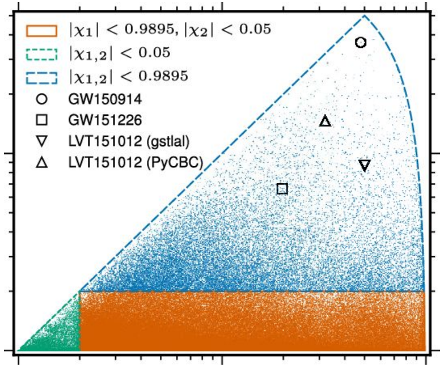
Gravitational waves from massive black hole binaries in non-circular orbits

Antoni Ramos-Buades

Fundamental Physics meets Waveforms with LISA meeting
September 5, 2024

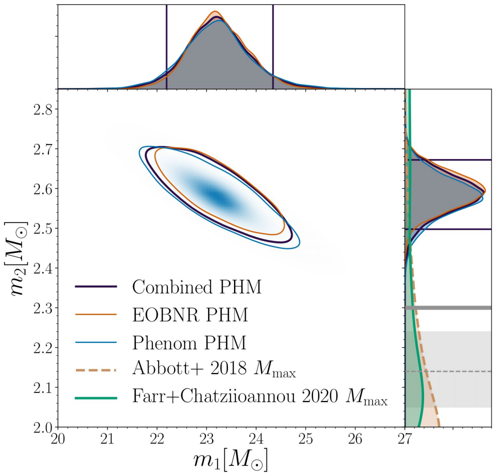
Waveform knowledge **CRUCIAL** for GW astronomy

Matched filter searches



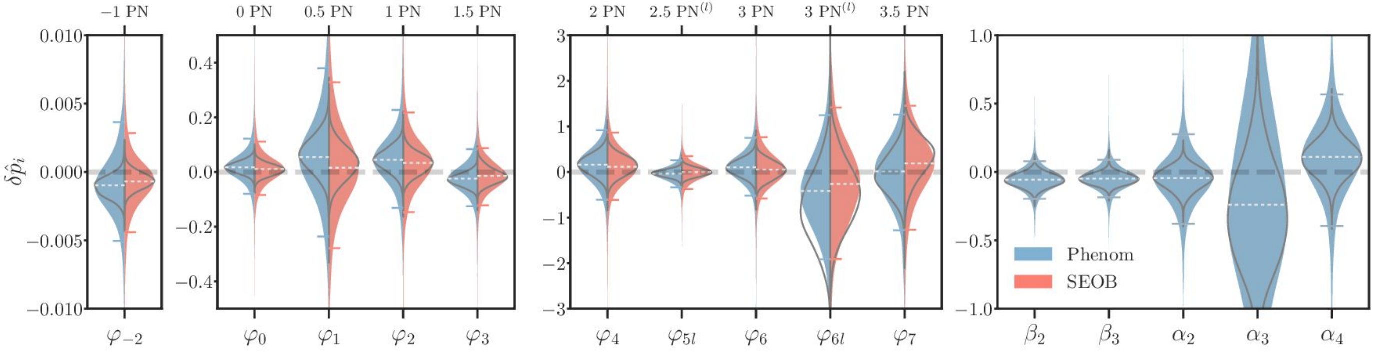
LIGO+Virgo, PRX2016 (1606.04856)

Parameter estimation



GW190814, LIGO+Virgo, AJL2020 (2006.12611)

Testing General Relativity



LIGO+Virgo+KAGRA PRD2021 (2010.14529)

Circularization in compact binaries

- Existence of GWs inferred from **Hulse-Taylor** (HT) **binary** in 1974.

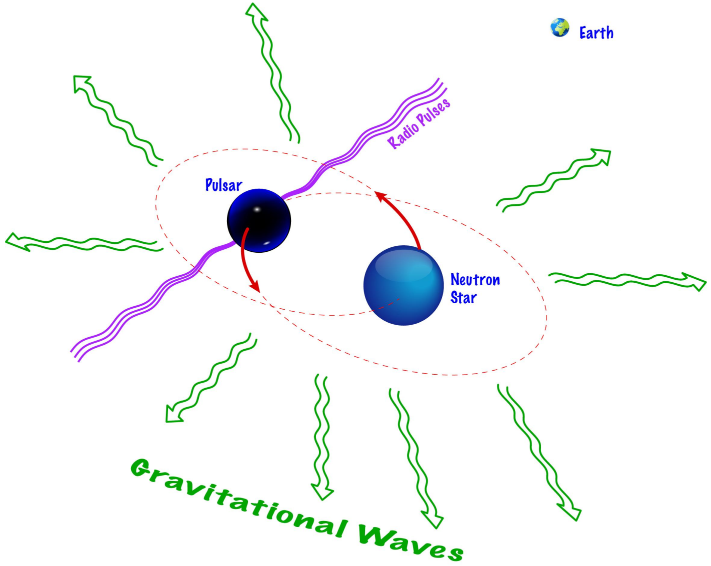


Figure from https://asd.gsfc.nasa.gov

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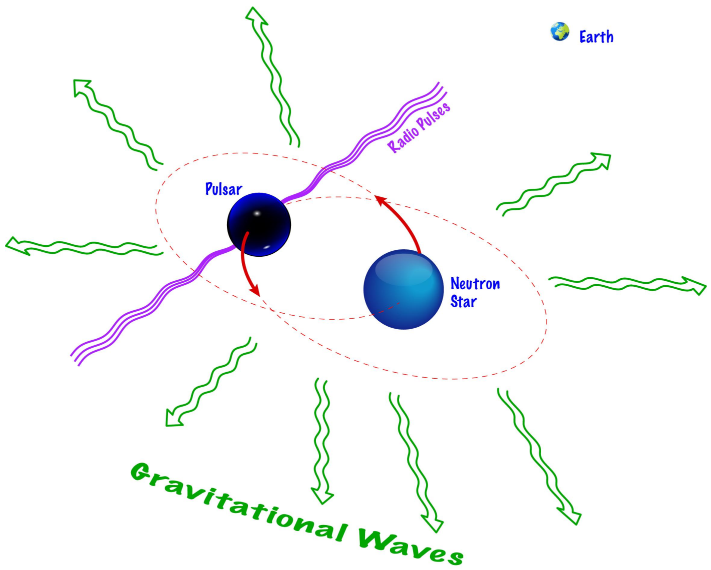
- Hulse-Taylor (HT) binary exists in an **elliptical orbit** with $e \sim 0.7$.



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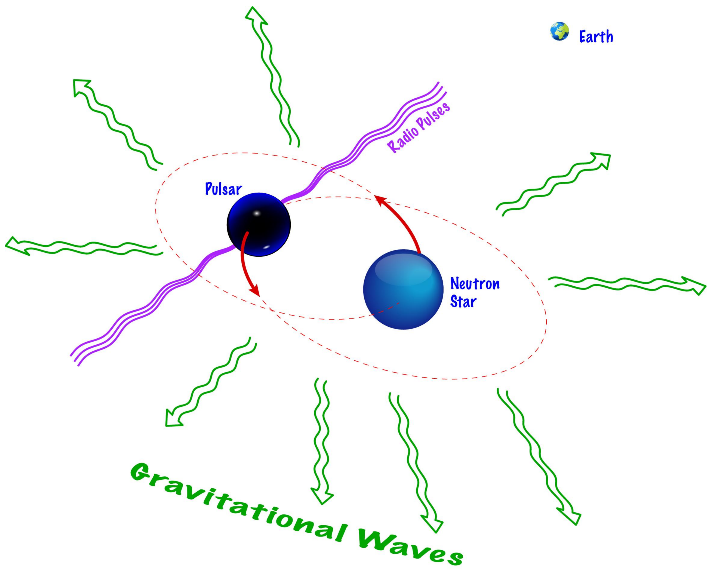

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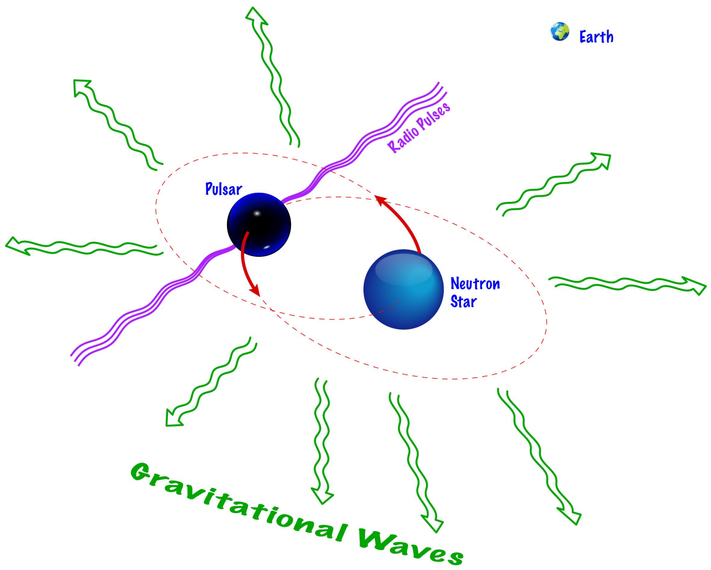

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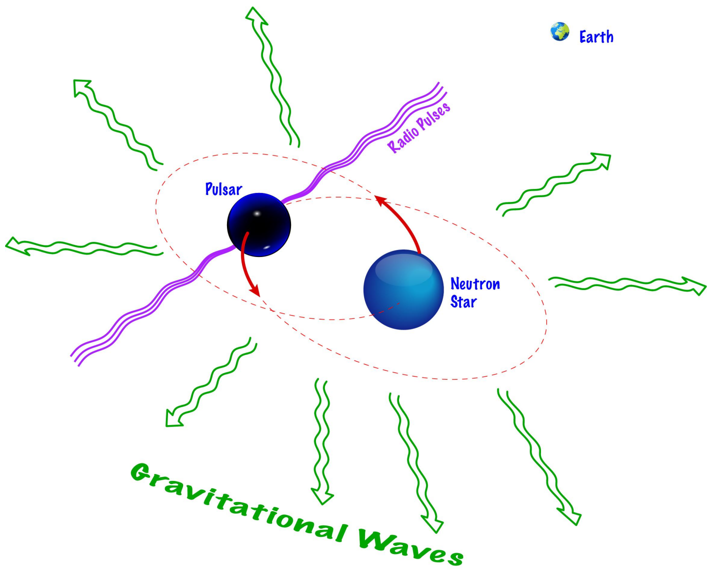

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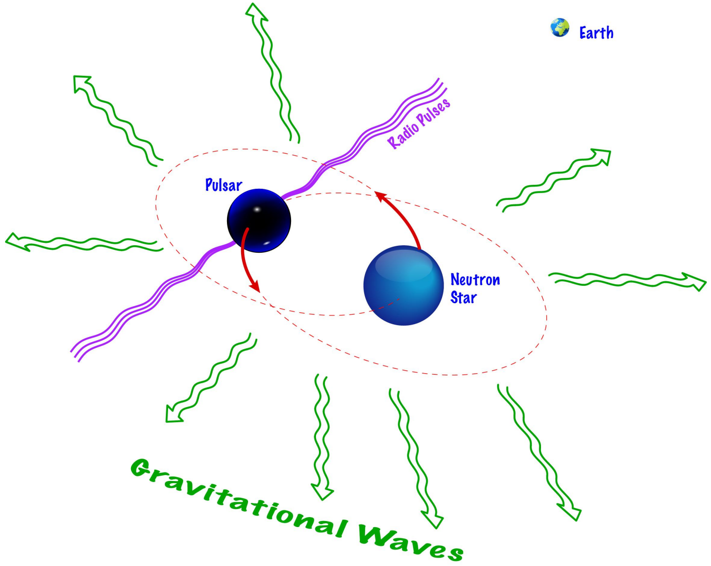
- **Broad picture: Widely separated binaries**, will have **circularised** by the time they enter the **frequency band** of **ground-based** **GW detectors**.
- **Population synthesis studies** [O'Leary+2009, Samsing+2014, ...]:

__Globular clusters__ and __galactic nuclei__ can host a __population of moderate and high eccentric binaries__ emitting in the band of ground-based detectors.



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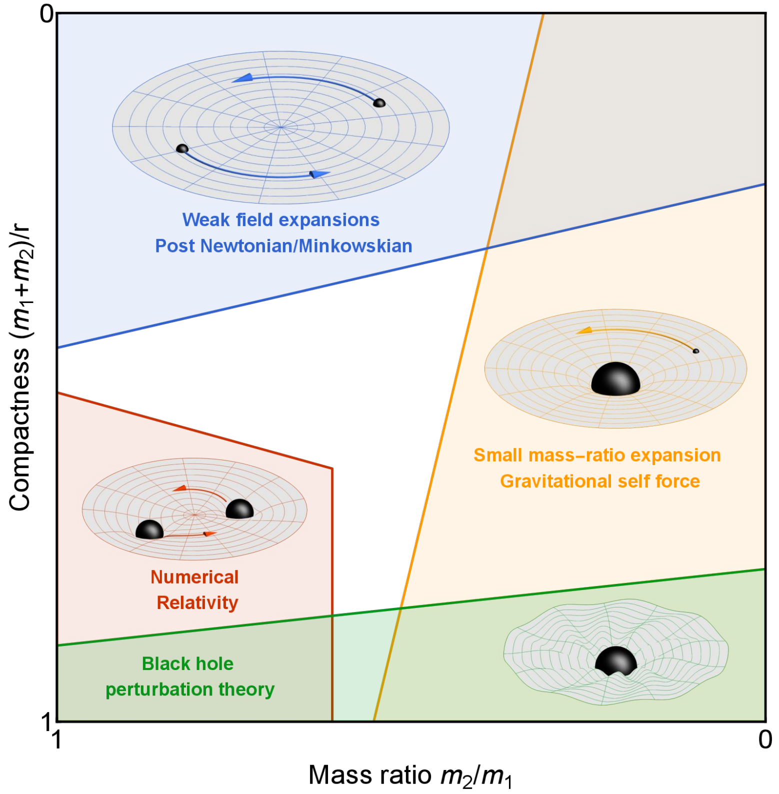
Approaches to the two-body problem

- Different approaches within General Relativity:
  - Numerical methods for inspiral-merger-ringdown signals
    - **Numerical relativity** (NR).
  - Analytical approaches for inspiral or ringdown:
    - **Post-Newtonian** theory, **post-Minkowskian** theory, **small-mass ratio** (SMR) expansion.
  - Semi-analytical methods for full waveforms:
    - **Effective-One-Body** (EOB) formalism .
    - **Phenomenological** framework.
    - **NR surrogate** models.



LISA Consortium Working group (2023) 2311.01300

Generic binary black holes

- Recent years a lot of effort modelling **quasicircular** BBHs including **higher order modes**.
  - **Aligned-spin** systems [*PhenomX/THM, SEOBNRv5HM, NRHybSur3dq8 ...*].
  - **Precessing-spin** systems [*PhenomXPHM/O4a/TPHM, SEOBNRv5PHM, NRSur7dq4 ...*]

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Colleoni, Hamilton, Pompili's talks for details

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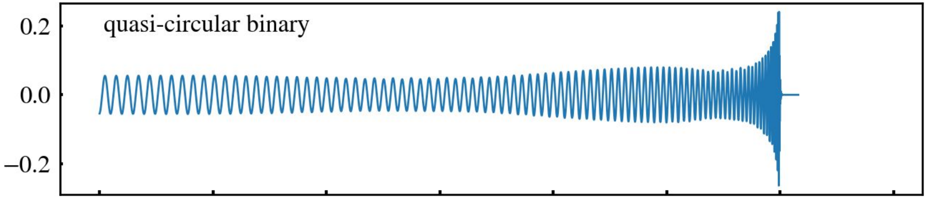
- **Quasicircular spin-precessing** binaries:
  - New timescale: $t_{\rm orb} < t_{\text{spin-prec}} < t_{\rm RR}$
  - 7 intrinsic parameters: $(\eta, \vec{S}_1, \vec{S}_2)$
  - Spin-spin and spin-orbit couplings $\rightarrow$ precession orbital plane



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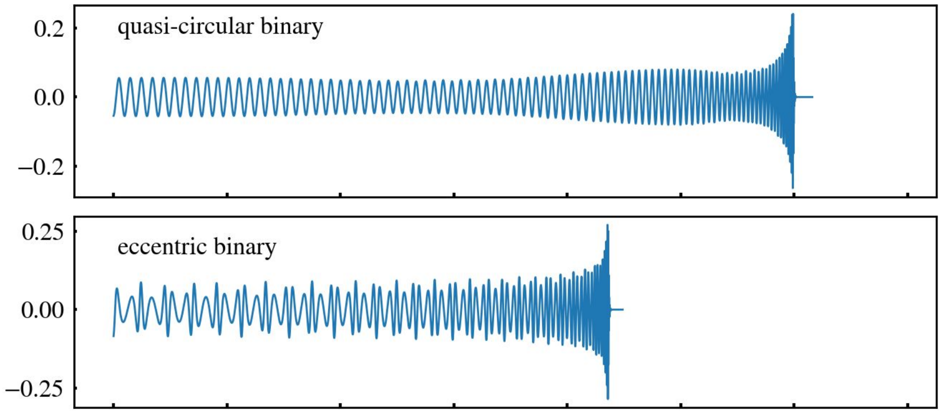
    - 5 intrinsic parameters $(\eta, S_{1z}, S_{2z}, e, \Omega)$.
    - **Eccentricity not uniquely defined in GR**!



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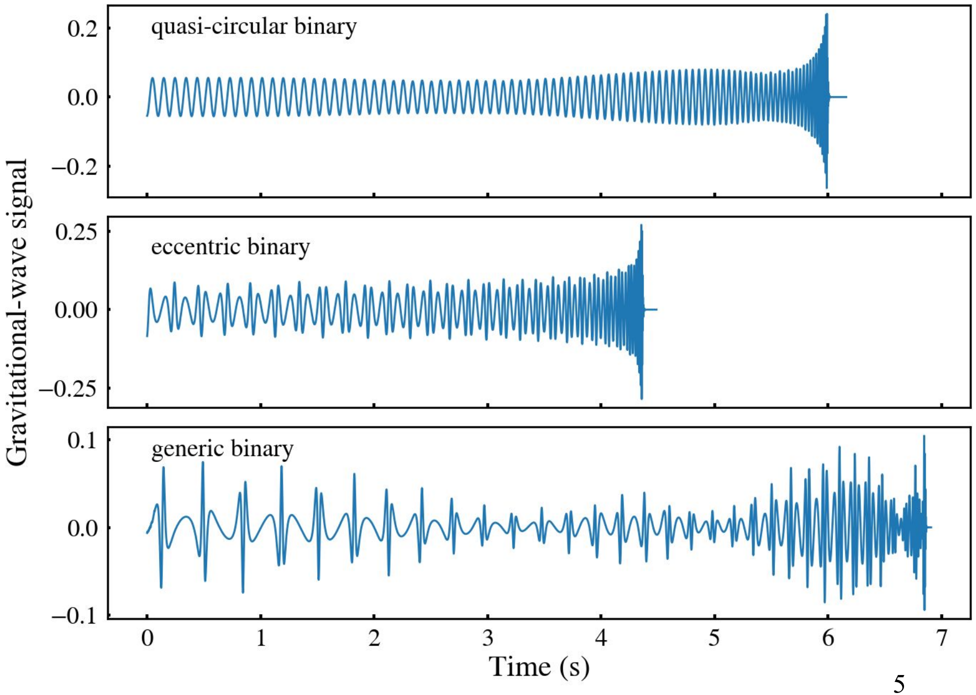

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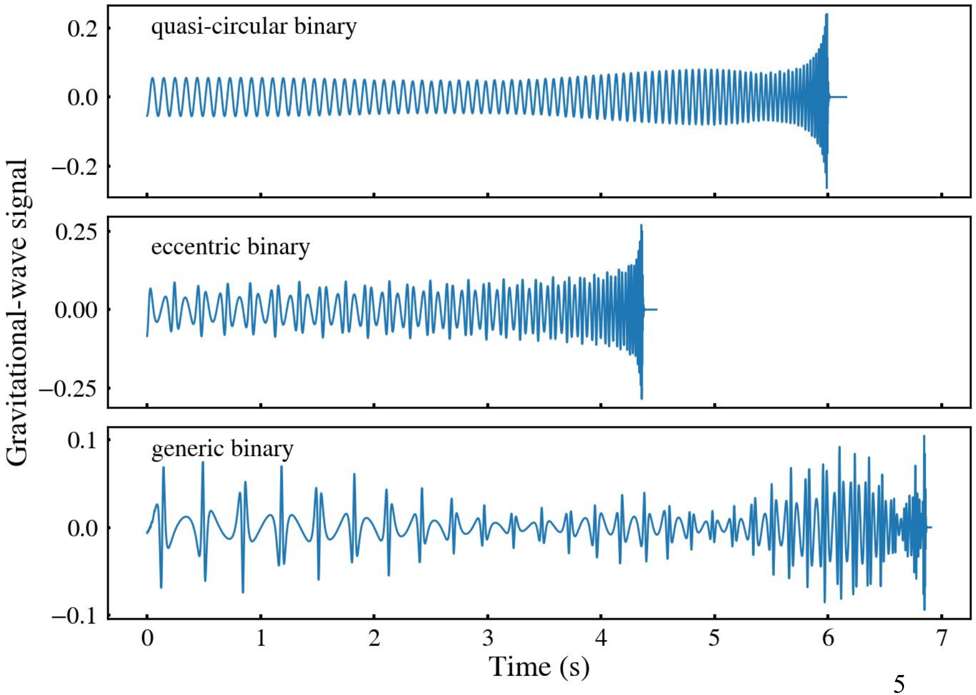
Long's talk→unbound case



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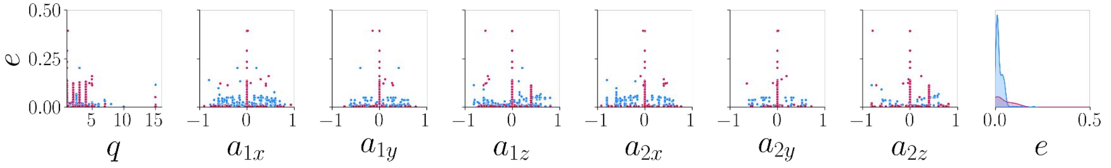
Numerical relativity progress

- No new numerical techniques needed for eccentricity.
- Several NR groups started covering eccentric BBH parameter space:
  - **RIT**, **Maya** catalog [Healy+2022, Ferguson+2023], **SXS** collaboration [Hinder+2017,Islam+21,RB+2022]
  - **Individual groups (ET)** [Huerta+2017,RB+2020,Joshi+2023,Andrade+2023, Bonino+2024]



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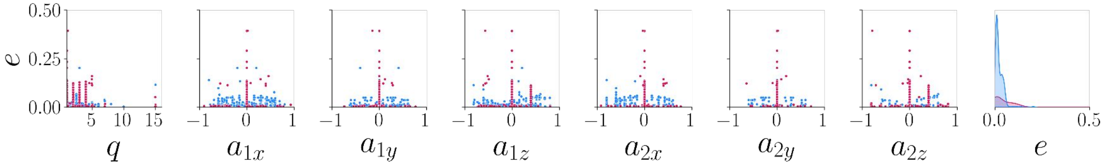

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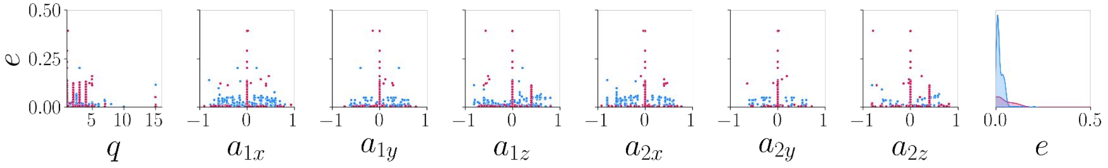

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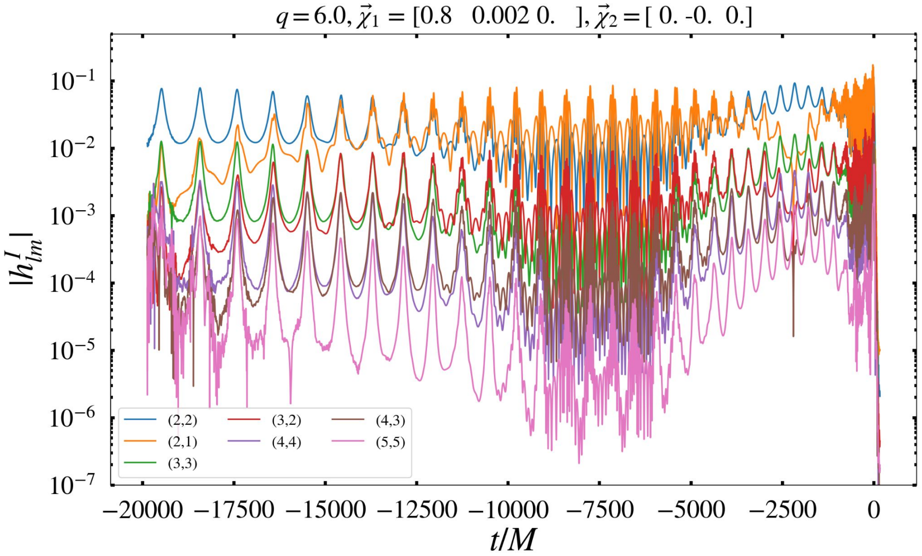

- Ongoing work within SXS to produce **long and accurate precessing-spin eccentric** simulations controlling initial parameters [Nee+2024,RB+2024].

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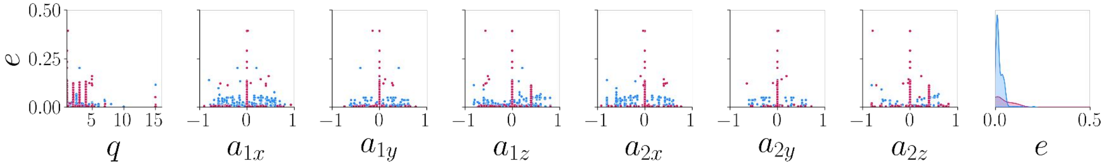
More about NR →Lovelace's talk



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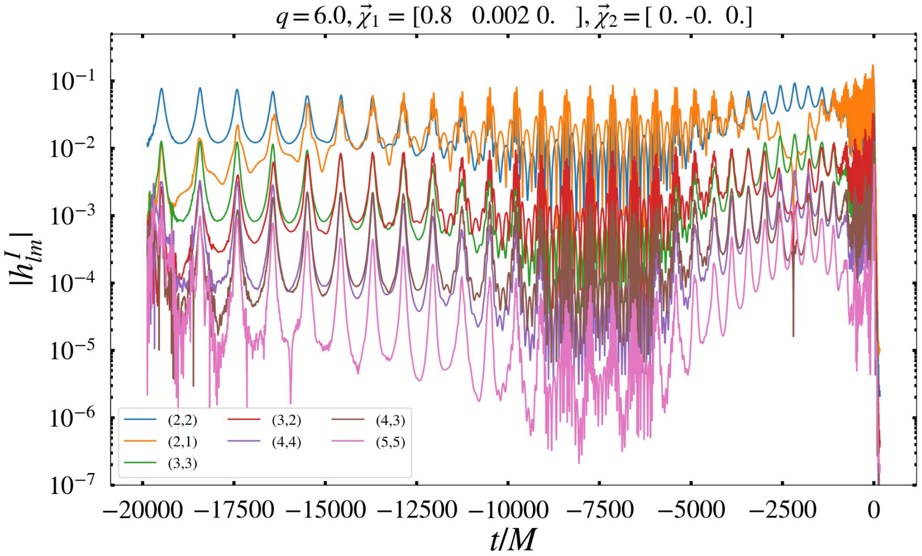


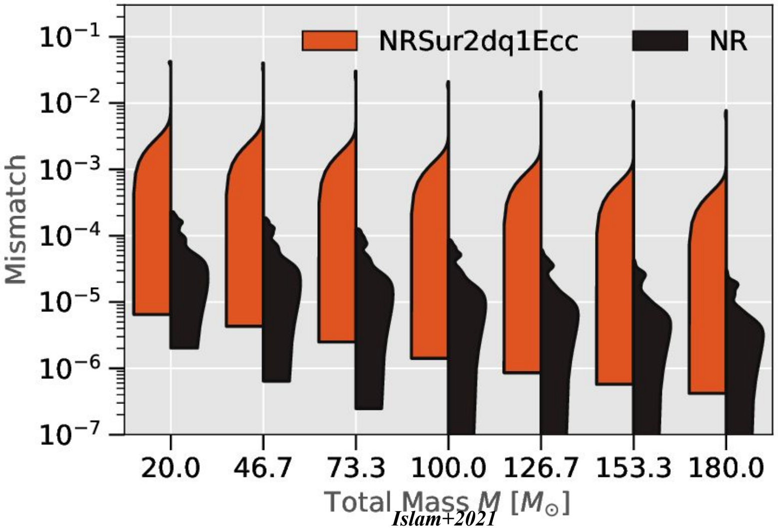
Eccentric surrogate models

- First non-spinning IMR eccentric models ENIGMA [Huerta+2017], Eccentric IMR [Hinder+2017].
  - Combining quasi-Keplerian parametrization + merger model.
  - No higher modes, no calibration to NR (accurate up to e_0 ~0.3).
  - Recently extended to spinning binaries with higher order modes ESIGMAHM [Kaushik+2024].

- NR calibrated models [Islam+2021, Setyawati+2021].

- First NR calibrated equal-mass non-spinning surrogate [Islam+2021]
  - Calibrated to 47 simulations up e_0 ~0.2.
  - Highly accurate.
  - Can be extended to mass ratio 3.
  - Limited by length of NR waveforms (~5500M).

- Ongoing work to extend surrogates to higher mass ratios and spins [Ravichadran+2024, Islam+2024, Nee+2024] .

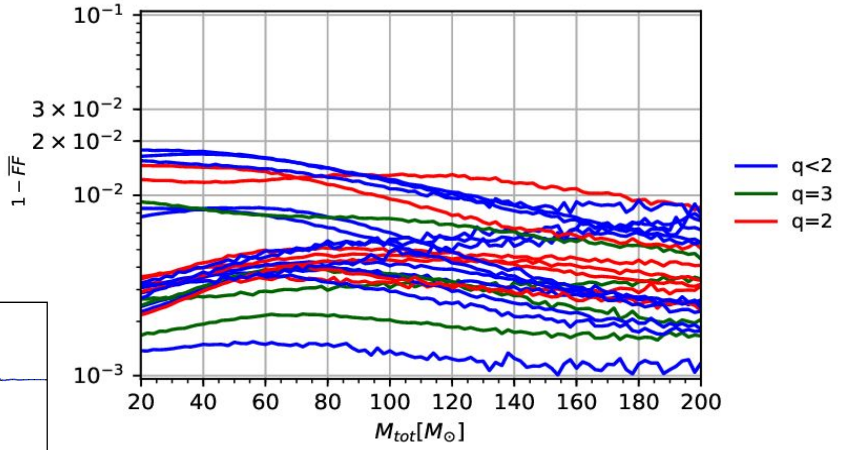


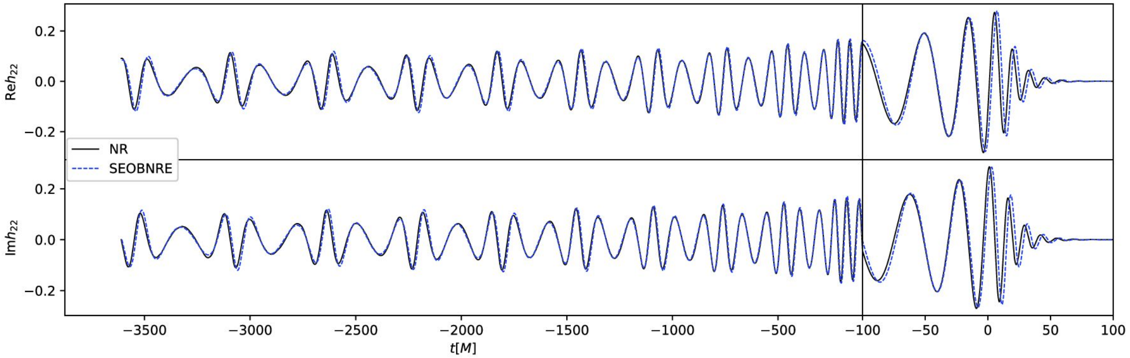
Islam+2021

Effective-One-Body eccentric models

- Lot of progress on constructing IMR **eccentric** models:
  - SEOBNRE : SEOBNREHM [Cao+2019, Liu+2021], SEOBNREPHM [Liu+2023]
  - TEOBResumS : TEOBResumS-Dali [Nagar+2021,Gamba+2023]
  - SEOBNR models : SEOBNRv4EHM [RBs+2021], SEOBNRv5EHM [Gamboa+2024]

- **SEOBNRE** models:
  - Extension of SEOBNRv4 models to eccentricity.
  - Non-precessing model accurate to e_0 ~0.3.
  - Derived eccentric corrections for waveform multipoles.
  - Recently extended to spin-precession models [Liu+2023].
    - Compared to 2 NR simulations.

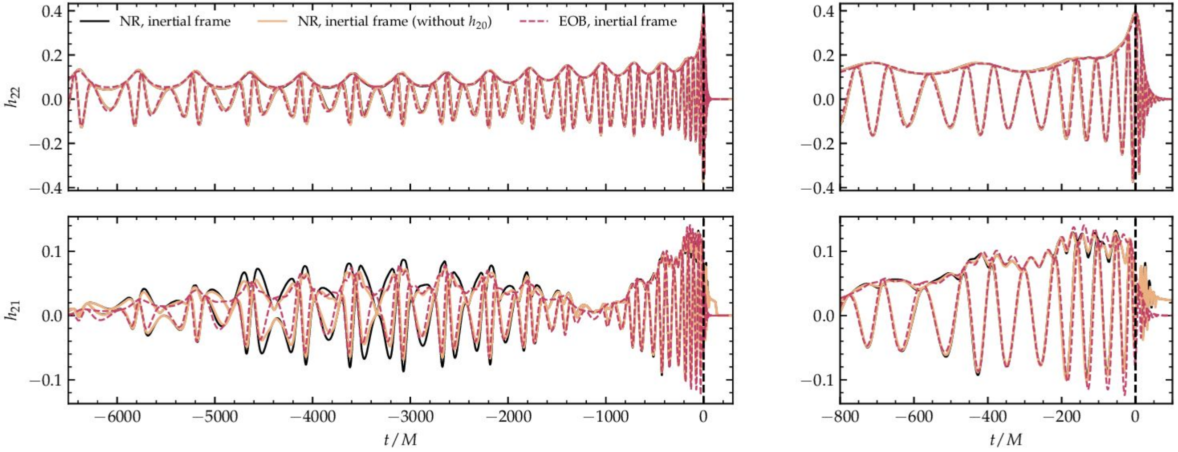


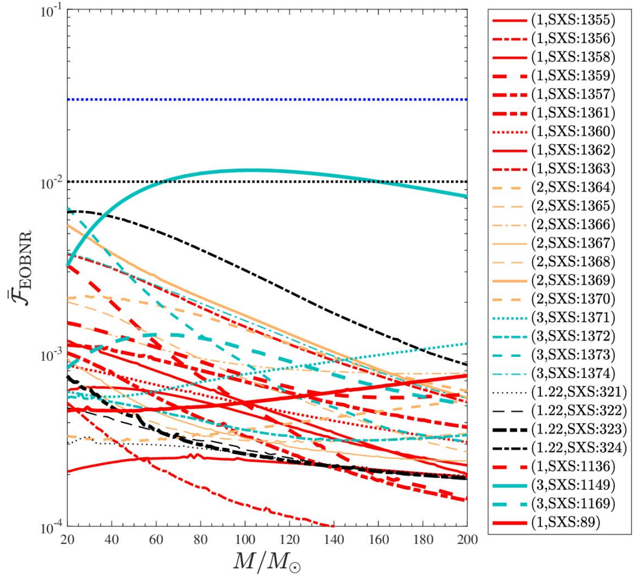


Effective-One-Body eccentric models. TEOBResumS-Dali

- Built upon resummed EOB analytical information [Chiaramello+2019, Placidi+2022,Nagar+2021/23/24].
- Initial parameters based upon **Keplerian parametrization**.
- Quasi-circular limit departs from TEOBResumS-GIOTTO [Nagar+2023].
- Highly accurate compared to public NR waveforms of e_0 ~0.3.

- Used for parameter estimation of some GW events [Iglesias+2023,Bonino+2023].
- Currently under LVK review.
- Recently extended to spin-precession [Gamba+2024].

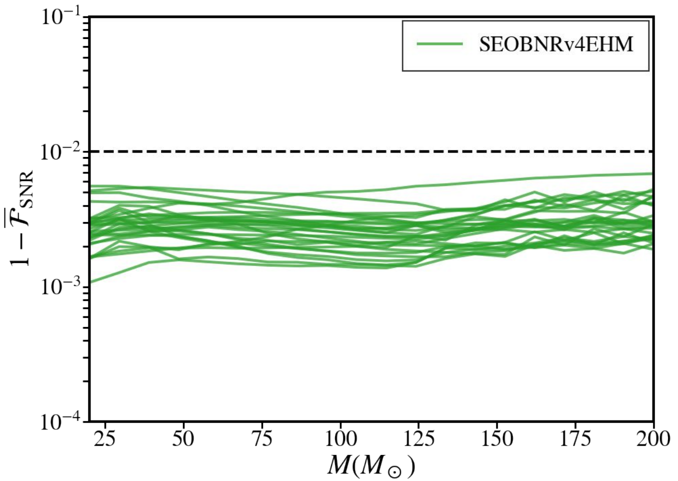




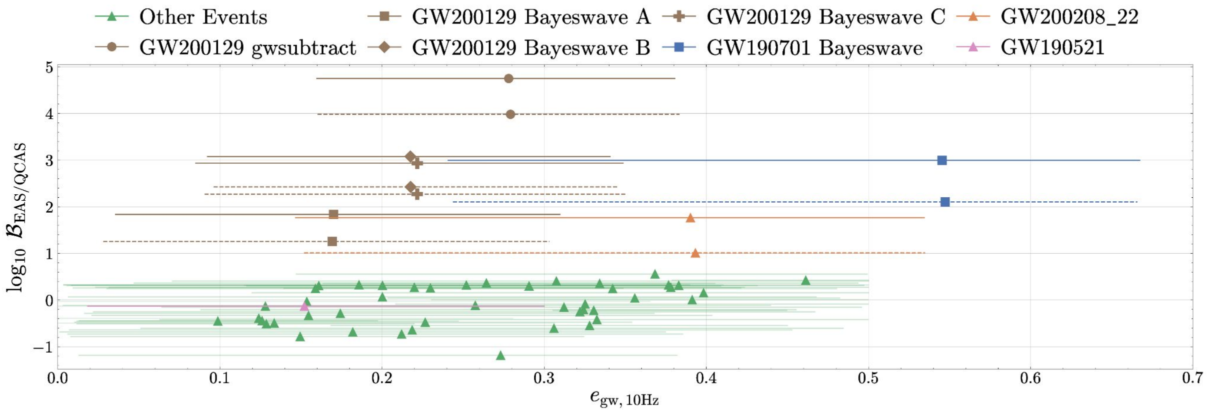
Effective-One-Body eccentric models. SEOBNRv4EHM

- Extension of quasi-circular **SEOBNRv4HM** [Bohe+2017, Cotesta+2018].
- Eccentricity corrections to waveform modes [Khalil+2021].
- Two-parameter initial conditions based on Keplerian parametrization.
- Highly accurate to public eccentric NR waveforms of e_0 ~0.3 [RB+2022].
- Parameter space coverage (at 20Hz):

$$q \in [1, 20], \chi_{1,2} \in (-0.99, 0.99), e_0 \in (0, 0.3),$$

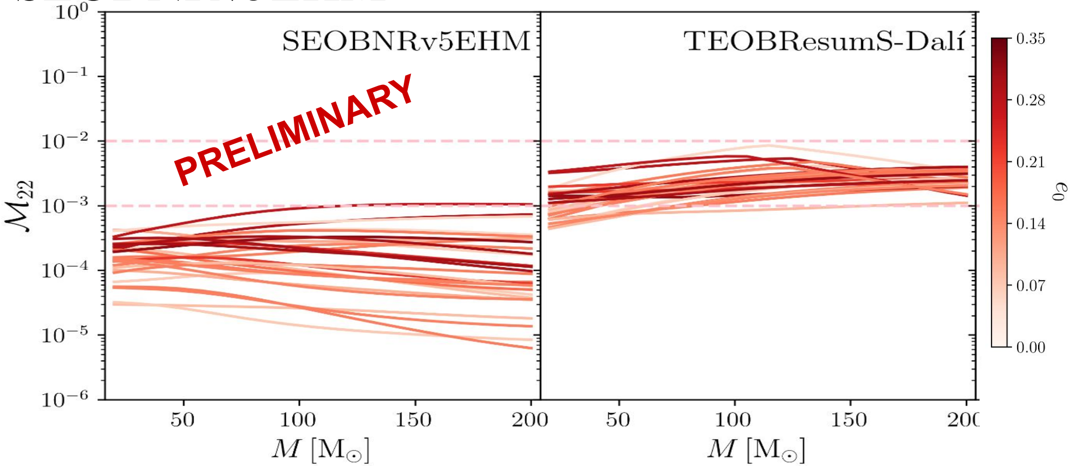


- Several applications in data analysis:
  - Systematic **parameter estimation** analysis [RB+2023] with **DINGO** [Gupte+2024].
  - **Search sensitivity** studies [Gadre+2023] and **population** studies [Vijaykumar+2023]

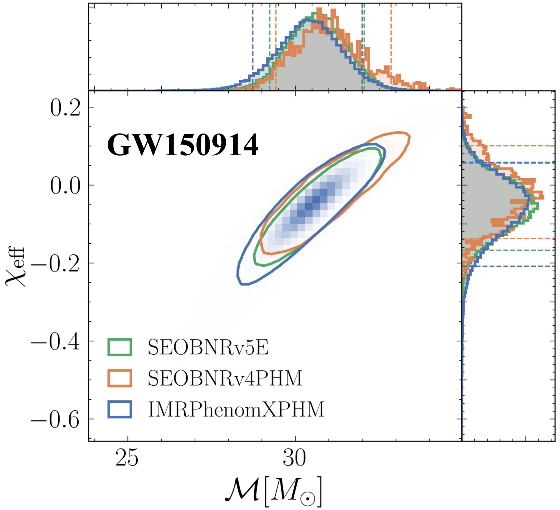


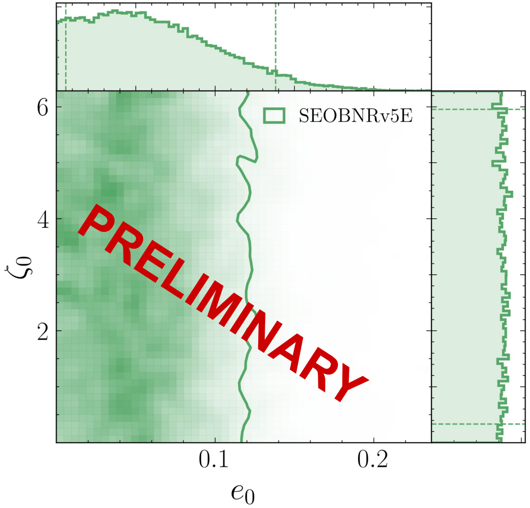
Effective-One-Body eccentric models. SEOBNRv5EHM

- Extension of quasi-circular **SEOBNRv5HM** [Pompili+2023].
- Eccentricity corrections to waveform modes and dynamics [Gamboa+2024].
- Highly accurate to public and private NR waveform eccentric NR waveforms of e_0 <0.8 [Gamboa+2024].



- Analysis of GW events ongoing.
- Currently under LVK review.





Phenomenological eccentric models

- Last years lack of development of development of eccentric models in phenomenological approach.

- Ongoing work to extend **IMRPhenomTHM** [Estelles+2020] and **IMRPhenomXHM** [Garcia-Quiros+2020] to eccentricity.
  - **IMRPhenomTEHM** [Planas+2024]: time-domain based on quasi-Keplerian parametrization.
  - **IMRPhenomXE** [RB+2024]: frequency-domain based on stationary phase approx.

- Both implemented in a new python infrastructure for PhenomX/T models ***phenomxpy*** [Garcia-Quiros+2024].

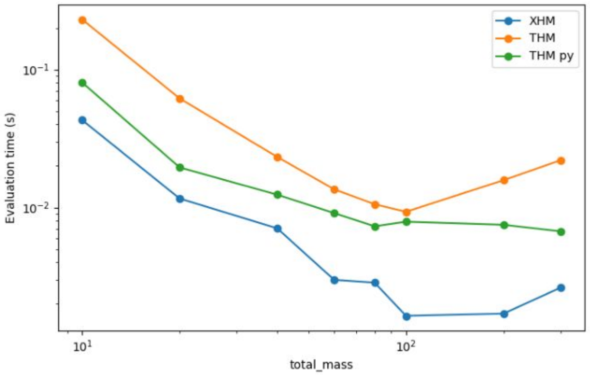
- Currently implemented: **IMRPhenomT*** and **IMRPhenomXAS** (Relative error with LAL waveforms of 10^{-13}-10^{-16}).

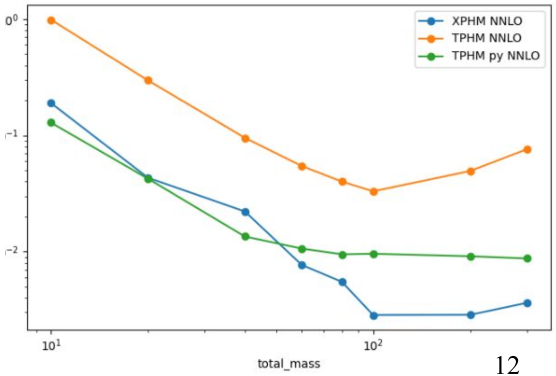
- Phenomxpy benefits from **python speed-up packages** (Numba routines and GPU support).

- Implemented conditioning routines from LALSuite.

- Speed up by a factor 1.2-2.5 / 4-9 compared to LAL (using just-in-time compilation).

Example (CPU only on CIT): q=3, f_{min}=20 Hz, srate=4K





Phenomenological eccentric models. IMRPhenomTEHM

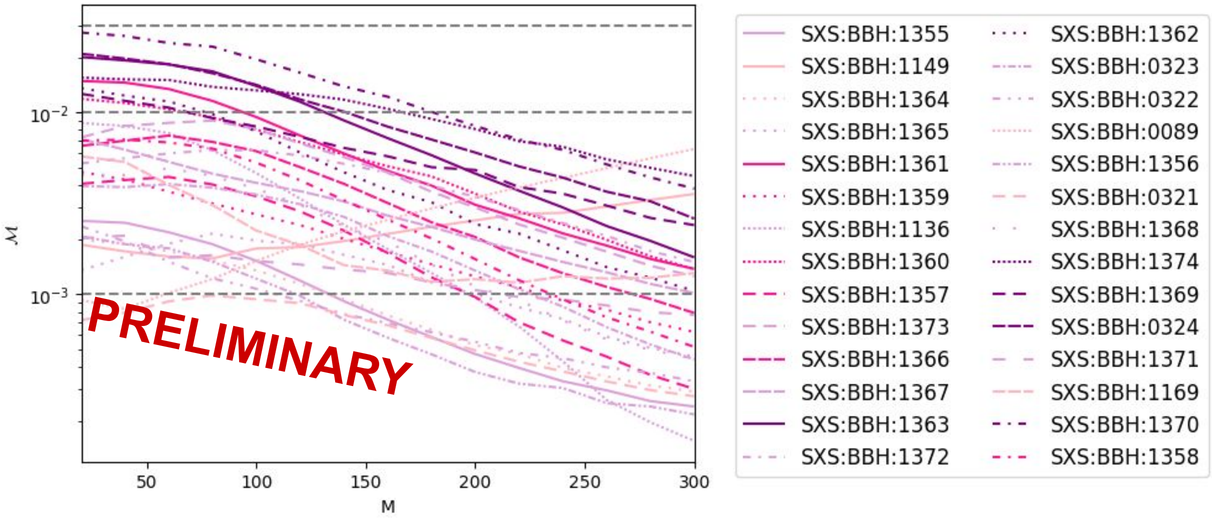
- Eccentric inspiral based on **quasi-Keplerian parametrization**:
  - 3PN radiation reaction equations + **quasicircular PhenomT frequency**.
  - 3PN spinning modes eccentricity expanded O(e^{12}) [no oscillatory memory].

- Eccentric corrections to (l,m)=[(2,2),(2,1),(3,3),(4,4),(5,5)] modes .

- Merger-ringdown consistent with quasicircular **IMRPhenomTHM**.

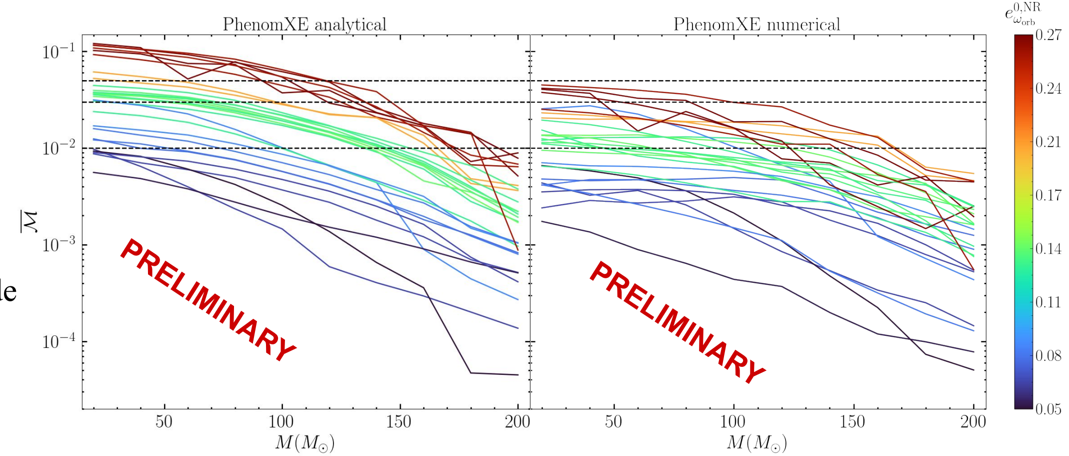
- No calibration to eccentric NR waveforms.

- Comparison to public eccentric NR waveforms mismatch <3% [Planas+2024].



Phenomenological eccentric models. IMRPhenomXE

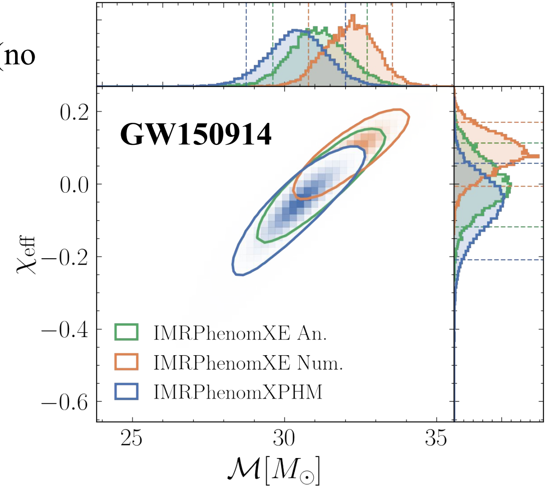
- **IMRPhenomXE**: Same eccentric inspiral as IMRPhenomTE + SPA approx.

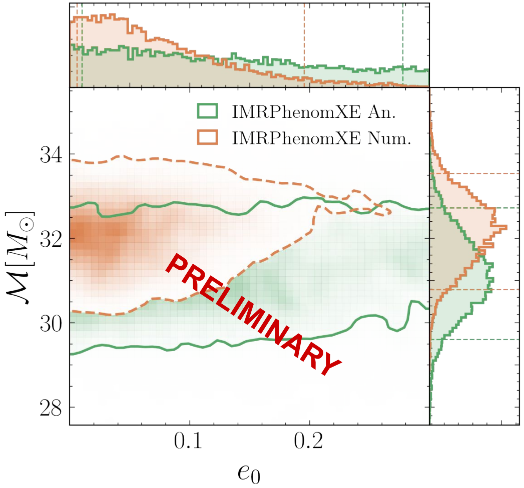


- Two models [RB+2024]:
    - **Analytical**: eccentricity expansion in amplitude and phase up to $\mathbf{O(e_0)^6}$.
    - **Numerical**: time-domain evolution + SPA approx. $\mathbf{O(e)^6}$.

- Comparison to public eccentric waveforms (no calibration).

- Efficient model GW150914 →PE with Bilby in a few hours.





Conclusions

- Different families are producing **ready-to-use multipolar eccentric non-precessing models**.
- Most models are compared to public eccentric NR waveforms up to e_0~0.3, and show **high accuracy**.
- **More non-precessing NR waveforms needed** for NR calibration.
- **First attempts to include spin-precession**. Main caveat: compared to 2 NR waveforms.
- Need more and longer eccentric precessing NR waveforms.
- Optimistically first **ready-to-use eccentric precessing models by 2025-2026** → Need to include NR-calibration for LISA.