LISA Response Function

Overview and challenges in modeling the measurement chain

University of Glasgow

Overview

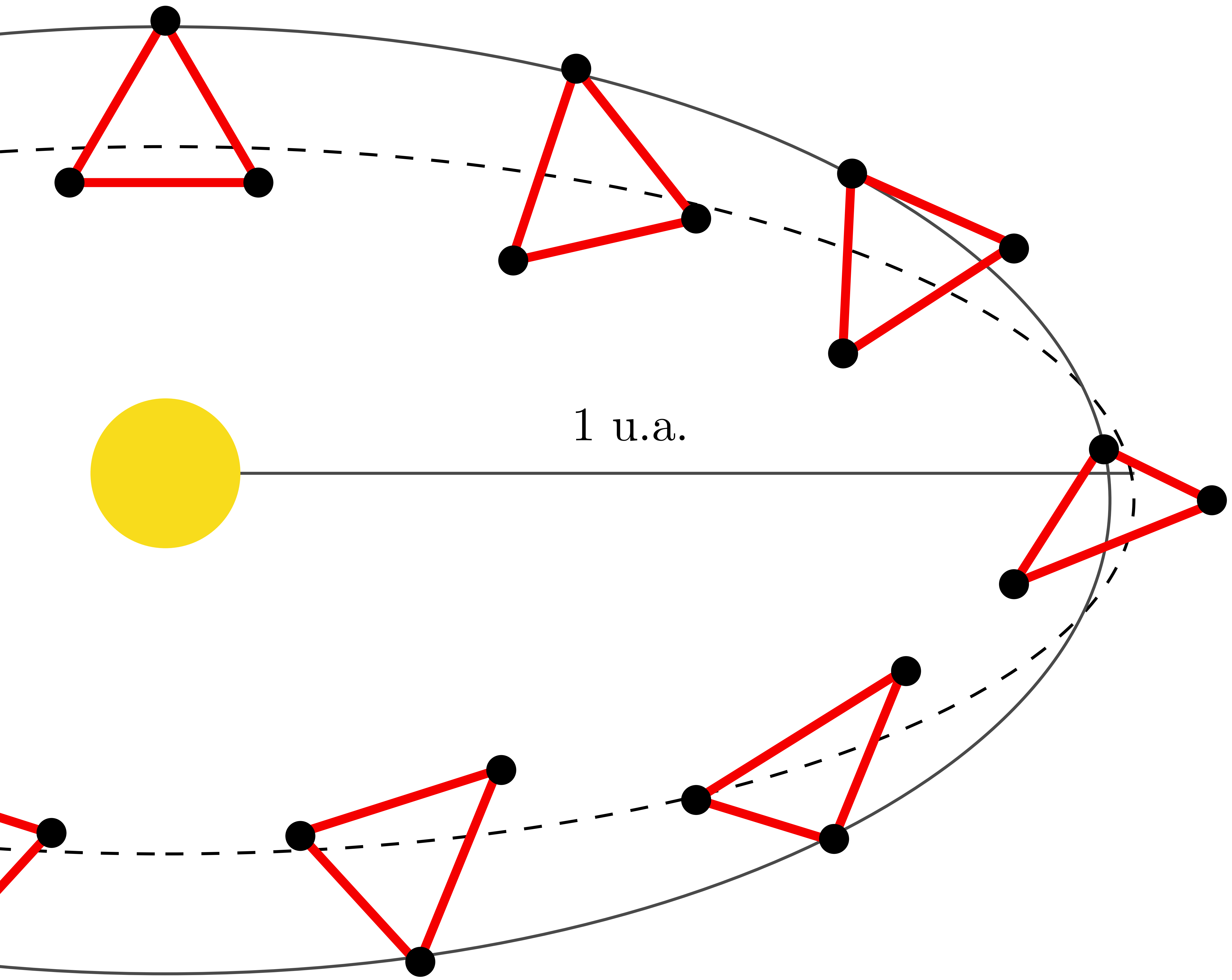
Measurement chain

Response function

Data processing

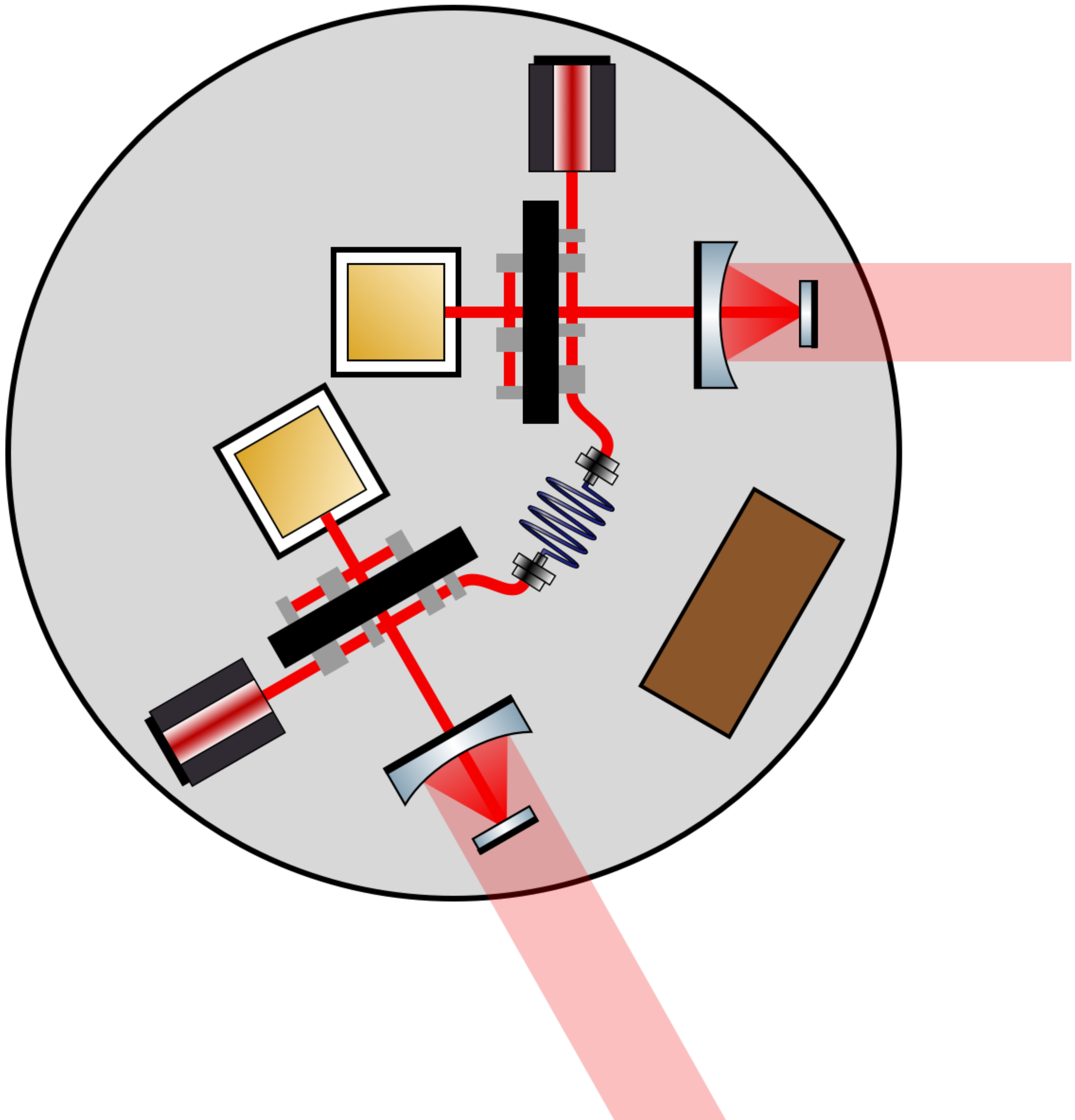
Challenges & takeaways

Measurement principles



- Monitor tidal forces (relative acceleration) between free-falling test masses using precision laser interferometry
- 3 pairs of test masses in equilateral triangular formation, cartwheeling in quasi-Keplerian heliocentric orbits (never far from Earth for communication)

Measurement principles



- Monitor tidal forces (relative acceleration) between free-falling test masses using precision laser interferometry
- 3 pairs of test masses in equilateral triangular formation, cartwheeling in quasi-Keplerian heliocentric orbits (never far from Earth for communication)
- Drag-free spacecraft (along sensitive axes) shield test masses from spurious, external forces

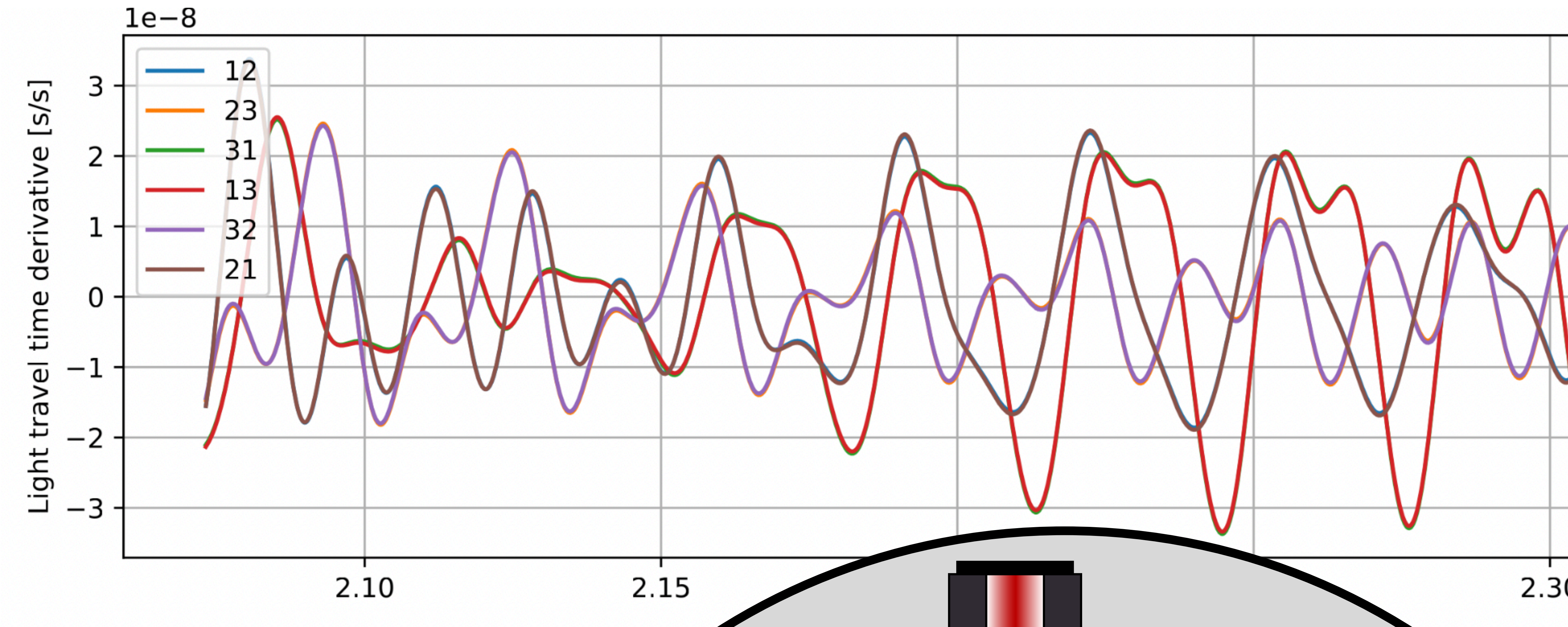
Interferometric measurements

- Interferometers compare the phases of a propagated laser beam (from distant laser) and a local laser beam:

$$\phi_{\mathrm{BN}} \propto \phi_{2\to1} - \phi_1 = \nu_2(t - L_{12}) + H_{2\to1} - \nu_1 t$$

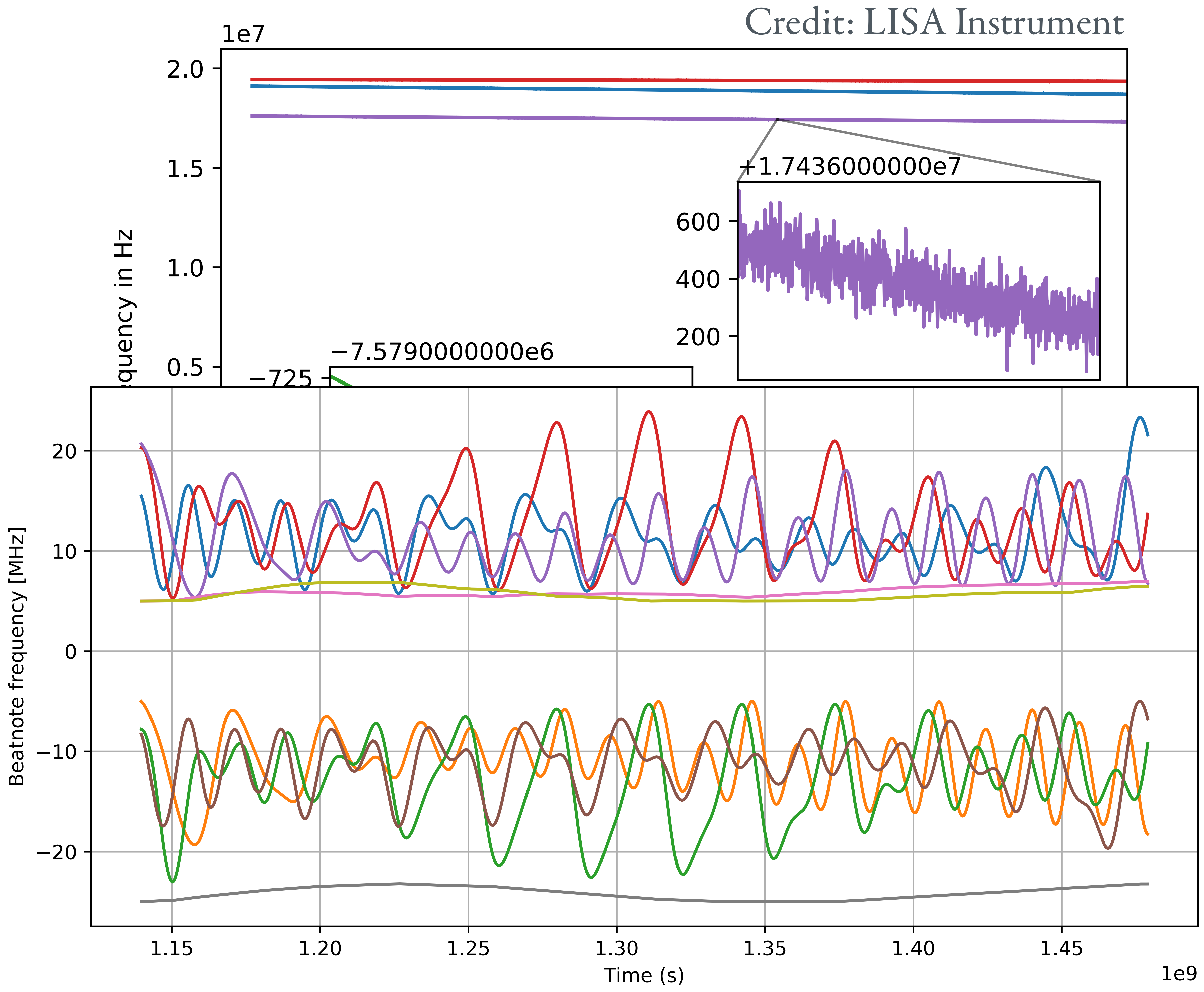
- Ground-based detectors use homodyne interferometers, with $\nu = \nu_1 = \nu_2$, and lock on a dark fringe: $\nu L_{12} = \pi/2$

- Orbital dynamics dictates that arm lengths cannot remain constant, ie. transmitted beam is Doppler-shifted



Credit: ESA / LISA Orbits

Interferometric measurements



- Orbital dynamics dictates that arm lengths cannot remain constant, ie. transmitted beam is Doppler-shifted
- LISA uses heterodyne frequency, where

$$\phi_{\mathrm{BN}} \propto (\nu_{2\to 1} - \nu_1)t + H_{2\to 1} = \nu_{\mathrm{BN}}t + H_{2\to 1}$$

Tens of MHz

+ noise at 100s of Hz

100s nHz

- 18 beatnotes (split interferometry) are the raw LISA measurements

Time-domain link response function

- Use geodesic equation to compute express ν_t from ν_e

$$\nu_{\text{BN}} = \nu_t - \nu_r = (\nu_e - \nu_r) + y_{re}\nu_e$$

where $y_{re} = (\nu_t - \nu_e)/\nu_e$ is the "overall Doppler shift"

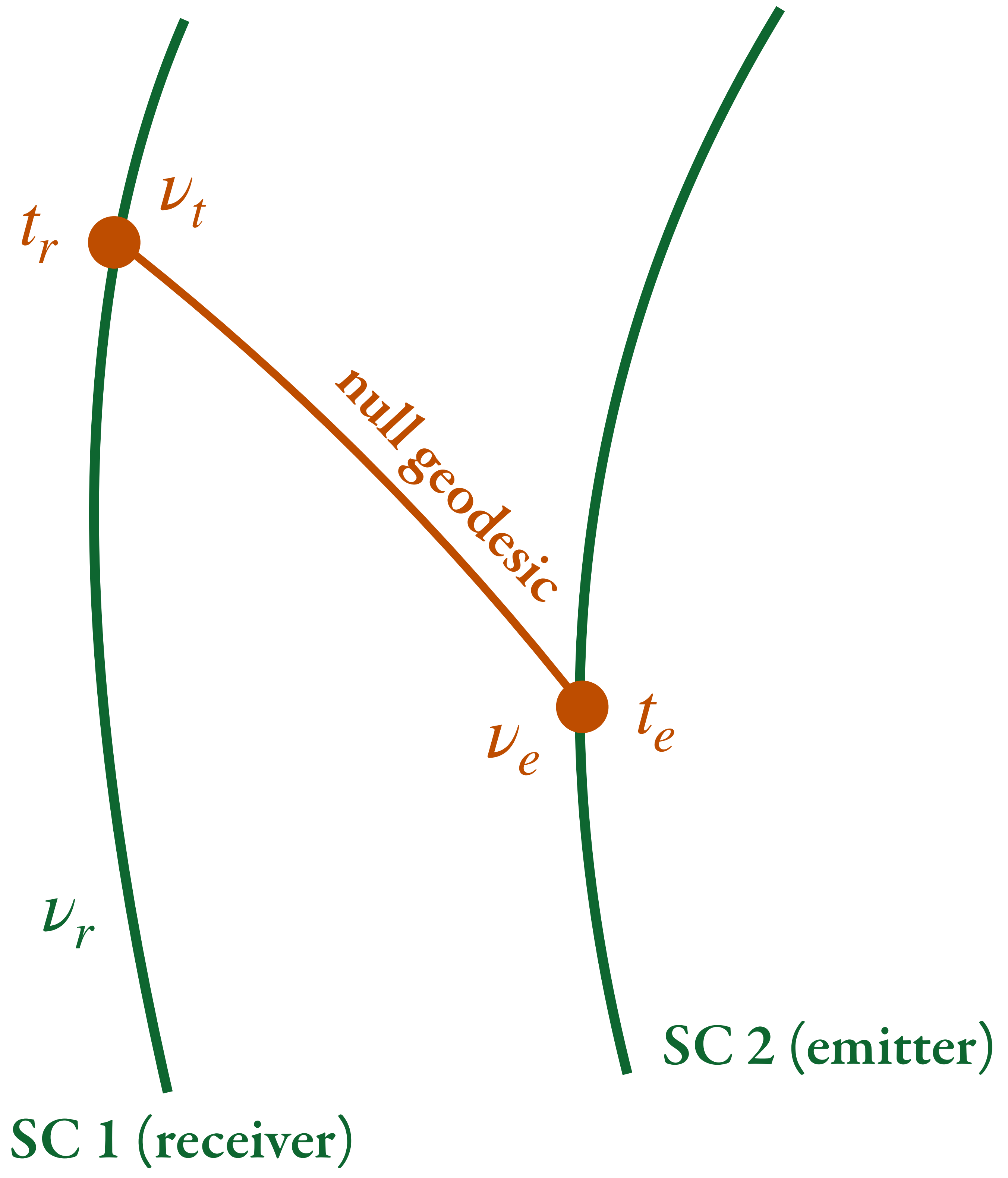
- Assume metric $g_{\mu\nu} = \eta_{\mu\nu} + h^{\text{SS}}_{\mu\nu} + h_{\mu\nu}$
- Assume plane wave GW, $h_{\mu\nu}(t, \mathbf{x}) = h_{\mu\nu}(t - \hat{\mathbf{k}} \cdot \mathbf{x})$
- Then

$$y_{re} = y^{\text{SS}}_{re} + y^{\text{GW}}_{re} + \mathcal{O}(h^{\text{SS}}_{\mu\nu} h_{\mu\nu})$$

Doppler shift from SS

Effect of GW

10^{-8} smaller than GW



ONE DERIVATION AMONGST MANY!

Credit: A. Hees, LISA Rosetta Stone (in prep.)

Time-domain link response function

- From [Blanchet+ 2001], GW-induced Doppler shift only depends on derivative of the coordinate light travel time

$$y_{re}^{\mathrm{GW}} = 1 - \frac{\nu_t}{\nu_e} \approx 1 - \frac{\mathrm{d}t_e}{\mathrm{d}t_e}$$

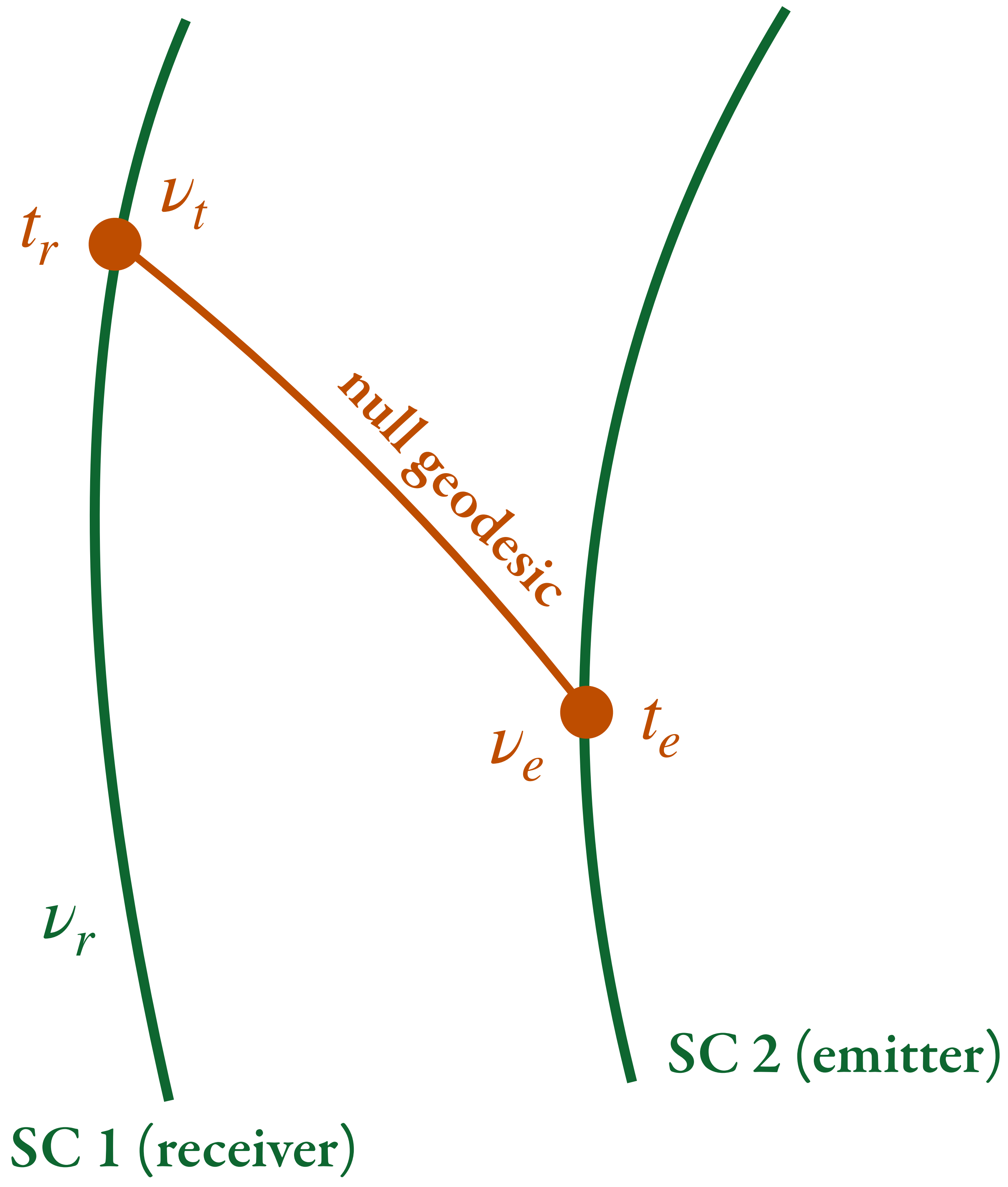
- Using the Time Transfer Function formalism [Teyssandier+ 2008], we find the implicit equation

$$t_r - t_e = \mathbf{r}_{re} + \frac{\mathbf{r}_{re}}{c}[\hat{\mathbf{r}}_{re} \otimes \hat{\mathbf{r}}_{re}] : \int_0^1 \mathbf{h}[x_r - \mu(x_r - x_e)]\,\mathrm{d}\mu$$

$\mathbf{x}_\mathbf{r}(t_r) - \mathbf{x}_\mathbf{e}(t_e)$

unit vector pointing to receiver

first-order in h



- Solve, neglecting terms in $h_{\mu\nu} v/c$ (10^{-4} smaller)

Time-domain link response function

- We find

"phase" at reception

$$t_r - t_e = \frac{1}{2}\frac{1}{1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{r}}_{\mathbf{re}}} \int_{\xi_e(t_r)}^{\xi_r(t_r)} [\hat{\mathbf{r}}_{re} \otimes \hat{\mathbf{r}}_{re}] : \mathbf{h}(\xi)\, \mathrm{d}\xi$$

"phase" at emission

antenna patterns

- Derivative wrt. t_r gives the usual expression for y^{GW}

$$y_{re}(t_r) = \frac{1}{2} \frac{H_{re}\left(t_r - L_{re} - \hat{\mathbf{k}} \cdot \mathbf{x_e}(t_r - L_{re})\right) - H_{re}\left(t_r - \hat{\mathbf{k}} \cdot \mathbf{x_r}(t_r)\right)}{1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{r}}_{\mathbf{re}}}$$

VALID DOWN TO 0.1% ERRORS

- Slowly varying (nonstationary) due to orbital effe
- Usual "algebraic" approximations include
  - Static (equal-arm) constellation: fix $\mathbf{x_e}$, $\mathbf{x_r}$, $\hat{\mathbf{r}}_{\mathbf{re}}$, and $L_{re} = L$
  - Low-frequency limit

LISA GW Response

https://pypi.org/project/lisagwresponse
https://doi.org/10.5281/zenodo.8321733

Fast LISA Response (GPU)

https://pypi.org/project/fastlisaresponse

LDC Software
(some additional approx.)

https://pypi.org/project/lisa-data-challenge
https://doi.org/10.5281/zenodo.7332221

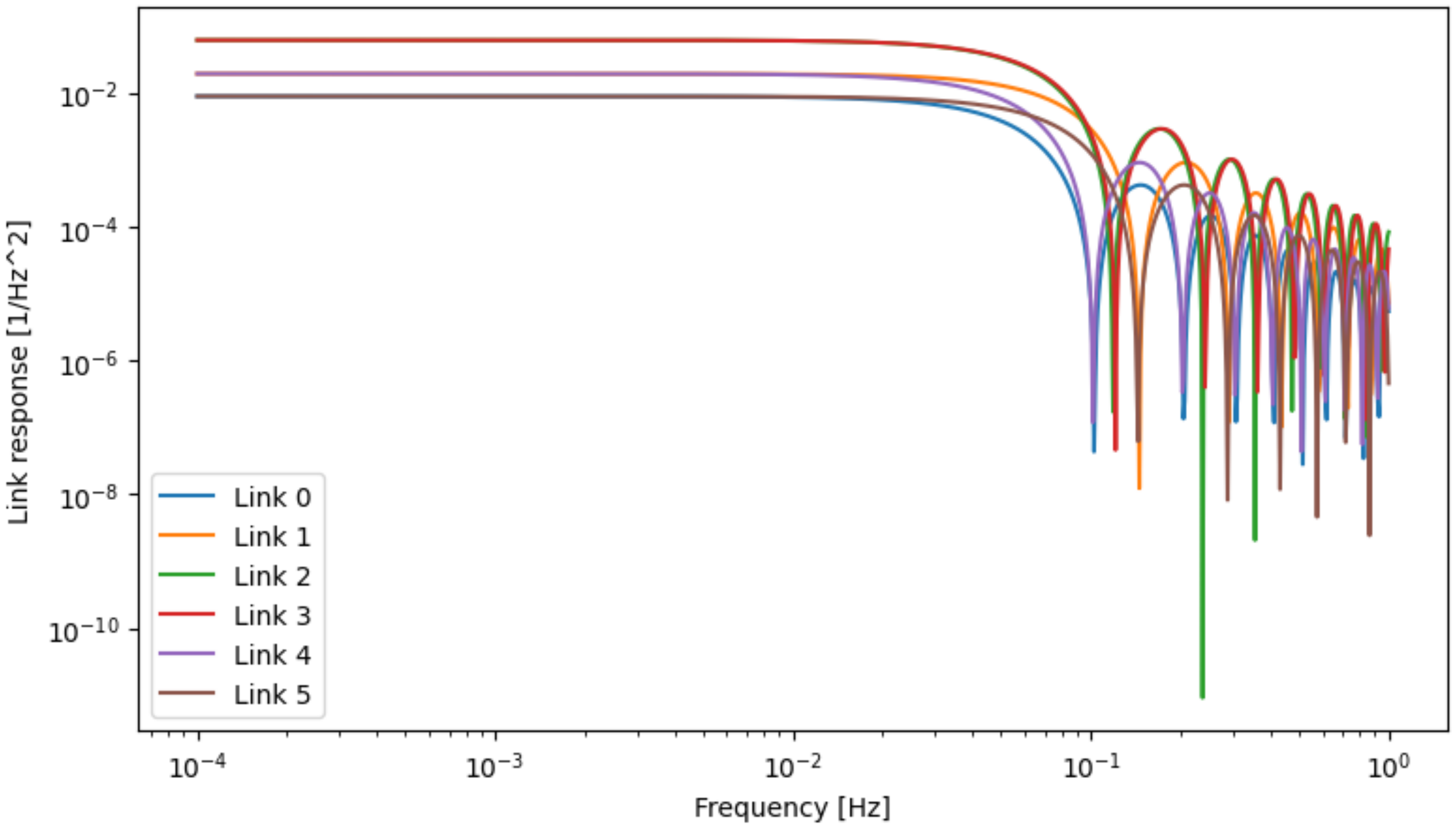
LISA Data Generation and Analysis Workshop

Oct 7 – 10, 2024, Online
https://indico.in2p3.fr/event/33255

Frequency-domain link response function

- "Locally stationary" response can be put in the form (e.g. [Cornish+ 2003])

$$\tilde{y}_{re}(f, t_r) = \frac{1}{2}\mathrm{sinc}\left[\pi f L_{re}(1 + \hat{\mathbf{k}} \cdot \hat{\mathbf{r}}_{\mathbf{re}})\right] e^{i2\pi f L_{re}(1+\hat{\mathbf{k}}\cdot\hat{\mathbf{r}}_{\mathbf{re}})}[\hat{\mathbf{r}}_{re} \otimes \hat{\mathbf{r}}_{re}] : \tilde{\mathbf{h}}(f)$$



- LISA will need a global fit for tens of thousands of sources, probably using block-Gibbs MCMC sampling
- Likelihood computation needs be computationally efficient (∼100 ms), includes waveform and response
- Various tricks around
  - Parallelization / hardware acceleration
  - Heterodyning [Cornish 2021]
  - …

Frequency-domain link response function

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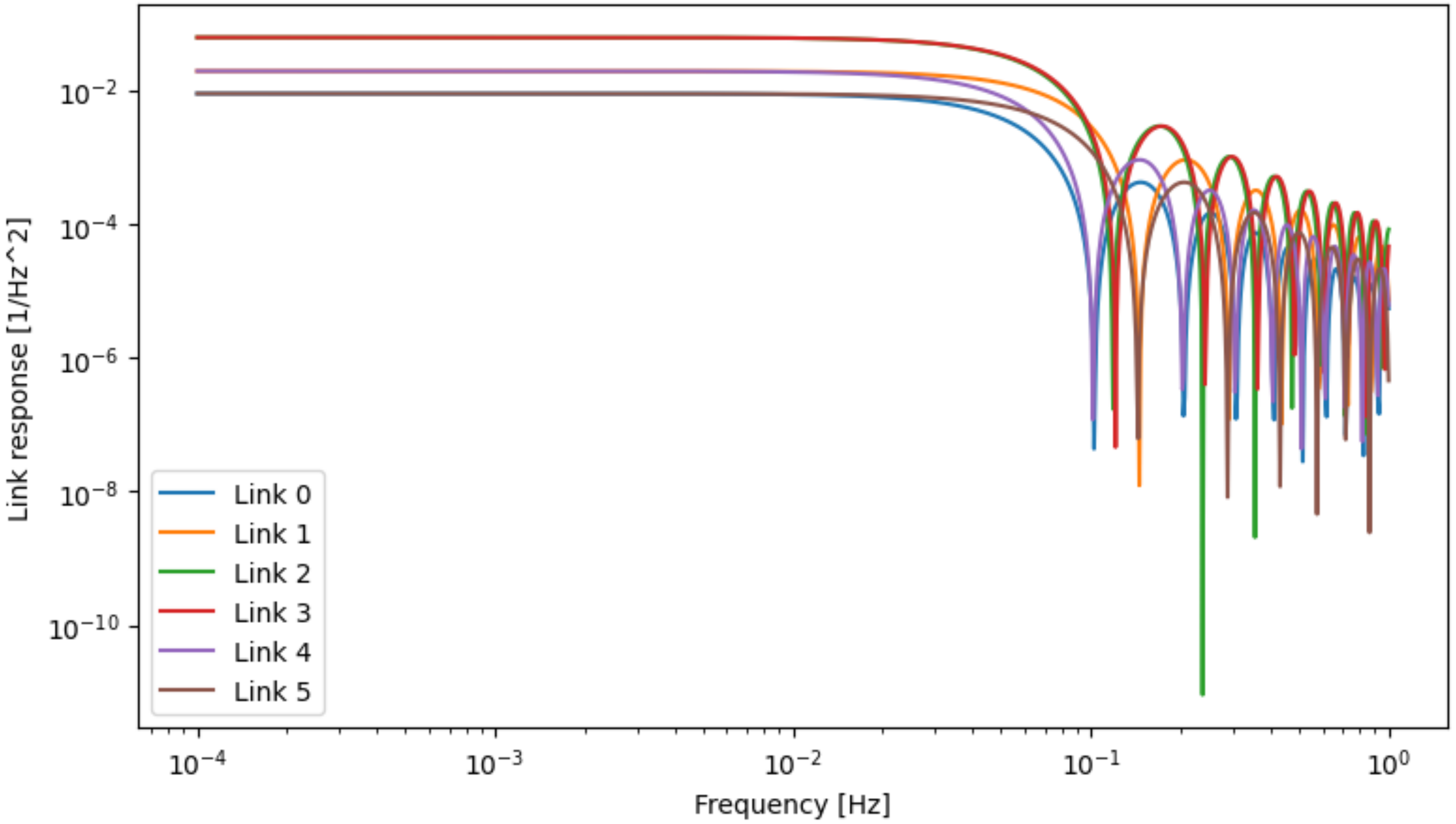
$$\tilde{y}_{re}(f, t_r) = \frac{1}{2}\mathrm{sinc}\left[\pi f L_{re}(1 + \hat{\mathbf{k}} \cdot \hat{\mathbf{r}}_{\mathbf{re}})\right] e^{i2\pi f L_{re}(1+\hat{\mathbf{k}}\cdot\hat{\mathbf{r}}_{\mathbf{re}})}[\hat{\mathbf{r}}_{re} \otimes \hat{\mathbf{r}}_{re}] : \tilde{\mathbf{h}}(f)$$



LDC Software

https://pypi.org/project/lisa-data-challenge
https://doi.org/10.5281/zenodo.7332221

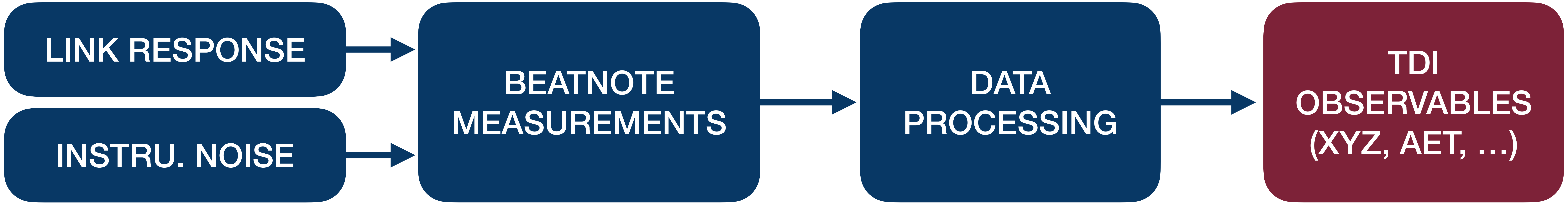
LISA Beta

https://pypi.org/project/lisabeta

LISA Analysis Tools

https://github.com/mikekatz04/LISAanalysistools
https://zenodo.org/records/10930980

Data processing



- Need to recombine beatnotes to construct TM-to-TM measurements
- Need to reduce non-suppressed, overwhelming laser noise (unequal-arm interferometer)

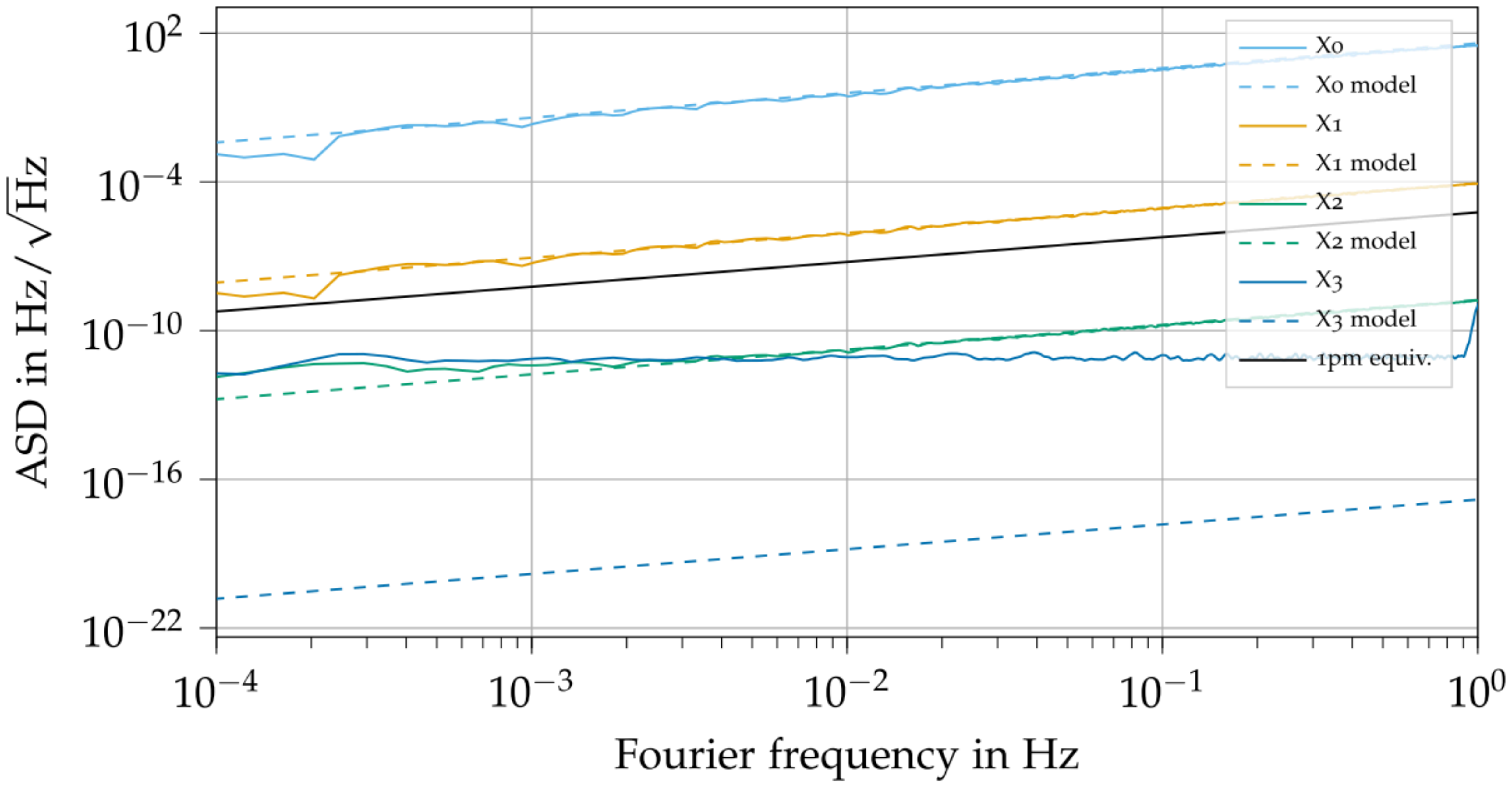
$$\nu_{\text{BN}} = p(t - 2L_1) - p(t - 2L_2) \neq 0$$

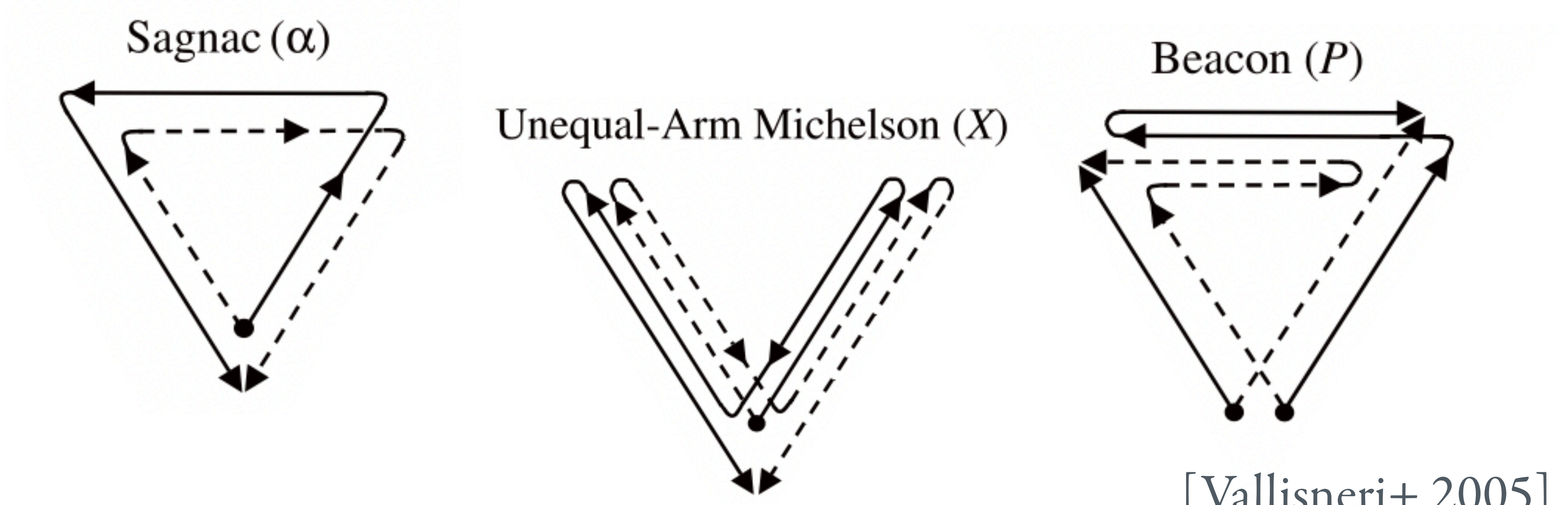
- Beatnotes in each spacecraft sampled on slightly different grid, need to synchronize the data (phase alignement)
- … and other calibration and noise-suppression steps

Time-delay interferometry

- Laser noise 8 orders of magnitude above expected signals, but measured coherently by different beatnotes
  - Find subspace free of laser noise, project beatnotes on this minimum variance space
- Solved algebraically for a static constellation (generation 1) with 4 or 6 “generating combinations” (Sagnac $\alpha, \beta, \gamma, \zeta$)
  - Insufficient for the LISA case
- Flexing constellation (generation 2+) not solved (non-commutative algebra is hard), but
  - Approximated by “promoting 1st-generation combinations” and hoping we cover all space
  - Time or frequency-domain linear algebra approaches (PCI or TDI-∞)

Credit: O. Hartwig

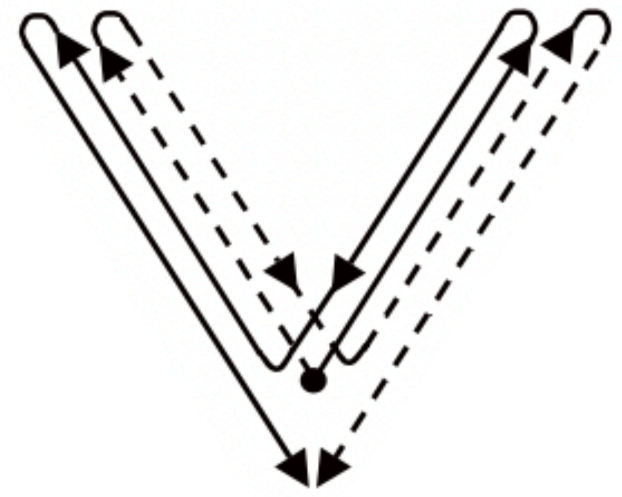




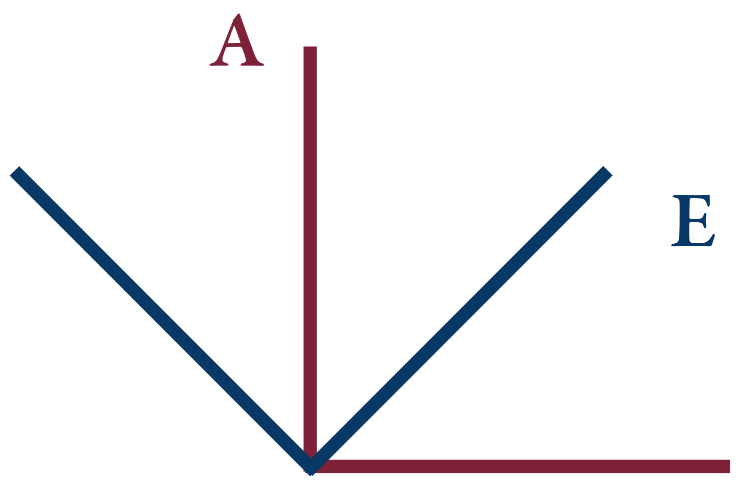
[Vallisneri+ 2005]

Time-delay interferometry

Unequal-Arm Michelson (X)



$$A = \frac{1}{\sqrt{2}}(Z - X) \, ,$$
$$E = \frac{1}{\sqrt{6}}(X - 2Y + Z) \, ,$$
$$T = \frac{1}{\sqrt{3}}(X + Y + Z) \, .$$

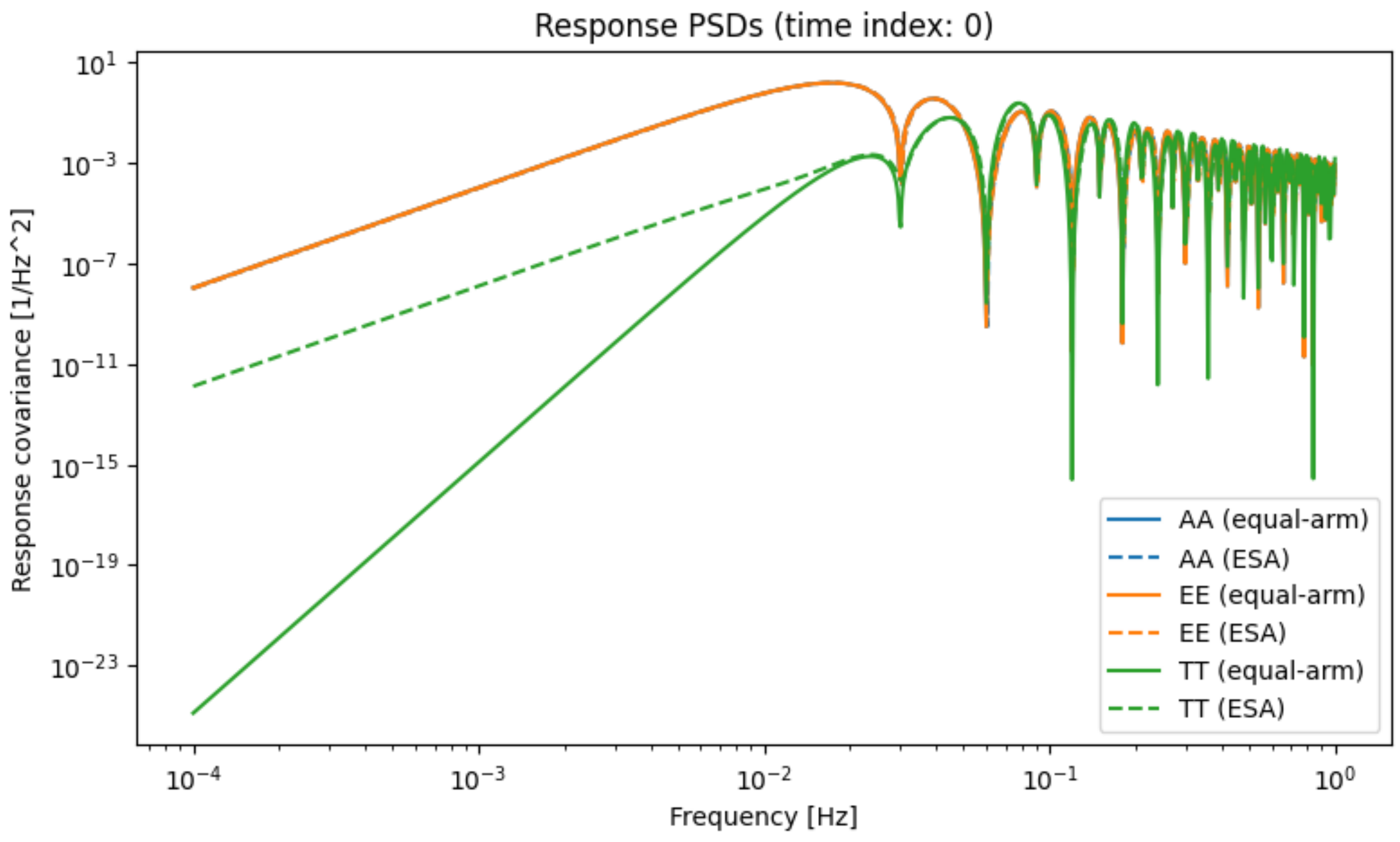


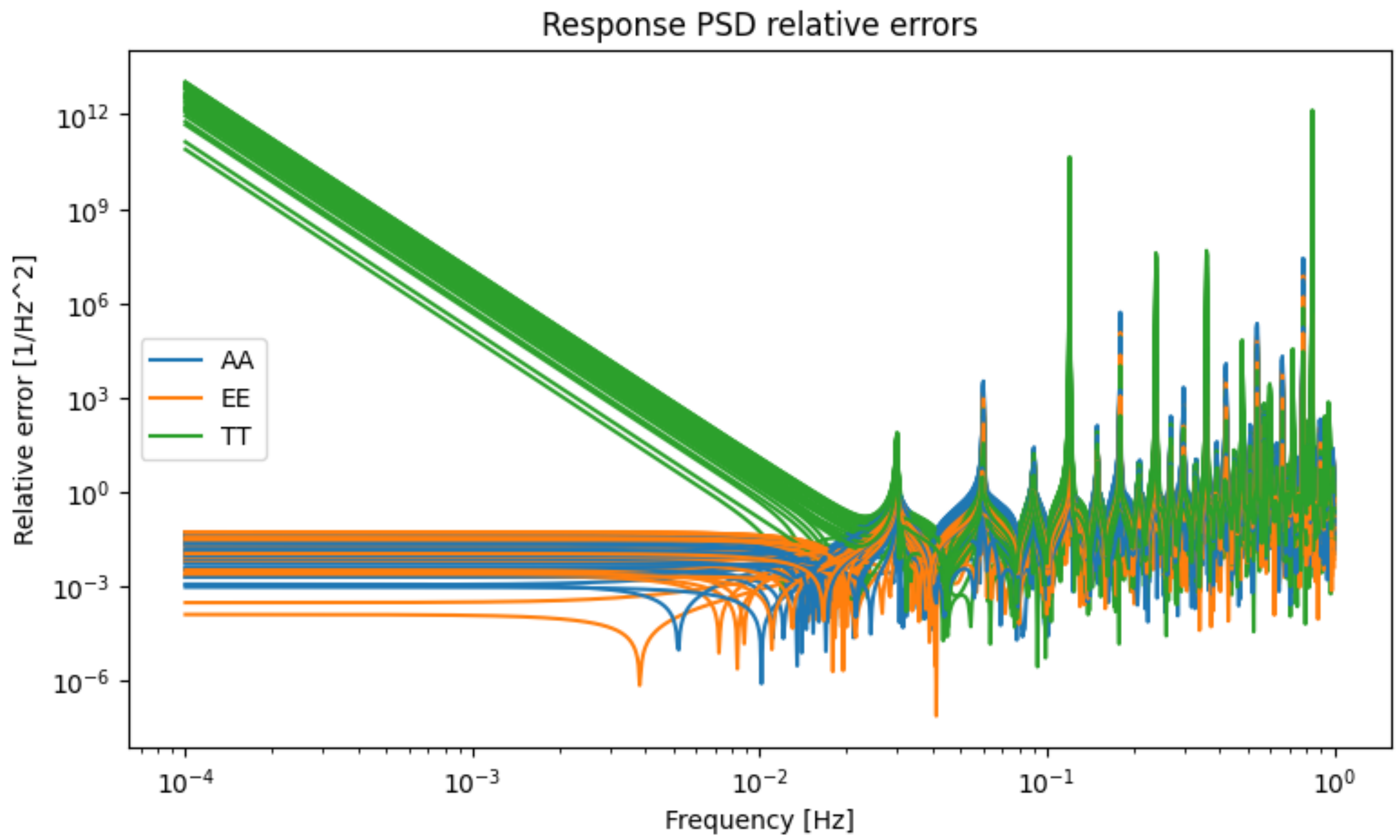
- "Almost everywhere" in the LISA band, 3 combinations are enough to span the laser-free space; pick your favorite set!
- TDI combinations are linear combinations of time-shifted beatnote measurements, therefore non-stationary over long timescales
  - Often approximated using constant, equal arms
- 1st-generation Sagnac generators have the least "zeros"

$$\alpha_1 = \eta_{13} + \mathbf{D}_{13}\eta_{32} + \mathbf{D}_{132}\eta_{21} - (\eta_{12} + \mathbf{D}_{12}\eta_{23} + \mathbf{D}_{123}\eta_{31})$$

- Michelson combinations XYZ are "rotationally symmetrical" and only involve 2 arms (resilient in case of link failure)
- Dominant secondary noises are test-mass and optical metrology noises. Assuming constant, equal arms and equal noise levels everywhere, one can find noise-diagonal AET combinations
  - Because of symmetries, AET is independent of noise levels and arm lengths
  - AET also diagonalize the low-frequency sky-averaged response (ie., T is a "null channel")

Note on “orthogonal channels”





- In realistic setup, the “diagonal” AET response (and noise covariance) assumption is only good up to a few percent level, probably not enough for most FP analyses
- No good reason to stick to AET in a realistic setup

Modeling challenges

- Errors in the response function
  - Be aware of approximations used, use better models when necessary
  - Marginalize over instrumental uncertainties if needed
  - Errors in calibration (eg. time sync.)
- [Savalle+ 2022] investigated calibration errors (amplitude & phase) using FIMs
  - Calibration errors $\sim<$ 0.1% to keep parameter uncertainties within x2 error
  - Using known binaries and EMRIs might constrain calibration errors at 0.01%
- Uncertainty in the noise
  - No true signal-free channel, noise must be estimated alongside sources in global fit
- Non-stationarity of noise and response
  - Orbital effects on response
  - Non-stationary “noises” (anisotropic population backgrounds, non-stationary transfer functions, time-dependent noises)
  - Non-stationary noise transfer functions
- Long-term coherence of the models
  - Operations (repointing, etc.) might change response function or noise coherence

Practical challenges

- Waveform, instrument modeling, and data analysis (and fundamental physics, and others…) communities will have to interact and share tools
- Improve interfaces through agreed-upon conventions (and maybe standardization?)
  - DDPC Convention task force just formed
- An important aspect is the interface between project activities (DDPC, NSGS, etc.) and the community at large (new symposium)

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LISA Analysis Tools Workshop

April 15-18 2024, Online

https://indico.physics.auth.gr/e/LATW