Exploring the no-hair theorem with LISA

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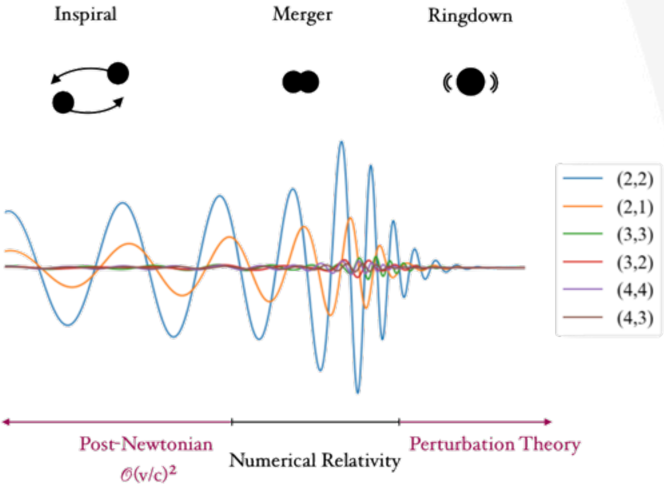
Fundamental Physics meets Waveforms with LISA - Potsdam

Outline

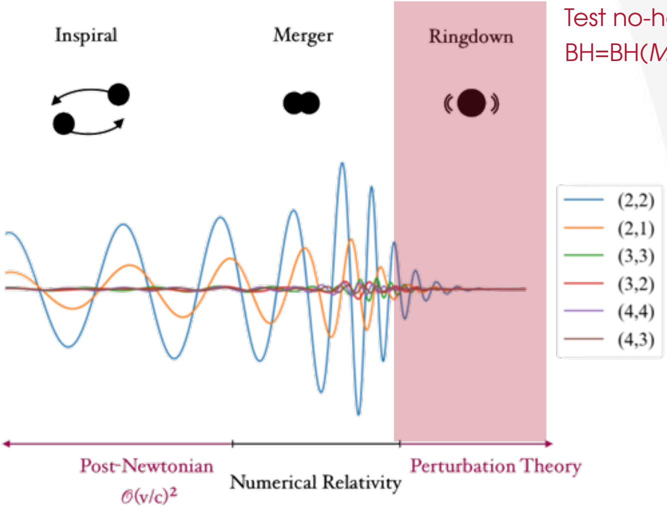
Introduction

Inspiral-merger-ringdown (IMR)



$$h^{IMR}(t,\Xi,\theta,\varphi) = \sum_{lm} A_{lm}(t,\Xi) e^{i\phi_{lm}(t,\Xi)} \; {}_{-2}Y^{lm}(\theta,\varphi)$$

Inspiral-merger-ringdown (IMR)



Test no-hair theorem

BH=BH(M_f, a_f)

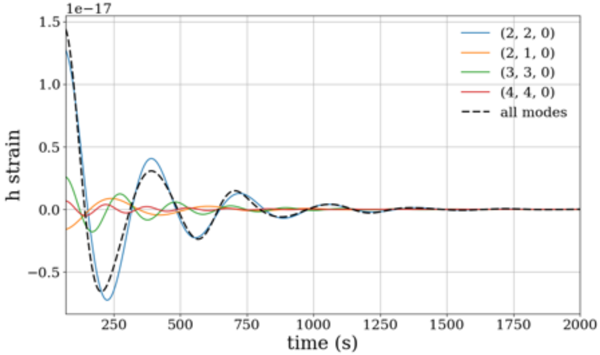
$$h^{IMR}(t, \Xi, \theta, \varphi) = \sum_{lm} A_{lm}(t, \Xi) e^{i\phi_{lm}(t,\Xi)} \, {}_{-2}Y^{lm}(\theta, \varphi)$$

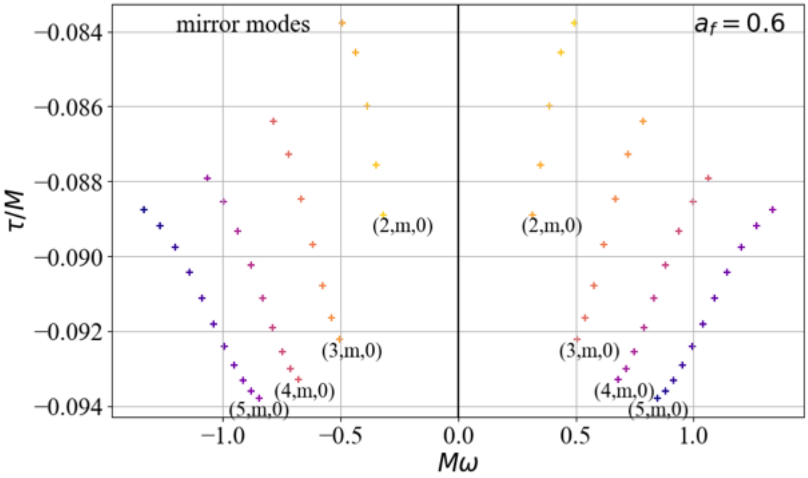
Sum of damped sinusoids → quasi-normal modes (QNMs)

$$h^{RD}(t, \Xi, \theta, \varphi) = \sum_{lmn} h_{lmn}(t, \Xi) \, {}_{-2}S^{lmn}(a_f \tilde{\omega}_{lmn}, \theta, \varphi)$$

$$h_{lmn}(t, \Xi) = A_{lmn}(\Xi) e^{-t/\tau_{lmn}} \cos(\omega_{lmn} t + \phi_{lmn}(\Xi))$$

$$\tilde{\omega}_{lmn} = \underbrace{\omega_{lmn}}_{\text{frequency}} (M_f, \, a_f) + i / \underbrace{\tau_{lmn}}_{\text{damping time}} (M_f, \, a_f)$$



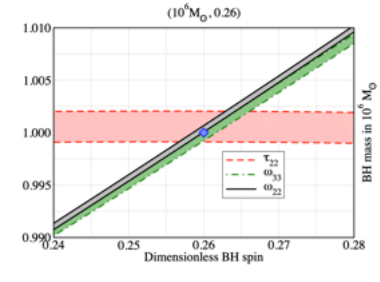

mirror modes
$a_f = 0.6$
−0.084
−0.086
−0.088
−0.090
−0.092
−0.094
τ/M
(2,m,0)
(3,m,0)
(4,m,0)
(5,m,0)
(2,m,0)
(3,m,0)
(4,m,0)
(5,m,0)
−1.0
−0.5
0.0
0.5
1.0
Mω

Testing the no-hair theorem with QNM

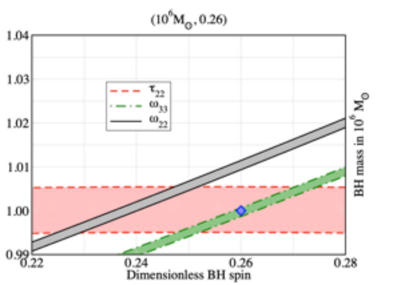
$$M\omega_{lmn} = f_1 + f_2(1 - a_f)^{f_3}$$
$$Q_{lmn} = q_1 + q_2(1 - a_f)^{q_3}$$
$$Q_{lmn} = \omega_{lmn}\tau_{lmn}/2$$

[Berti+,2006]

$$\omega_{lmn}^{nonGR} = \omega_{lmn}^{GR}(1 + \delta\omega_{lmn})$$
$$\tau_{lmn}^{nonGR} = \tau_{lmn}^{GR}(1 + \delta\tau_{lmn})$$



Mass = 10^6 $M_\odot$, a_f = 0.26



Mass = 10^6 $M_\odot$, a_f = 0.26

[Gossan+,2012]

Methodology

Analysis of QNMs with LISA

- Generation of the RD waveform and response of LISA in time domain → **Lisaring**
- TD, as the cut of the ringdown is cleaner. Different t_{start} are allowed
- Covariance matrix → Toeplitz matrix
- Different methods: fixing $\beta, \lambda, t_{start}$
  - Agnostic approach → PE = (4 dim × modes)

$$h(t) = \sum_k A_k e^{i\phi_k} e^{-it(\omega_k + i/\tau_k)}$$

  - GR + deviations: → PE =(4 dim × modes)

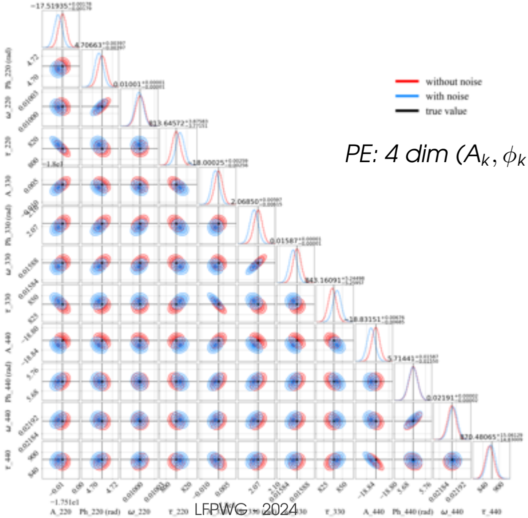
$$h(t) = \sum_{lmn} A_{lmn} e^{i\phi_{lmn}} e^{-it(\omega_{lmn}(Mf, af)*(1+\delta\omega_{lmn}) + i/(\tau_{lmn}(Mf, af)*(1+\delta\tau_{lmn})))}$$

- Injection GR + deviations approach:
  A_{lmn}, ϕ_{lmn} → [London et al. 2014, 2020]
  $\{m_1, m_2, \chi_1, \chi_2\}, \quad \tilde{\omega}_{lmn} = [(2,2,0), (3,3,0), (4,4,0)]$
  $\delta\omega_{lmn} = [0.0, 0.01, 0.03], \quad \delta\tau_{lmn} = [0.0, 0.05, 0.1]$

Results

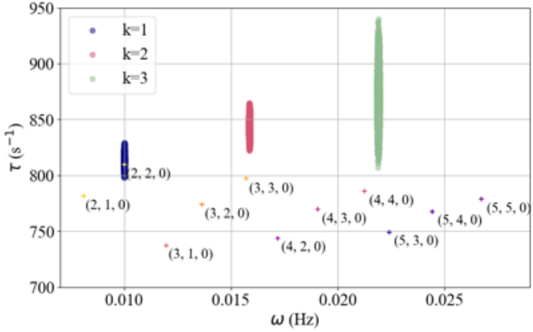
Agnostic approach

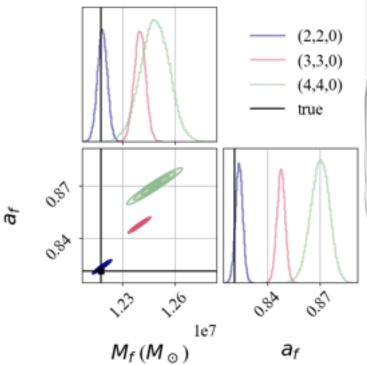
$$h(t) = \sum_{k}^{3} A_k e^{i\phi_k} e^{-it(\omega_k + i/\tau_k)}$$



PE: 4 dim $(A_k, \phi_k, \omega_k, \tau_k) \times k$

Agnostic approach



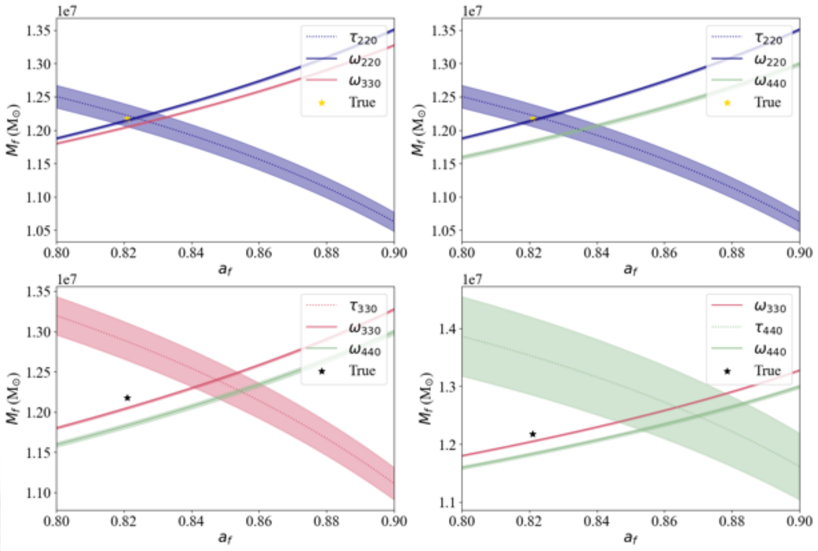


⟶ We can compute M_f, a_f

$$M\omega_{lmn} = f_1 + f_2(1 - a_f)^{f_3}$$

$$\omega_{lmn}\tau_{lmn}/2 = q_1 + q_2(1 - a_f)^{q_3}$$

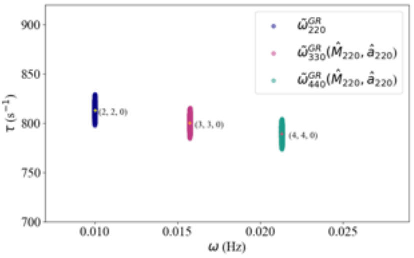
Agnostic approach

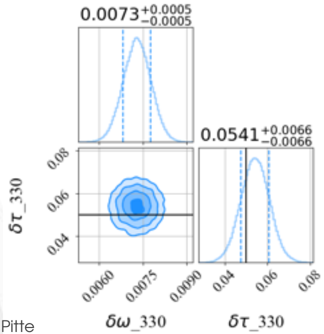


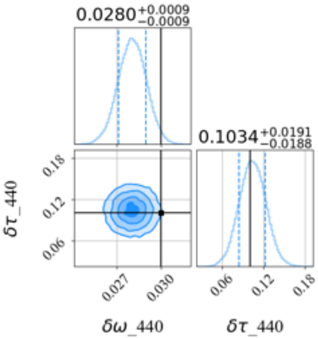
Agnostic approach

⇒

- assuming that (2,2,0) does not exhibit a deviation from GR. We can compute the GR spectrum for those parameters (M_f, a_f)

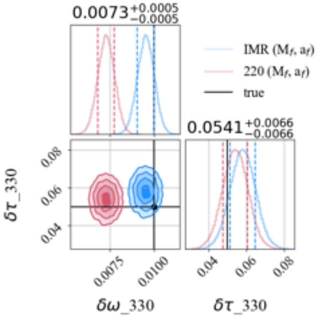




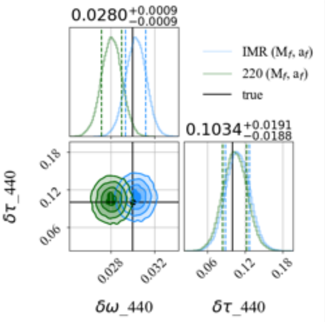


Fractional deviations with *agnostic appr.*: IMR vs QNMs M_f, a_f

true value: $\delta\omega = 0.01$, $\delta\tau = 0.05$



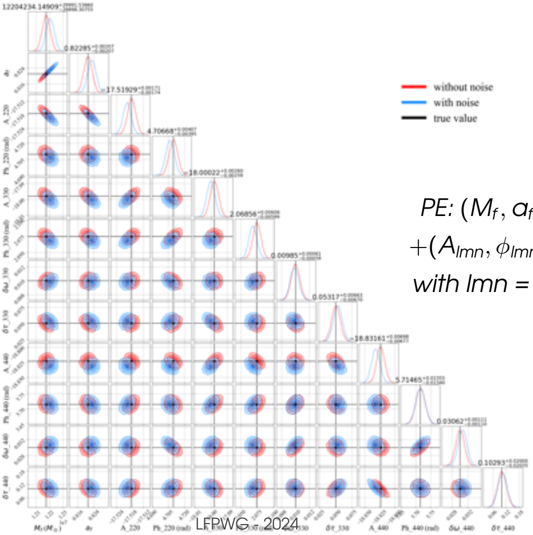
true value: $\delta\omega = 0.03$, $\delta\tau = 0.1$



- Small fluctuation in the (2,2,0) mode are translated into fluctuations in the final mass and spin
- Values of M_f, a_f come from a parametrization with $\sim 1-3\%$ error
- We can identify a deviation from GR, but we can not characterize it correctly since the recovered values do not agree with the injected values

GR + deviations

$$h(t) = \sum_{lmn} A_{lmn} e^{i\phi_{lmn}} e^{-it(\omega_{lmn}(M_f,a_f)*(1+\delta\omega_{lmn})+i/(\tau_{lmn}(M_f,a_f)*(1+\delta\tau_{lmn})))}$$



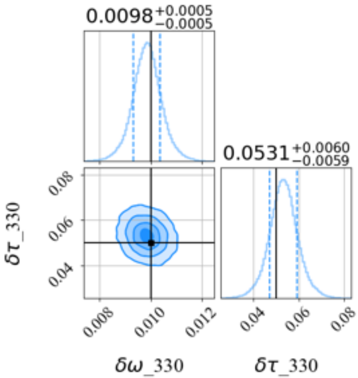
PE: $(M_f, a_f, A_{220}, \phi_{220})$

$+(A_{lmn}, \phi_{lmn}, \delta\omega_{lmn}, \delta\tau_{lmn})$

with lmn = (3,3,0), (4,4,0)

GR + deviations

inj value: $\delta\omega = 0.01$, $\delta\tau = 0.05$



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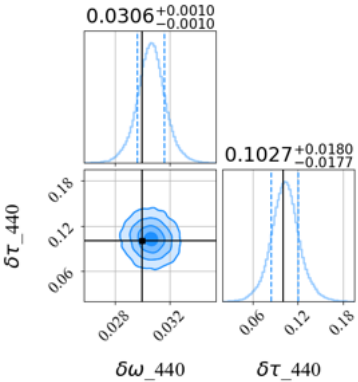
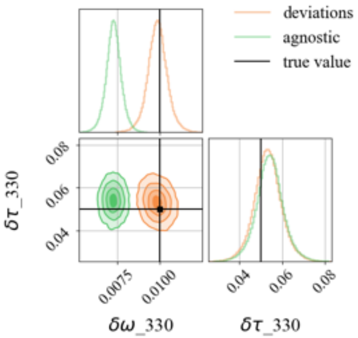


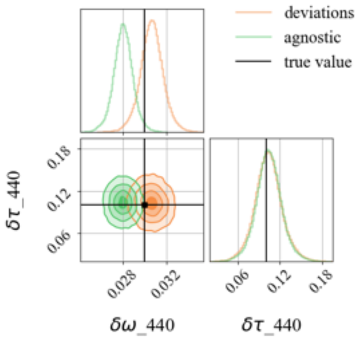
- We can recover the injected value with high accuracy
- Caveat: several assumptions have been made

Which method to use

$\Rightarrow$

$$\omega_{lmn}^{nonGR} = \omega_{lmn}^{GR}(1 + \delta\omega_{lmn}), \qquad \tau_{lmn}^{nonGR} = \tau_{lmn}^{GR}(1 + \delta\tau_{lmn})$$





- With the *agnostic appr.* we can identify which QNMs are present
- With *deviations appr.* we can recover the injected value and hence characterize the deviation
- The combination of both would be the optimal method*

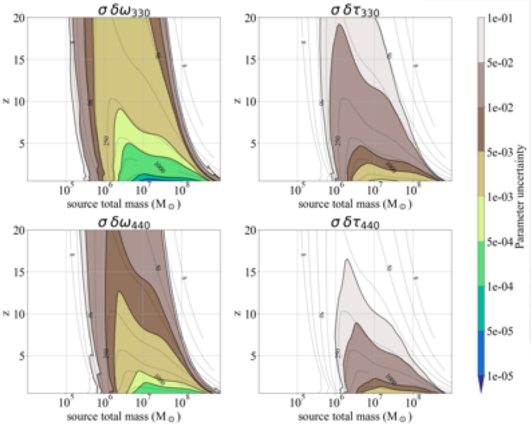
Standard deviation

With the *deviations* template we can study the precision on fractional deviations $(\delta\omega, \delta\tau)$

$$\sigma_{\theta_i} = \sqrt{\Gamma_{ii}^{-1}}$$

where Γ_{ij} is the Fisher matrix computed as the inner product

$$\Gamma_{kk} = \sum_{i,j=0}^{N-1} \partial_{\boldsymbol{\theta}_k} h_i(\boldsymbol{\theta}_k) \mathbf{C}_{ij}^{-1} \partial_{\boldsymbol{\theta}_k} h_j(\boldsymbol{\theta}_k$$



One can estimate the uncertainty on the fractional deviations throughout the universe.

Conclusion and prospects

Conclusions & prospects

Conclusions:

- Exploration of two different approaches $\rightarrow$ agnostic, deviation
- We inferred the fractional deviation for each template
- The *deviation* approach is crucial to confidently find the injected deviations. Need prior knowledge of QNMs.
- Combination of both method is what inform us the better
  - agnostic case to find which QNMs and raw estimation on (M_f, a_f)
  - deviations case to find deviations in each QNM
- Fractional deviation sensitivity for fixed parameters (q, $\chi_1, \chi_2, \iota, \phi, \beta, \lambda$) at different masses and distances

Prospects:

- Inclusion of $\beta, \lambda, t_{start}$ in the PE $\rightarrow$ better understanding of the robustness of the analysis
- Inclusion of nonlinear effects (QQNMs)
- Analysis with higher number of QNMs
- Sensitivity of LISA to NR waveforms
- Introduction of glitches or gaps

Conclusions & prospects

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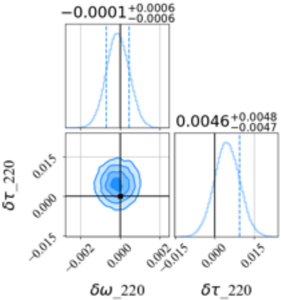
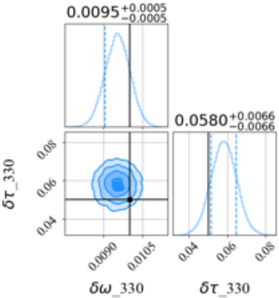
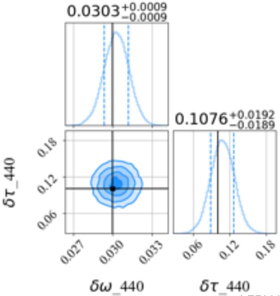
Thanks for your attention!

Back up slides

Agnostic IMR estimation

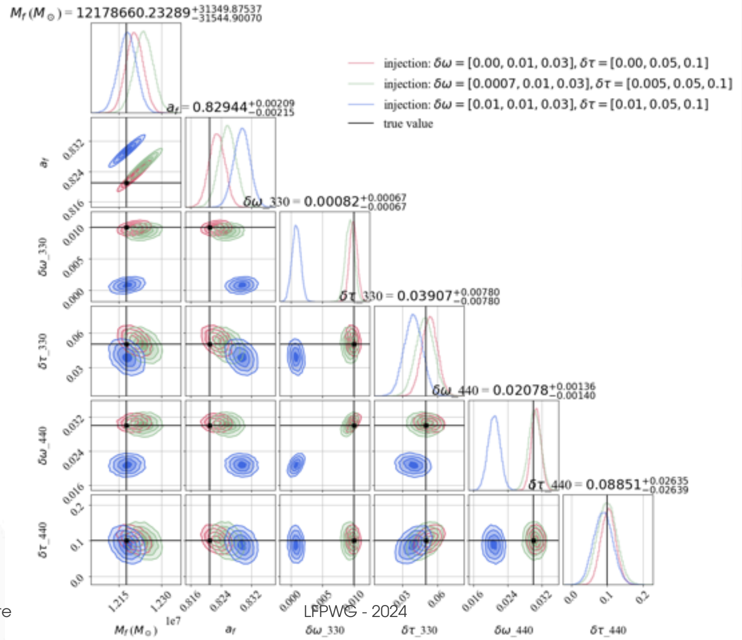




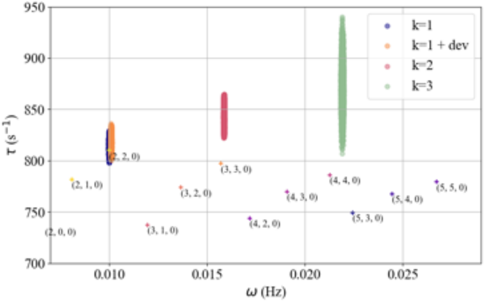


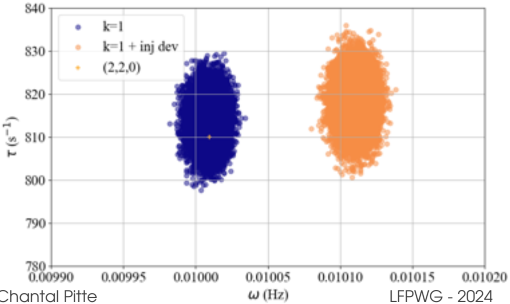
- Strongly depends on the estimated mean values of (M_f, a_f) from IMR $\rightarrow$ very precise waveform templates

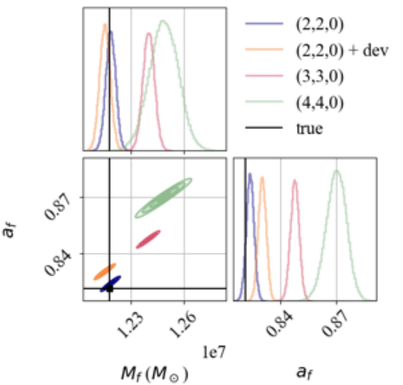
$\delta\tilde{\omega}_{220} = (0.01, 0.01)$, **no deviation assumed with *deviations appr.***



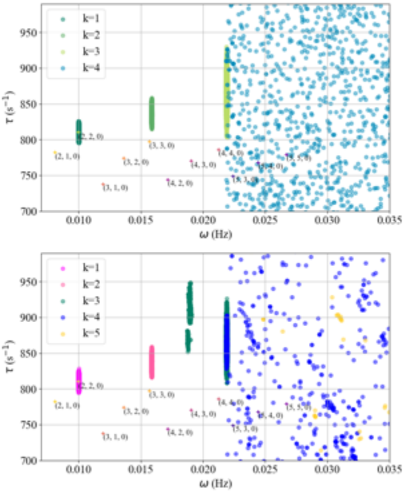
Agnostic appr. with deviation in 220







Search for more qnms



$log\,\mathcal{B} \sim 2.5, 1.24$

Teukolsky equation

⇐

$$\left[\frac{(r^2+a^2)^2}{\Delta} - a^2\sin^2\theta\right]\frac{\partial\Psi^2}{\partial_t^2} + \frac{4Mar}{\Delta}\frac{\partial\Psi^2}{\partial_t\partial_\phi} + \left[\frac{a^2}{\Delta} - \frac{1}{\sin^2\theta}\right]\frac{\partial\Psi^2}{\partial_\phi^2}$$
$$-\Delta^{-s}\frac{\partial}{\partial_r}\left(\Delta^{s+1}\right)\frac{\partial\Psi}{\partial_r} - \frac{1}{\sin\theta}\frac{\partial}{\partial_\theta}\left(\sin\theta\frac{\partial\Psi}{\partial_\theta}\right) - 2s\left[\frac{a(r-M)}{\Delta} - \frac{i\cos\theta}{\sin^2\theta}\right]\frac{\partial\Psi}{\partial_\phi}$$
$$-2s\left[\frac{M(r^2-a^2)}{\Delta} - r - ia\cos\theta\right]\frac{\partial\Psi}{\partial_t} + \left(s^2\cot^2\theta - s\right)\Psi = 4\pi\rho\mathbf{T}$$

Separable equation

Can be decoupled into a radial and angular part:

$$\Psi(t, r, \theta, \phi) = e^{-i\omega t} e^{im\phi} R(r) S(\theta)$$

where $R(r)$ and $S(\theta)$ satisfy

$$\Delta^{-s} \frac{\partial}{\partial_r} \left(\Delta^{s+1} \right) \frac{\partial R}{\partial_r} + \left(\frac{K^2 - 2is(r-M)K}{\Delta} + 4is\omega r - \lambda \right) R = 0,$$

$$\frac{1}{\sin\theta} \frac{\partial}{\partial_\theta} \left(\sin\theta \frac{\partial S}{\partial_\theta} \right)$$
$$+ \left(a^2\omega^2 \cos^2\theta - \frac{m^2}{\sin^2\theta} - 2a\omega s \cos\theta - \frac{2ms\cos\theta}{\sin^2\theta} - s^2 \cot^2\theta + s + \mathcal{A} \right) S = 0.$$

Where, $K \equiv (r^2 + a^2)\omega - am$, $\lambda \equiv \mathcal{A} + a^2\omega^2 - 2am\omega$ and $\mathcal{A}$ is the *separation constant*. For fixed values of s, m, $a\omega$ the eigenvalues are label by l.

Bayes factor

⟶ Bayesian method:

$$P(\Theta|D,M) = \frac{\overbrace{P(D|\Theta,M)}^{likelihood}\,\overbrace{P(\Theta|M)}^{prior}}{\underbrace{P(D|M)}_{evidence}}$$

⟶ Focus on estimating the evidence

$$\mathcal{Z} = \int_{\Omega_\Theta} P(D|M)d\Theta = \int_{\Omega_\Theta} \mathcal{L}(\Theta)\pi(\Theta)d\Theta = \int_0^1 \underbrace{\mathcal{L}(X)}_{iso-lkh} dX$$

where

$$\mathcal{L} = \frac{1}{\sqrt{\det(2\pi\mathbf{C})}} e^{-\frac{1}{2}(d-h(\boldsymbol{\theta}))^{\dagger}\mathbf{C}^{-1}(d-h(\boldsymbol{\theta}))}.$$

Bayes factor:

$$\mathcal{B} = \frac{\mathcal{Z}_1}{\mathcal{Z}_2} = \frac{\int_{\Omega_\Theta} \mathcal{L}_1(\Theta)\pi_1(\Theta)d\Theta}{\int_{\Omega_\Theta} \mathcal{L}_2(\Theta)\pi_2(\Theta)d\Theta}$$