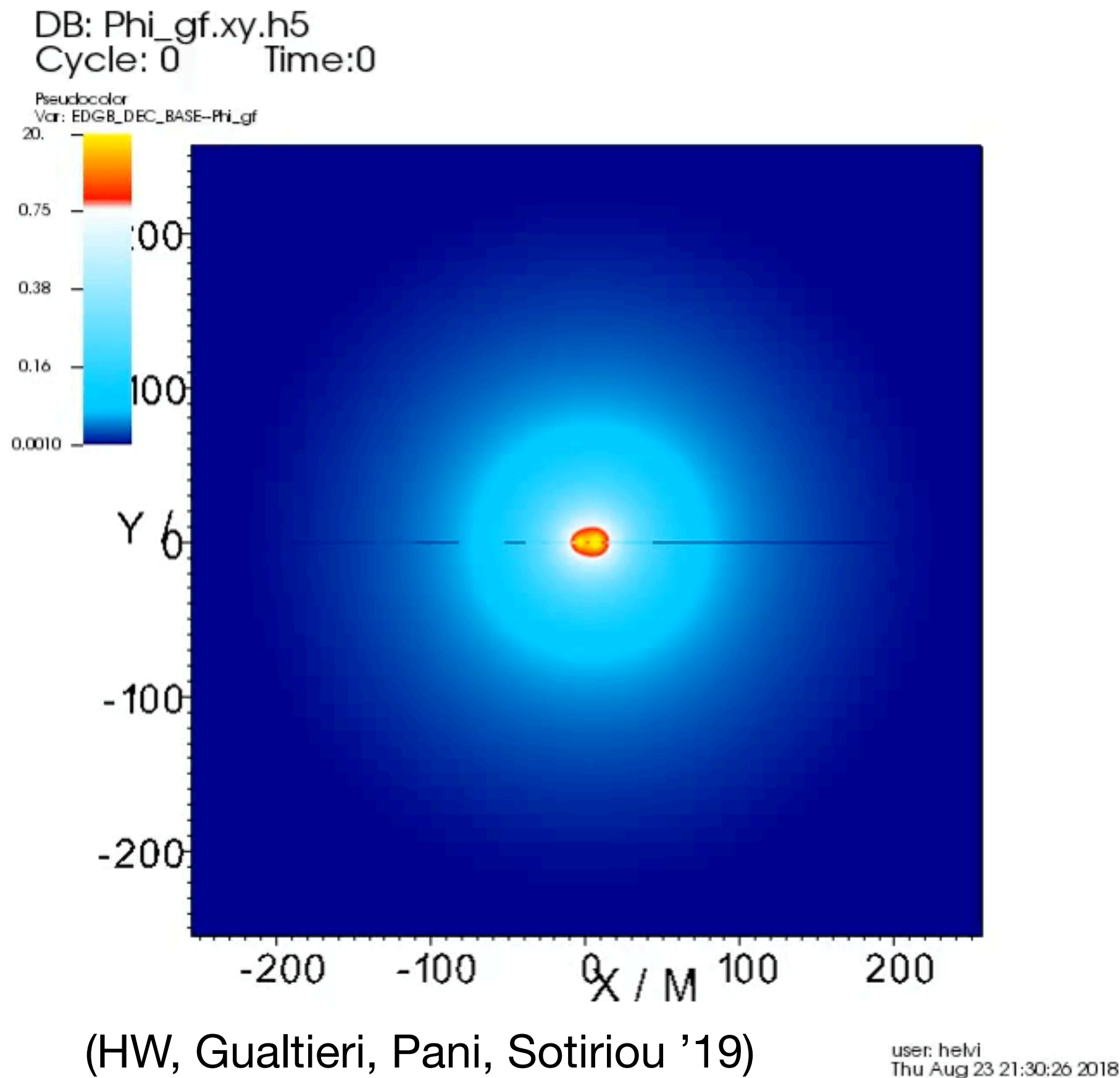


Merging

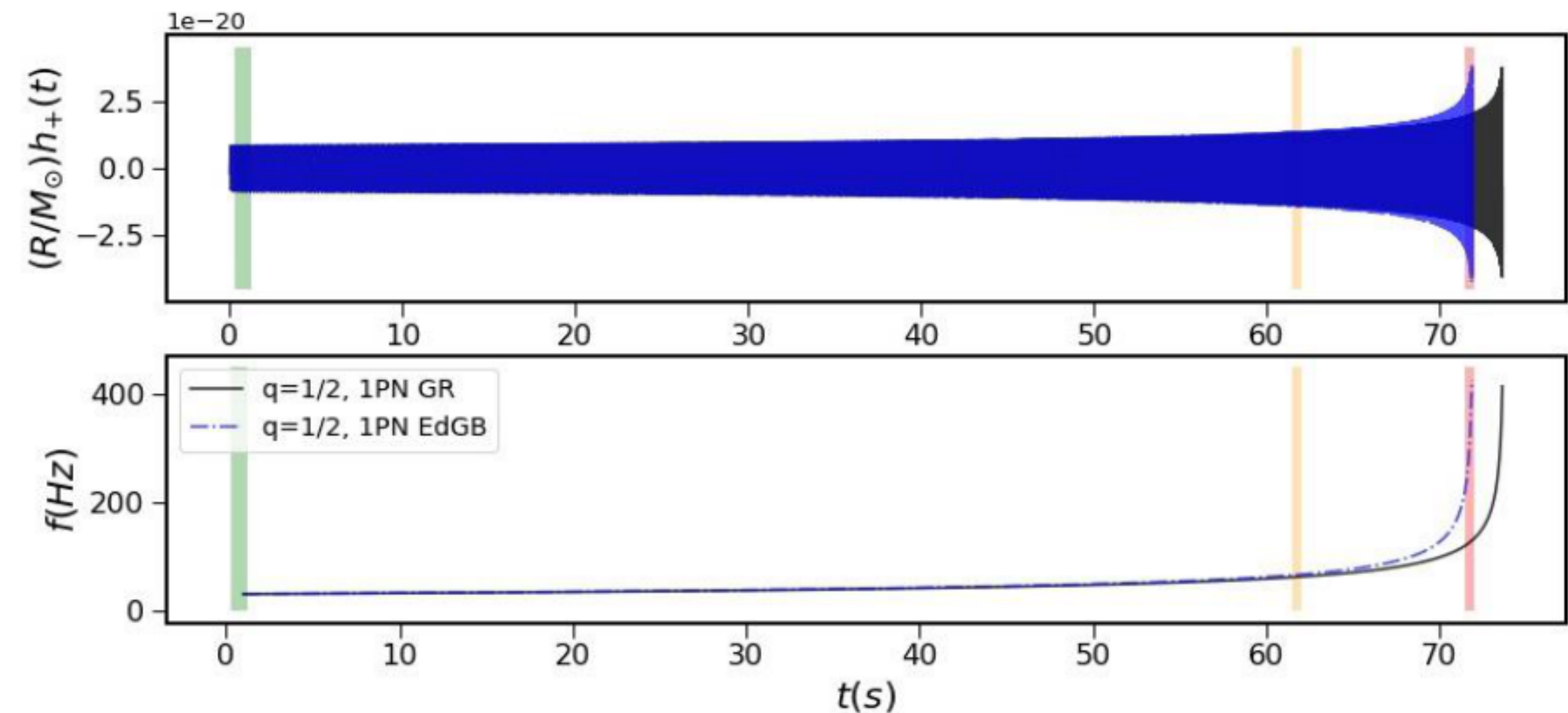
black holes beyond GR

Scalar Gauss-Bonnet gravity I

(HW, Gualtieri, Pani, Sotiriou '19; Shiralilou, Hinderer, Nissanke, Ortiz, HW '20, '21)



- Coalescence of black holes with scalar hair (“charge”)
 - scalar dipole radiation
 - Gravitational wave phase shift



PN Waveform (Shiralilou, Hinderer, Nissanke, Ortiz, HW '20, '21)

[See also: Yagi+ '11, Julie+ '18, '19, Lyu+ '22]

Canuda

Open-source numerical relativity code
for fundamental physics

DOI 10.5281/zenodo.7791842

Witek, Helvi; Zilhao, Miguel; Bozzola, Gabriele; Cheng, Cheng-Hsin; Dima, Alexandru; Elley, Matthew; Ficarra, Giuseppe; Ikeda, Taishi; Luna, Raimon; Richards, Chloe; Sanchis-Gual, Nicolas; Silva, Hector

<https://bitbucket.org/canuda/>

Canuda Numerical Relativity

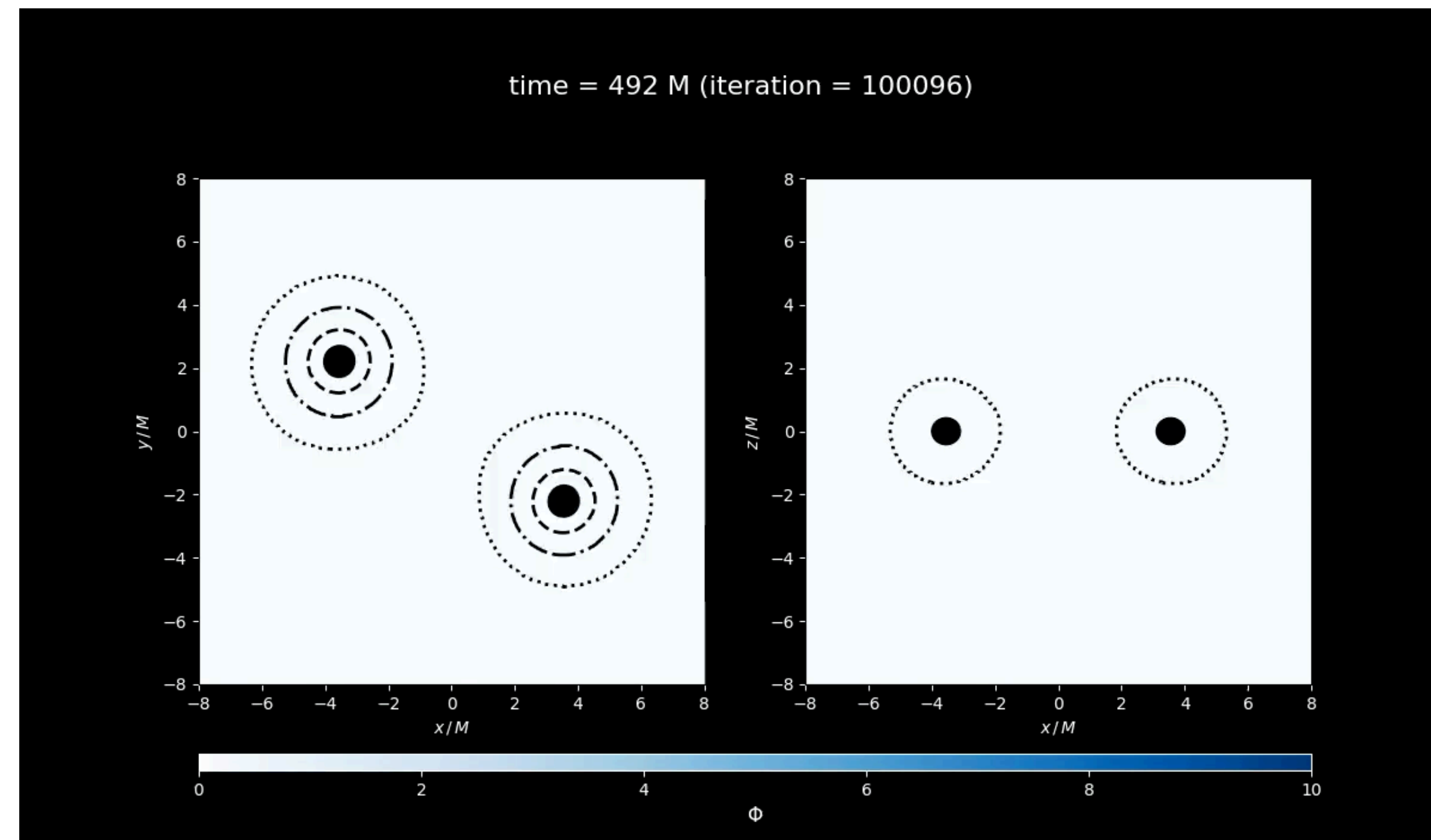
[Equivalent numerical approach in dCS: Okounkova et al '17-'19]

Scalar Gauss-Bonnet gravity II: dynamical (de-)scalarization

(Silva, Elley, HW, Yunes '21, '22)

- Consider quadratic coupling of scalar field
- Scalarization of single black holes if $\mu_{eff}^2 < 0$
- Coalescence \rightarrow dynamical scalarization or desclarization
- Nonlinear phenomenon may affect only merger and postmerger!

$$(\square - \mu_{eff}^2)\Phi = 0 \quad \mu_{eff}^2 \sim -\lambda \mathcal{G}$$

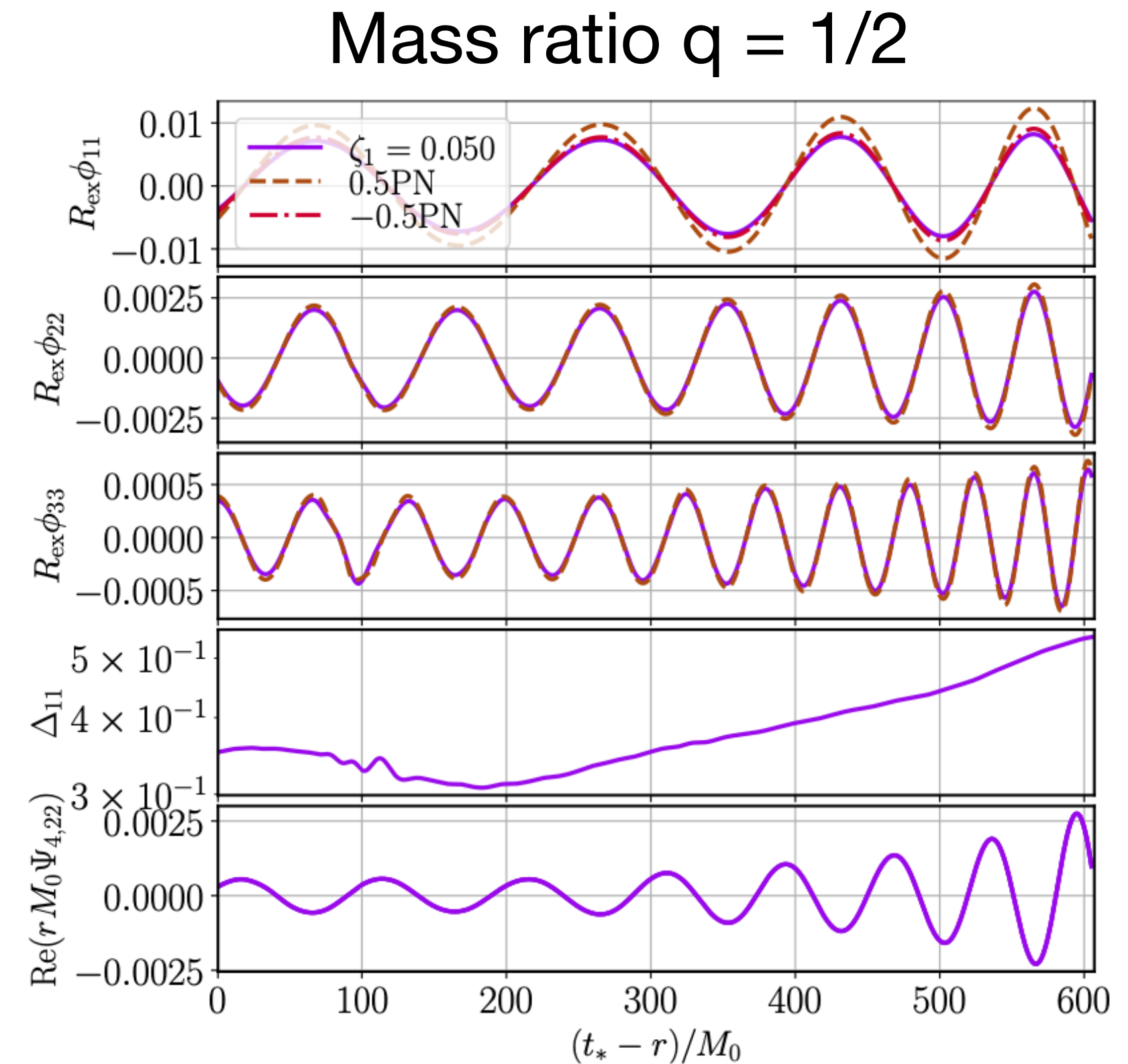
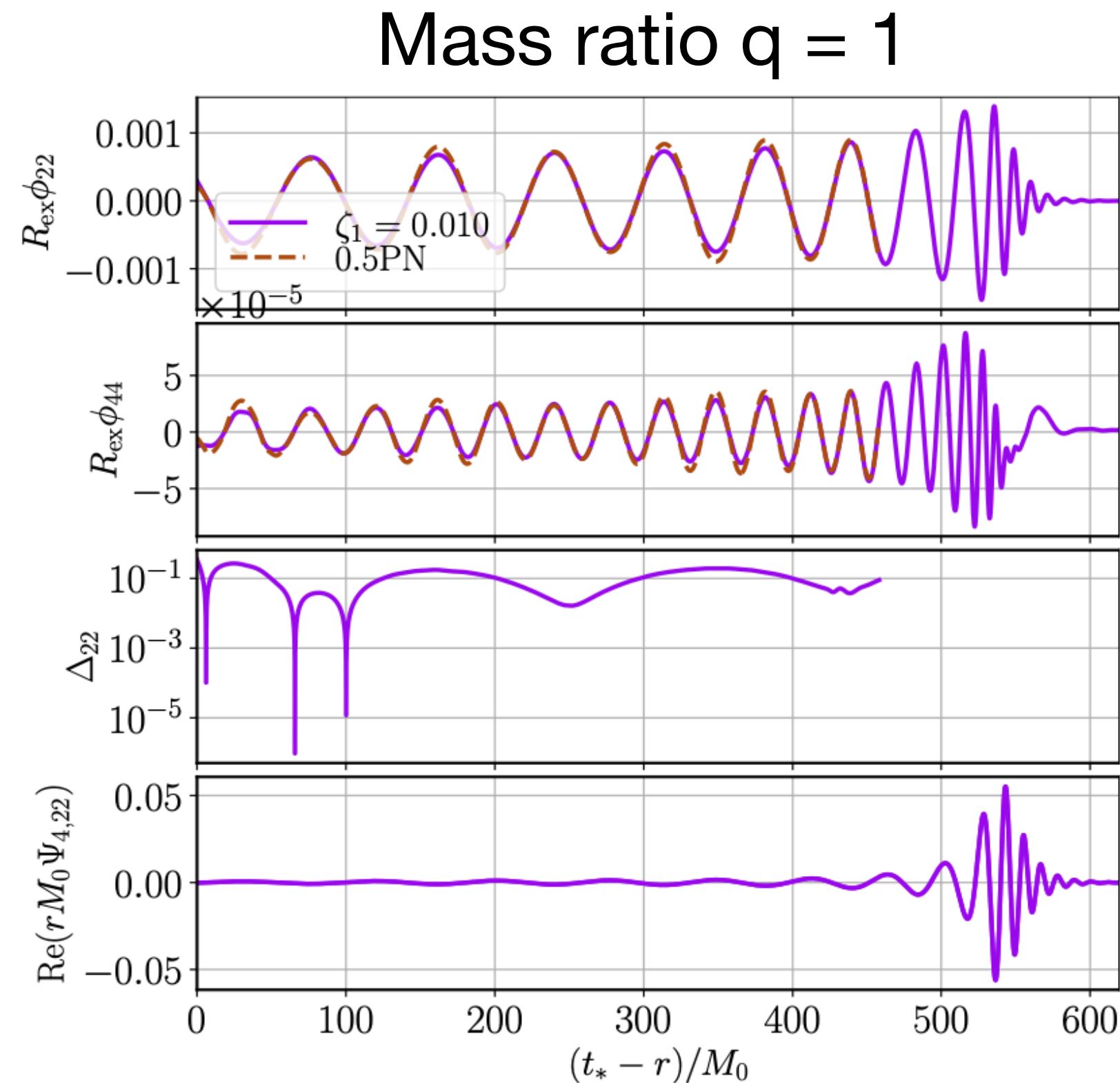


[Single BHs: Silva et al '17, Doneva et al '17, Antoniou et al '17, Blazquez-Salcedo '18, Macedo et al '19, Ripley & Pretorius '20, Spin-induced scalarization: Dima et al '20, Hod '20, Doneva et al '20, Herdeiro et al '20, Berti et al '20, Binaries: Silva, HW, Elley, Yunes '21; East, Ripley '21; Review: Doneva et al '22]

Scalar Gauss-Bonnet gravity w/ modified generalized harmonic gauge

(Corman, Ripley, East '22)

- Scalar Gauss-Bonnet gravity $\mathcal{L} = R - \frac{1}{2}(\nabla\Phi)^2 + \frac{\alpha_{GB}}{4}\Phi\mathcal{G}$
- Modified generalized harmonic gauge
- Breakdown near merger for some parameters



[See also Kovacs & Reall '20, East & Ripley '21, '22]

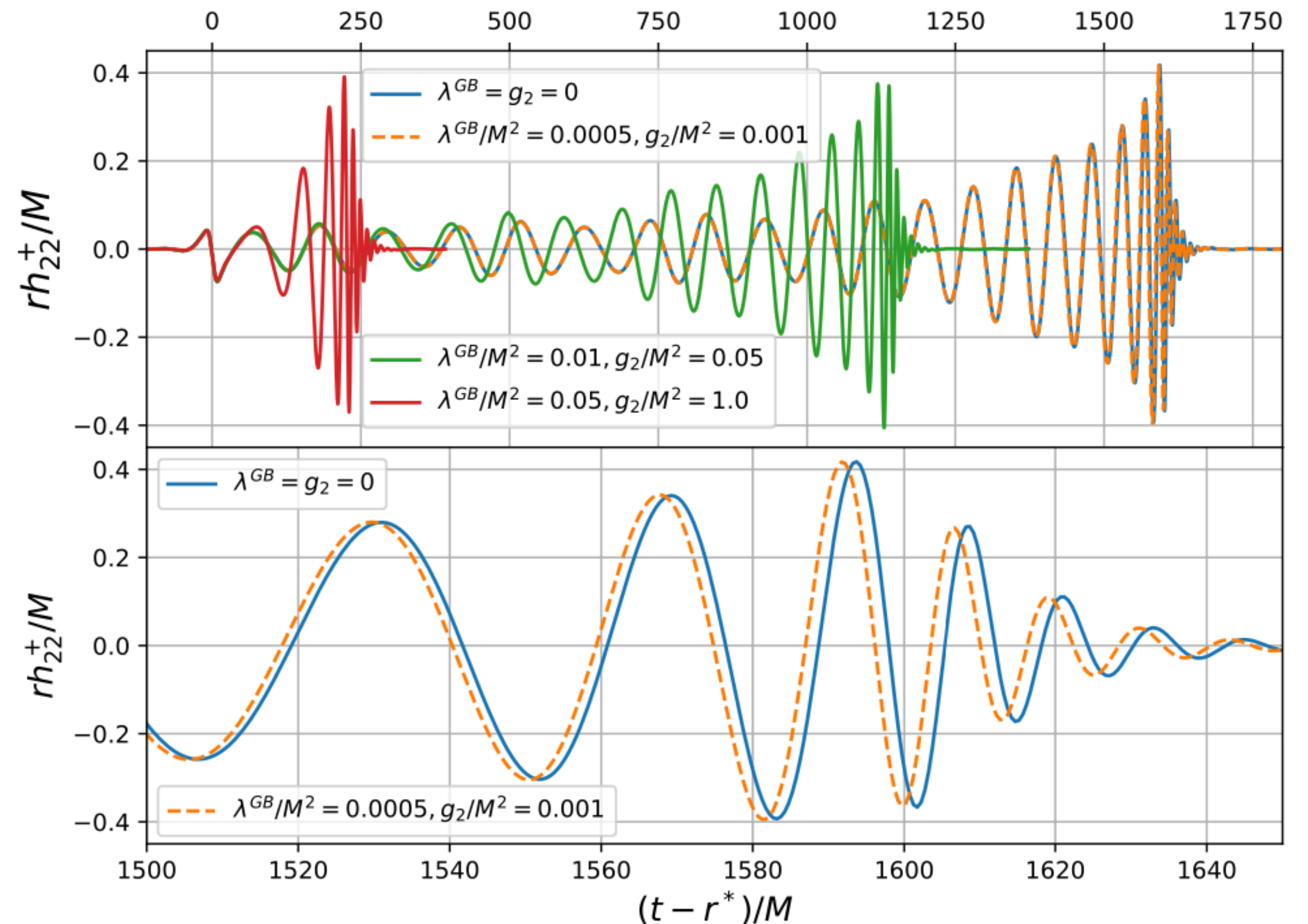
Four derivative scalar-tensor theory

(Sole, Clough, Figueras '22)

- General scalar-tensor theory w/ up to four derivatives

$$\mathcal{L} = R - \frac{1}{2}(\nabla\Phi)^2 + \frac{\lambda_{GB}}{4}\Phi\mathcal{G} + g_2(\Phi)(\nabla\Phi)^4$$

- Using ccZ4 w/ modified puncture gauge (inspired by modified generalized harmonic gauge)
- Binary black holes w/ $q=1$, $d=11M$



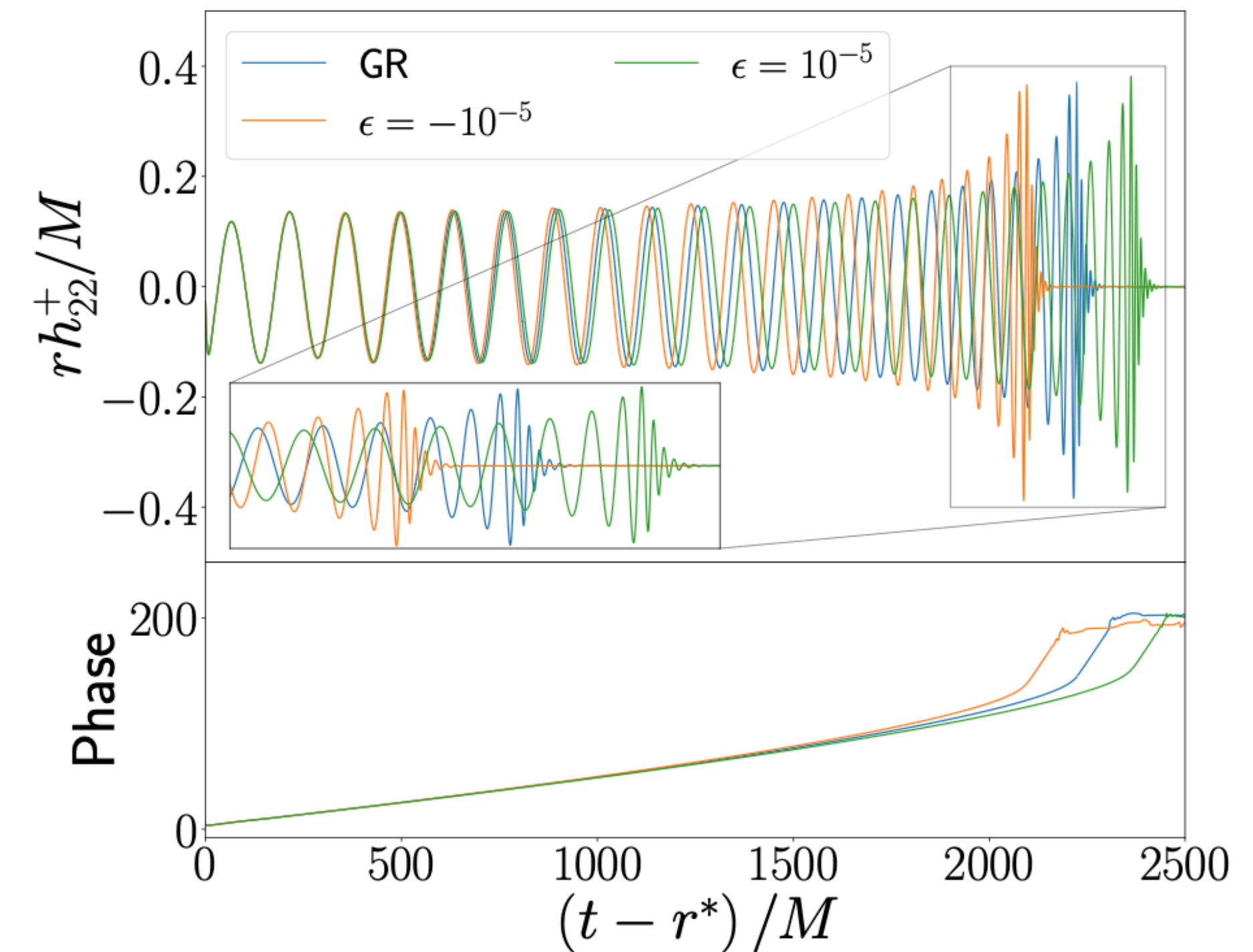
Fourth order effective field theory of gravity

(Cayuso, Figueras, Franca, Lehner '23)

- Effective field theory of gravity (parity even and in vacuum)

$$\mathcal{L} = R - \frac{1}{\Lambda_6} (R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma})^2 + \dots$$

- Using
 - i. order reduction
 - ii. fixing equations a la Israel-Stewart
- Binary black holes w/ $q=1$, $d = 12M$



[J. Cayuso, Ortiz, Lehner '17, Allwright & Lehner '18, Franchini et al '22, Lara et al '22, Gerhardinger et al '22]

Note: order reduction refers to inserting the eoms for the Ricci tensor, thus shifting its contribution to higher-order.

Sometimes this terminology is used for the order-by-order expansion, but not here

What about degeneracies with

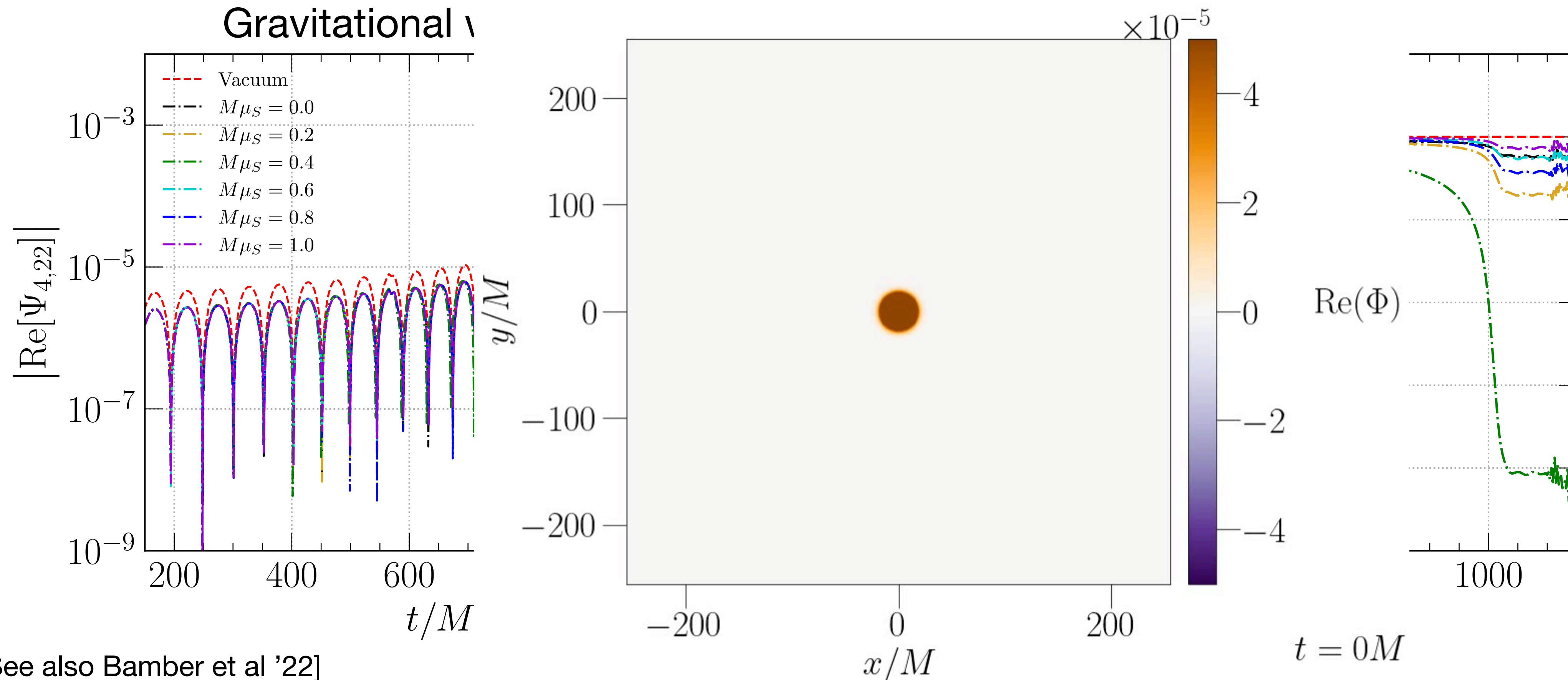
- **Unmodeled GR effects?**
- **Environment?**
- ...

Black holes binaries in dark matter environments

(C.-H. Cheng, Ficarra, HW in prep.)

- Consistent evolution of massive scalar cloud around binary black hole
 - Goal 1: new gravitational wave probe for new particles
 - Goal 2: toy model for environmental effects during merger
- Constraint satisfying initial data and eccentricity reduction

$$\square \Phi - \mu^2 \Phi = 0$$



[See also Bamber et al '22]

Summary

Status update in Numerical Relativity beyond GR (focus: higher derivative gravity)

- Proof-of-principle simulations of compact binary coalescence in classes of beyond GR model
- Scalar charges typical yields scalar (dipole/quadrupole) radiation
- Dephasing of gravitational wave signal
- Surprisingly simple merger signal — not as nonlinear as may have been expected

Where do we go from here?

- Synergy w/ PN and constructing full IMR models -> Waveform catalogs?
- Dictionary with theory-agnostic models for the plunge+merger (e.g. EoB)
- Concerns?
 - Robustness: waveform comparison and benchmark from different approaches and codes
 - Eccentricity reduction of initial data
 - Degeneracies with unmodelled GR or environmental effects

Thank you!

Some advertisement:



2nd Workshop “New Frontiers in Strong Gravity” 2022

“New Frontiers in Strong Gravity III”

Science Center Benasque / Spain

7-20 July 2024

<https://www.benasque.org/general/cgi-bin/years.pl?ano=2024>

