Quasi-normal modes of rotating black holes beyond General Relativity

Nicola Franchini

Laboratoire Astroparticule & Cosmologie (APC)
Université Paris Cité

Fundamental Physics meets Waveforms with LISA

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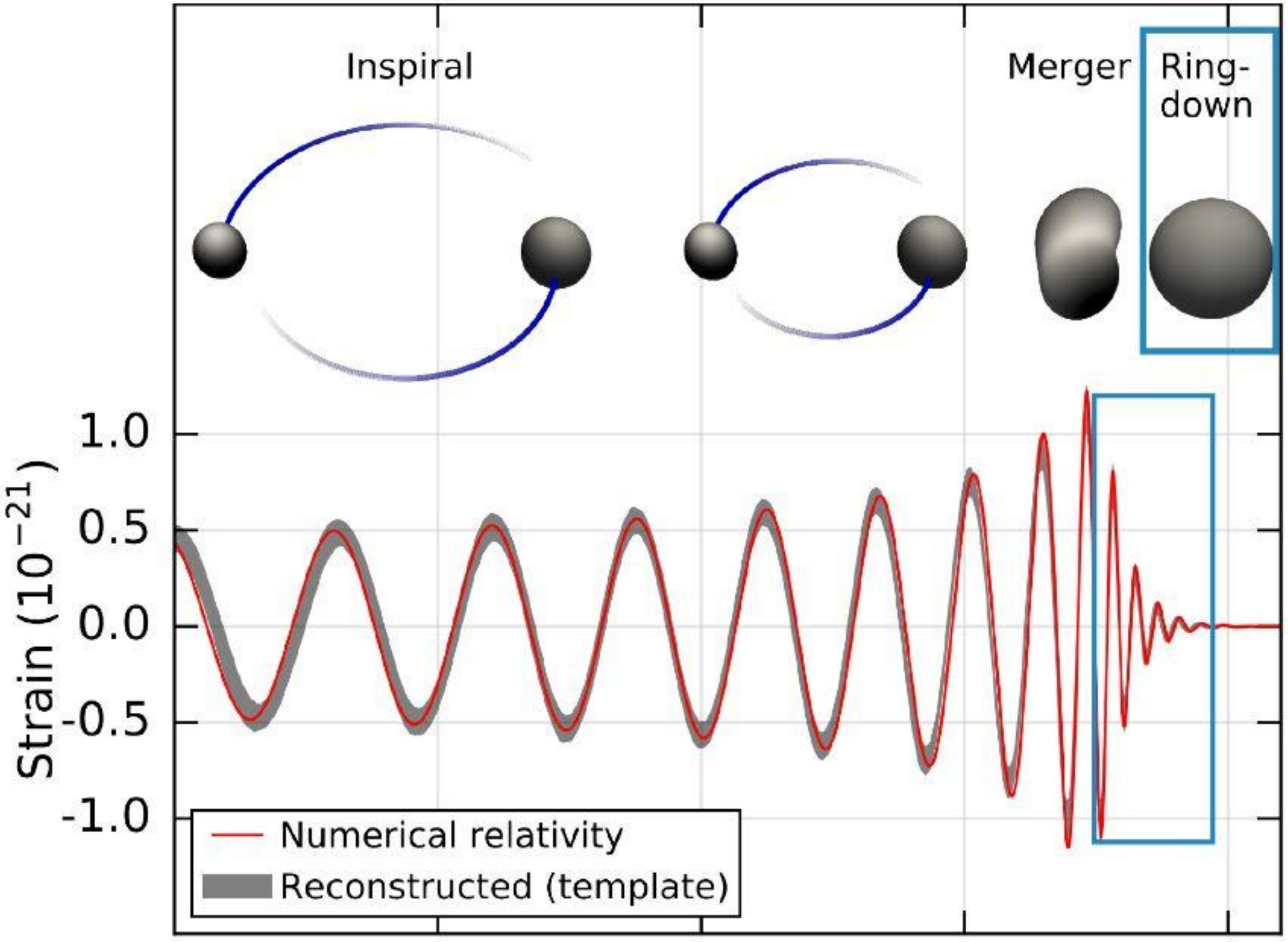
Black hole ringdown

- Final stage of black hole binary mergers
- Described by perturbation theory
- Tool to test general relativity



credit: LIGO-Virgo Collaboration 2016

Black hole ringdown

Superposition of damped sinusoids

$$h_+ - i h_\times = -\frac{M}{r} \sum_{\ell,m,n} A_{\ell mn} S(\theta) e^{-i\,\omega_{\ell mn} t} e^{im\varphi}$$

Complex frequencies, determined by mass and spin

Black hole ringdown

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Complex frequencies, determined by mass and spin

How do we compute QNMs?

BH perturbation in spherical symmetry

$$g_{ab} = g_{ab}^{(0)} + \varepsilon g_{ab}^{(1)}$$

$$\downarrow$$

$$R_{ab} = R_{ab}^{(0)} + \varepsilon R_{ab}^{(1)}$$

Let's assume vacuum:

$R_{ab}^{(0)} = 0 \longrightarrow g_{ab}^{(0)} = g_{ab}^{\mathrm{Sch}}$ is the Schwarzschild metric

BH perturbation in spherical symmetry

$$\frac{\mathrm{d}^2\Psi^\ell_\pm}{\mathrm{d}r_*^2} + \left(\omega^2 - V^\ell_\pm\right)\Psi^\ell_\pm = 0$$

Regge-Wheeler potential for axial tensor perturbation

$$V^\ell_- = \left(1 - \frac{1}{r}\right)\left[\frac{\ell(\ell+1)}{r^2} - \frac{3}{r^3}\right]$$

Zerilli potential for polar tensor perturbation, $\lambda = \ell(\ell+1) - 2$

$$V^\ell_+ = \left(1 - \frac{1}{r}\right)\left[\frac{\ell(\ell+1)}{r^2} - \frac{3}{r^3}\frac{r^2\lambda(\lambda+4) + 6r - 3}{(3 + r\lambda)^2}\right]$$

Regge, Wheeler 1957, Zerilli 1970

BH perturbation for Kerr

Kerr metric perturbations do not separe → Teukolsky formalism

- Based on Newman-Penrose decomposition
- Angular equation → Spin-weighted spheroidal harmonics
- Radial equation → Complex, long ranged potential
- *m* degeneracy breaking
- Frequencies depend on mass *M* and spin *a*

Teukolsky 1972

Extracting the spectrum

We solve an equation of the form $$\frac{d^2\Psi}{dr_*^2} + \left[\omega^2 - V_\ell(r)\right]\Psi = 0$$

with boundary conditions $$\lim_{r_* \to \pm\infty} \Psi = A_{(\pm)} e^{\pm i \omega r_*}$$

We are interested in the eigenvalues $$\omega_{n\ell m}$$

Overtone number

Angular momentum

Azimuthal number

Kokkotas, Schmidt 1999; Nollert 1999; Berti, Cardoso, Starinets 2009

Black hole ringdown beyond GR?

Status of QNMs in alternative theories

Possible deviations due to:

- different background metric
- different dynamics
- couplings with additional fields
- modified boundary conditions

} Modifications in the equations of the perturbations

Extended discussion in **NF, Volkel 2023**

Status of QNMs in alternative theories

Some common difficulties:

- Analysis mostly limited to non-rotating or slowly-rotating case only
  Blazquez-Salcedo *et al.* (2016), Langlois, Noui, Roussille (2022), Volkel, NF, Barausse (2022)
- Non-separability of the equations
  Dias, Godazgar, Santos (2022);
- No RW/Zerilli-like equations
  Blazquez-Salcedo *et al.* (2016), Langlois, Noui, Roussille (2022)
- Difficult to compute overtones
  ***e.g.* Molina, Pani, Cardoso, Gualtieri (2011)**
- Boundary conditions less clear
  Langlois, Noui, Roussille (2021)

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In this talk

Black hole ringdown beyond GR

Overall assumption #1: small coupling expansion $\zeta \ll 1$

$$\mathrm{GR} + \mathrm{fields} + \zeta\ \mathrm{interaction}$$

Many examples in literature can fit this definition: scalar-tensor theories, higher derivative gravity, Einstein-Gauss-Bonnet, dynamical Chern-Simons, Einstein-Maxwell and more…

Black hole ringdown beyond GR

Overall assumption #2: slow-rotation for background corrections

$$g_{ab} = g_{ab}^{\mathrm{Kerr}} + \sum_{i=0}^{N} \sum_{j=1}^{M} a^i \zeta^j g_{ab}^{(ij)}$$

In most cases, these corrections are analytic functions of *r* and θ

Black hole ringdown beyond GR

$$g_{ab} = g_{ab}^{\rm Kerr} + \sum_{i=0}^{N}\sum_{j=1}^{M} a^i \zeta^j g_{ab}^{(ij)}$$

Simultaneous small-spin and small-coupling expansions

- Low order spin expansion (up to *N=2*), high order coupling expansion (taking *M* as large as possible)
- Low order coupling expansion, (up to *M=1*), high order spin expansion (taking *N* as large as possible)

* See also a different approach solving numerically the full system of perturbed EoMs

K.W. Chung, Yunes (2023,2024)

BH perturbation in slow rotation (in GR)

$$g_{ab} = g_{ab}^{(0)} + \varepsilon g_{ab}^{(1)}$$

$$\downarrow$$

$$R_{ab} = R_{ab}^{(0)} + \varepsilon R_{ab}^{(1)}$$

Let’s assume vacuum with slow rotation:

$$R_{ab}^{(0)} = 0 \longrightarrow g_{ab}^{(0)} = g_{ab}^{\mathrm{Sch}} + a g_{ab}^{\mathrm{SR},1} + a^2 g_{ab}^{\mathrm{SR},2}$$

BH perturbation in slow rotation (in GR)

$$\frac{\mathrm{d}^2\Psi^\ell_\pm}{\mathrm{d}r_*^2} + \left(\omega^2 - V^\ell_\pm\right)\Psi^\ell_\pm = 0$$

Regge-Wheeler (-)
Zerilli (+)

$$\mathrm{d}r_* = \frac{\mathrm{d}r}{(1-1/r)\left(1 + a f_1 + a^2 f_2\right)}$$

$$V^\ell_\pm = V^\ell_{\pm,0} + aV^\ell_{\pm,1} + a^2 V^\ell_{\pm,2}$$

In the end, there is just a spin modification to tortoise coordinate and potential

Pani 2013, NF 2023

BH perturbation in slow rotation (beyond GR)

Background

$$g_{ab}^{(0)} = \sum_{i=0}^{2} \sum_{j=0}^{M} a^i \zeta^j g_{ab}^{(ij)}$$

Eqs of motion

$$R_{ab} - \frac{1}{2} g_{ab} R = T_{ab}^{\mathrm{bGR}}$$

+ other fields eoms

$a \ll 1$ $\zeta \ll 1$

Perturbation equation

$$\frac{\mathrm{d}^2 \Psi_\pm}{\mathrm{d}r_*^2} + \left(\omega^2 + V_\pm\right) \Psi_\pm = A_\pm \Phi$$

BH perturbation in slow rotation (beyond GR)

Status in the literature:

$$\frac{\mathrm{d}^2\Psi_\pm}{\mathrm{d}r_*^2} + \left(\omega^2 + V_\pm\right)\Psi_\pm = A_\pm\Phi$$

Breaking of isospectrality

- Higher derivative gravity
  - 1st order in spin, 1st order in coupling

  Cano, Fransen, Hertog, Maenaut (2020,2021);

- dynamical Chern-Simons
  - 1st order in spin, 1st order in coupling

  Wagle, Yunes, Silva (2021); Srivastava, Chen, Shankaranarayanan (2021)

- scalar-Gauss-Bonnet
  - 2nd order in spin, 6th order in coupling

  Pierini, Gualtieri (2021, 2022);

BH perturbation in small coupling

Modified Teukolsky equation

$$\mathcal{D}^{s}_{GR}\psi = \zeta \mathcal{D}^{s}_{nonGR}\psi^{\,*}$$

* In general the additional terms contain metric perturbations rather than Weyl scalars, but they can be brought to this form with the so-called *metric-reconstruction*

BH perturbation in small coupling

Modified Teukolsky equation

$$\mathcal{D}^{s}_{GR}\psi = \zeta \mathcal{D}^{s}_{nonGR}\psi$$

Depends on:

M BH mass

a BH spin

s Spin of the perturbation

Other quantities!

BH perturbation in small coupling

Li, Wagle, Chen, Yunes *2022*
Hussain, Zimmerman *2022*
Cano, Fransen, Hertog, Maenaut *2023*

Modified Teukolsky equation

$$\mathcal{D}^s_{GR}\psi = \zeta\mathcal{D}^s_{nonGR}\psi$$

With ansatz $\psi = e^{-i\omega t}e^{im\varphi}R(r)S(\theta)$

the equation does not decouple into radial and angular part

BH perturbation in small coupling

Li, Wagle, Chen, Yunes *2022*
Hussain, Zimmerman *2022*
Cano, Fransen, Hertog, Maenaut *2023*
Gosh, NF, Volkel, Barausse *2023*

Modified Teukolsky equation

$$\mathcal{D}^s_{GR}\psi = \zeta \mathcal{D}^s_{nonGR}\psi$$

With ansatz $\psi = e^{-i\omega t} e^{im\varphi} R(r) S(\theta)$

the equation does not decouple into radial and angular part

- Expand angular function in a spheroidal harmonics basis
- Use completeness relation of spheroidal harmonics $\langle S_\ell | S_{\ell'} \rangle = \delta_{\ell\ell'}$

BH perturbation in small coupling

Radial Teukolsky equation

$$\frac{1}{\Delta^s R(r)} \frac{d}{dr} \left[\Delta^{s+1} R'(r) \right] + V(r) = 0$$

where

$$V(r) = 2\mathrm{i}s \frac{\mathrm{d}K}{\mathrm{d}r} - \lambda_{\ell m} + \frac{1}{\Delta} \left(K^2 - \mathrm{i}sK \frac{\mathrm{d}\Delta}{\mathrm{d}r} \right)$$

$$\Delta = r^2 - r + a^2 , \qquad K = (r^2 + a^2)\omega - am ,$$

$$\lambda_{\ell m} = B_{\ell m} + a^2\omega^2 - 2am\omega .$$

BH perturbation in small coupling

Modifications to the radial equation

$$\frac{1}{\Delta^s R(r)}\frac{d}{dr}\left[\Delta^{s+1}R'(r)\right] + V(r) + \delta V(r) = 0$$

corrections depending on the theory

where

$$V(r) = 2\mathrm{i}s\frac{\mathrm{d}K}{\mathrm{d}r} - \lambda_{\ell m} + \frac{1}{\Delta}\left(K^2 - \mathrm{i}sK\frac{\mathrm{d}\Delta}{\mathrm{d}r}\right)$$

$$\Delta = r^2 - r + a^2\,, \qquad K = (r^2 + a^2)\omega - am\,,$$

$$\lambda_{\ell m} = B_{\ell m} + a^2\omega^2 - 2am\omega\,.$$

BH perturbation in small coupling

$$\frac{1}{\Delta^s R(r)} \frac{d}{dr} \left[\Delta^{s+1} R'(r)\right] + V(r) + \delta V(r) = 0$$

We take agnostic modification

$$\delta V(r) = \frac{1}{\Delta} \sum_{k=-K}^{4} \alpha^{(k)} \left(\frac{r}{r_+}\right)^k$$

Cano, Capuano, NF, Maenaut, Volkel *2024-A*

BH perturbation in small coupling

$$\frac{1}{\Delta^s R(r)}\frac{d}{dr}\left[\Delta^{s+1}R'(r)\right]+V(r)+\delta V(r)=0$$

We take agnostic modification

$$\delta V(r)=\frac{1}{\Delta}\sum_{k=-K}^{4}\alpha^{(k)}\left(\frac{r}{r_+}\right)^k$$

All these coefficients assumed proportional to $\zeta \ll 1$

Cano, Capuano, NF, Maenaut, Volkel *2024-A*

BH perturbation in small coupling

We take agnostic modification leading to shifts of frequencies and separation constants

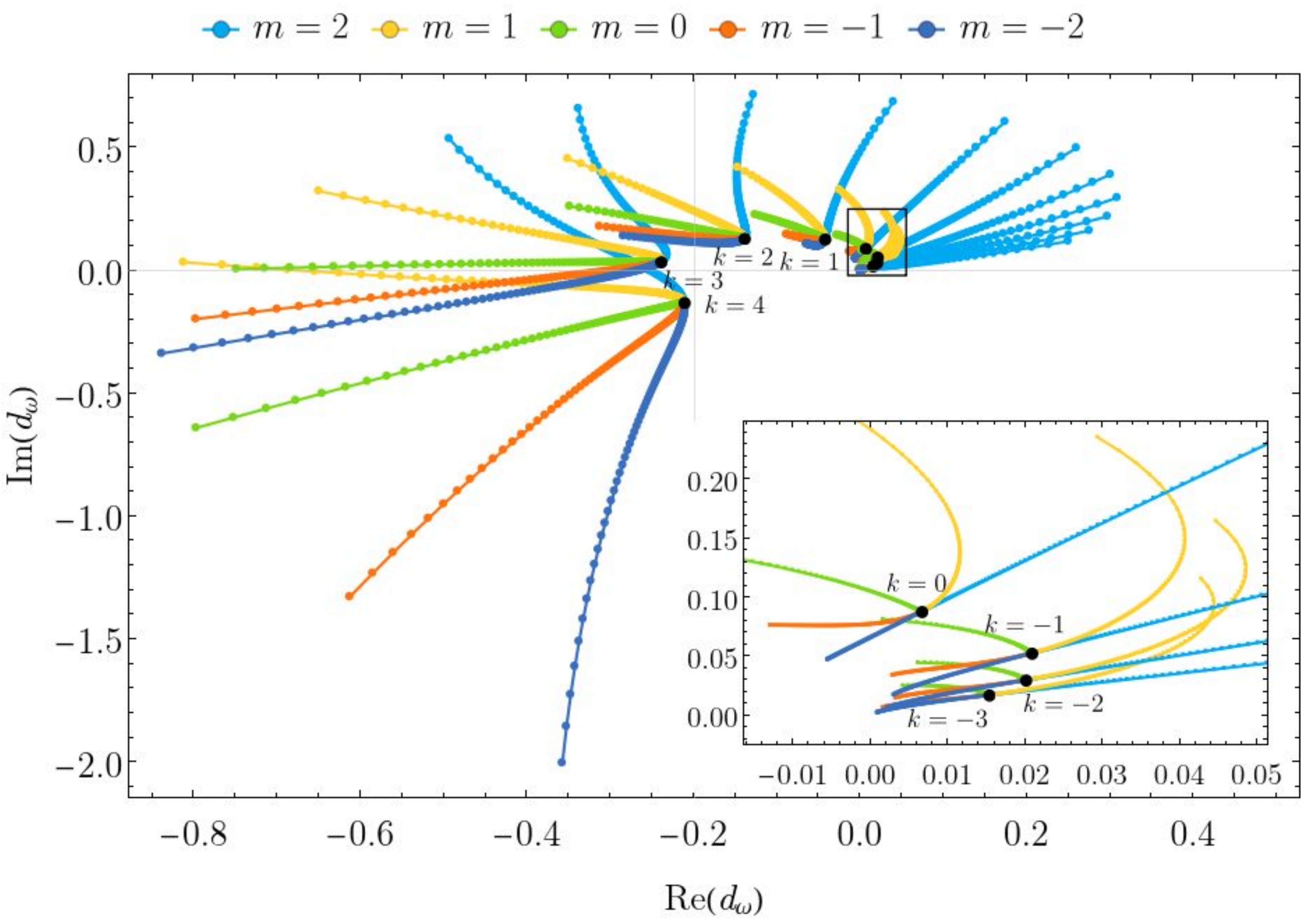
$$\omega_{n\ell m} \simeq \omega^0_{n\ell m} + \sum_k d^{(k)}_{\omega,n\ell m}\alpha^{(k)}$$

universal coefficients

Cano, Capuano, NF, Maenaut, Volkel *2024-A*

BH perturbation in small coupling

Frequency coefficients for *n=0, l=2*



Available with tutorial on github

Cano, Capuano, NF, Maenaut, Volkel *2024-A*

BH perturbation in small coupling: application

QNMs of rotating BHs in Higher Derivative Gravity

$$S_{\rm HDG}=\frac{1}{16\pi}\int d^4x\sqrt{g}\left[R+\lambda_{\rm ev}R_{ab}{}^{cd}R_{cd}{}^{ef}R_{ef}{}^{ab}\right]$$

↓ Manipulation of the equations to get

$$\delta\omega^{\pm}=\frac{\omega^{\pm}-\omega^{\rm KERR}}{\lambda}=\sum_{k\in k^{\rm HD}}\alpha_{\pm}^{(k)}d_{(k)}$$

$$k^{\rm HD}=[-2,0,1,2]$$

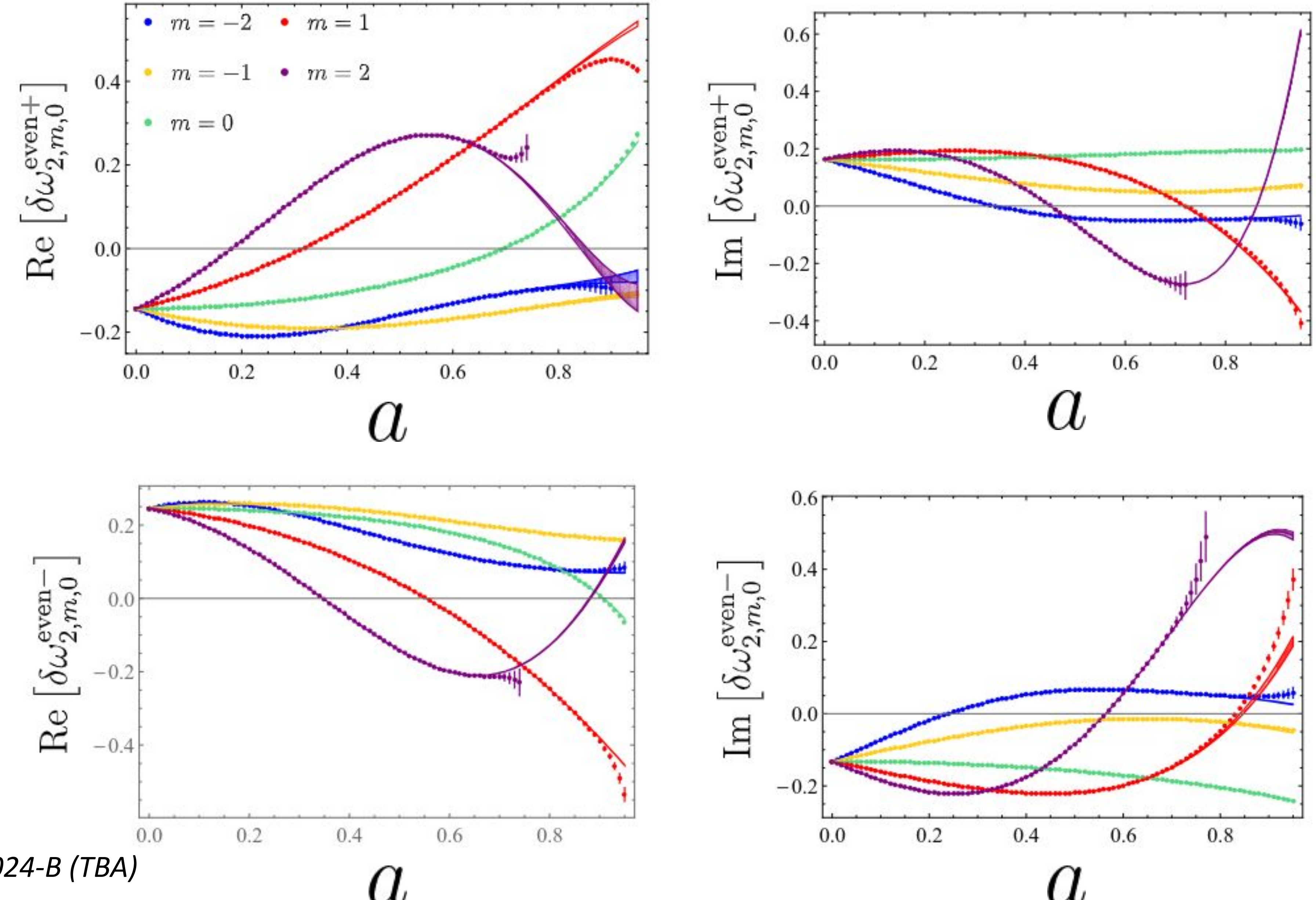
Up to 18th order in the spin + Pade spin resummation

Cano, Capuano, NF, Maenaut, Volkel *2024-B (TBA)*

BH perturbation in small coupling: application

First accurate computation of beyond-GR QNMs

- *n=[0,2] l=[2,4]*
- up to *a=[0.7,0.9]*
- Ready for tests against real data



Cano, Capuano, NF, Maenaut, Volkel *2024-B (TBA)*

Conclusions

- Analysis limited to non-rotating or slowly-rotating case only: beyond Teukolsky formalism to compute QNMs of fully rotating solutions
- Non-separability of the equations: solved assuming small-coupling and slow-rotation
- Slow rotation to obtain RW/Zerilli-like equations
- Difficult to compute overtones: not mentioned in the talk, we managed to find an extension of continued fraction method, stable for $n>0$