Alexandre Toubiana

Max-Planck-Institut für Gravitationsphysik
Albert-Einstein-Institut

Fundamental physics meets waveforms with LISA
02/10/2024

Quasi-normal mode measurements with LISA and the impact of systematics

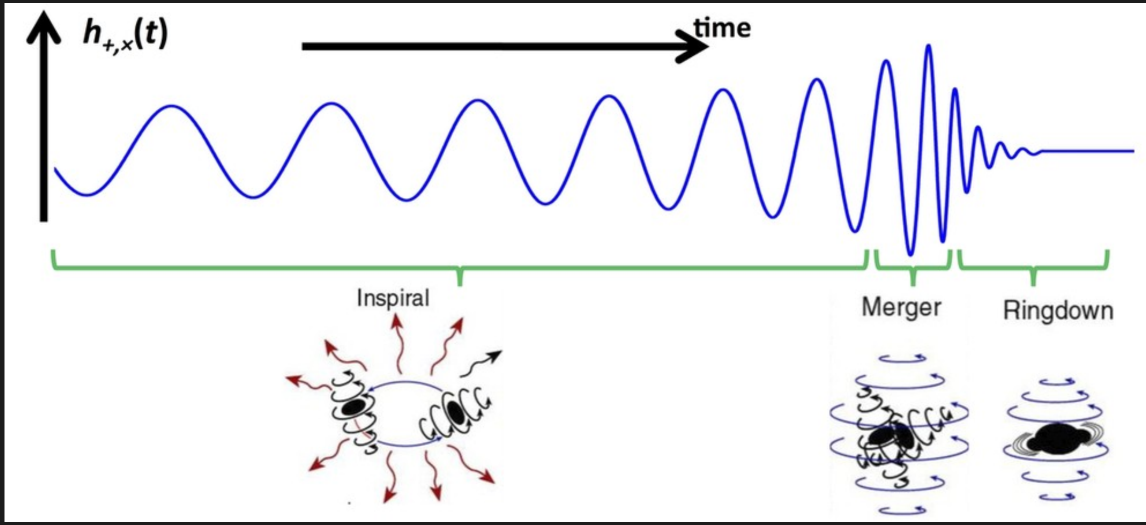
Toubiana, Pompili, Buonanno, Gair, Katz PRD 2024
Toubiana, Gair arXiv:2401.06845

Challenges to test GR with GWs

- GW computations are lengthy and difficult
- Number of proposed extensions to GR is very large, but little observational guidance
- Few full computations and simulations in modified gravity theories
- Resort to phenomenological approach

→ **Parametrised tests**

Ringdown tests



M. Favata, SXS, K. Thorne

- Quasi-normal modes (QNMs) depend only on remnant properties → no hair tests (see morning session)
- But:
  - Amplitudes and phases depend on the binary properties (mass ratio, spins…)
  - Analysis is sensitive to the chosen starting time of the ringdown
  - Loose SNR from inspiral-merger

IMR approach to ringdown tests

Model the whole signal and introduce additional parameters to capture deviations to the QNMs with pSEOBNRv5HM

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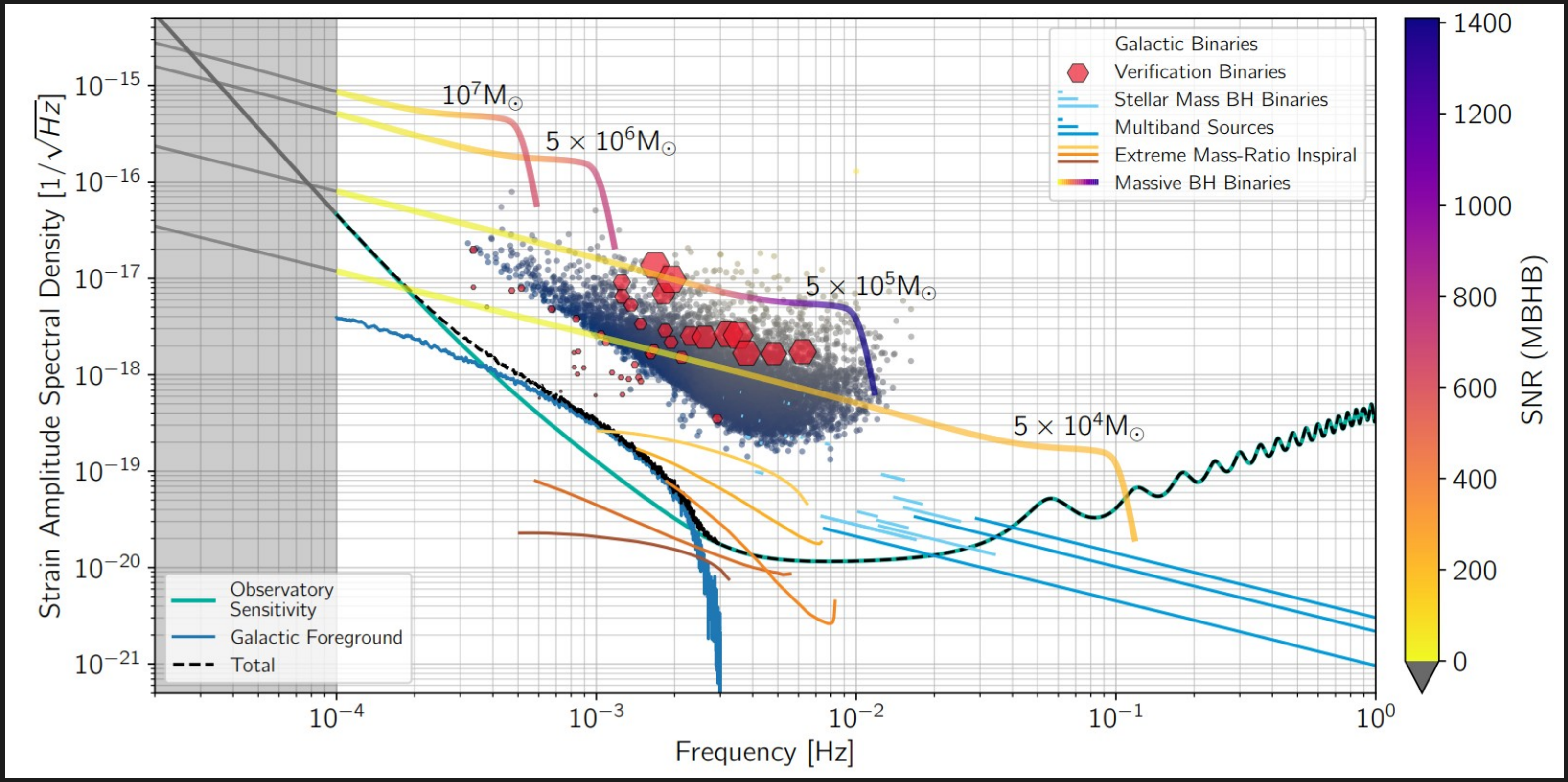
Model the whole signal and introduce additional parameters to capture deviations to the QNMs with pSEOBNRv5HM

Caveat: accuracy of the inspiral-merger model impacts the analysis

LISA sources



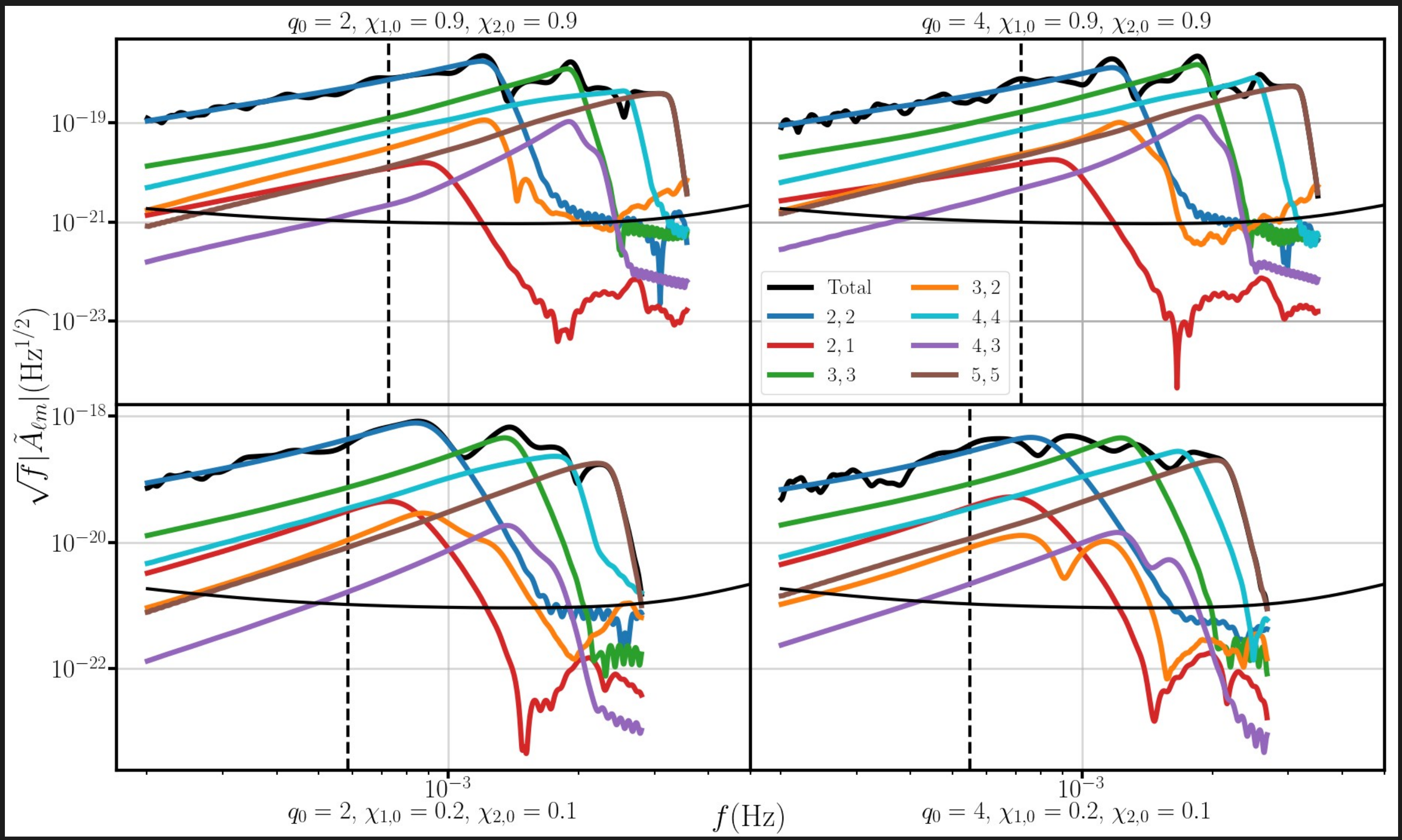
Credits: LISA Definition Study Report

Massive black hole binaries (MBHBs) best candidates for ringdown tests with LISA

Goals

- Quantify the accuracy of ringdown tests using the IMR approach with LISA
- Assess the impact of systematics

Considered systems



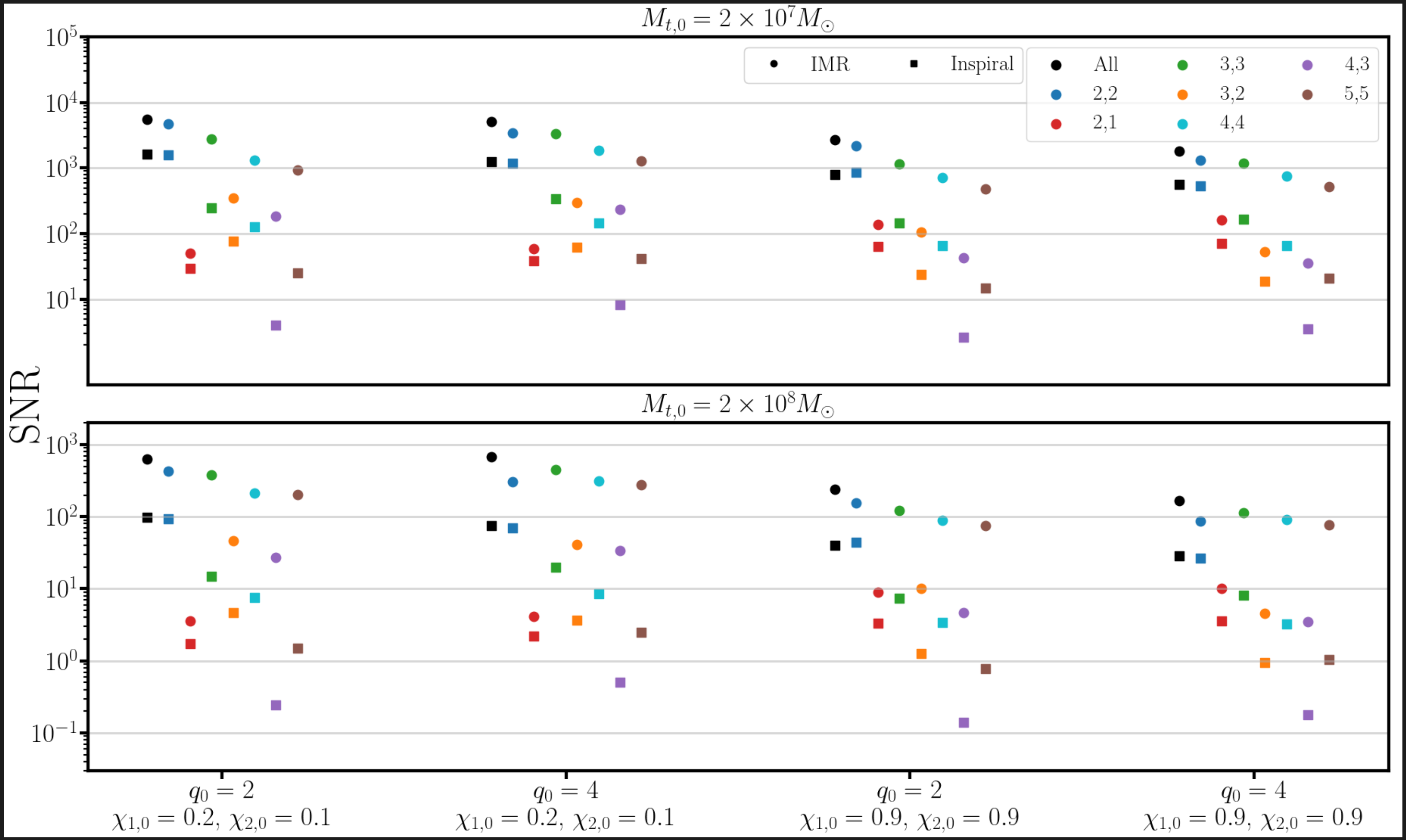
$$M_t = 2 \times 10^7 M_\odot$$

$$z = 2.2$$

- Consider also systems with $M_t = 2 \times 10^8 M_\odot$ and/or $z = 3.7$
- “Heavy seed” systems
- Use “long-wavelength” approximation

Signal-to-noise ratios (SNRs)

- At $z = 2.2$:



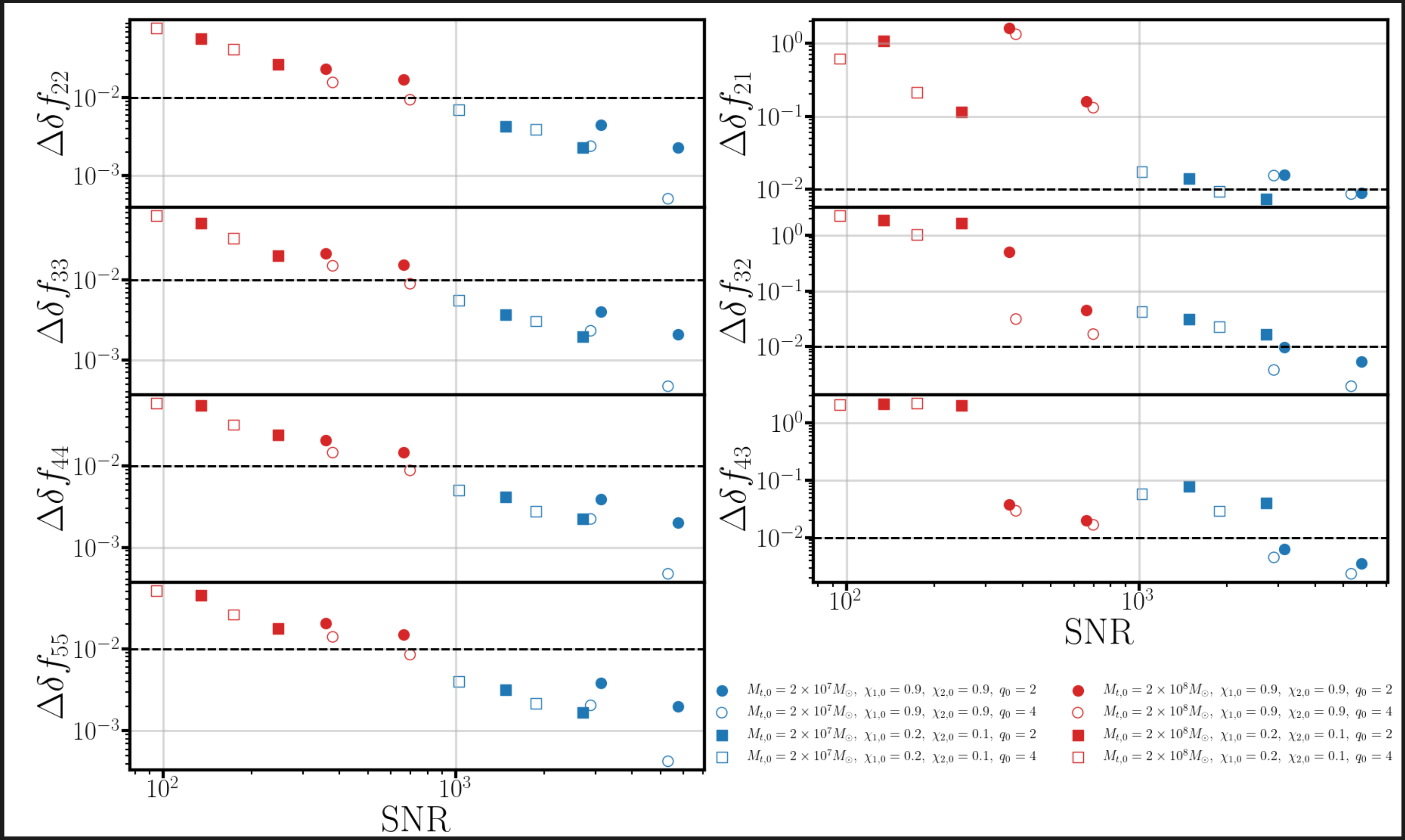
- At $z = 3.7$, rescale by 0.54

Analyses

- Simulate injections of MBHBs and do Bayesian analysis:
  - GR injection, GR templates
  - GR injection, non-GR templates
  - Non-GR injection, non-GR templates

non-GR injection, non-GR templates

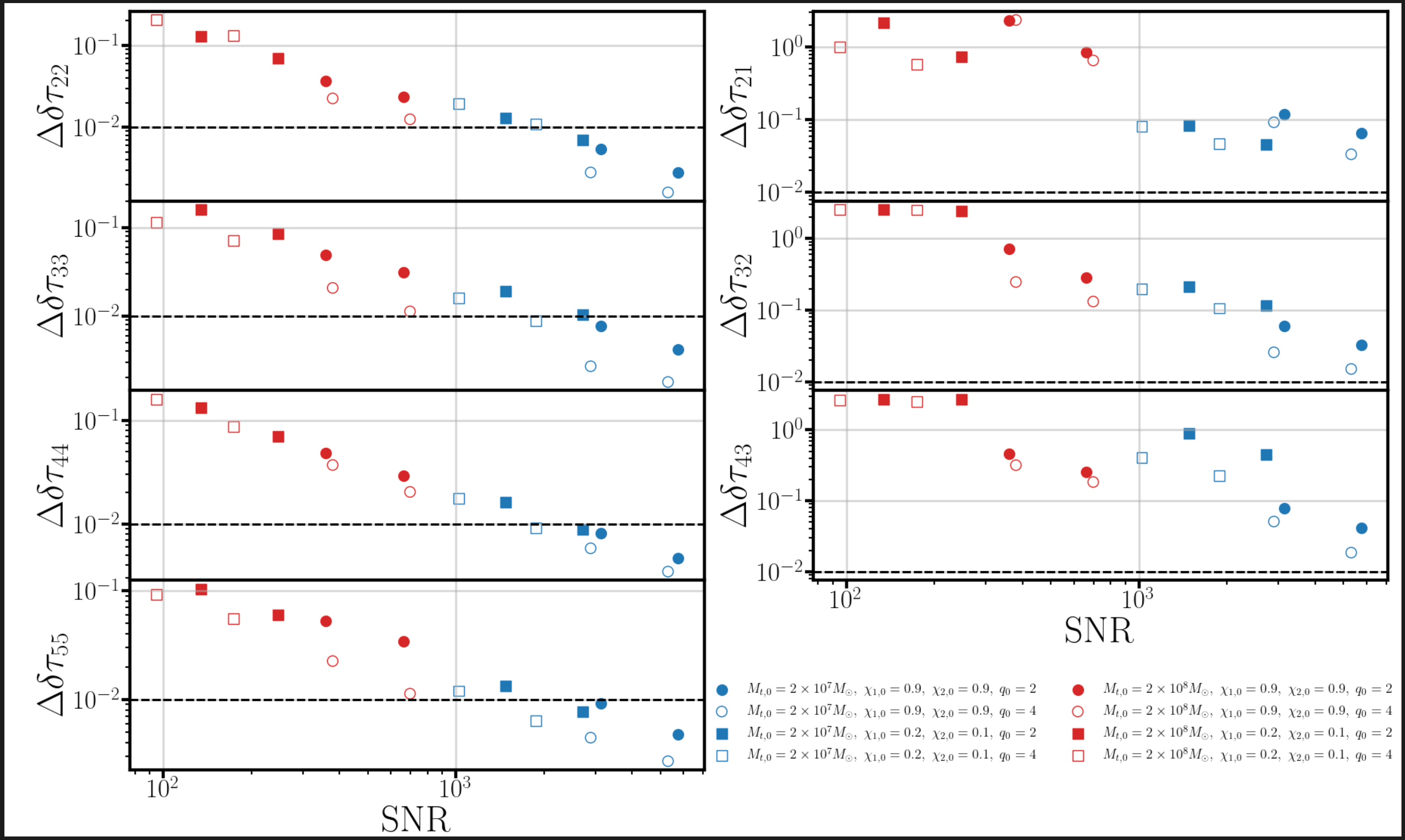
$$\delta f_{lm,0} = \delta \tau_{lm,0} = 0.01$$



- Measurement error depends little on the injected modification

non-GR injection, non-GR templates

$$\delta f_{lm,0} = \delta\tau_{lm,0} = 0.01$$



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Impact of systematics

- So far used EOB waveforms to generate mock signal and analyse it
- Could mismodelling lead to erroneous detection of GR deviation?

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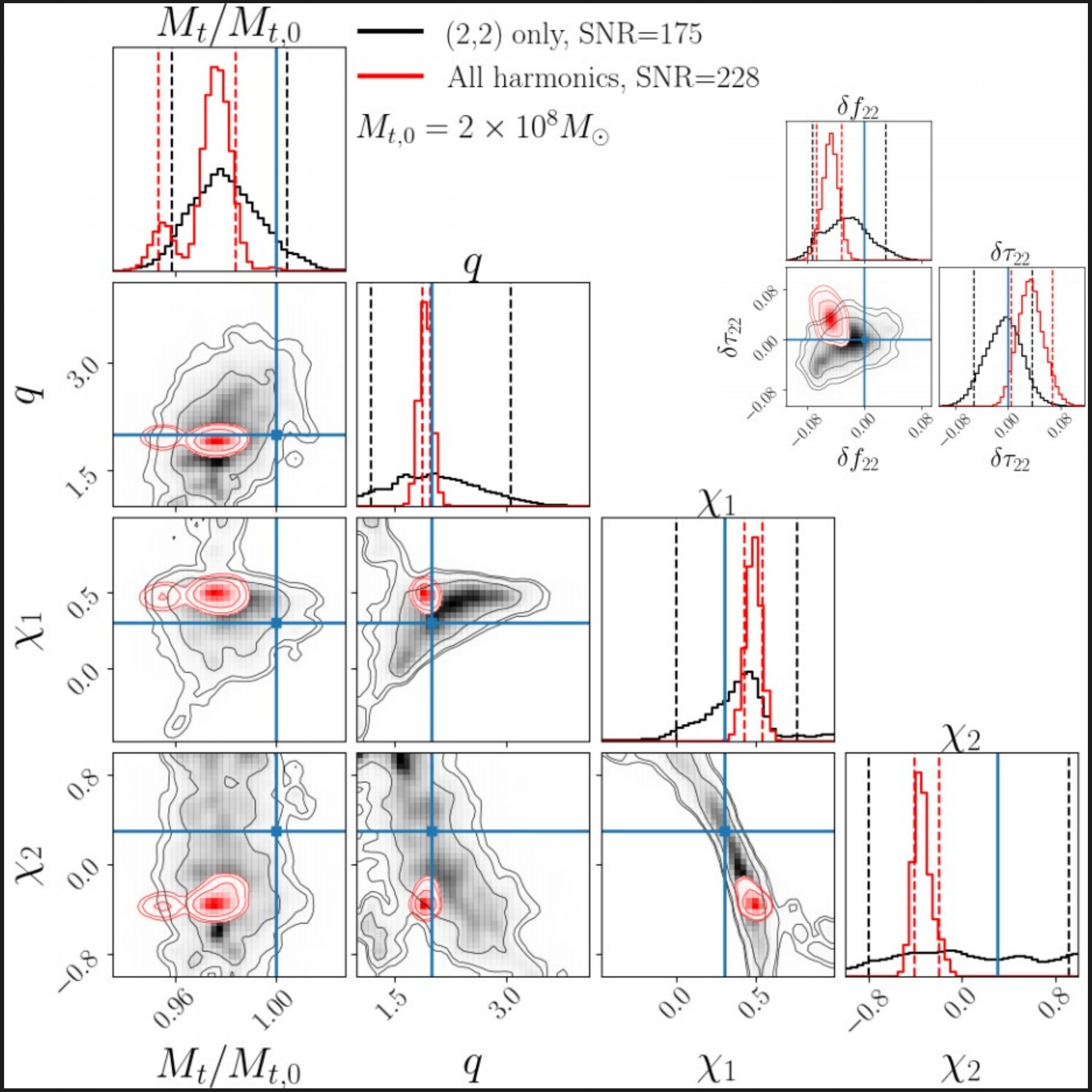
- Could mismodelling lead to erroneous detection of GR deviation?

→ Use numerical relativity (NR) for the mock signal $q = 2,\ \chi_1 = \chi_2 = 0.3$

SXS:BBH:2125:

$M_t = 2 \times 10^7 \text{ or } 2 \times 10^8 M_\odot$ $z = 2.2 \text{ or } 3.7$

Impact of systematics



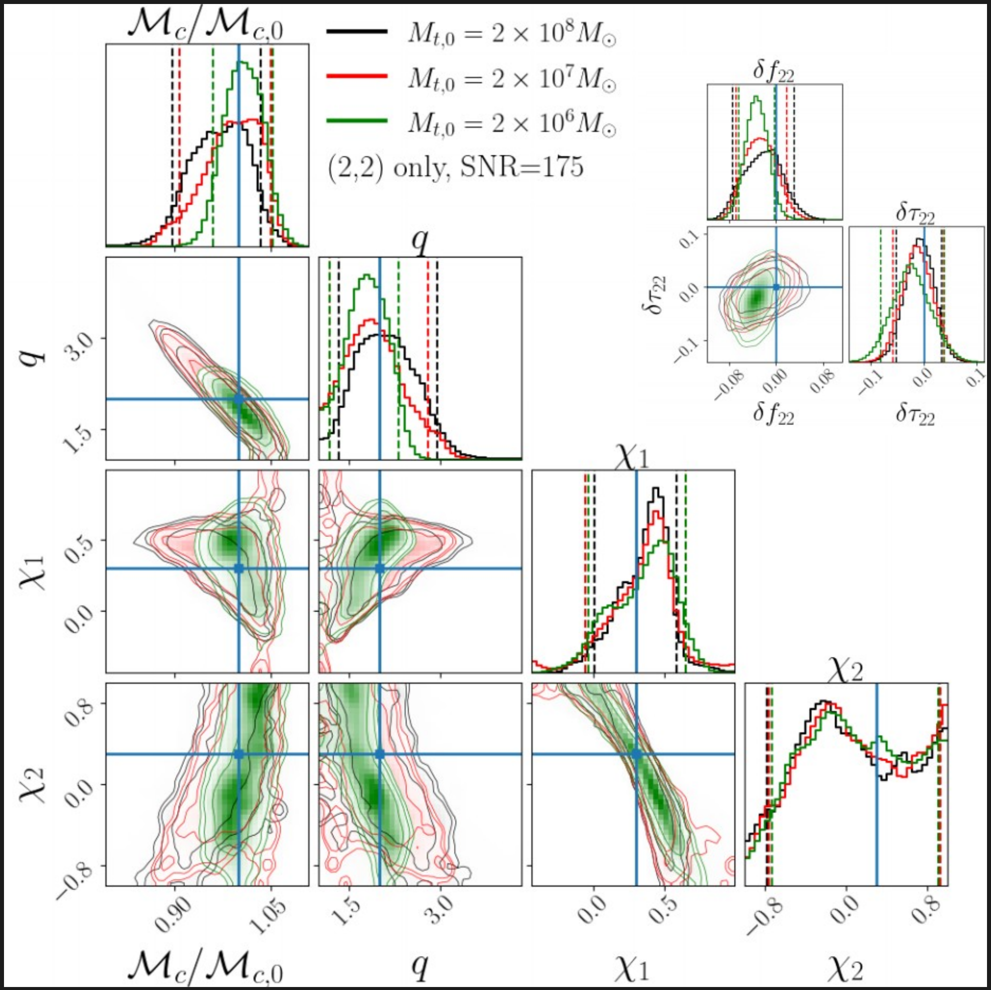
$$M_{t,0} = 2 \times 10^8 M_\odot$$

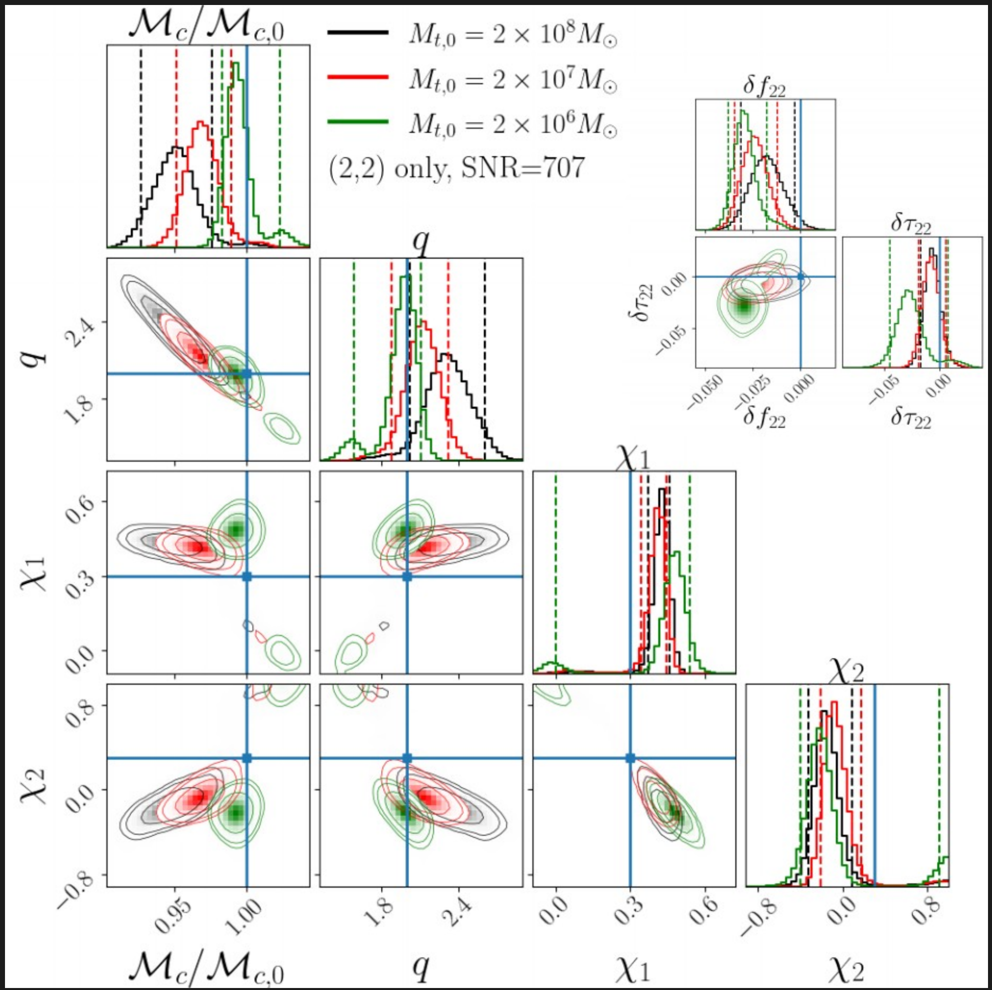
$$z = 2.2$$

Exploring systematic effects

- Systematics will be important for astrophysical sources
- How/why do they appear and from which SNR?

Impact of systematics





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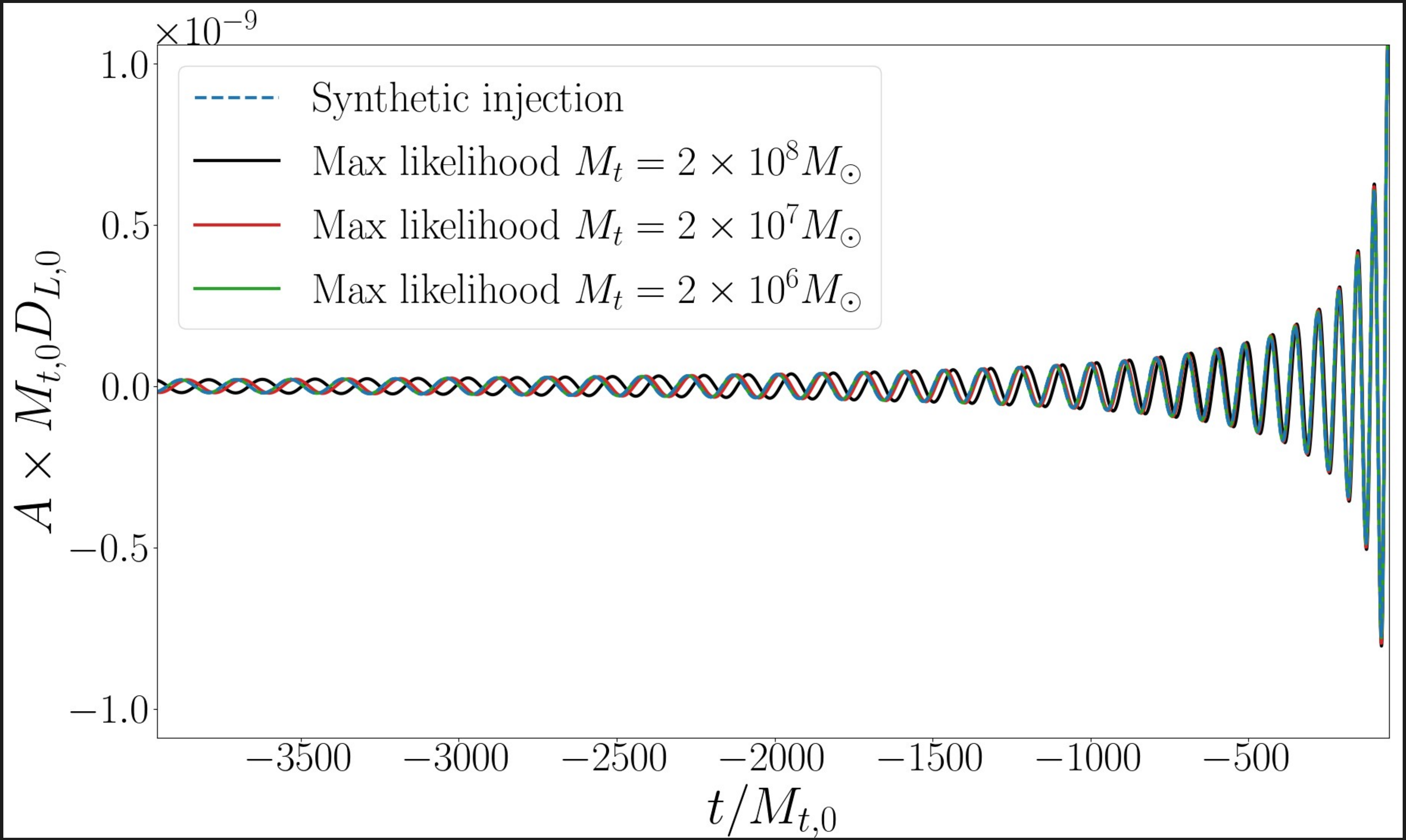
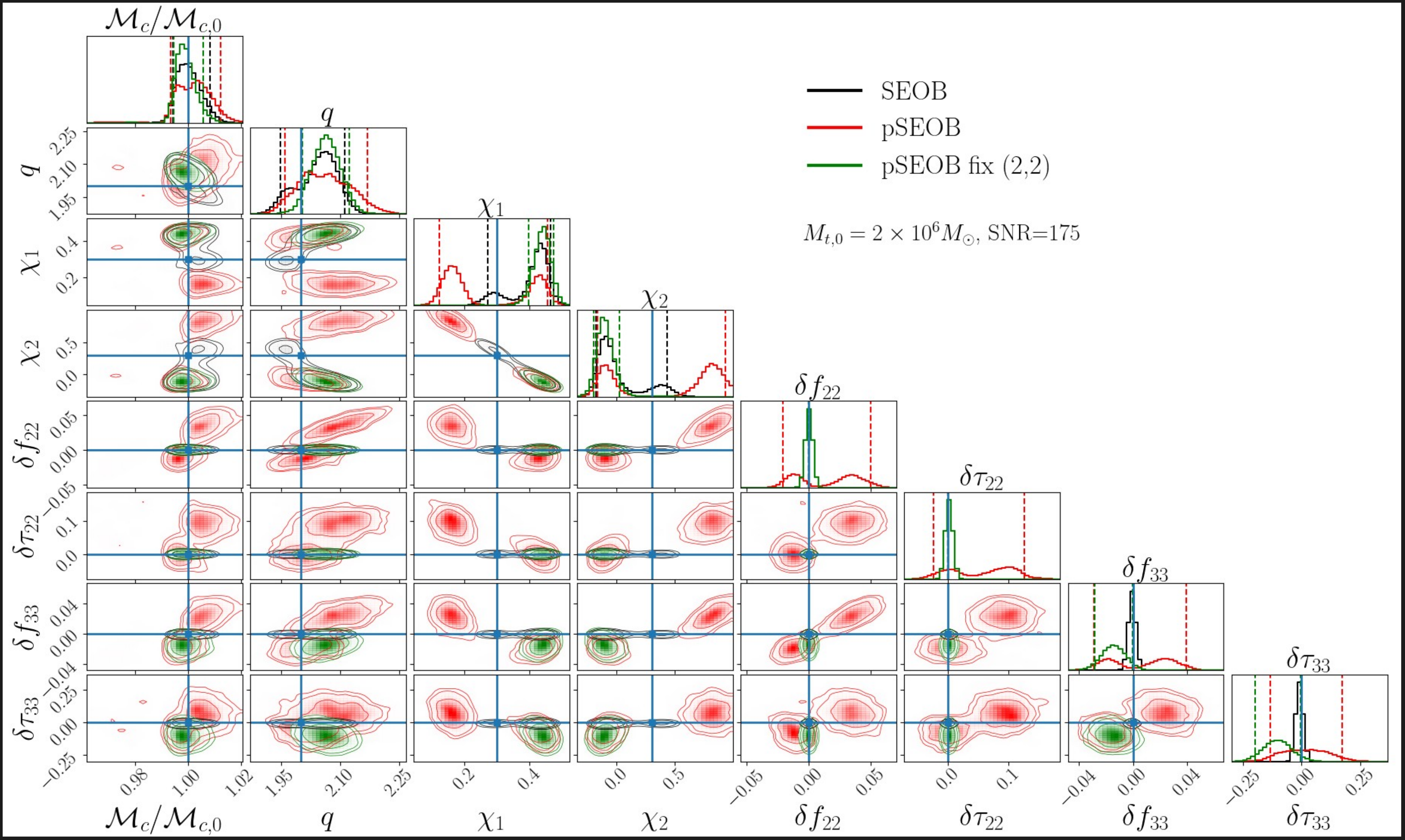
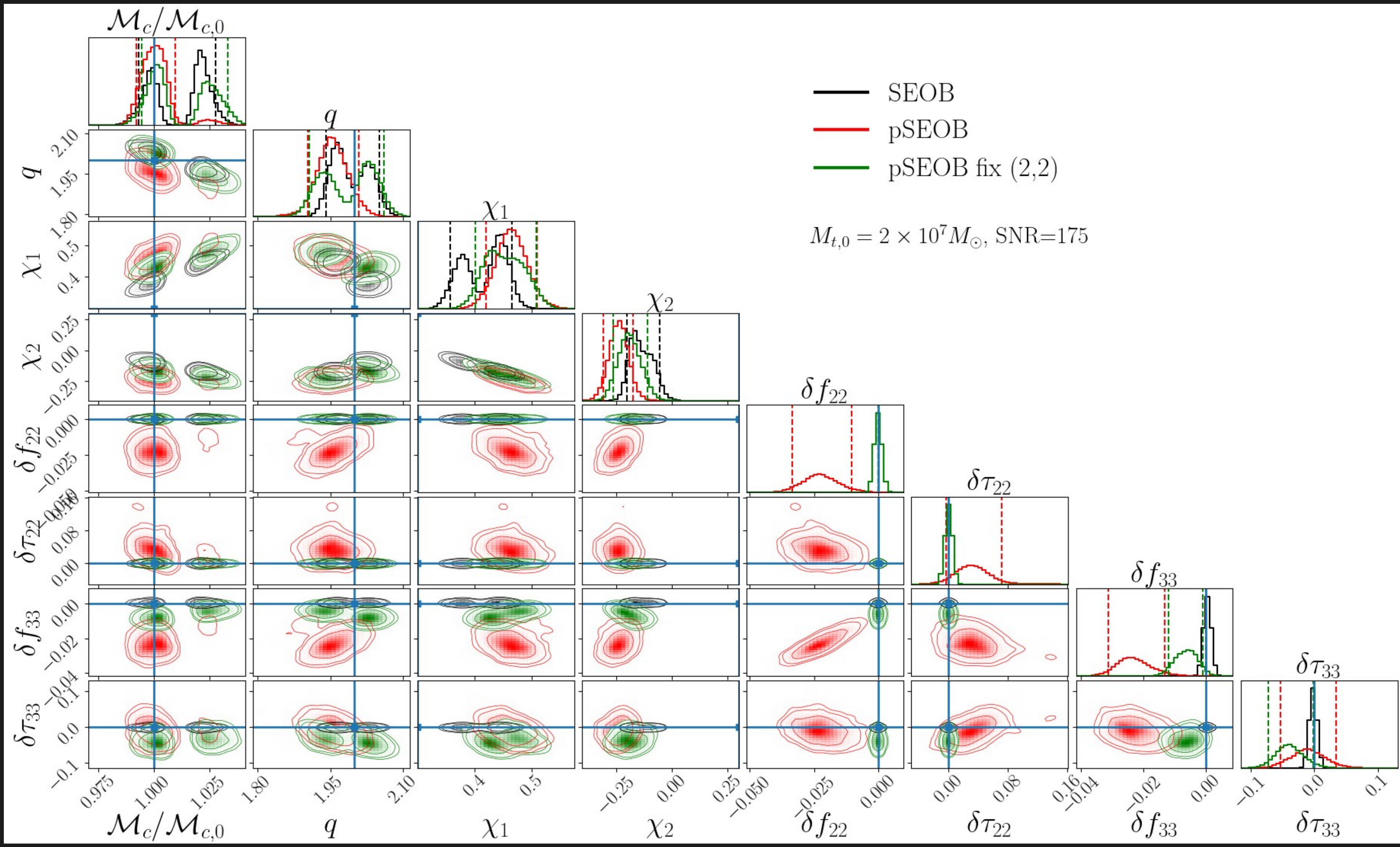


Including higher harmonics



Including higher harmonics



Interpretation

- GR deviation coefficients can accommodate more than deviations from GR
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- One of the main sources of error is the alignment between harmonics

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- GR deviation coefficients can accommodate more than deviations from GR
- Inspiral-merger-ringdown tests are very sensitive to details of modelling
- One of the main sources of error is the alignment between harmonics
- “Simple” criterion for bias?

Revisited criterion for bias

$$1-\mathcal{O}(s_0,h(\theta_0)) \leq \frac{n_p\left(1-\frac{2}{9n_p}+1.3\sqrt{\frac{2}{9n_p}}\right)^3}{2\mathrm{SNR}_T^2}+1-\mathcal{O}_{\max}(s_0,h)$$

- For single detector: $\mathcal{O}(d_1,d_2)=\frac{(d_1|d_2)}{\sqrt{(d_1|d_1)\,(d_2|d_2)}}$
- Accounts for non-perfectness of templates
- Statement about the full posterior

Proposed criterion

- Akaike information criterion:

$$\mathrm{AIC} = 2n_p - 2\ln\hat{\mathcal{L}}$$

- Bayes' factor:

$$\ln\mathcal{B} = -\frac{1}{2}(\mathrm{AIC}_1 - \mathrm{AIC}_2)$$

- Compare Bayes' factor when fixing a set of parameters θ^1 to "true" value vs when varying them:

$$\ln\mathcal{B} = \ln\hat{\mathcal{L}}(\theta^1 = \theta^1_0) - \ln\hat{\mathcal{L}} + n^1_p$$

Practical implementation

- Likelihood scaling with SNR:

$$\ln \mathcal{L} = \ln \mathcal{L}_{\mathrm{SNR}_0} \left(\frac{\mathrm{SNR}}{\mathrm{SNR}_0} \right)^2$$

- Under Gaussian approximation:

$$\ln \hat{\mathcal{L}} =< \ln \mathcal{L} > + \frac{n_p}{2}$$

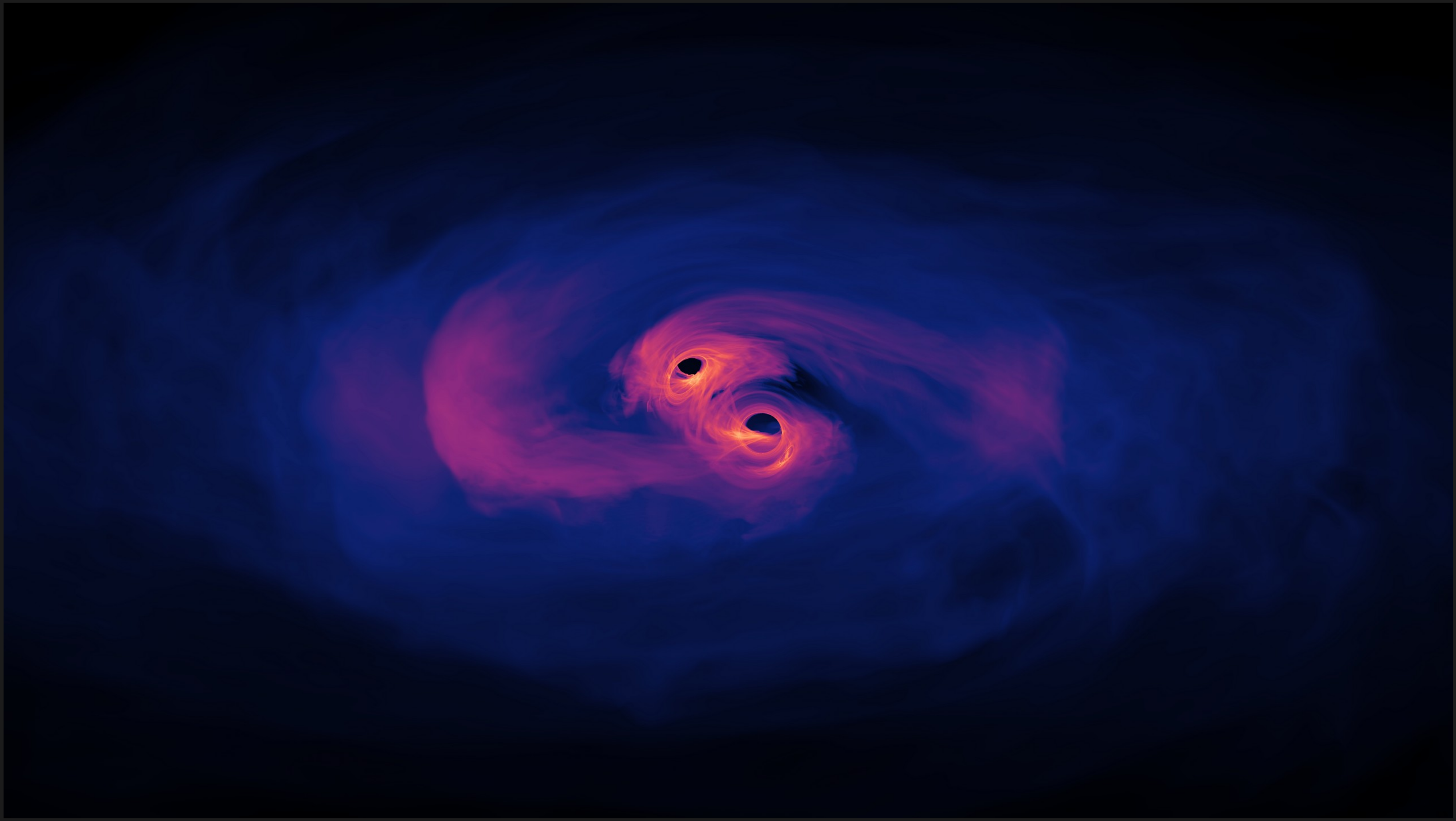
Accuracy limit

- SNR limits to favour GR deviations ($\ln \mathcal{B} > 3$):
  - $M_t = 2 \times 10^8 M_\odot$: 977 with (2,2) only, 68 all harmonics
  - $M_t = 2 \times 10^7 M_\odot$: 598 with (2,2) only, 93 all harmonics
  - $M_t = 2 \times 10^6 M_\odot$: 330 with (2,2) only, 214 all harmonics
- Indicative for LVK as well (with appropriate mass rescaling)

Conclusions

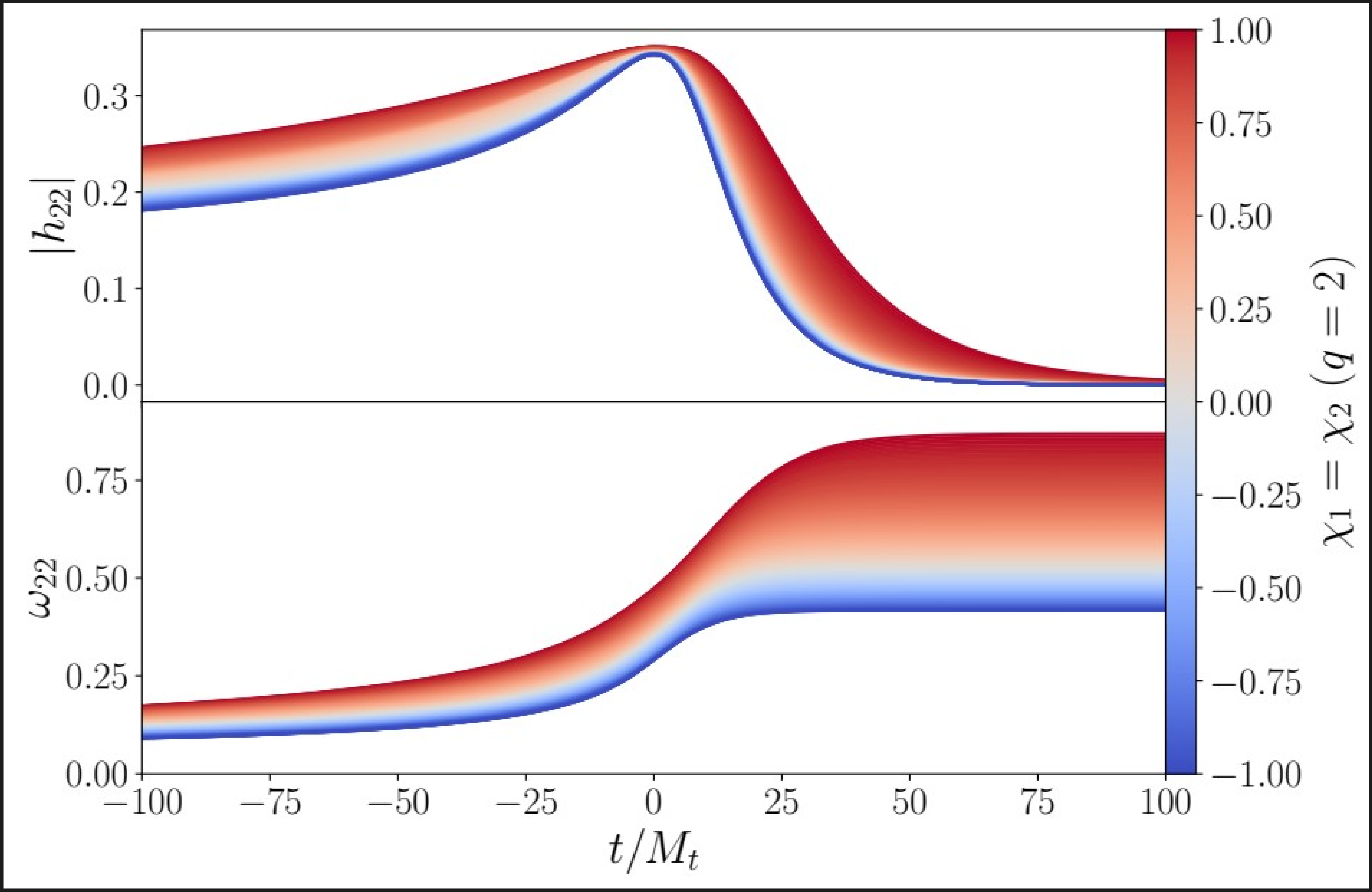
- LISA will observe MBHBs with SNRs up to 1000s both in inspiral and merger-ringdown:
  - Can measure the source parameters with great accuracy
  - Perform exquisite tests of GR, probing fractional deviations to the QNMs down to 0.001
- But… current waveform models are not accurate enough for these high SNRs

Thank you for your attention!



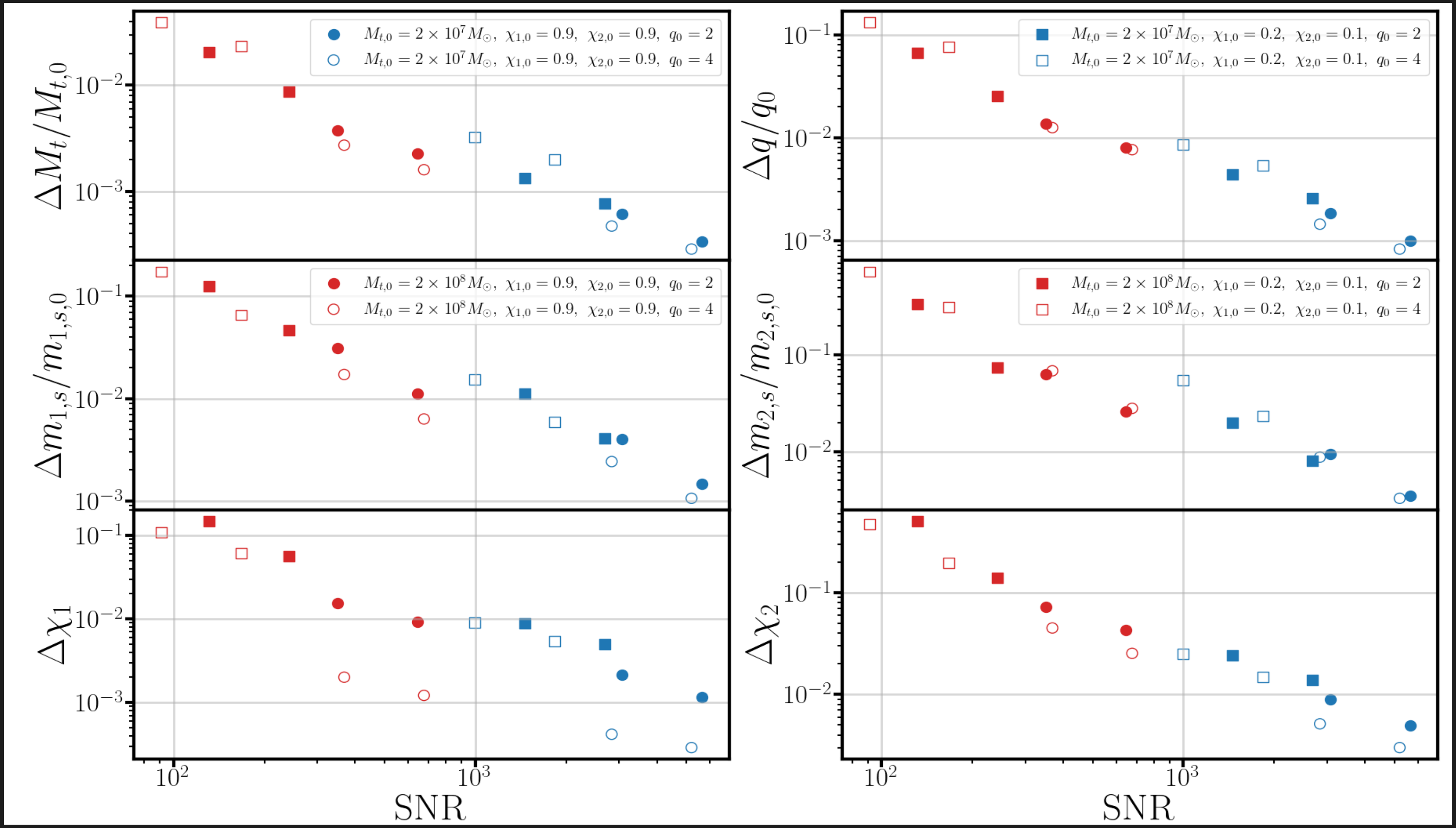
Credits: NASA's Goddard Space Flight Center

Dependence on parameters

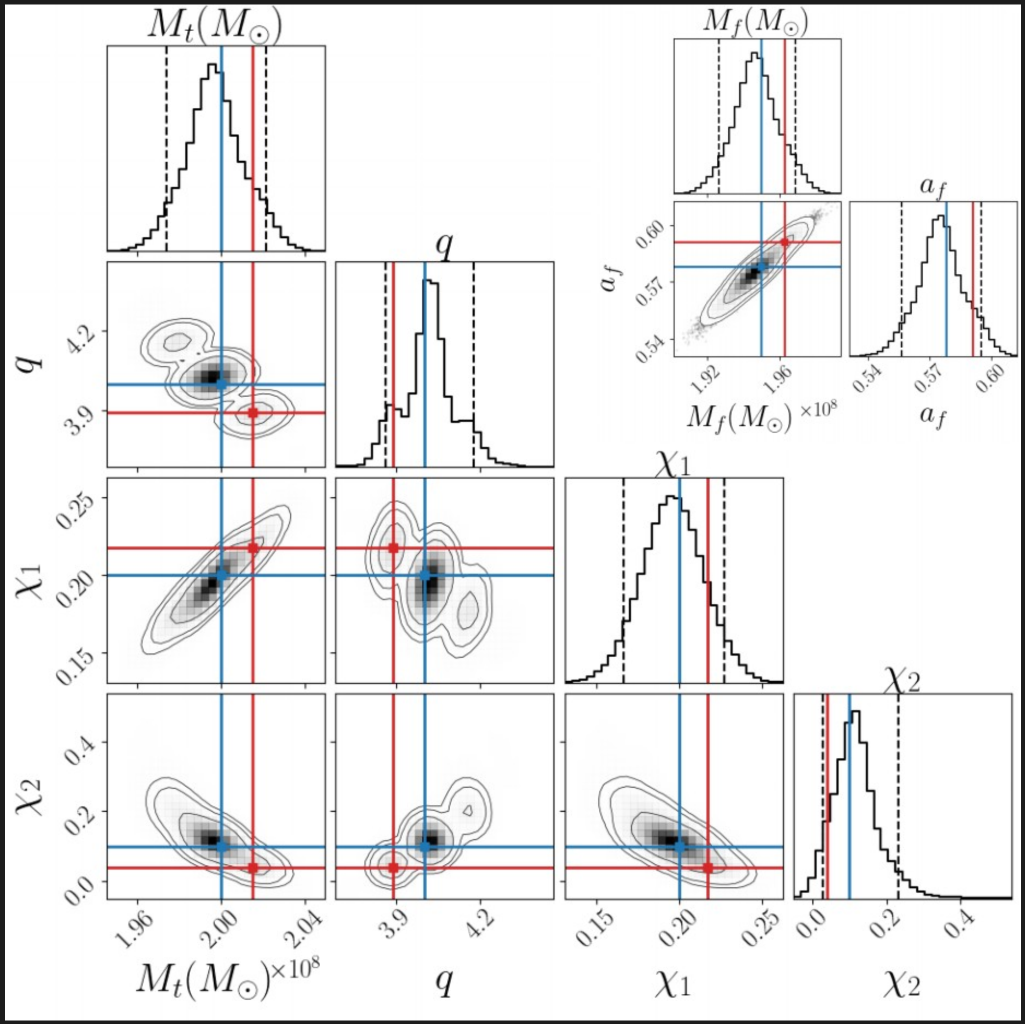


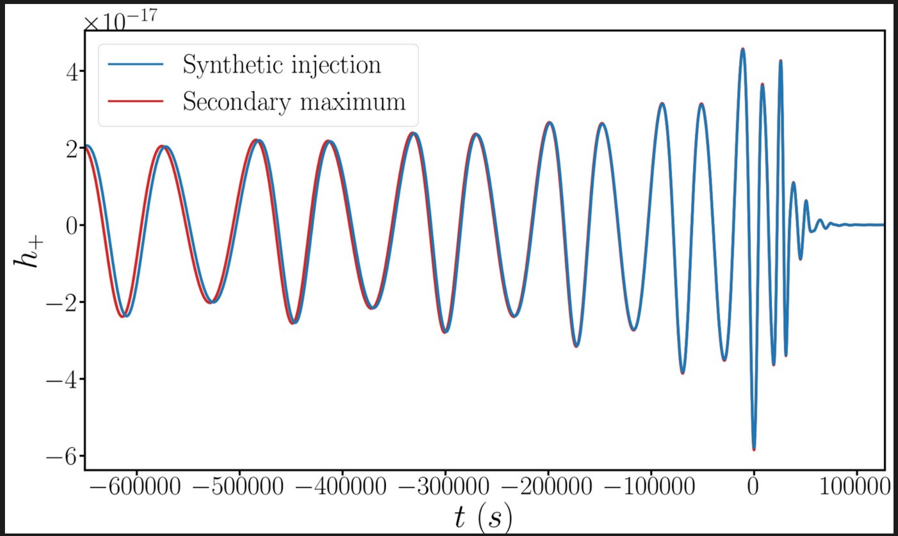
GR injection, GR templates

- Width of 90% confidence intervals:

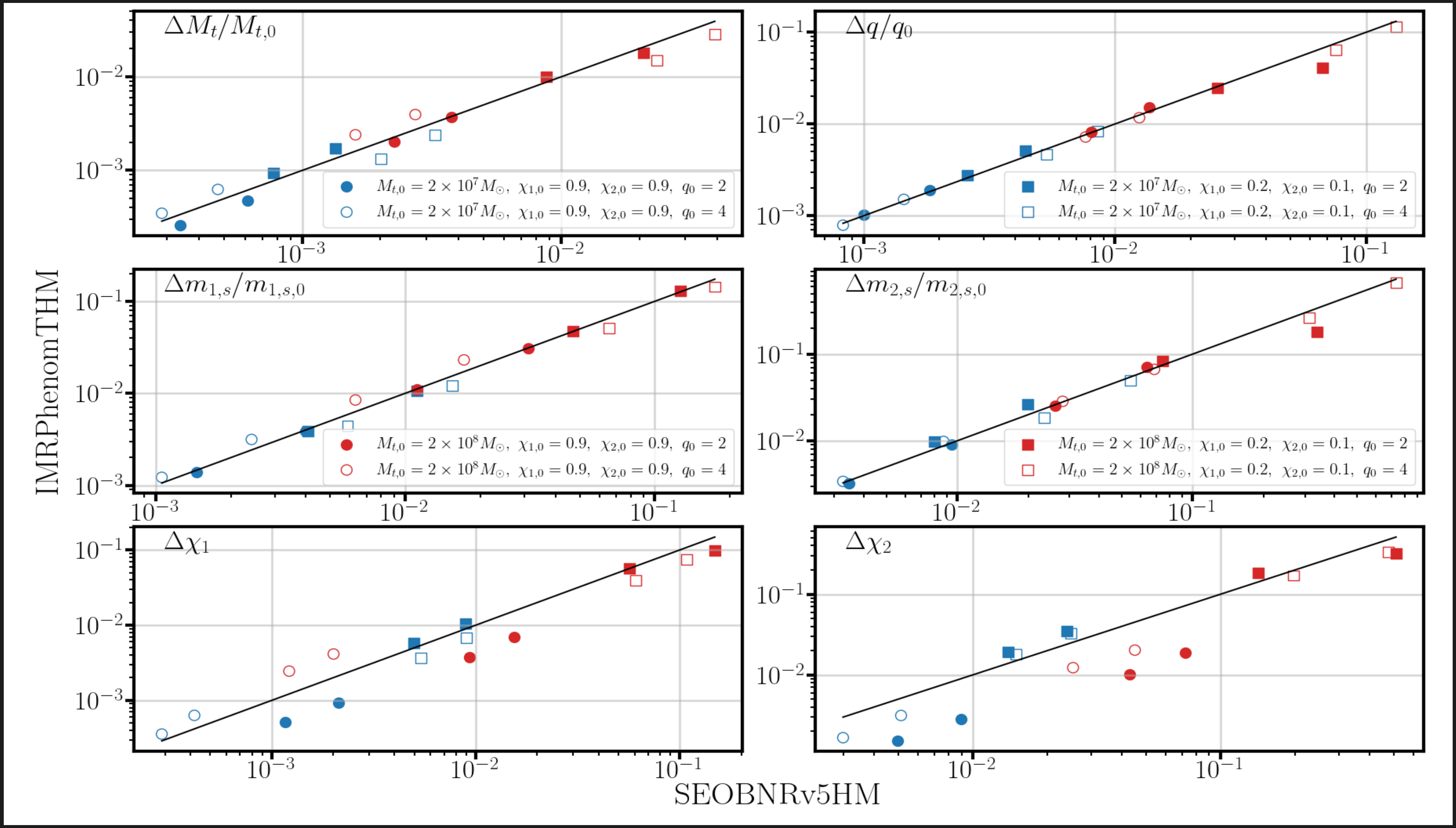


Multimodality



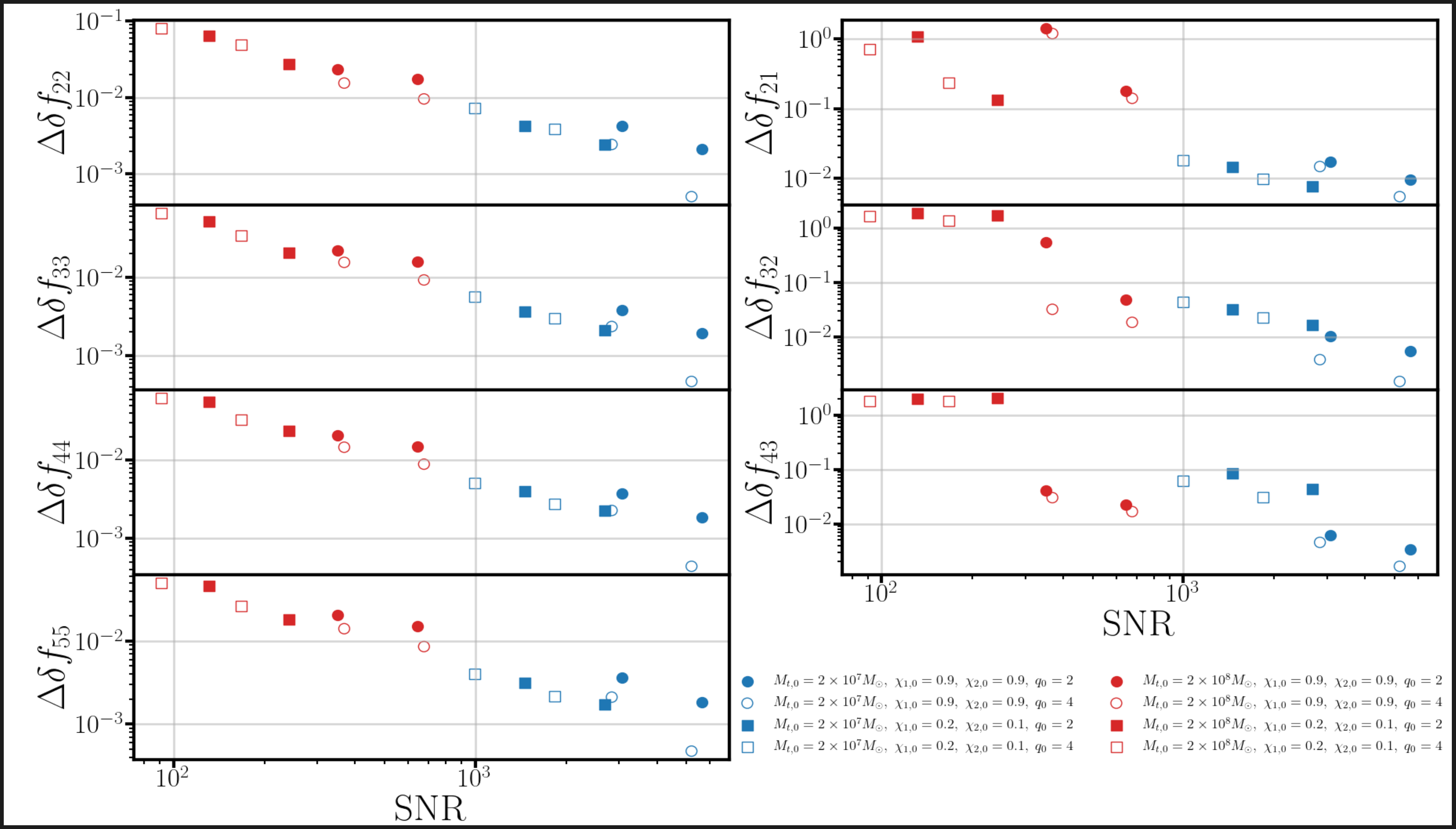


EOB vs Phenom



GR injection, non-GR templates

$$\delta f_{lm,0} = \delta\tau_{lm,0} = 0$$



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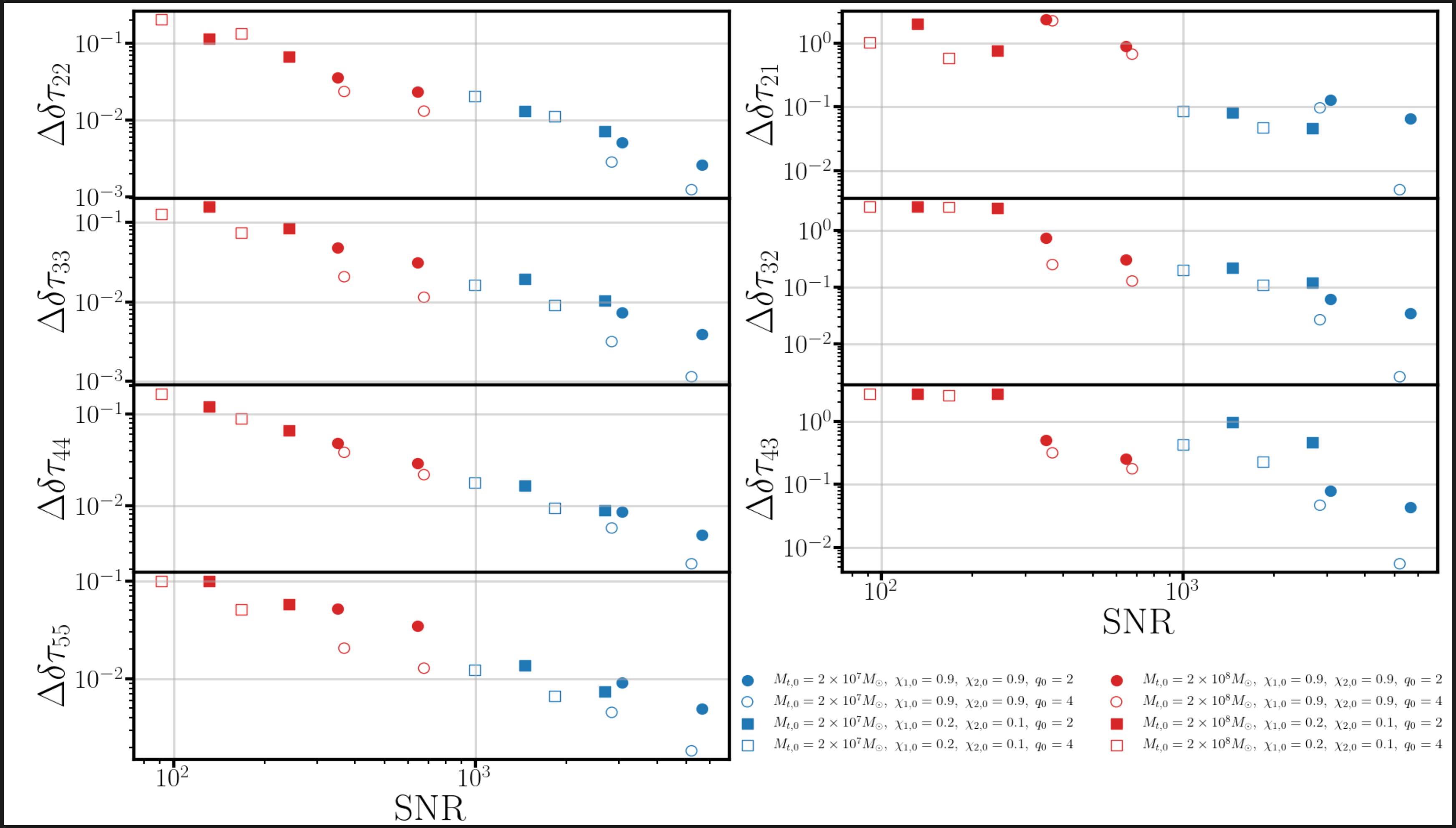
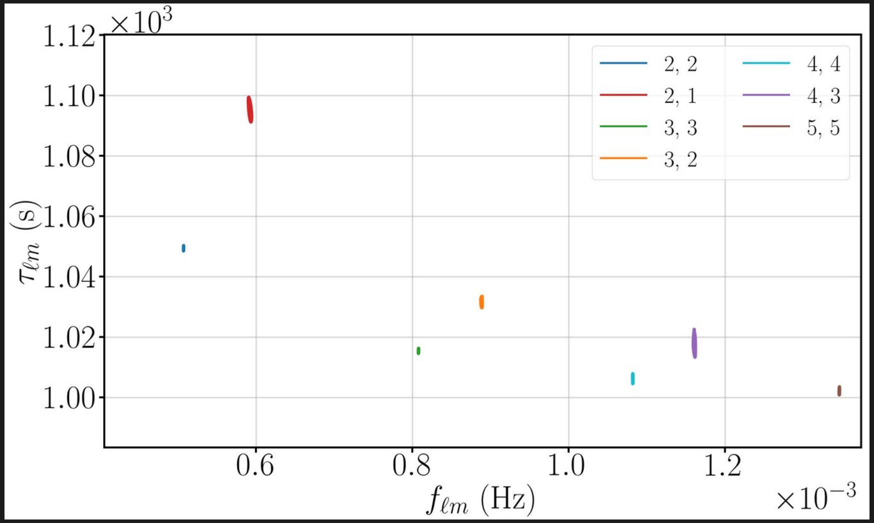
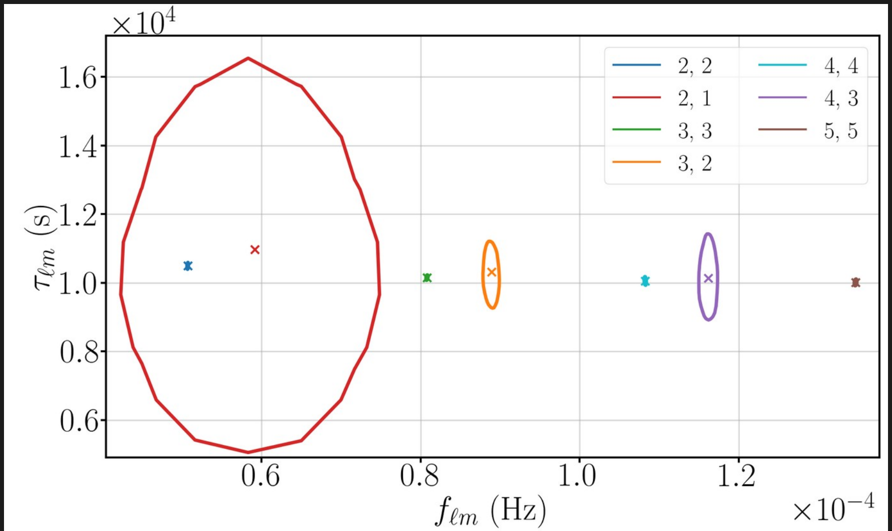


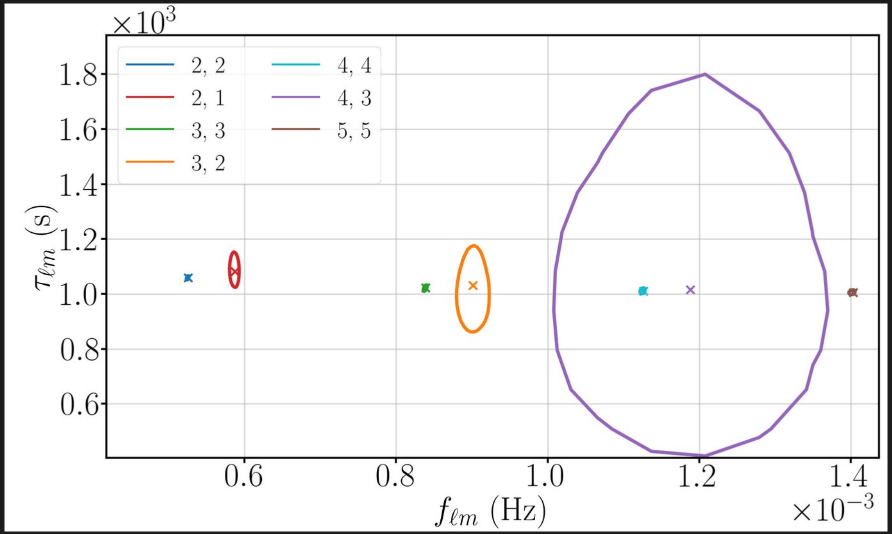
QNMs measurement



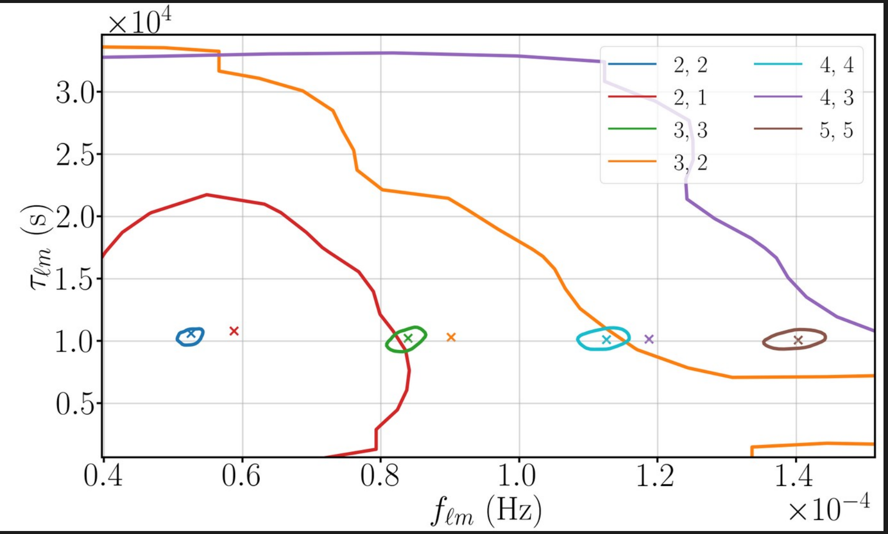
$M_{t,0} = 2 \times 10^7 M_\odot,\ \chi_{1,0} = \chi_{2,0} = 0.9,\ q_0 = 4\ z_0 = 3,\ \mathrm{SNR} = 3659$



$M_{t,0} = 2 \times 10^8 M_\odot,\ \chi_{1,0} = \chi_{2,0} = 0.9,\ q_0 = 4\ z_0 = 3,\ \mathrm{SNR} = 475$

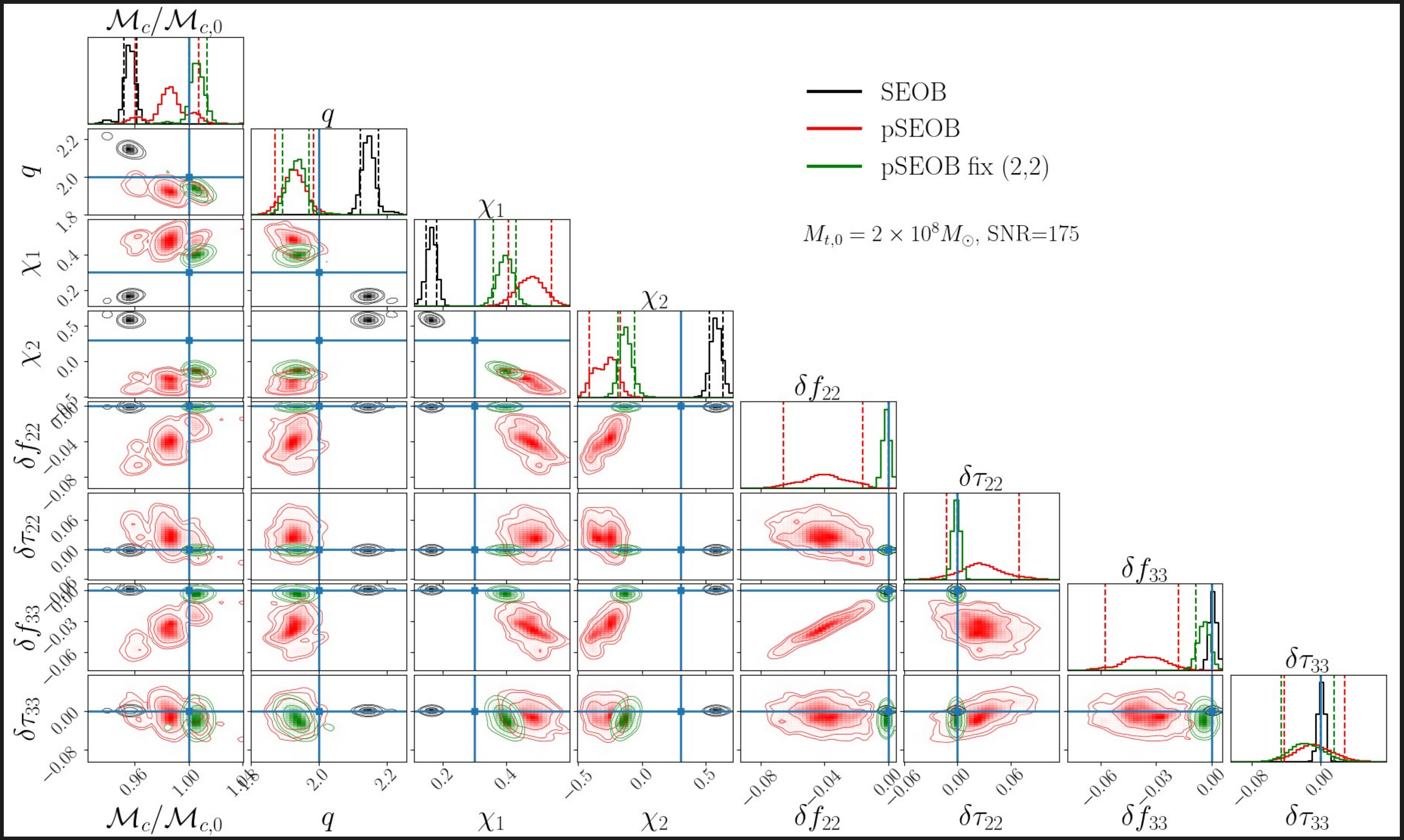


$M_{t,0} = 2 \times 10^7 M_\odot,\ \chi_{1,0} = 0.2,\ \chi_{2,0} = 0.1,\ q_0 = 2,\ z_0 = 5,\ \mathrm{SNR} = 1030$



$M_{t,0} = 2 \times 10^8 M_\odot,\ \chi_{1,0} = 0.2,\ \chi_{2,0} = 0.1,\ q_0 = 2,\ z_0 = 5,\ \mathrm{SNR} = 93$

Impact of systematics



Parameter estimation in a nutshell

Treat the parameters of the source, θ, as random variables

Bayes' theorem:

$$p(\theta|d,\mathcal{H}) = \frac{p(d|\theta,\mathcal{H})p(\theta|\mathcal{H})}{p(d|\mathcal{H})}$$

Likelihood:

$$\prod_o \exp\left[-\frac{1}{2}(d_o - h_o(\theta)|d_o - h_o(\theta))\right]$$

Large dimensions function (7-17), need efficient way to compute the posterior, e.g. Markov Chain Monte Carlo (MCMC)

pSEOBNRv5HM

- Waveform:

$$h_+(\boldsymbol{\Theta}, \iota, \varphi_0; t) - i h_\times(\boldsymbol{\Theta}, \iota, \varphi_0; t) = \frac{1}{D_L} \sum_{\ell,m} {}_{-2}Y_{\ell m}(\iota, \varphi_0)\, h_{\ell m}(\boldsymbol{\Theta}; t)$$

$$h_{\ell m}(\boldsymbol{\Theta}, t) = h_{\ell m}(\boldsymbol{\Theta}, t)^{\text{insp-plunge}}\, \theta(t^{\ell m}_{\text{match}} - t) + h_{\ell m}(\boldsymbol{\Theta}, t)^{\text{merger-RD}}\, \theta(t - t^{\ell m}_{\text{match}})\,,$$

$$h^{\text{merger-RD}}_{\ell m}(t) = \nu\, \tilde{A}_{\ell m}(t)\, e^{i\tilde{\phi}_{\ell m}(t)}\, e^{-i\sigma_{\ell m 0}(t - t^{\ell m}_{\text{match}})}$$

$$\sigma_{lm0} = 2\pi f_{lm0} + \frac{i}{\tau_{lm0}}$$

$$(\ell, m) = (2,2),\ (3,3),\ (4,4),\ (5,5),\ (2,1),\ (3,2),\ (4,3)$$

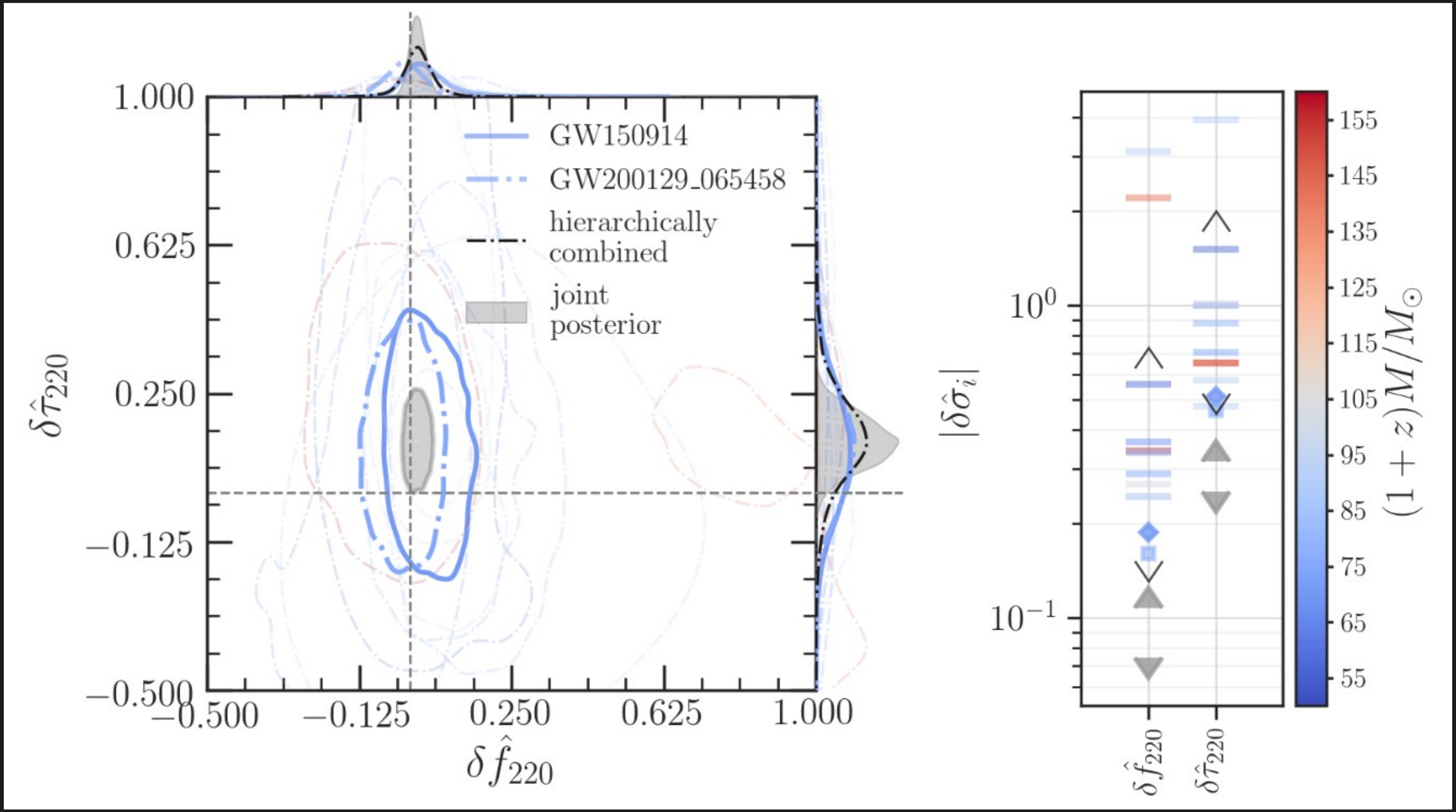
- GR deviation:

$$f_{\ell m 0} \to f_{\ell m 0}\,(1 + \delta f_{\ell m})$$

$$\tau_{\ell m 0} \to \tau_{\ell m 0}\,(1 + \delta \tau_{\ell m})$$

Applied to GWTC-3

- With pSEOBNRv4HM:



- Still compatible with GR.