

EMRIs: does (Dark) Matter matter?

Francisco Duque



3rd September 2024, *Fundamental Physics meets Waveforms with LISA*



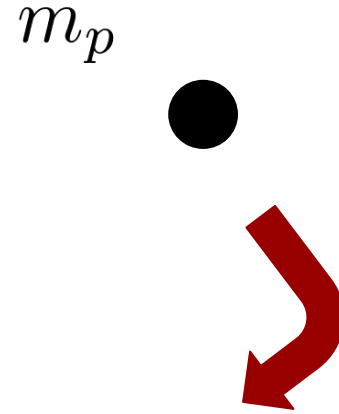
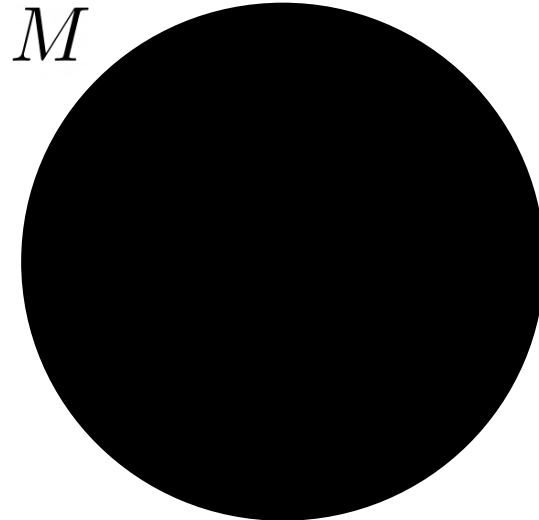
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EMRIs: 10^5 orbital cycles in LISA

$\left(\frac{\Delta M}{M}\right)_{\text{obs}}$	$\Delta\chi_1$	
$3.5 \times 10^{-5+6 \times 10^{-5}}$ -1.9×10^{-5}	$3.1 \times 10^{-5+1.5 \times 10^{-4}}$ -1.1×10^{-5}	<i>LISA "Red Book", arXiv:2402.07571</i>

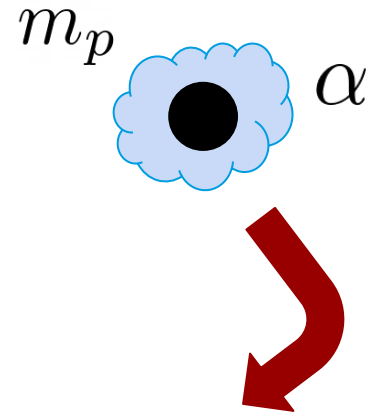
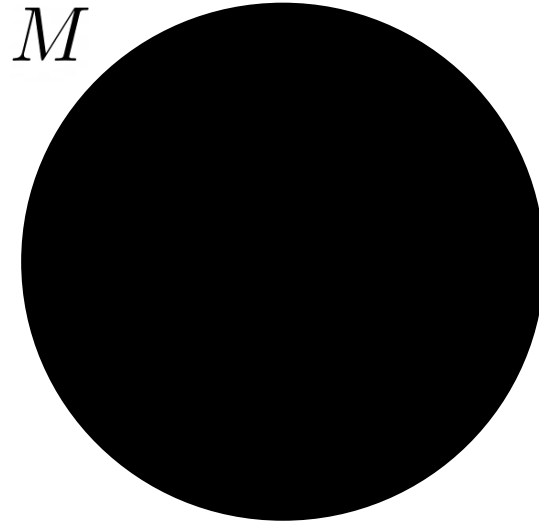


$$g_{\mu\nu}^{\text{exact}} = g_{\mu\nu} + \epsilon h_{\mu\nu}^{(1)} + \epsilon^2 h_{\mu\nu}^{(2)} + \mathcal{O}(\epsilon^3)$$

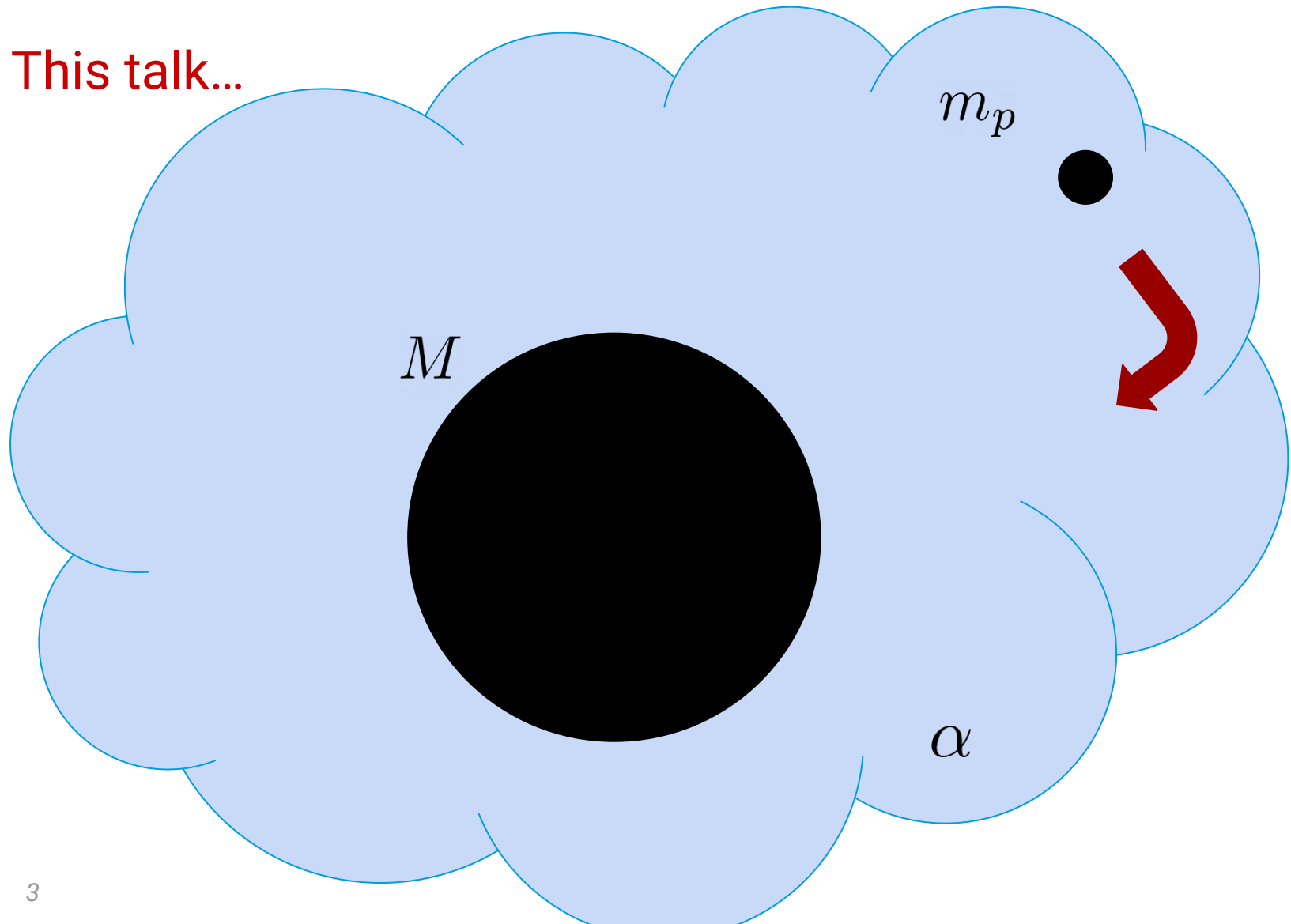
(Sorry...)

$$\epsilon = q = m_p/M$$

Susanna's talk...



This talk...



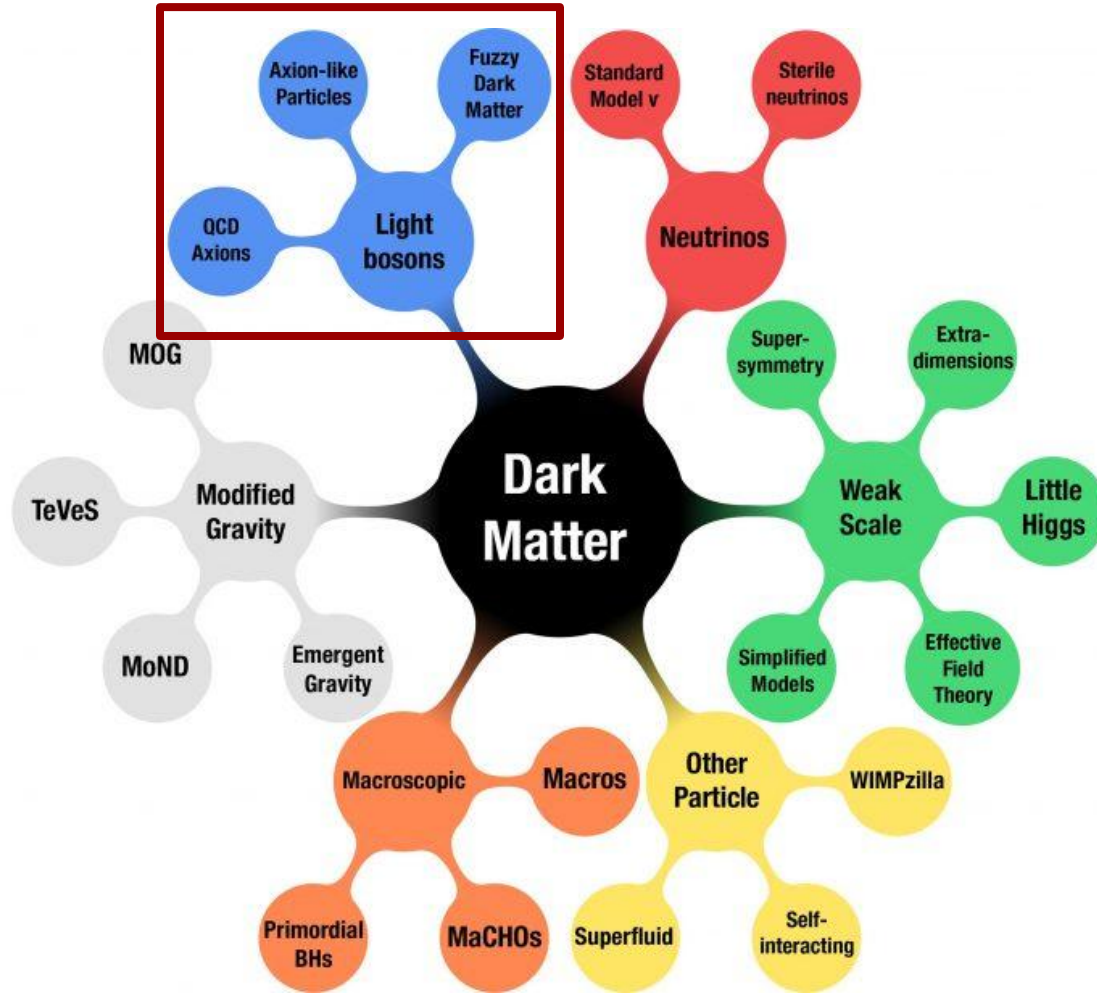


Image Credit: G. Bertone and T. M. P. Tait

de Broglie wavelength

$$\lambda_{\text{dB}} = \frac{2\pi}{m_{\Phi} v} = 0.48 \text{kpc} \left(\frac{10^{-22} \text{ eV}}{m_{\Phi}} \right) \left(\frac{250 \text{ km/s}}{v} \right)$$

No. particles in de Broglie volume λ_{dB}^3

$$N_{\text{dB}} \sim \left(\frac{34 \text{ eV}}{m_{\Phi}} \right)^4 \left(\frac{250 \text{ m/s}}{v} \right)^3$$

$$m_{\Phi} \lesssim 10 \text{ eV}$$



Classical Waves

$$r \gtrsim 1 \text{ kpc}$$

Particle



$$r \lesssim 1 \text{ kpc}$$

Wave

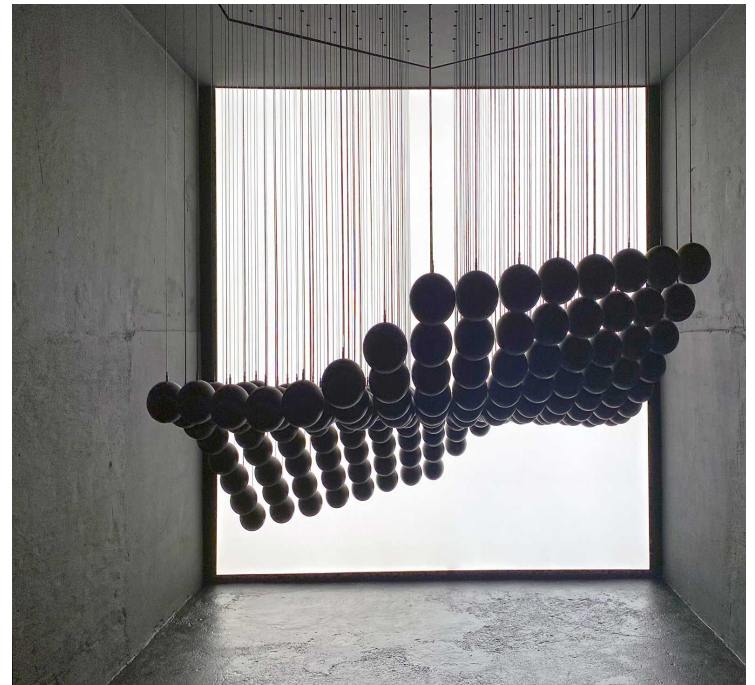
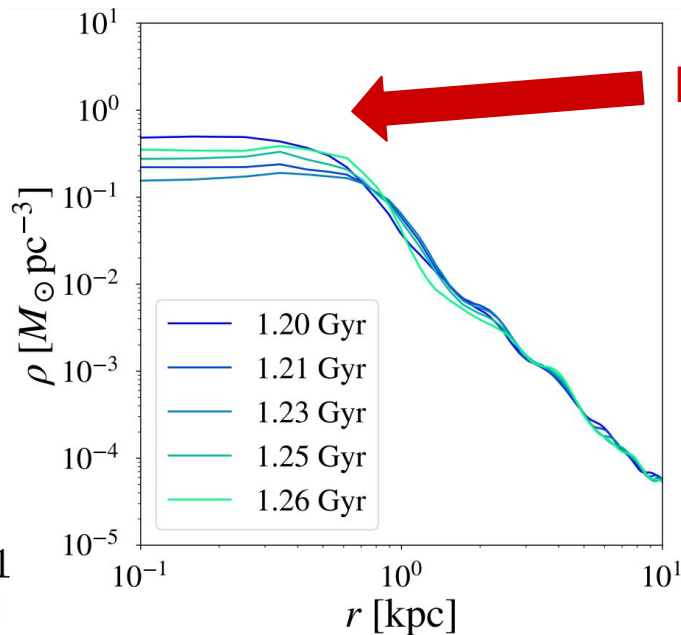


Image Credit: DARK MATTER Berlin

Boson Stars: self-gravitating (compact) objects



Flat profile: no cuspy-halo!

$$\mu = m_{\Phi} c / \hbar = \lambda_c^{-1}$$

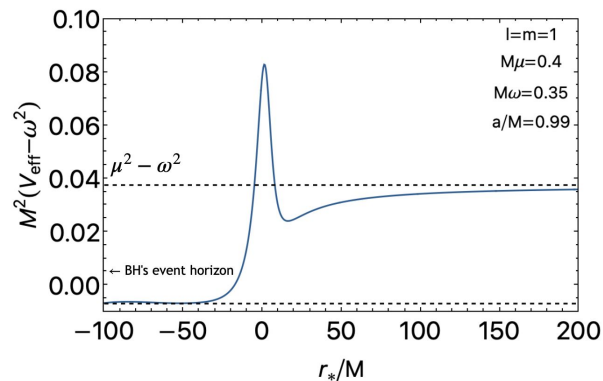
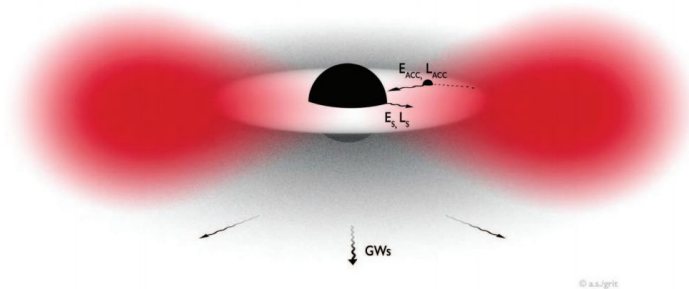
$$\frac{M_{\text{BS}}}{10^9 M_{\odot}} \sim \frac{1 \text{ kpc}}{R_{\text{BS}}} \left(\frac{10^{-22} \text{ eV}}{\mu} \right)^2$$

$$\tau_{\text{accr}} \sim 30 \text{ Gyr } f(\nu_0) \left(\frac{10^{10} M_{\odot}}{M_{\text{BS},0}} \right)^5 \left(\frac{10^{-22} \text{ eV}}{\mu} \right)^6$$

Superradiant clouds: dominated by BH gravity

$$\omega < m\Omega_H$$

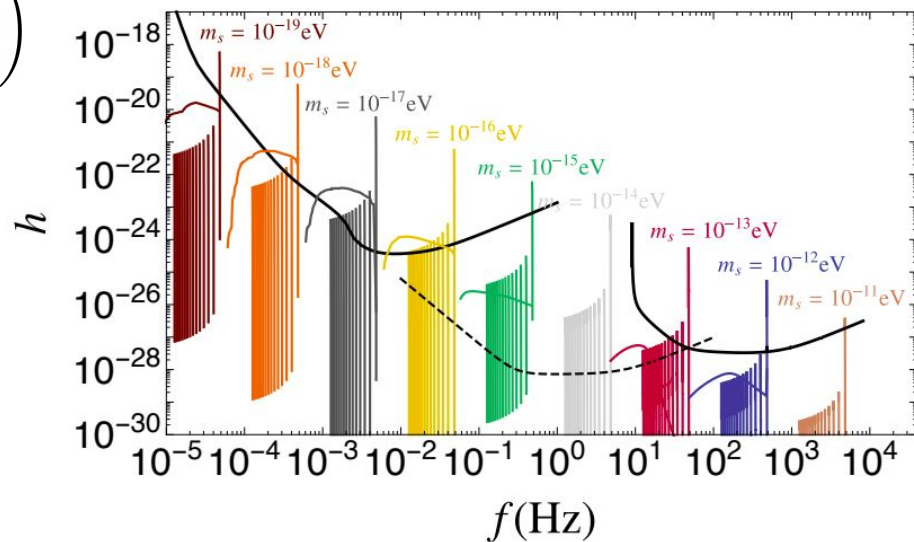
$$\alpha = M\mu \sim \mathcal{O}(1)$$



$$\tau_{\text{inst}}^s \approx 30 \text{ days} \left(\frac{M}{10M_\odot} \right) \left(\frac{0.1}{M\mu} \right)^9 \left(\frac{0.9}{J/M^2} \right)$$

$$\tau_{\text{inst}}^{\text{vec}} \approx 280 \text{ s} \left(\frac{M}{10M_\odot} \right) \left(\frac{0.1}{M\mu} \right)^7 \left(\frac{0.9}{J/M^2} \right)$$

$$\tau_{\text{GW}}^s \approx 10^5 \text{ yr} \left(\frac{M}{10M_\odot} \right) \left(\frac{0.1}{M\mu} \right)^{15} \left(\frac{0.5}{\Delta (J/M)^2} \right)$$



EMRI surrounded by bosonic environment



Binary interacts w/ matter



Trajectory changes w.r.t. vacuum



GW signature changes



What can we (not) learn?

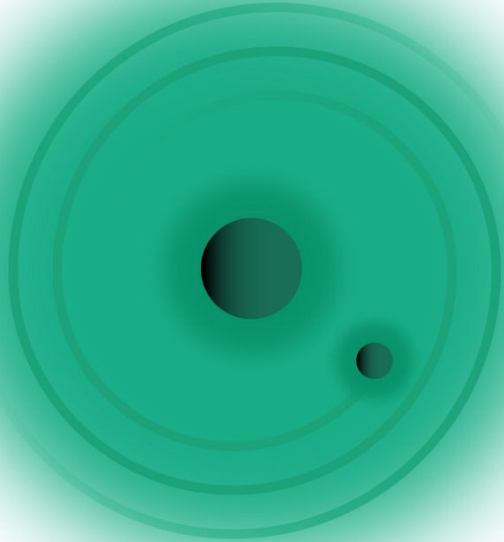


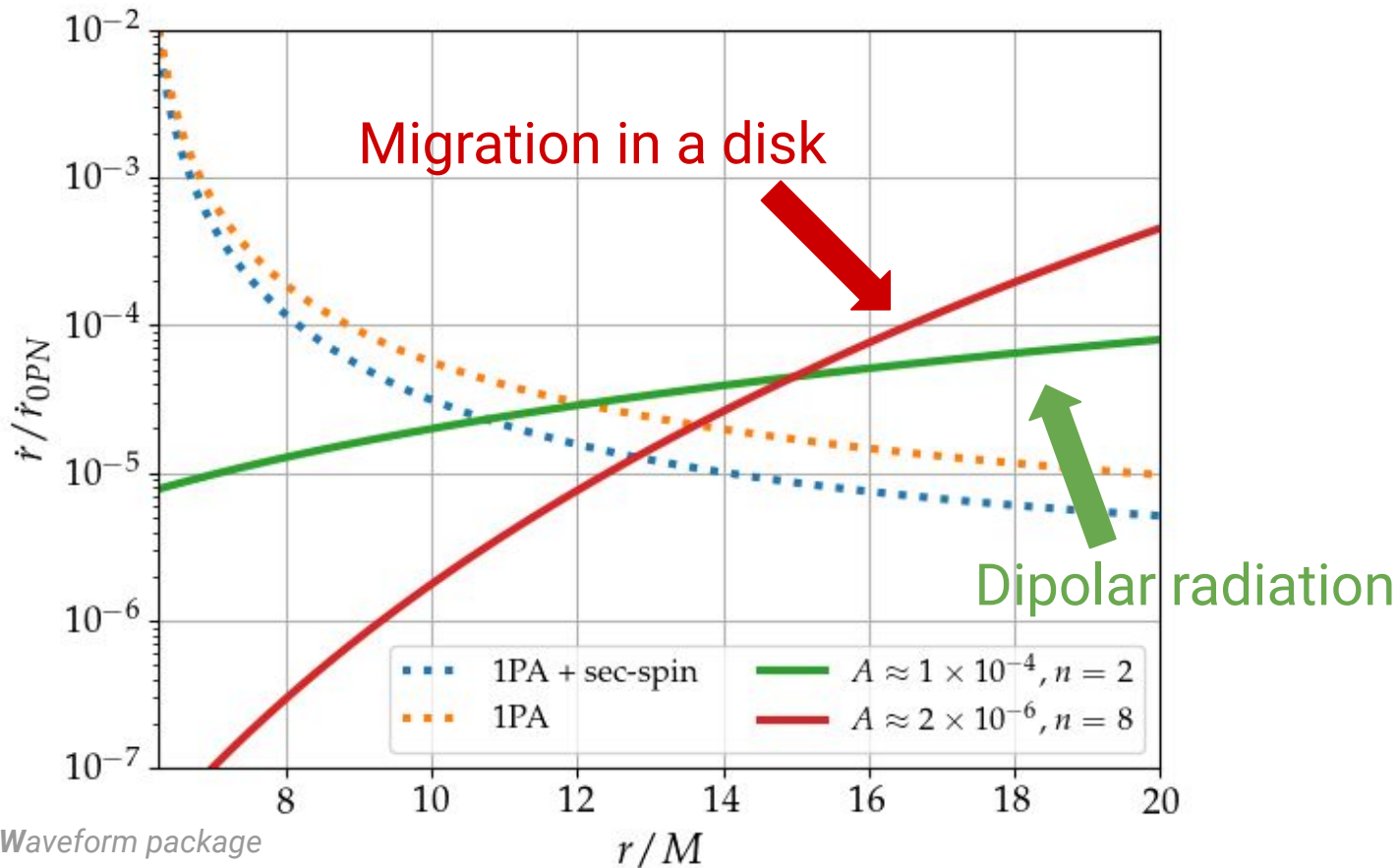
Image Credit: Beatriz Oliveira

3	SCIENCE OBJECTIVES	27
3.1	SO1: Study the formation and evolution of compact binary stars and the structure of the Milky Way Galaxy	28
3.1.1	Formation and evolution pathways of dark compact binary stars in the Milky Way and in neighbouring galaxies	29
3.1.2	The Milky Way mass distribution	31
3.1.3	The interplay between gravitational waves and tidal dissipation	32
3.2	SO2: Trace the origins, growth and merger histories of massive Black Holes	34
3.2.1	Discover seed Black Holes at cosmic reionisation	36
3.2.2	Study the growth mechanism and merger history of massive Black Holes from the epoch of the earliest quasars	38
3.2.3	Identify the electromagnetic counterparts of massive Black Hole binary coalescences	40
3.3	SO3: Probe the properties and immediate environments of Black Holes in the local Universe using EMRIs and IMRIs	42
3.3.1	Study the properties and immediate environment of Milky Way-like MBHs using EMRIs	45

EMRIs are a novel probe of stellar populations in the close vicinity of MBHs. A typical EMRI observation will provide a measurement of the sBH mass to $\sim 1\%$ and the eccentricity and inclination of the sBH orbit to 10^{-5} and 10^{-4} respectively. The emerging picture is that environmental effects will be detectable in a variety of realistic astrophysical scenarios. Even a single successful measurement would provide invaluable information on the presence of matter in the form of stars, gas or dark matter, only a few Schwarzschild radii from the MBH horizon.

$$\dot{L} = \dot{L}_{\text{GW}} + \dot{L}_{\text{BGR}}$$

$$\dot{L}_{\text{BGR}}/\dot{L}_{\text{GW}}^{(0)} = A \left(\frac{r}{10M} \right)^n$$



Uses the *FastEMRIWaveform* package

Beyond-vacuum GR effects compete with 2nd-order SF

Tackle the problem with **BH perturbation theory**

Duque et al., arXiv:2312.06767 (accepted @ PRL) + Brito & Shah, Phys. Rev. D 108, 084019 (2023)

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi} - \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi^* - \frac{1}{2} V(\Phi^* \Phi) + \mathcal{L}^m \right)$$

U(1) symmetry

$$J_Q^\mu = 2g^{\mu\nu} \text{Im} [\Phi^* \partial_\nu \Phi]$$

$$Q = \int d^3x \sqrt{-g} J_Q^t$$

Einstein-Klein-Gordon

$$G_{\mu\nu} = T_{\mu\nu}^\Phi + T_{\mu\nu}^m$$

$$\square_g \Phi = \frac{\partial V}{\partial \Phi^*} \approx \mu^2 \Phi$$

\mathcal{L}^m models the point-particle  perturbative scheme

$$g_{\mu\nu}^{\text{exact}} = g_{\mu\nu} + \epsilon h_{\mu\nu} + \mathcal{O}(\epsilon^2)$$

$$\Phi^{\text{exact}} = \Phi + \epsilon \delta\Phi + \mathcal{O}(\epsilon^2)$$

Susanna's talk

vs

Now

$$G_{\mu\nu} = T_{\mu\nu}^m$$

$$\square_g \Phi = \alpha T_{\mu\nu}^m$$

$$G_{\mu\nu} = T_{\mu\nu}^\Phi + T_{\mu\nu}^m$$

$$\square_g \Phi = \frac{\partial V}{\partial \Phi^*} \approx \mu^2 \Phi$$

Vacuum GR background

No vacuum background

No scalar background

No direct coupling of particle
to scalar

$$\Phi^{\text{exact}} = \text{✗} + \epsilon \delta \Phi + \mathcal{O}(\epsilon^2)$$

Exploit spherical symmetry to expand perturbations in spherical harmonics

Group in **polar vs axial** depending on how they transform under parity transformations

Massage perturbation equations to get (in some *gauge*) a coupled set of evolution equations for metric + scalar

Pick the bosonic environment + particle motion

Solve E.O.M. ("**hard**") and obtain GW/Scalar flux + waveform

Motion of the point particle

$\mathcal{O}(\epsilon^0)$: geodesics on background spacetime

Two-timescale expansion *Hinderer & Flanagan, PRD 78.064028 (2008)*

Orbital Timescale

$$T_o \sim M$$

\ll

Inspiral Timescale

$$T_i \sim M^2/m_p$$

Orbital energy evolves much slower than the orbital phase

$\mathcal{O}(\epsilon^1)$: adiabatic flow over a succession of geodesics

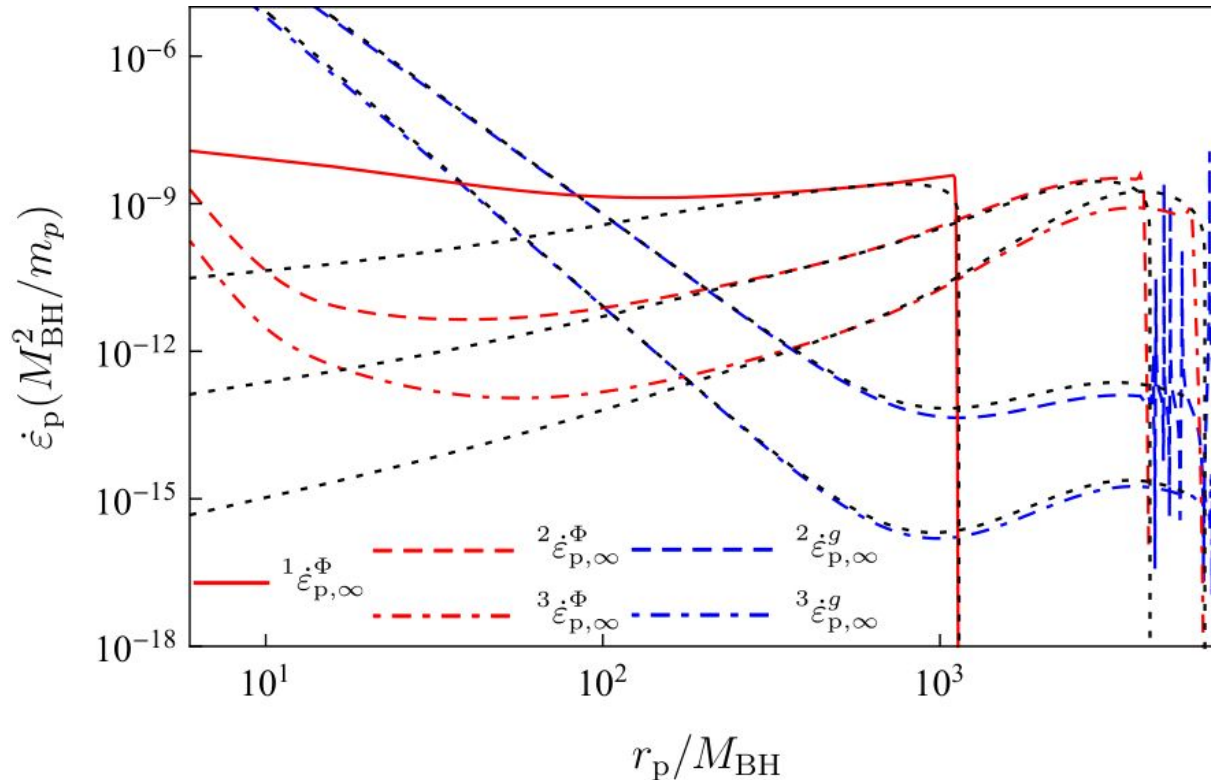
$$\frac{dE_p}{dt} = -\dot{E}_\infty^g - \dot{E}_H^g - (\dot{E}_\infty^\Phi - \omega\dot{Q}_\infty) - (\dot{E}_H^\Phi - \omega\dot{Q}_H)$$

Boson stars: circular orbits

Duque et al., arXiv:2312.06767 (accepted @ PRL) + Annulli, Vicente & Cardoso PRD 102, 063022 (2020)

Self-gravity of the scalar configuration is **not** negligible

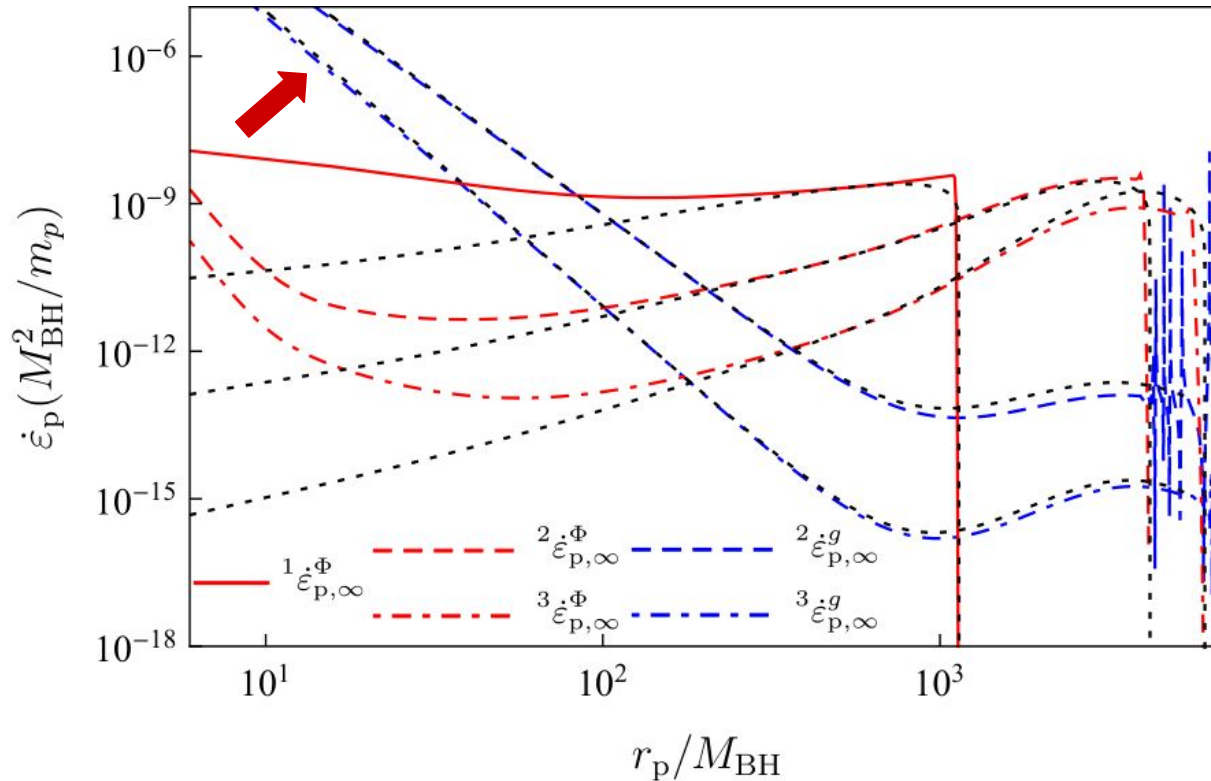
Must solve **coupled** scalar + gravity sector **@ same time**



Boson stars: circular orbits

Duque et al., arXiv:2312.06767 (accepted @ PRL)

GWs are shifted w.r.t. to vacuum prediction

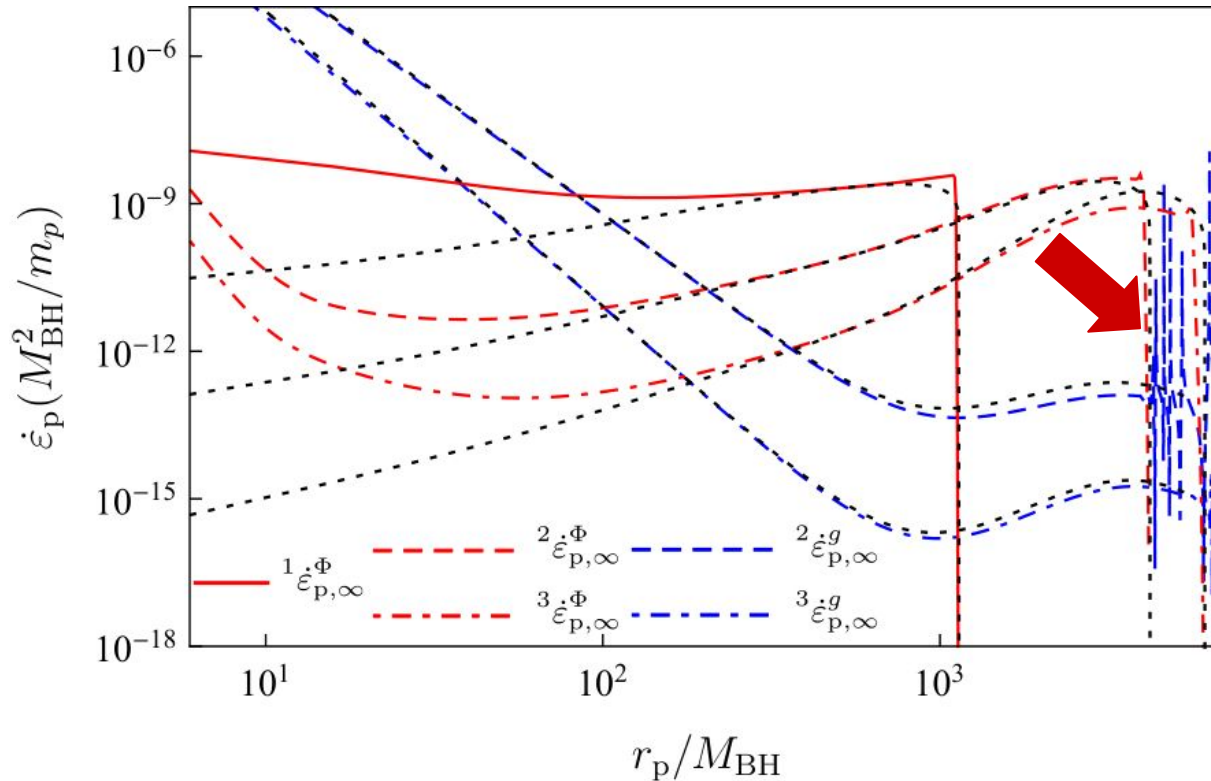


Boson stars: circular orbits

Duque et al., arXiv:2312.06767 (accepted @ PRL)

GWs are shifted w.r.t. to vacuum prediction

Resonances at low frequencies in the gravitational sector

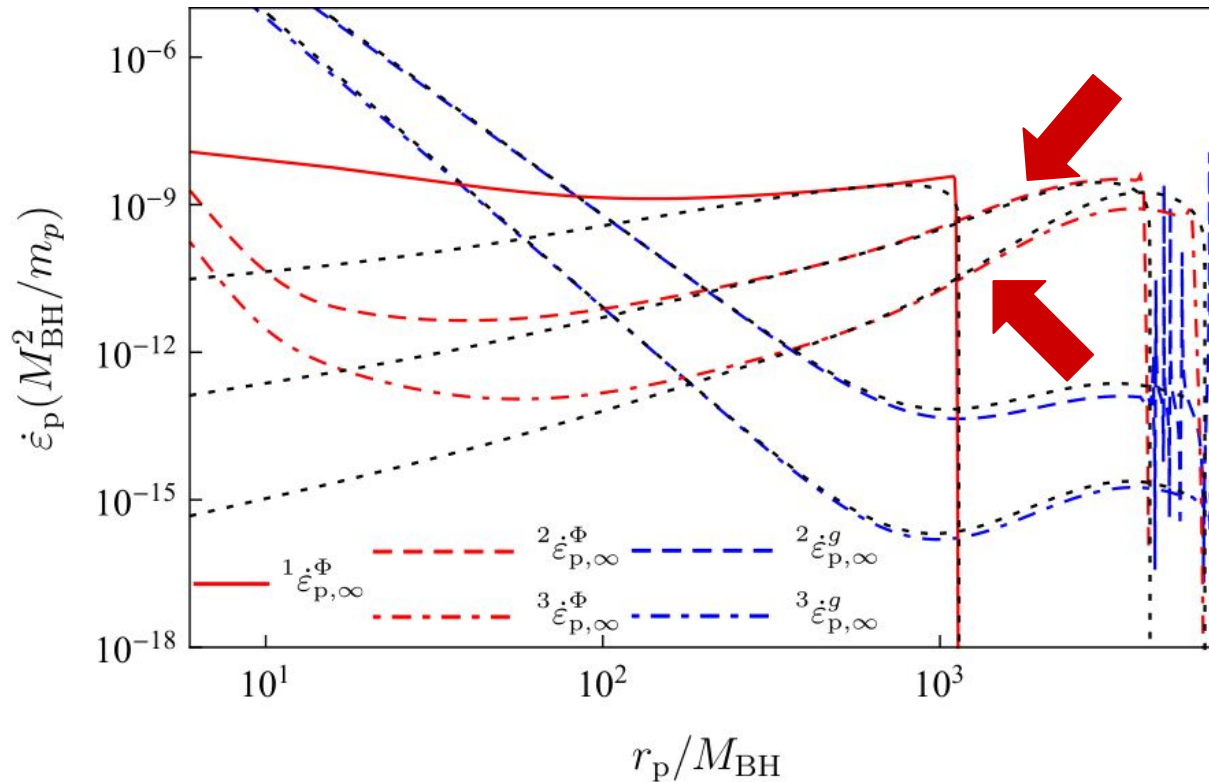


Boson stars: circular orbits

Duque et al., arXiv:2312.06767 (accepted @ PRL)

Scalar flux agrees with Newtonian, analytic predictions @ $\Omega_p \ll \mu$

Annuli, Vicente & Cardoso PRD 102, 063022 (2020)

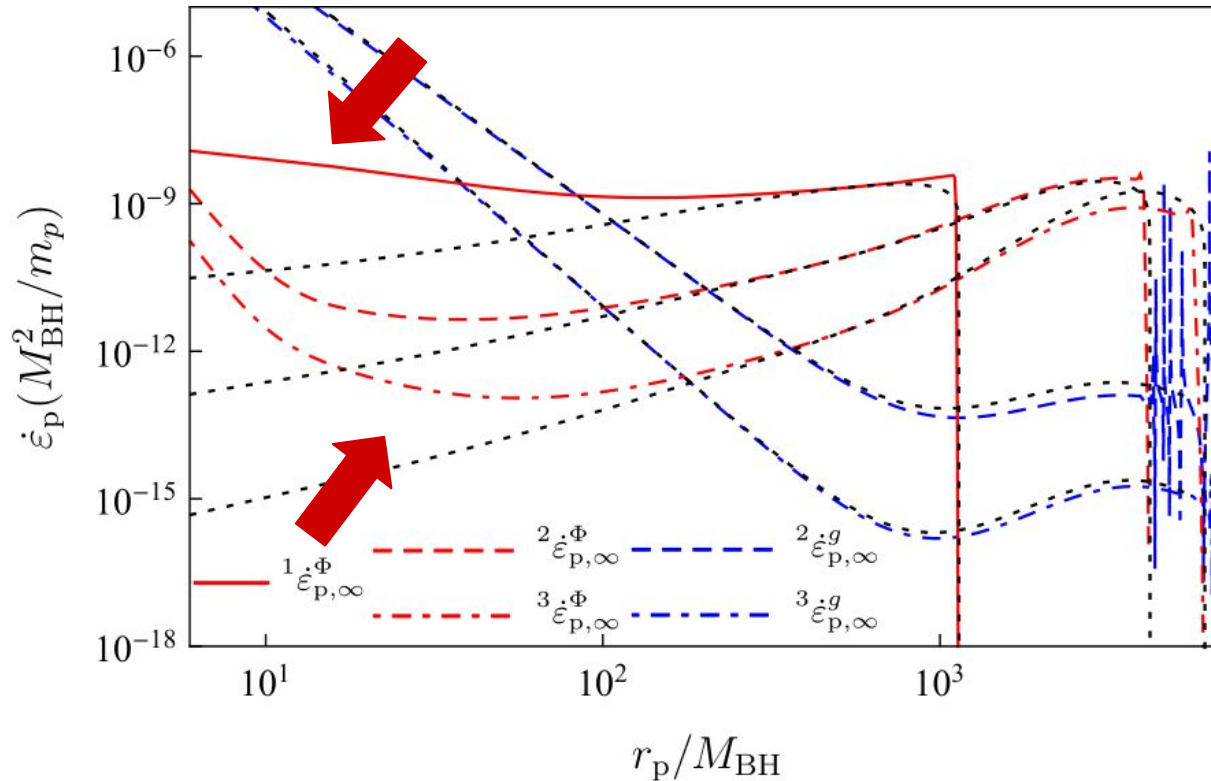


Boson stars: circular orbits

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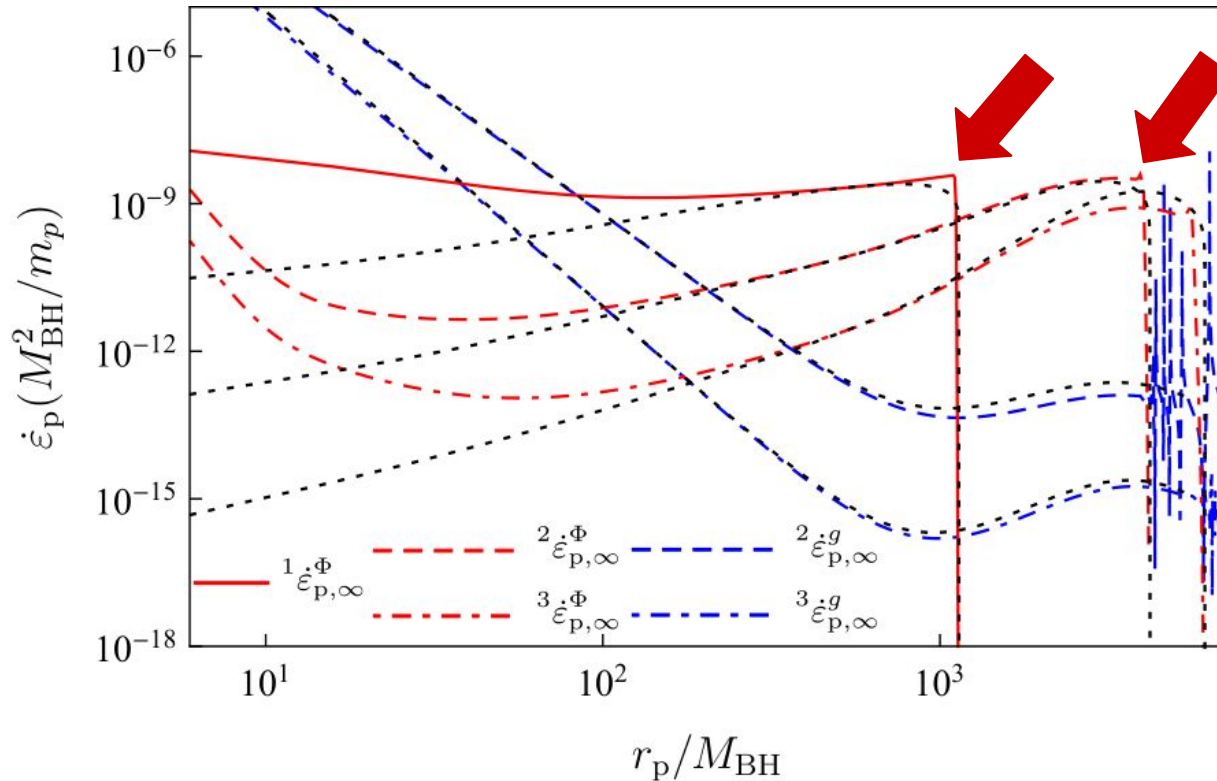
But **fails completely** @ $\Omega_p \gg \mu$



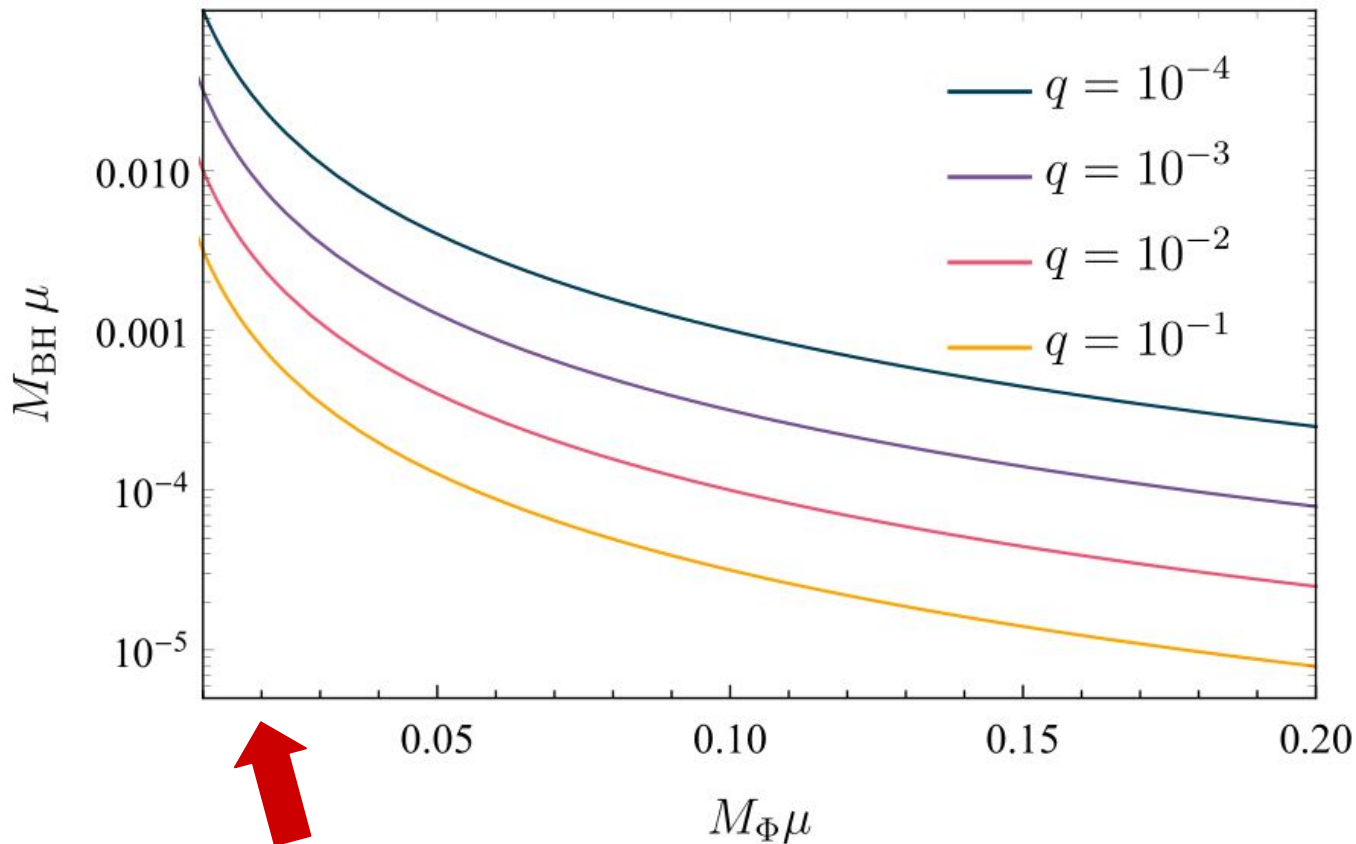
Boson stars: circular orbits

Duque et al., arXiv:2312.06767 (accepted @ PRL)

Ionization: scalar emission only activated for $m\Omega_p \geq \mu - \omega$



Dephasing with 4 years of observation close to merger



Astrophysical Halos

Bosonic Clouds

Test-Field: $G_{\mu\nu} = T_{\mu\nu}^{\Phi} + T_{\mu\nu}^{\text{m}} \quad \square_g \Phi = \mu^2 \Phi$

"Vacuum" background + cloud w/ hydrogen atom structure

$\alpha = M\mu$ is the "gravitational fine-structure" constant

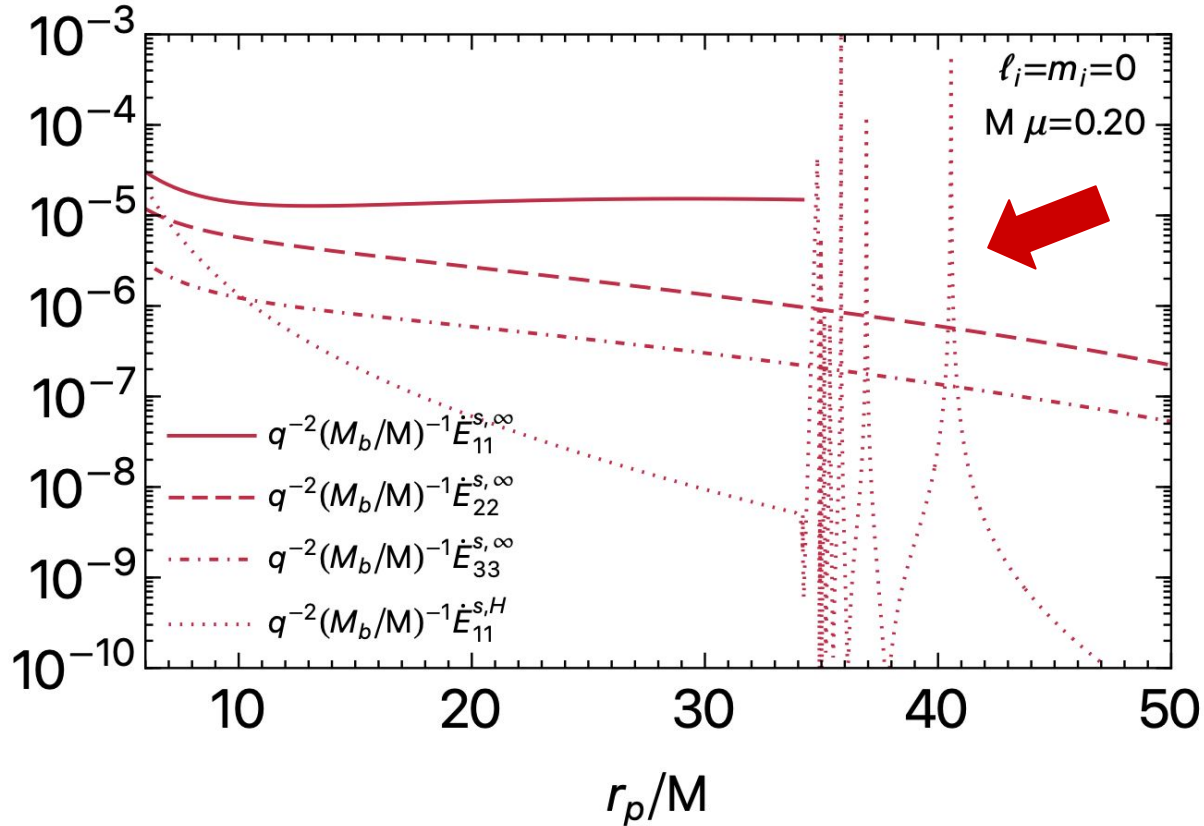
For $M\mu \ll 1$, solutions in Laguerre polynomials

$$\Phi(r) \approx \sqrt{\frac{M_{\Phi}}{\pi M_{\text{BH}}}} (M_{\text{BH}}\mu)^2 \left(1 - \frac{2M_{\text{BH}}}{r}\right)^{-2i\mu M_{\text{BH}}} e^{-M_{\text{BH}}\mu^2 r}$$

$$\omega = \mu \left[1 - \frac{1}{2} (M\mu)^2\right]$$

Bosonic Clouds: circular orbits

Resonances at low frequencies in the scalar flux @ BH horizon



The Gravitational Atom

Baumann et al., JCAP 12.006 (2019) + Tomaselli et al., arXiv:2403.03147 (2024)

Emission only for $m\Omega_p > \mu - \omega$ **Photoelectric effect**

For our system $\Omega_p = \sqrt{M/r_p^3} \approx 4 \times 10^{-3}$ **vs** $\mu - \omega = 4 \times 10^{-3}$

Cloud is in the fundamental $\ell = m = 0$ state

But there are overtones/states w/ different ℓ m

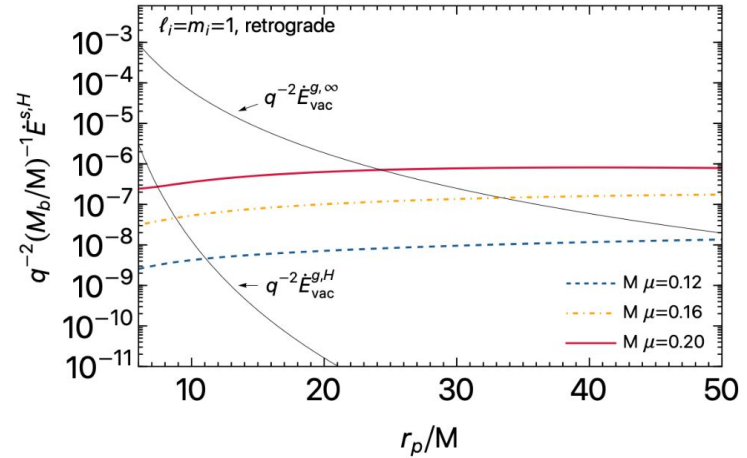
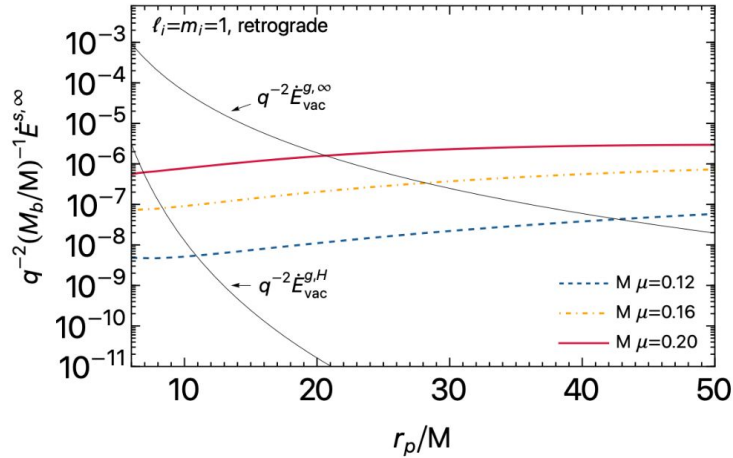
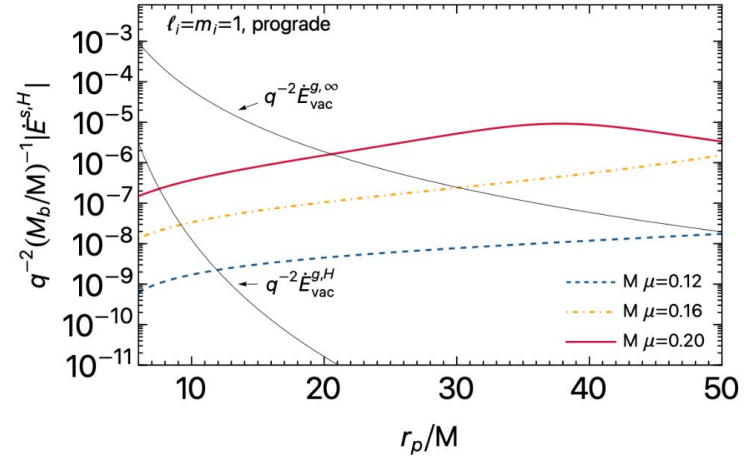
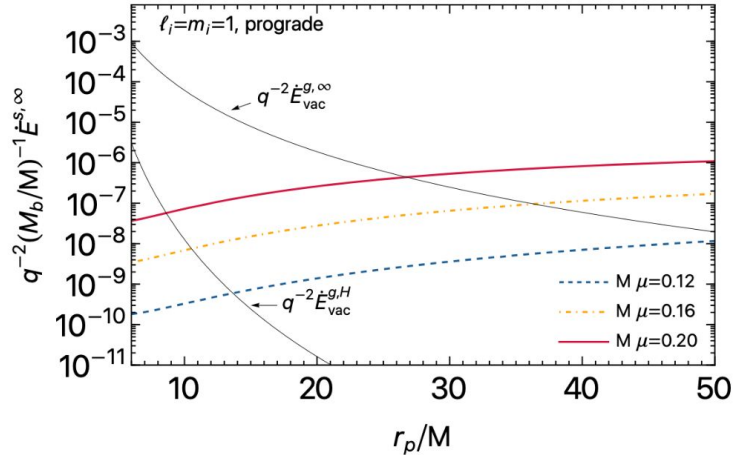
$$\omega_{n\ell m} = \mu \left(1 - \frac{\alpha^2}{2n^2} - \frac{\alpha^4}{8n^4} - \frac{(3n - 2\ell_i - 1)\alpha^4}{n^4(\ell_i + 1/2)} + \frac{2(a/M)m_i\alpha^5}{n^3\ell_i(\ell_i + 1)(\ell_i + 1/2)} + \mathcal{O}(\alpha^6) \right)$$

If $m\Omega_p = \Delta\omega$ binary induces resonant transitions

Non-Spherical Clouds

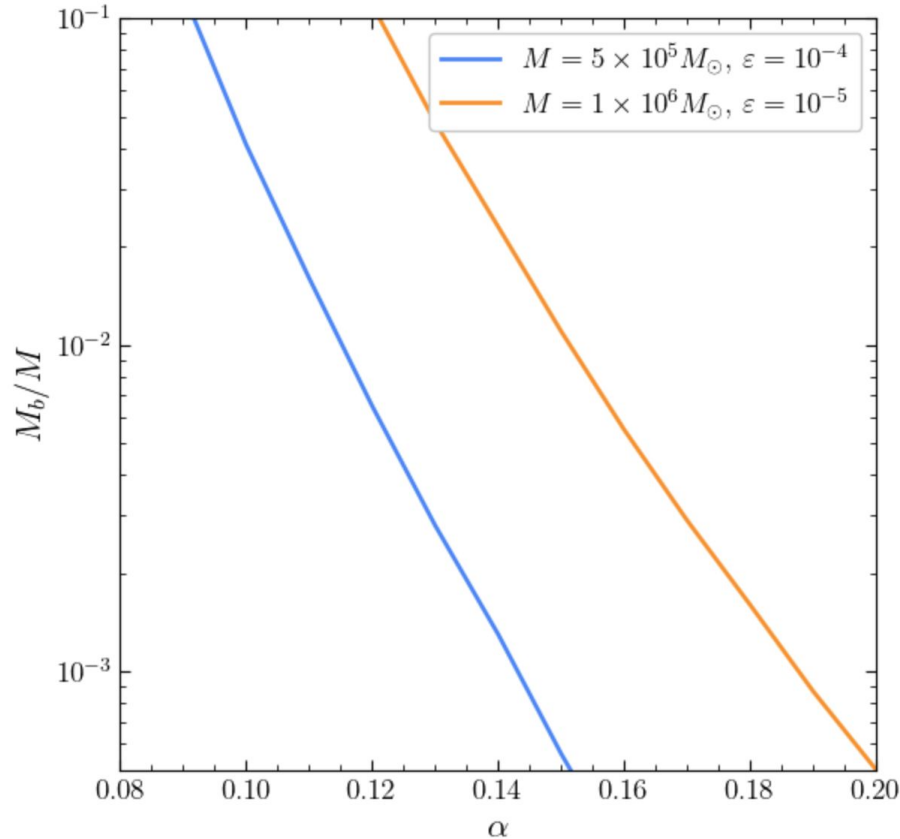
Brito & Shah, Phys. Rev. D 108, 084019 (2023)

$$M_b = M_\Phi$$



Dephasing with 4 years of observation before merger

$$M_b = M_\Phi$$



Khalvati + Santini et al., in prep

$M_\Phi/M \lesssim 0.1$ **→** $M_\mu \gg 0.1$ could be probed

For the details...



Day 2 (school), Tuesday, July 2nd

Chair: Valentin Boyanov

09:00-10:00 Eugen Radu: **BH uniqueness and dirty BHs**

10:00-10:30 Coffee break

10:30-11:30 Maria Alessandra Papa: **GWs from monochromatic sources: data analysis_Lec1**

11:30-12:30 Francisco Duque: **EMRIs or evolution of binaries in fundamental fields** [lecture note_EMRIEMRI_BosonCloud](#)



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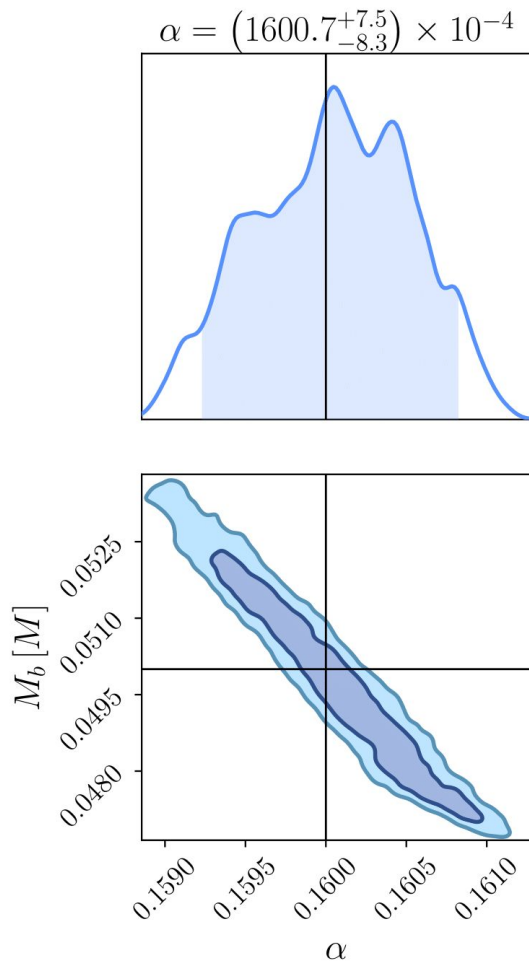
Here you can find routines that can be used to compute the flux in gravitational waves emitted by a scalar cloud around a Kerr black hole. Some computations make use of the Black Hole Perturbation toolkit (<https://bhptoolkit.org/>). We also use results from [arXiv:gr-qc/0306120](https://arxiv.org/abs/gr-qc/0306120), [arXiv:0705.2880](https://arxiv.org/abs/0705.2880) and [arXiv:1312.2326](https://arxiv.org/abs/1312.2326). The data generated by this routine served as input for the python package [gwaxion](https://pypi.org/project/gwaxion/) (<https://pypi.org/project/gwaxion/>).

Gravitational waves from boson clouds

Description	References	Download
Computation of gravitational-wave flux from a scalar cloud	Brito et al. [arXiv:1706.06311] and [arXiv:1706.05097]	Notebook
Scalar and gravitational fluxes from EMRIs in scalar environments	Duque et al. [arXiv:2312.06767]	File

P.E. w/ FastEMRIWaveforms

Katz et al., PRD 104.064047 (2021)



$$M \sim 4 \times 10^5 M_\odot$$

$$m_p = 20 M_\odot$$

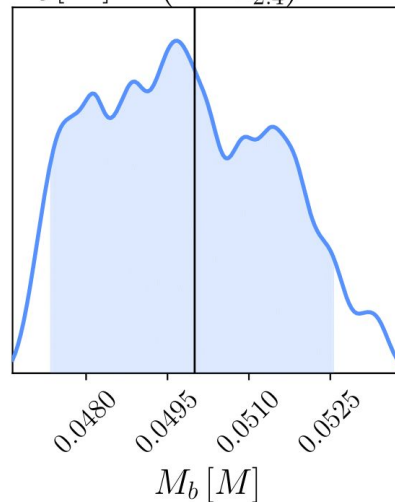
$$a/M = 0.6$$

$$p_0 = 19.9 M$$

$$T_{\text{obs}} = 4 \text{ yrs}$$

$$\text{SNR} = 50$$

$$M_b [M] = (49.7^{+2.9}_{-2.4}) \times 10^{-3}$$



Khalvati + Santini et al., in prep
+

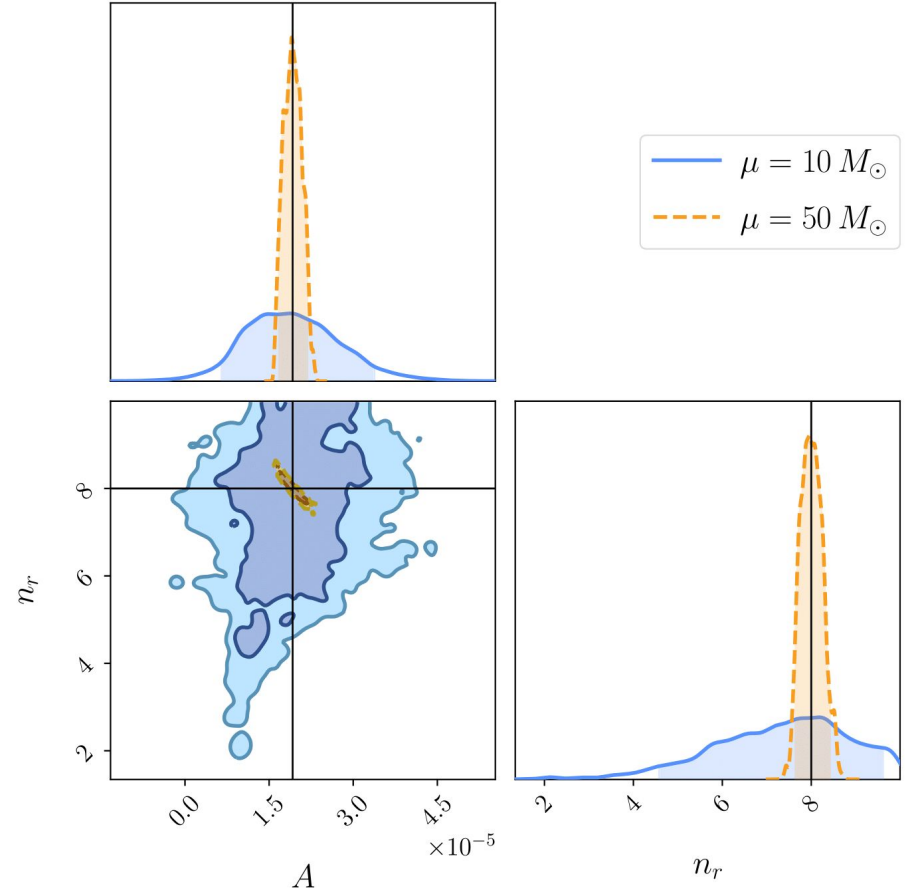
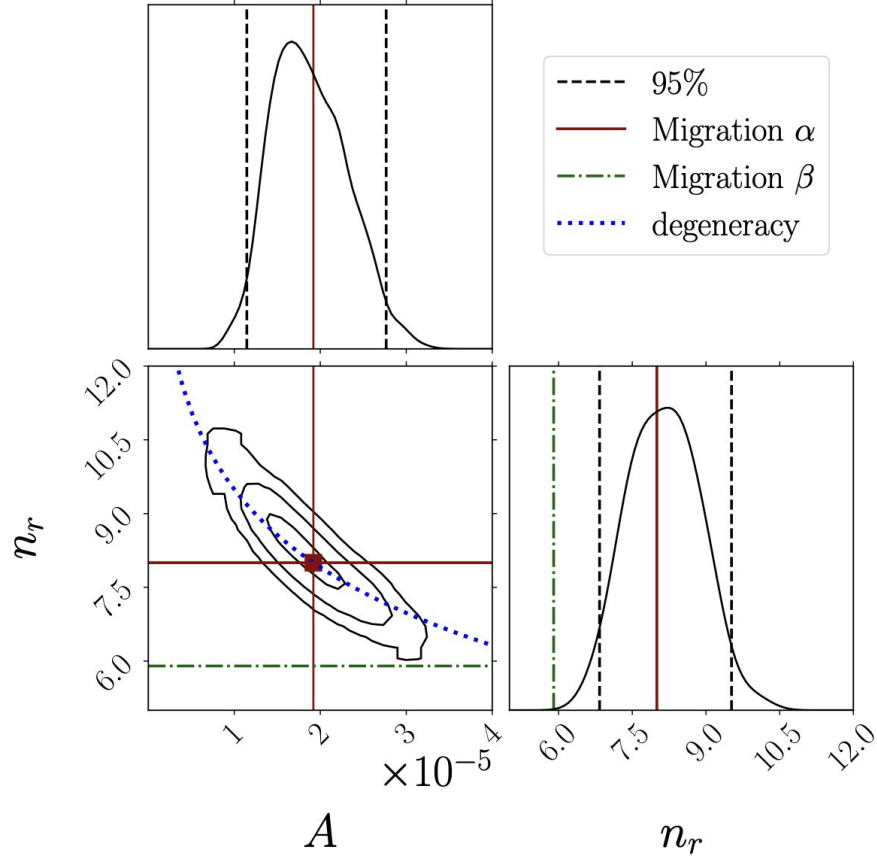
Cole et al. Nature Astron. 7
(2023) 8, 943-950

$$\dot{L}_{\text{BGR}}/\dot{L}_{\text{GW}}^{(0)} = A \left(\frac{r}{10M} \right)^{n_r}$$

AAK waveforms

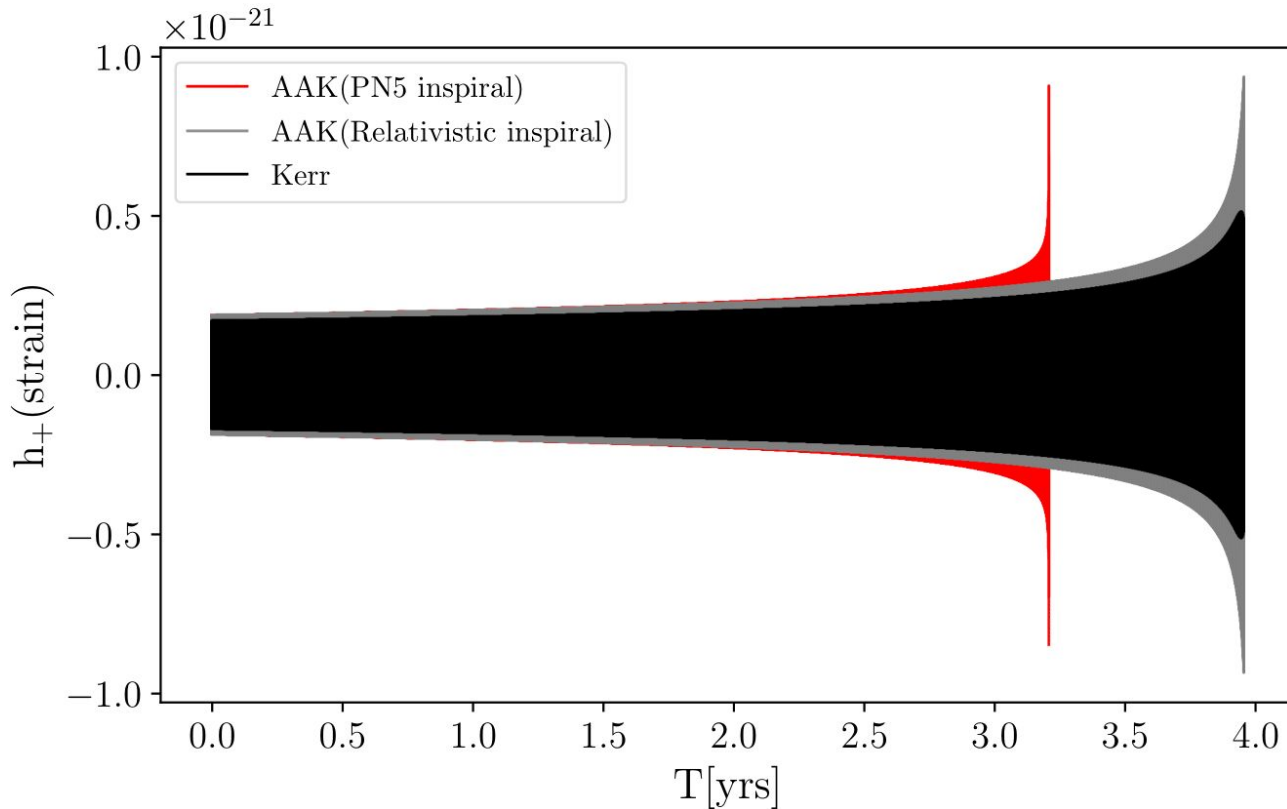
vs

Relativistic waveforms



Same EMRI system for $a/M = 0.99$

Khalvati + Santini et al., in prep



AAK underestimates relative power on early inspiral

Where environmental effects are **more important!**

Possible Extensions

1. Numerical Relativity *Katy Clough's talk tomorrow*
2. Eccentric/Inclined orbits *Tomaselli et al., arXiv:2403.03147 (2024)*
3. Vector fields
4. Spinning BHs *(someone is doing it...)*
5. Real fields

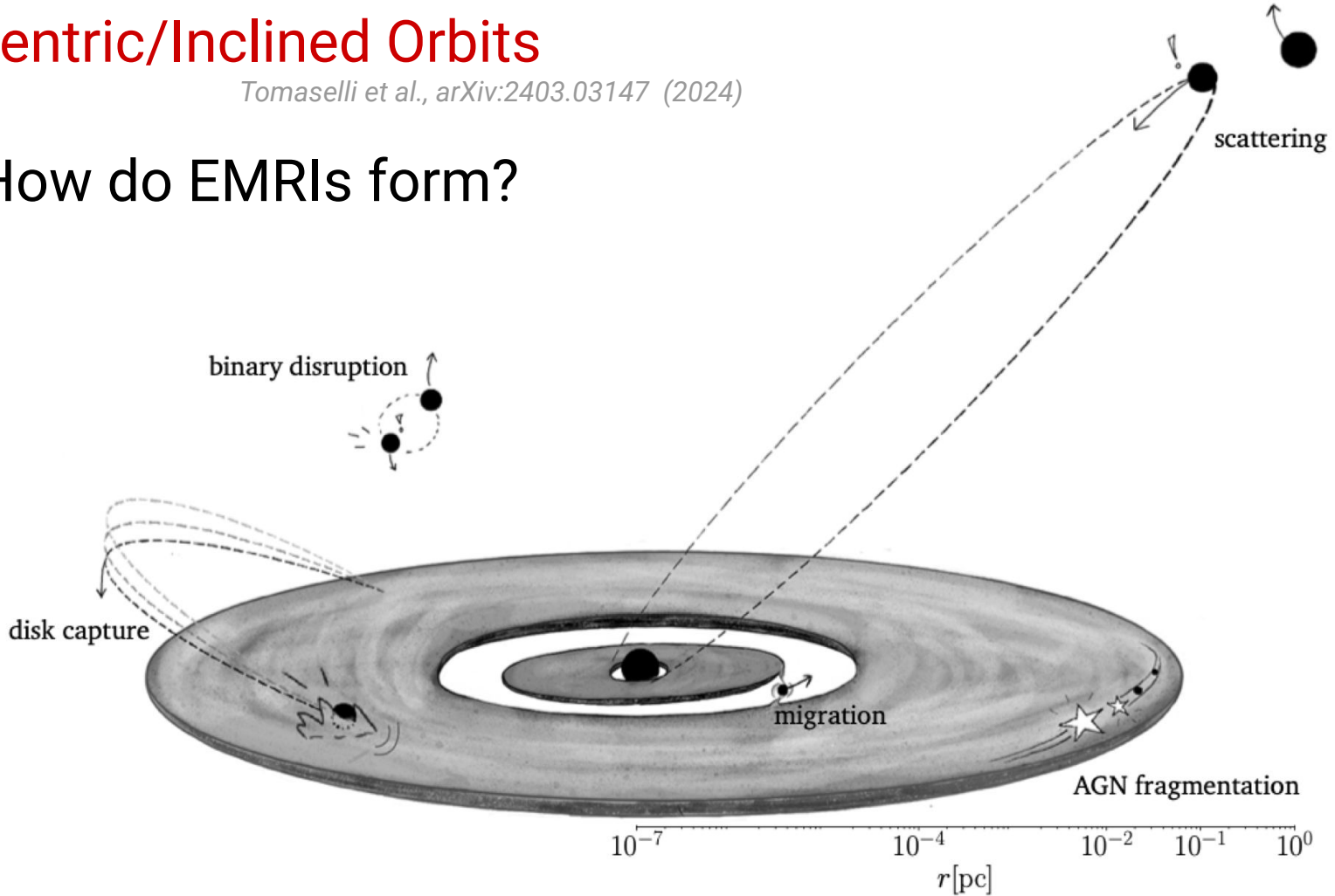
Possible Extensions

1. Numerical Relativity *Katy Clough's talk tomorrow*
2. *Eccentric/Inclined orbits* *Tomaselli et al., arXiv:2403.03147 (2024)*
3. Vector fields
4. Spinning BHs *(someone is doing it...)*
5. *Real fields*

Eccentric/Inclined Orbits

Tomaselli et al., arXiv:2403.03147 (2024)

Q: How do EMRIs form?

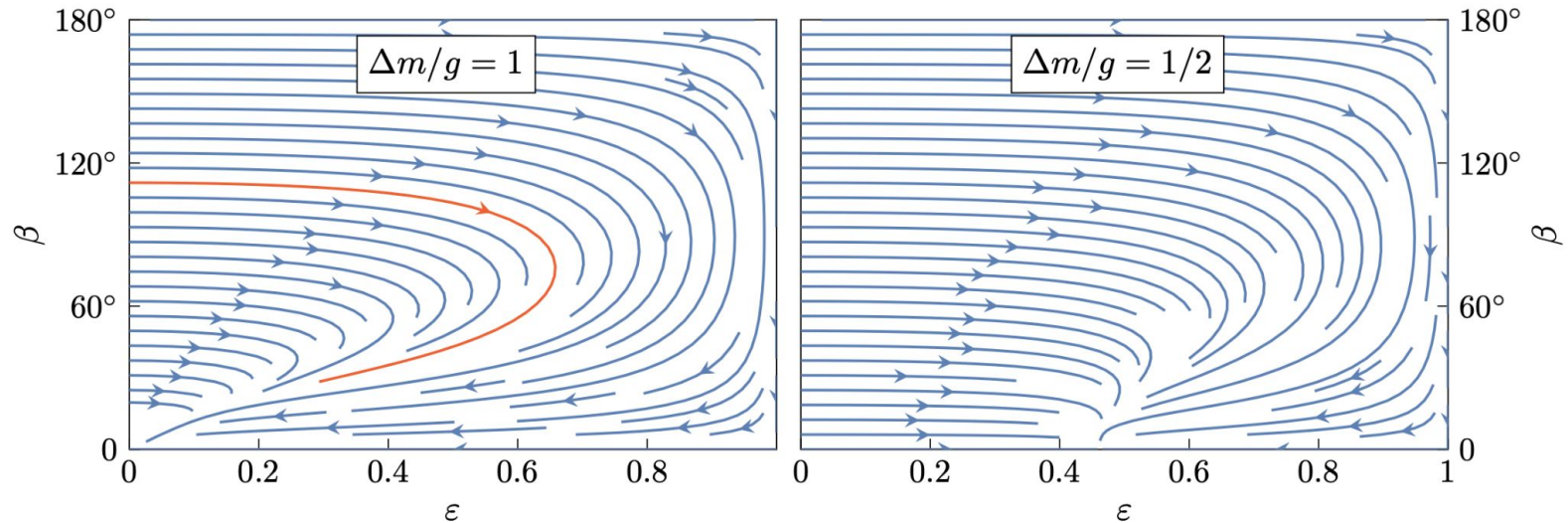


Eccentric/Inclined Orbits

Tomaselli et al., arXiv:2403.03147 (2024)

Q: Does the cloud survive when binary enters in band?

A: More chance for retrograde orbits

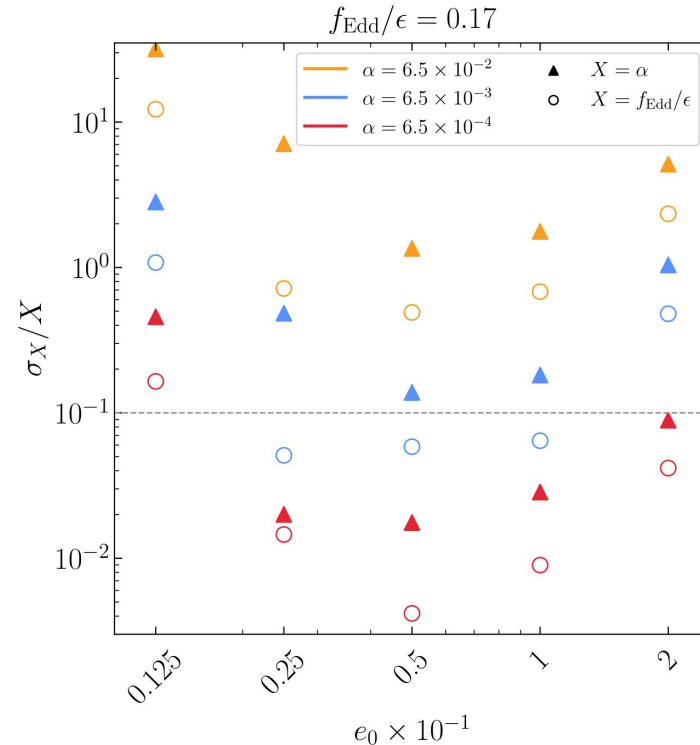


Resonances lead to **fixed points** in eccentricity and inclination

Fixed points also observed for Accretion Disks

Breaking degeneracies: more detailed/better models should help

Example: eccentric EMRI in accretion disk *Duque, Kejriwal et al., in prep*



$$M = 10^6 M_{\odot}$$

$$m_p = 50 M_{\odot}$$

$$p_0 = 16.83 M$$

$$\text{SNR} = 50$$

$$T_{\text{obs}} = 4 \text{ yrs}$$

Can constrain accretion rate **and** viscosity **simultaneously**

Not possible for circular motion w/ migration torques

Real Fields: $T_{\mu\nu}^{\Phi}$ is no longer time independent $\longrightarrow Q$

Cloud emits GWs \longrightarrow Different Energy-Balance law

Environment introduces new **timescales:** T_{relax} , $T_{\text{Env Reac}}$

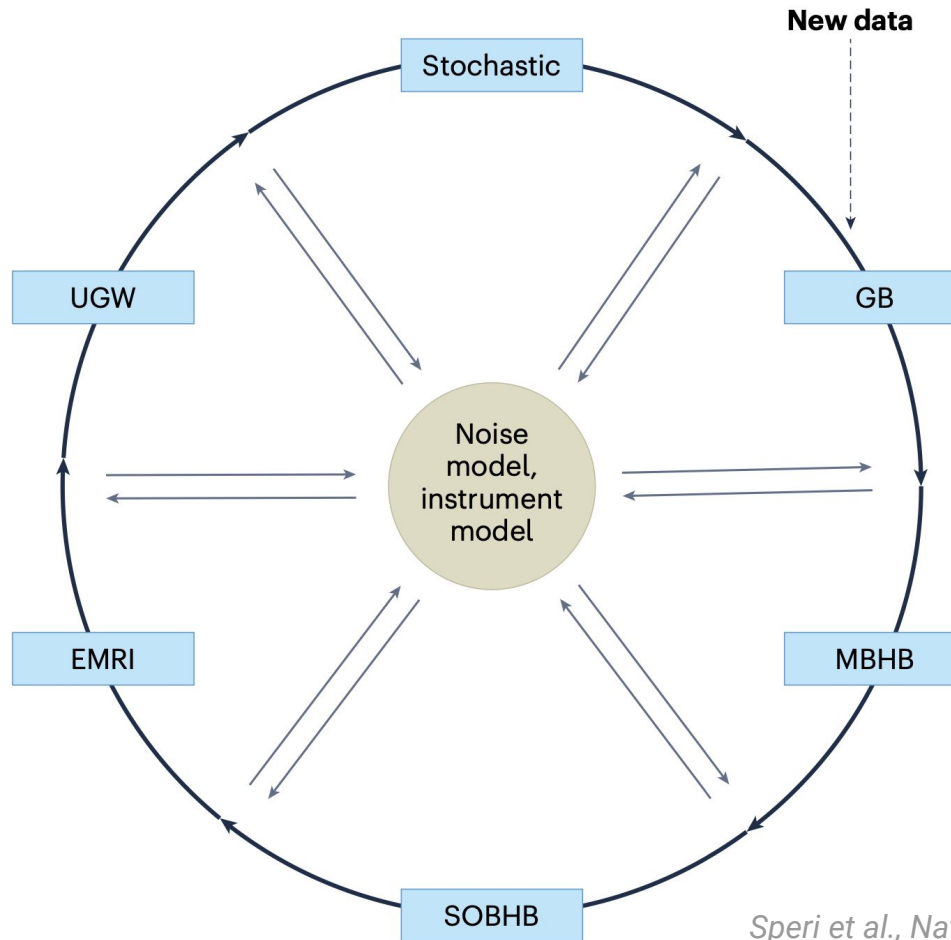
Q: How to integrate this in the (Post-)Adiabatic way?

Warburton: "Give us a force and we (SF comm.) know how to do it"

Environmental self-force \longleftrightarrow Multi-timescale expansion (?)

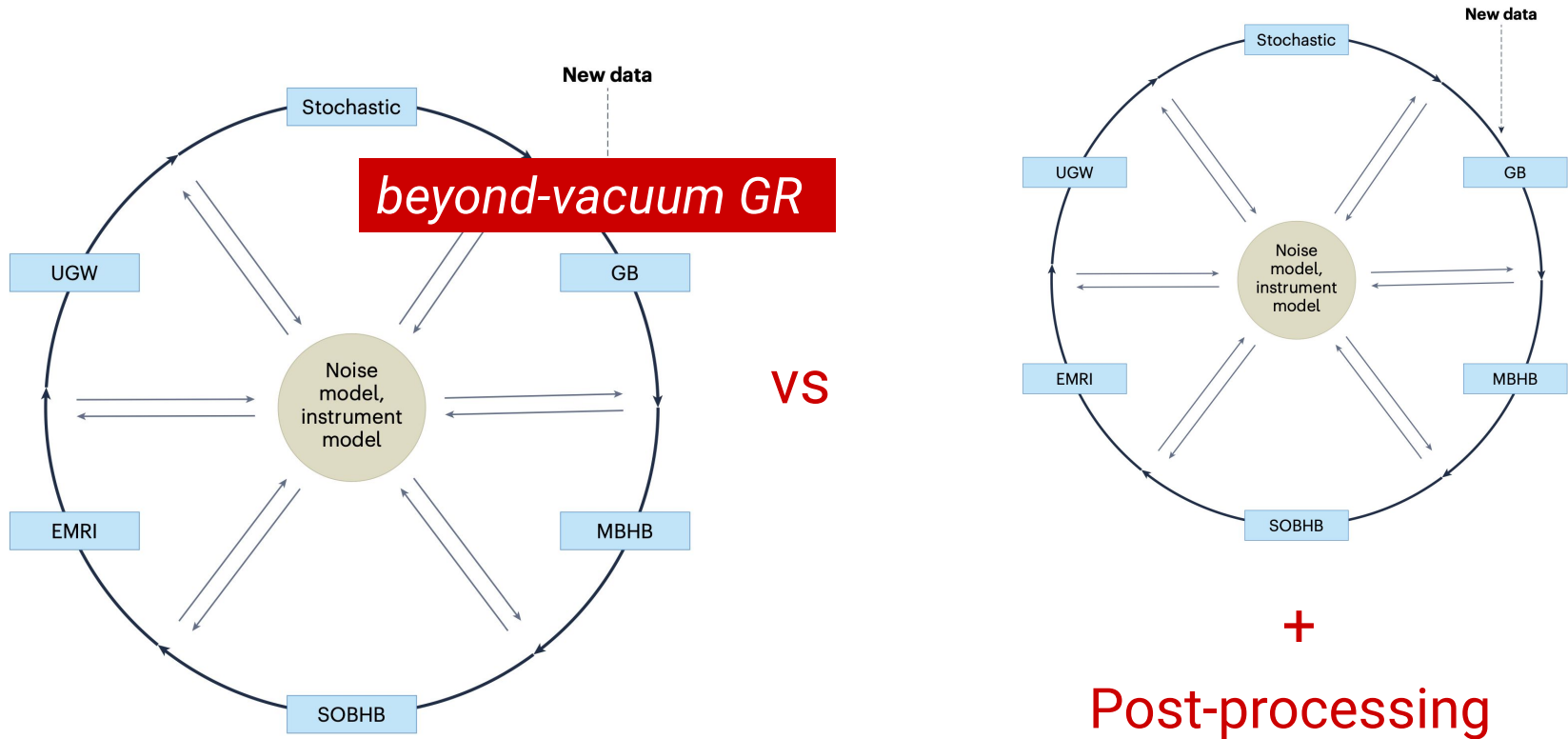
e.g.: $T_{\text{orb}} \ll T_{\text{relax}} \ll T_{\text{Rad Reac}} \ll T_{\text{Env Reac}}$

Global Fit: how to study **beyond-vacuum GR** physics w/ LISA?



Speri et al., Nature Astr. 6, p. 1356–1363 (2022)

Global Fit: how to study **beyond-vacuum GR** physics w/ LISA?



Take-home message

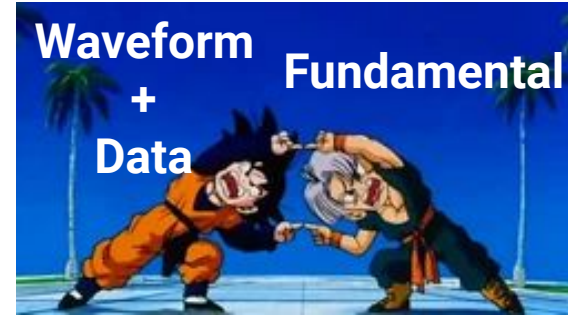
EMRIs in relativistic environments is **still** quite unexplored *but so far...*

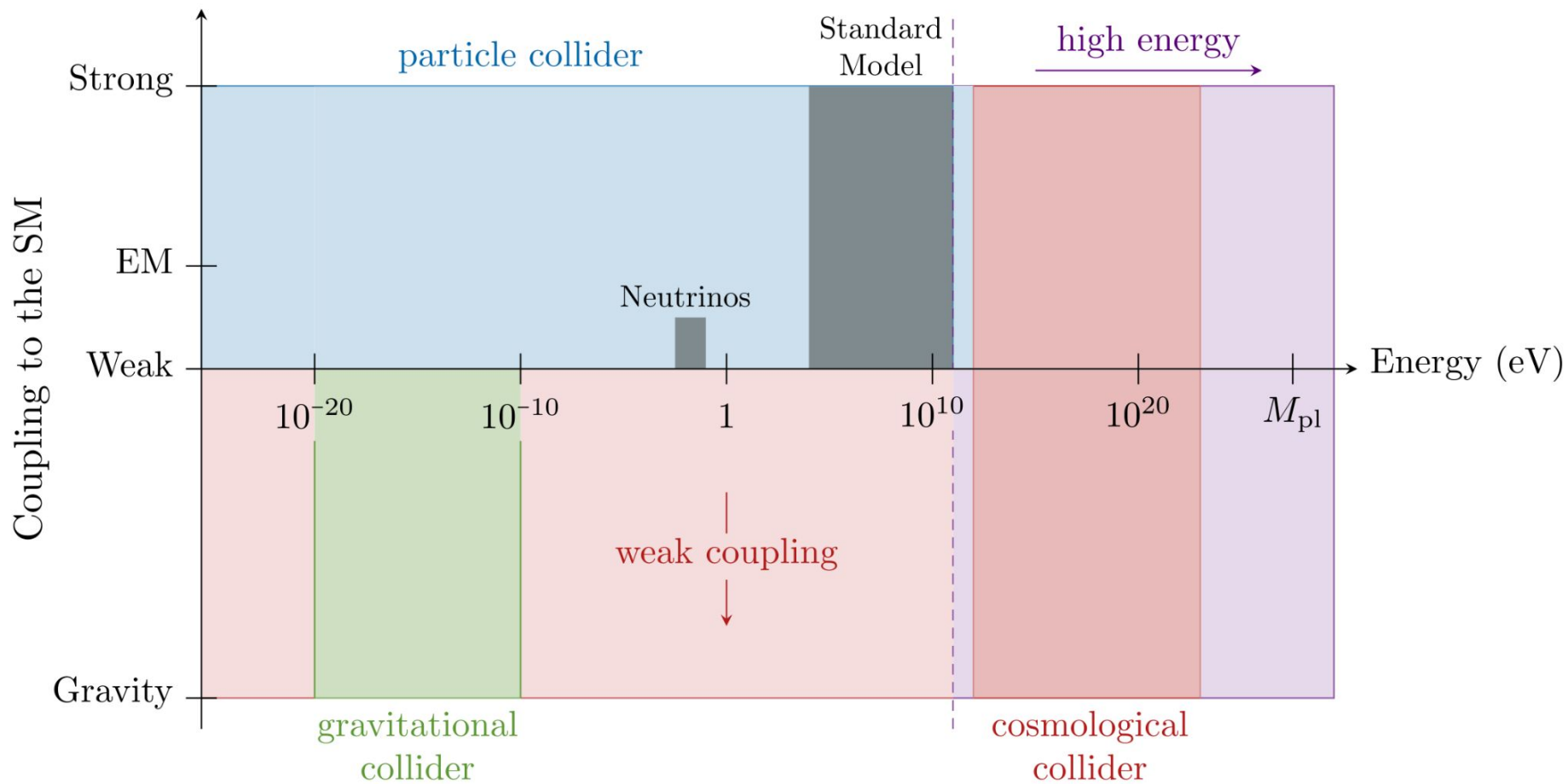
Matter does seem to matter

+ models  + phenomenology  + distinguishability (?)

But how to do it for LISA?

But how to do it for LISA?





Baumann et al. PRD 101, 083019 (2020)

Inner region α - disk: $\Sigma(r) = \Sigma_0 \left(\frac{r}{10M} \right)^{3/2}$ $h(r) = h_0 \left(\frac{10M}{r} \right)$

$$e < h$$

$$e > h$$

Subsonic

Supersonic

(Global) Migration Torques

(Local) Dynamical Friction

$$\langle \dot{a} \rangle \propto -(\Sigma_0/h_0^2) a^5$$

$$\langle \dot{a} \rangle \propto (\Sigma_0/h_0) a^4 / e$$

$$\langle \dot{e} \rangle \propto -(\Sigma_0/h_0^4) a^6 e$$

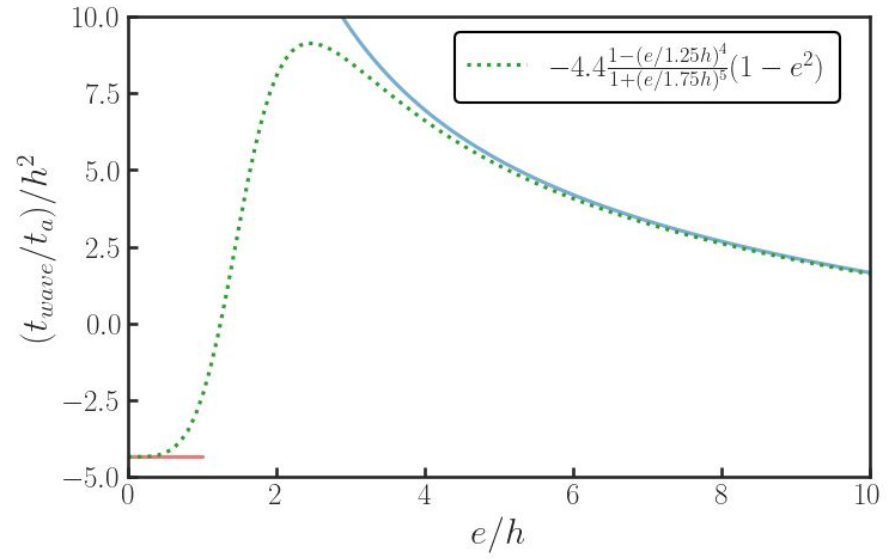
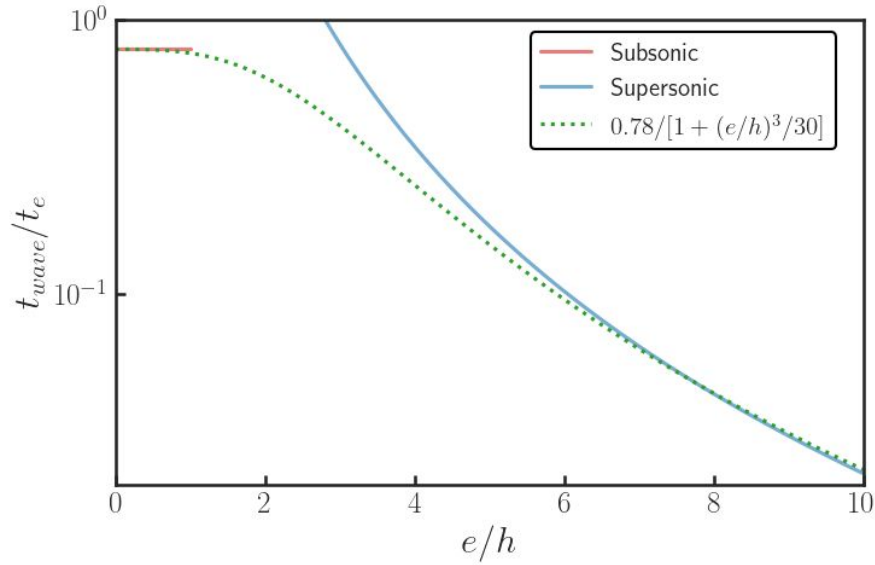
$$\langle \dot{e} \rangle \propto -(\Sigma_0/h_0) a^3 / e^{7/4}$$

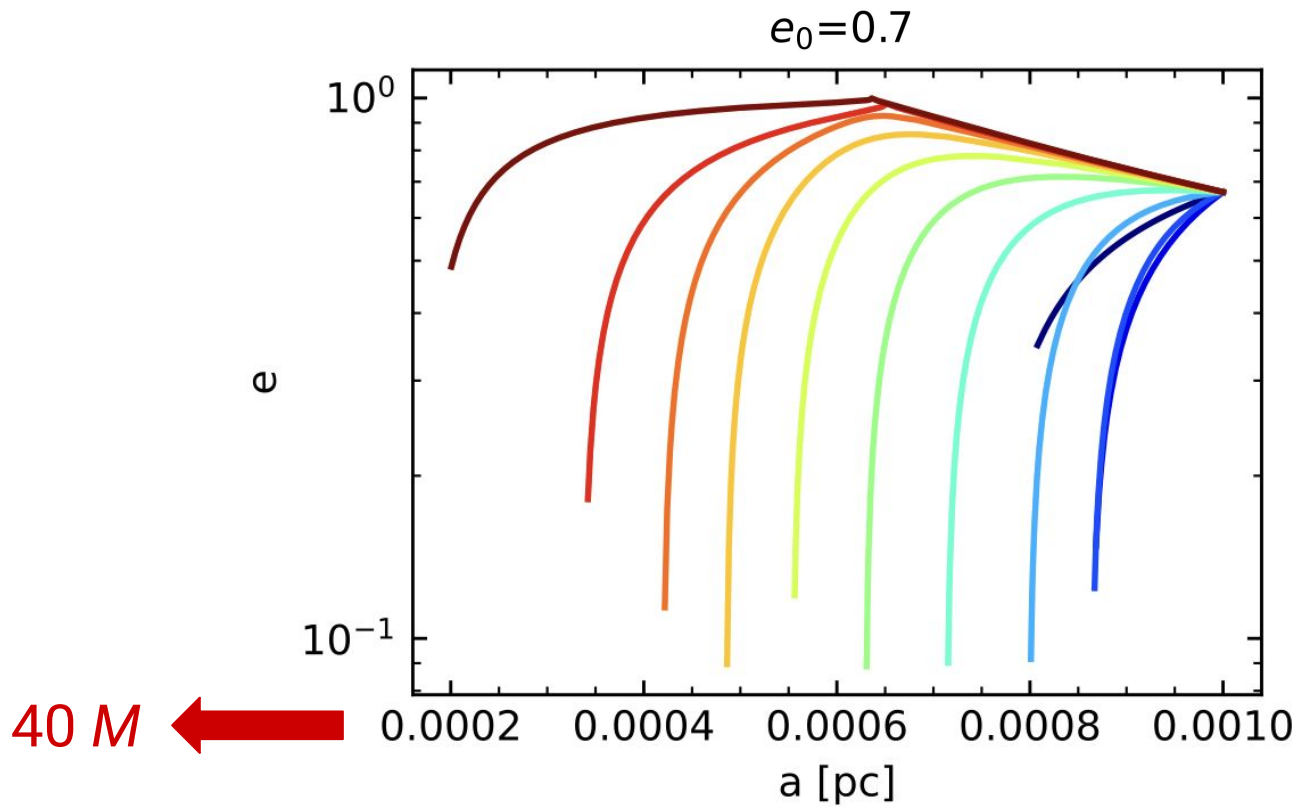
$$t_x = X / \langle \dot{X} \rangle \quad t_e / t_a \sim h^2 \ll 1$$

$$t_e / t_a \sim (e/h)^2 \gg 1$$

For supersonic motion, migration timescale \ll damping of eccentricity

$$t_{\text{wave}}^{-1} = \varepsilon \left(\frac{\Sigma r^2}{M} \right) \frac{\Omega_K}{h^4}$$





Since the eccentricity damping may not be as efficient as inclination damping, there might be some eccentricity residual on captured BHs