# Beyond GR: from no-hair theorems to nonlinear dynamics 

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## Motivation

A host of current (LVK, EHT, PTAs, NICER, ...) and future (LISA, Athena, ET, Cosmic Explorer, PTAs, ...) observations will reveal the nature of black holes and neutron stars
$\rightarrow$ Can we use them to search for new fundamental physics?

Key points:

* New physics $\rightarrow$ New fields
* Theories predict structure and dynamics of compact objects


## Case study: scalar fields and BHs

Light scalars are ubiquitous in extensions of GR or the Standard Model

* Can new physics leave an imprint on BHs?
$\rightarrow$ Can BHs have scalar hair?
* Which observations are more sensitive to new physics?
$\rightarrow$ Which BHs give best measurement of scalar charge?
* Are all black holes the same?
$\rightarrow$ Can scalar hair appear only for some black holes?


## No-hair theorems

Asymptotically flat black holes have no scalar hair
-. Minimally coupled, Brans-Dicke; stationary
S.W. Hawking, Comm. Math. Phys. 25, 152 (197ß)
-. Self-interacting, Scalar-tensor theories; stationary
T. P. S. and V. Faraoni, Phys. Rev. Lett. 108, 081103 (2012)
-8. Shift-symmetric; static, spherically symmetric/slowly rotating, assumptions on the current
L. Hui, A. Nicolis, PRL 110, 241104 (2013)
T.P.S. and S.-Y. Zhou, PRL112, 251102 (2014)

## No difference from GR?

Actually there is...
.f Perturbations are different
E. Barausse and T.P.S., Phys. Rev. Lett. 101, 099001 (2008)
-. ...but hard to excite in astrophysical setting!
-. Interesting exceptions exist, e.g. superradiance for axions
A. Arvanitaki and S. Dubovksy, Phys. Rev. D 83, 044026 (2011)
R. Brito, V. Cardoso and P. Pani, Lect.Notes Phys. 906, 1 (2015)

## Hairy black holes

Consider the action
T.P.S. and S.-Y. Zhou, PRL 112, 251102 (2014);

Phys. Rev. D 90, 124063 (2014).

$$
S=\frac{m_{P}^{2}}{8 \pi} \int d^{4} x \sqrt{-g}\left(\frac{R}{2}-\frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi+\alpha \phi \mathcal{G}\right)
$$

The corresponding scalar equation is

$$
\square \phi+\alpha \mathcal{G}=0
$$

At small coupling/weak field identical to exponential coupling of dilaton in string theory
P. Kanti et al., Phys. Rev. D 54, 5049 (1996) N. Yunes and L. Stein, Phys. Rev. D 83, 104002 (2011)

Solve to first order in the coupling
-. metric is Schwarzschild,

$$
\phi^{\prime}=\alpha \frac{16 M^{2}-C r^{3}}{r^{4}(r-2 M)}
$$

## Scalar charge

Regularity on the horizon implies $C=2 / M$

$$
\phi^{\prime}=-\frac{2 \alpha\left(r^{2}+2 M r+4 M^{2}\right)}{M r^{4}}
$$

The scalar charge is fixed to be

$$
P=\frac{2 \alpha}{M}
$$

Confirmed with numerical scalar collapse
R. Benkel, T.P.S. and H. Witek, Phys. Rev. D 94 (R), 121503 (2016);

Class. Quant. Grav. 34, 064001 (2017)
More generally, for shift-symmetric scalars

$$
P \propto \alpha \int_{\mathcal{H}} n_{a} \mathcal{G}^{a} \quad \mathcal{G}=\nabla_{a} \mathcal{G}^{a}
$$

M. Saravani \& T.P.S., Phys. Rev. D 99, 12, 124004 (2019)

## NR beyond GR


H. Witek, L. Gualtieri, P. Pani and T.P.S., Phys. Rev. D 99, 064035 (2019)

L. Salo, K. Clough, P. Figueraas, arXiv:2208.14470

In need of a well-posed IVP problem:

- Perturbation in the coupling
R. Benkel, T.P.S. and H. Witek, Phys. Rev. D 94 (R), 121503 (2016)
M. Okounkova et al., Phys. Rev. D 96, 044020 (2017)
-. Israel-Stewart-like approach
J. Cayuso, N. Ortiz, and L. Lehner, Phys. Rev. D 96, 084043 (2017)
\&. Novel 3+1 formulations
J. L. Ripley, and F. Pretorius, Phys. Rev. D 99, 084014 (2019)
A. D. Kovacs and H. S. Reall, PRL 124, 221101 (2020)

A new frontier in Numerical Relativity!

## Light scalars and GW

## BH Binaries



* dipole emission
* additional energy flux, i.e. change in orbital dynamics
* Modified waveform


## EMRIs

* Small mass ratio $q=m_{p} / M \ll 1$
* very long inspiral
* Precise mapping of spacetime of the primary


## BH mass and charge

So far we've seen two cases
\& Theories that are covered by no-hair theorems
-. An exception in which the scalar charge is fixed to be

$$
P=\frac{2 \alpha}{M}
$$

More generally, for shift-symmetric scalars

$$
\begin{aligned}
P \propto \alpha \int_{\mathcal{H}} n_{a} \mathcal{G}^{a} \mathcal{G}=\nabla_{a} \mathcal{G}^{a} \\
\text { M. Saravani \&e T.P.S., Phys. Rev. D 99, 12, } 124004(\text { (2019 ) }
\end{aligned}
$$

If $M$ is the only relevant scale for the BH

$$
\alpha \ll M^{2} \rightarrow P / M \ll 1
$$

and large (enough) BHs are effectively Kerr BHs!

## Probes for massless scalars

## BH Binaries



A. Maselli, N. Franchini, L. Gualtieri, and T.P.S, PRL 125, 14, 141101 (2020)
A. Maselli, N. Franchini, L. Gualtieri, T.P.S, S. Barsanti, P. Pani, Nature Astronomy (202Z)

Ultra-light scalars: same except
S. Barsanti, A. Maselli , T.P.S, L. Gualtieri, PRL (accepted), arXiv:2212.03888 [gr-qc]

* superradiance-powered clouds
* spin-induced scalarization


## Black hole scalarization

No-hair theorem for the action

$$
S=\frac{m_{P}^{2}}{8 \pi} \int d^{4} x \sqrt{-g}\left(\frac{R}{2}-\frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi+f(\phi) \mathcal{G}\right)
$$

provided that $\quad f^{\prime}\left(\phi_{0}\right)=0, \quad f^{\prime \prime}\left(\phi_{0}\right) \mathcal{G}<0$
That is, for the equation

$$
\square \phi=-f^{\prime}(\phi) \mathcal{G}
$$

trivial solutions are unique if admissible, if the effective mass squared is positive

$$
\text { H. O. Silva, J. Sakstein, L. Gualtieri, T.P.S, and E. Berti, PRL 120, } 131104 \text { (2018) }
$$

\& But if it is negative then there can be scalarization!

## Tachyonic instability

T. Damour and G. Esposito-Farese, PRL 70, 22ఙ0 (1993)

Massive scalar

$$
\square \phi-\mu^{2} \phi+\mathrm{NL}=0
$$

Perturbation around Schwarzschild

$$
\begin{aligned}
& \delta \phi=\sum_{\ell m} \frac{\psi_{\ell m}(r)}{r} Y_{\ell m}(\theta, \varphi) e^{-i \omega t} \\
& \frac{d^{2} \psi}{d r_{*}^{2}}+\left[\omega^{2}-V_{\mathrm{eff}}(r)\right] \psi=0, \\
& d r / d r_{*}=1-2 M / r
\end{aligned}
$$



Taken from G. Esposito-Farese, arXiv:gr-cq/040200ry

$$
\int_{2 M}^{\infty} V_{\mathrm{eff}} d r_{*} \leqslant 0
$$

$$
V_{\mathrm{eff}}=\left(1-\frac{2 M}{r}\right)\left[\frac{\ell(\ell+1)}{r^{2}}+\frac{2 M}{r^{3}}+\mu^{2}\right]
$$

## Black hole scalarization

$$
\mathcal{G}_{\mathrm{Kerr}}=\frac{48 M^{2}}{\left(r^{2}+\chi^{2}\right)^{6}}\left(r^{6}-15 r^{4} \chi^{2}+15 r^{2} \chi^{4}-\chi^{6}\right)
$$

- For $\chi=0$ : Schwarzschild and $\mathcal{G}>0$

Scalarization for $f^{\prime \prime}\left(\phi_{0}\right)>0$
H. O. Silva, J. Sakstein, L. Gualtieri, T.P.S, and E. Berti, PRL 120, 131104 (2018)
D. D. Doneva and S. S. Yazadjiev, PRL 120, 131103 (2018)

ヶ. For $\chi \neq 0: \mathcal{G}$ can change sign near the horizon
Spin-induced scalarization when $f^{\prime \prime}\left(\phi_{0}\right)<0$
A. Dima, E. Barausse, N. Franchini, and T.P.S, PRL 125, 231101 (2020)
C. A. R. Herdeiro, E. Radu, H. O. Silva, T.P.S., and N. Yunes, PRL 126, 011103 (2021)
E. Berti, L. G. Collodel, B. Kleihaus, and J. Kunz, PRL 126, 011104 (2021)

## Perspectives

- Scalarization "screens" new physics at low curvatures
\&. Linear instability in strong field, quenched nonlinearly

$$
\tilde{g}^{\mu \nu}\left[g_{\mu \nu}, \phi\right] \nabla_{\mu} \nabla_{\nu} \phi=m_{\text {eff }}^{2}\left[g_{\mu \nu}, \phi\right] \phi+\text { nonlinear terms }
$$

-. Charge controlled by nonlinear interactions, e.g. $\phi^{4}, \phi^{2} R$
C. F. B. Macedo et al., Phys. Rev. D 99, 104041 (2019)
G. Antoniou, A. Lehebel, G. Ventagli, and T.P.S, Phys. Rev. D 104, 044002 (2021)
\& Others fields? Vectorisation, tensorization
F. M. Ramazanoglu, Phys. Rev. D 96, 064009 (2017)
H. Silva, A. Coates, F. M. Ramazanoglu, and T. P. S., Phys. Rev. D 105, 024046 (2021)
-. Other instabilities?
F. M. Ramazanoglu, Phys. Rev. D 97, 024008 (2018)
C. A. R. Herdeiro and E. Radu, Phys. Rev. D 99, 084039 (2019)
D. D. Doneva and S. S. Yadzadjiev, arXiv:2107.01738 [gr-qc]

## Perspectives

\& Dynamical scalarization
C. Palenzuela et al., Phys. Rev. D 89, 044024 (2014)
H. O. Silva et al., PRL 127, 031101 (2021)
M. Ealey et al., arXiv: 2205.06240 [gr-qc]
\& Cosmological evolution
T. Anson, E. Babichev, C. Charmousis, S. Ramazanov, JCAP 06023 (2019)
G. Antoniou, L. Bordin, and T.P.S, PRD 103, 024012 (2021)
c. Non-linear evolution and well-posedness
W. E. East and J. L. Ripley, PRL 127, 101102 (2021)
F. Thaalba, M. Bezares, N. Franchini, and T.P.S., arXiv:2306.01695 [gr-qc]
-. Stability of scalarized black holes
J. L. Blazquez-Salcedo, D. D. Doneva, J. Kunz, S. S. Yazadjiev, Phys. Rev. D 98, 084011 (2018)
C. F. B. Macedo et al., Phys. Rev. D 99, 104041 (2019)
G.Antoniou, C. F. Macedo, R. McManus, and T.P.S., arXiv:2R04.01684 [gr-qc]
B. Kleihaus, J. Kunz, T. Utermoehlen, E. Berti, Phys. Rev. D 107, L081501 (2023)
M. Minamitsuji and S. Mukohyama, arXiv:2305.05185 [gr qc]
...a mechanism that wants to become a theory.
Review: D. D. Doneva, F. M. Ramazanoglu, H. O. Silva, T. P.S., S. S. Yazadjiev, arXiv:R211.01766 [gr-qc]

## BHs and Ricci coupling

G. Antoniou, L. Bordin, and T.P.S, PRD 103, 024012 (2021)
G. Antoniou, A. Lehebel, G. Ventagli, and T.P.S, Phys. Rev. D 104, 044002 (2021)
G.Antoniou, C. F. Macedo, R. McManus, and T.P.S., Phys. Rev. D 106, 2, 024029 (202ん)

$\hat{M}$

## BHs and Ricci coupling

F. Thaalba, M. Bezares, N. Franchini, and T.P.S., arXiv:2306.01695 [gr-qc]



## Summary and conclusions

* Black holes beyond GR could reveal new fundamental physics
* No-hair theorems help identify interesting scenarios (as exceptions)
* Studying them brings on new challenges
$>$ Finding better EFTs and understanding their range of validity
> Developing numerical formulations and implementations for nonlinear evolution
> Extending perturbative techniques beyond GR

