Fundamental Physics Meets Waveforms With LISA
September 5 2024

SEOBNRv5: advancements in effective-one-body gravitational waveforms towards LISA

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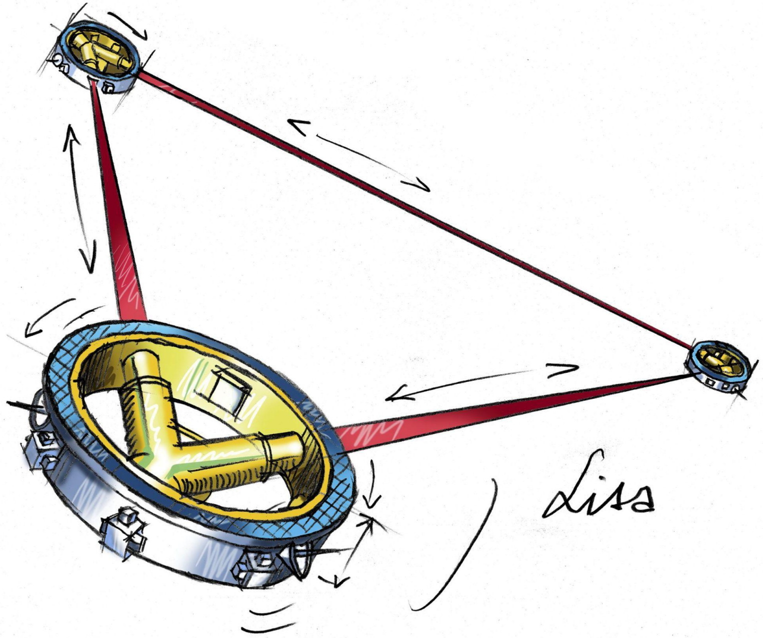
Max Planck Institute for Gravitational Physics,
Albert Einstein Institute, Potsdam

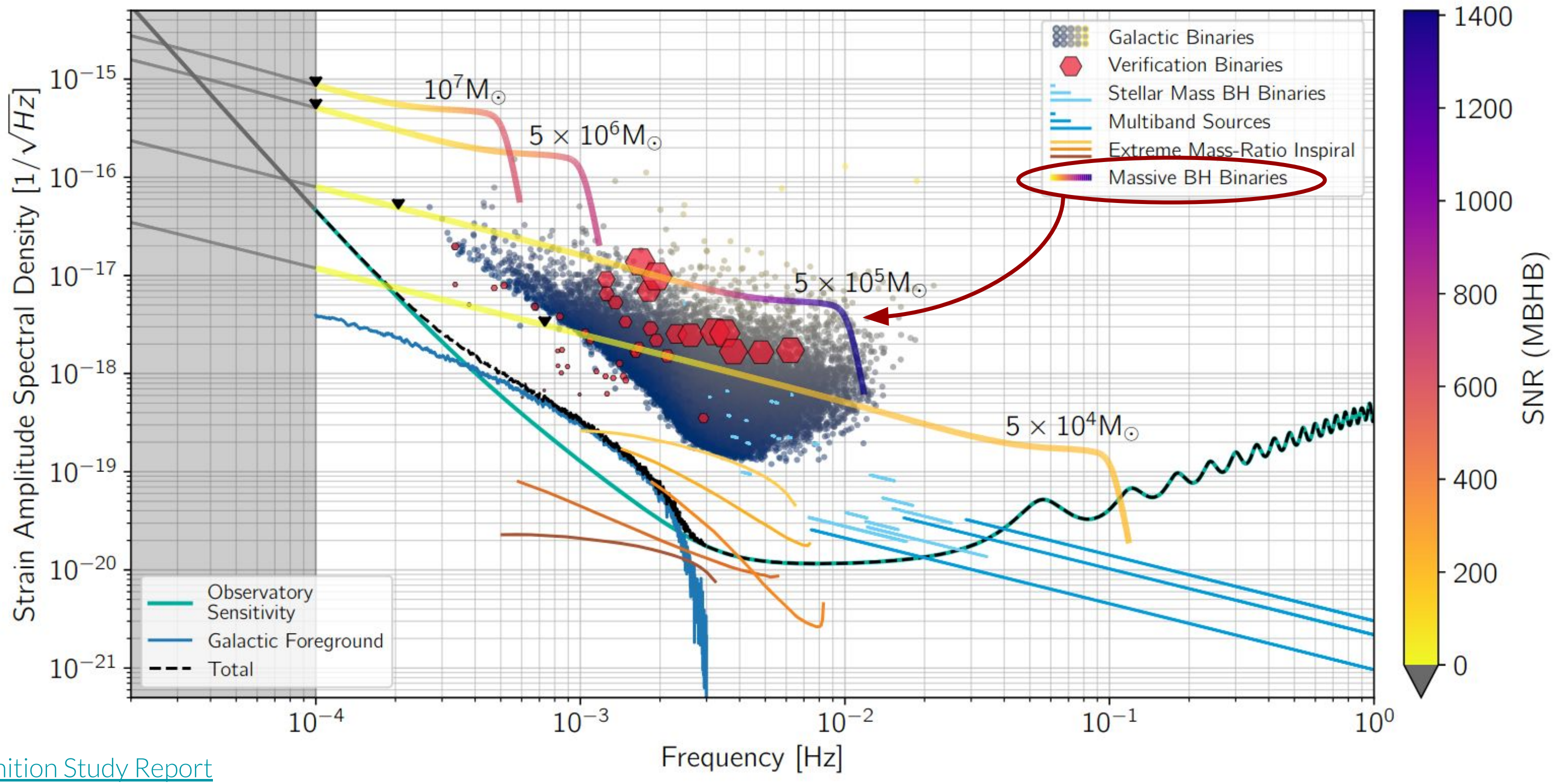
Overview

- The effective-one-body (EOB) approach to the two-body problem and the SEOBNRv5 family of waveform models
- Progress and challenges towards LISA
  - Accuracy
  - Physical completeness
  - Efficiency



Credit: ESA - C. Vijoux

LISA binary sources



LISA Definition Study Report

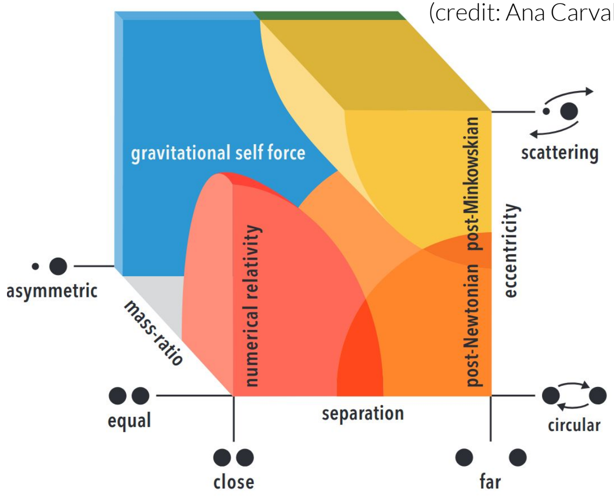
Inspiral-merger-ringdown waveforms

Einstein's equations can be solved:

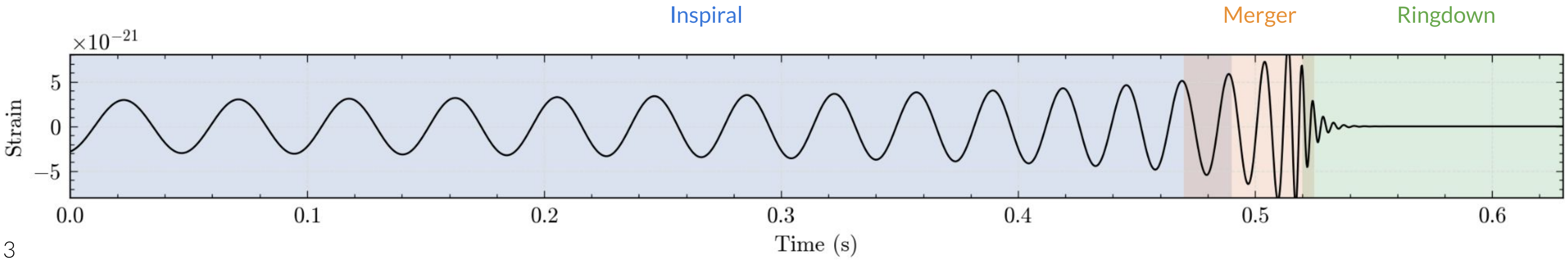
- Analytically
  - post-Newtonian (large separation, slow motion): expansion in $v^2/c^2 \sim GM/r$
  - post-Minkowskian (large separation, fast motion): expansion in G
  - gravitational self-force (small mass-ratio): expansion in m_2/m_1
- Numerically

State-of-the-art GW models used for searches, parameter estimation and tests of GR based on synergy between analytical and numerical relativity.

(credit: Ana Carvalho)



Effective-one-body (EOB) theory combines results from all methods to provide waveforms for the entire coalescence.



Effective-one-body waveforms

Main ingredients of EOB waveforms:

1. EOB Hamiltonian describing the conservative binary dynamics

Buonanno & Damour 99
Damour 00
Buonanno, Chen & Damour 05 Damour, Jaranowski & Schafer 08 Barausse, Racine & Buonanno 10
Barausse & Buonanno 11
Damour & Nagar 14
Balmelli & Damour 15
Khalil, Steinhoff, Vines & Buonanno 20
Khalil, Buonanno+ inc. LP 23

EOB Hamiltonian

$$H^{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\mu}\right)}$$

Effective-one-body waveforms

Main ingredients of EOB waveforms:

1. EOB Hamiltonian describing the conservative binary dynamics

2. Radiation reaction (RR) force to account for loss of energy and angular momentum via emission of GWs

Damour, Iyer & Sathyaprakash 98
Buonanno & Damour 00
Damour & Nagar 07
Damour, Iyer & Nagar 09
Pan, Buonanno, Fujita+ 11
Taracchini, Pan, Buonanno+ 12

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Radiation reaction

$$\mathscr{F} = \frac{\Omega}{16\pi}\frac{p}{L}\sum_{\ell,m} m^2 |h_{\ell m}|^2$$

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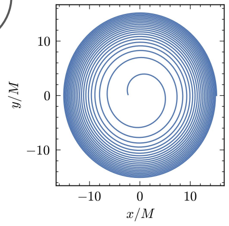
Equations of motion (ODE)

$$\dot{\vec{r}} = \frac{\partial H^{\rm EOB}}{\partial \vec{p}}$$

$$\dot{\vec{p}} = -\frac{\partial H^{\rm EOB}}{\partial \vec{r}} + \vec{\mathcal{F}}$$

Orbital dynamics

$$\vec{r}, \vec{p}$$



Effective-one-body waveforms

Main ingredients of EOB waveforms:

1. EOB Hamiltonian describing the conservative binary dynamics

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3. Gravitational waveform modes for inspiral, merger, and ringdown

Damour, Iyer, Jaranowski+ 03
Damour & Nagar 07
Damour, Iyer & Nagar 09
Pan, Buonanno, Fujita+ 11
Taracchini, Pan, Buonanno+ 12
Damour & Nagar 14
Nagar & Shah 16
Cotesta, Buonanno, Bohe+ 18
Nagar, Pratten, Riemenschneider+ 19
LP, Buonanno, Estellés+ 23

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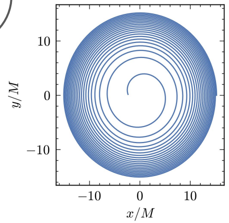


Waveform modes

$$h_{\ell m}^{\mathrm{insp-plunge}} = h_{\ell m}^{\mathrm{Newt}} S_{\mathrm{eff}} T_{\ell m} f_{\ell m} e^{i\delta_{\ell m}} h_{\ell m}^{\mathrm{NQC}}$$

$$h_{\ell m}^{\mathrm{merg\text{-}RD}} \quad \tilde{A}_{\ell m} e^{i\tilde{\phi}_{\ell m}} e^{-i\sigma_{\ell m0}(t - t^{22}_{\mathrm{peak}})}$$

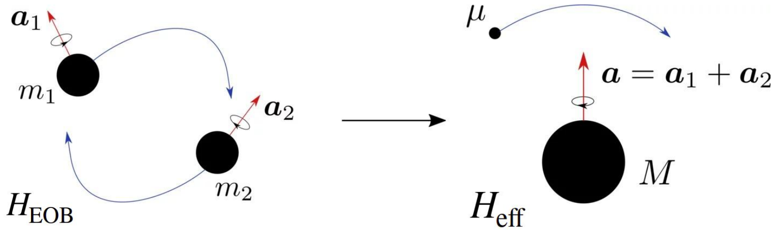
$$h_{\ell m} = \begin{cases} h_{\ell m}^{\mathrm{insp-plunge}}, & t < t^{22}_{\mathrm{peak}}, \\ h_{\ell m}^{\mathrm{merg-RD}}, & t > t^{22}_{\mathrm{peak}}. \end{cases}$$

Effective-one-body Hamiltonian

Two-body dynamics mapped into dynamics of one effective body moving in deformed BH spacetime, deformation being the mass ratio ν.

$$H_{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}}{\mu} - 1\right)}$$

$$M = m_1 + m_2$$
$$\mu = m_1 m_2 / M$$
$$\nu = \mu / M$$



(credit: Mohammed Khalil)

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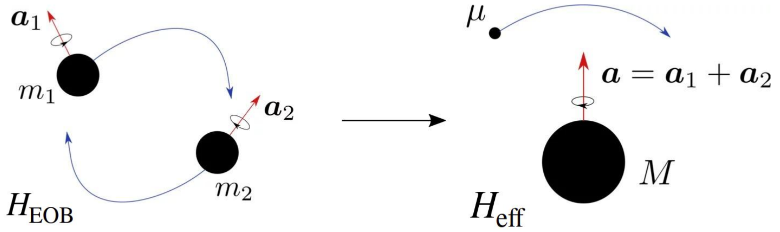

(credit: Mohammed Khalil)

$$H_{\rm eff} = \sqrt{p_{r_*}^2 + A(r)\left[\mu^2 + \frac{p_\phi^2}{r^2} + Q\left(r, p_{r_*}\right)\right]}$$

In the nonspinning ν → 0 limit reduces to the Hamiltonian of a test mass in a Schwarzschild background.

$$A(r) = 1 - \frac{2}{r} + \frac{2\nu}{r^3} + \nu\left(\frac{94}{3} - \frac{41\pi^2}{32}\right)\frac{1}{r^4} + \frac{a_5}{r^5} + \left[\nu\, a_6 + \nu\left(\frac{144\nu}{5} + \frac{7004}{105}\right)\ln r\right]\frac{1}{r^6}$$

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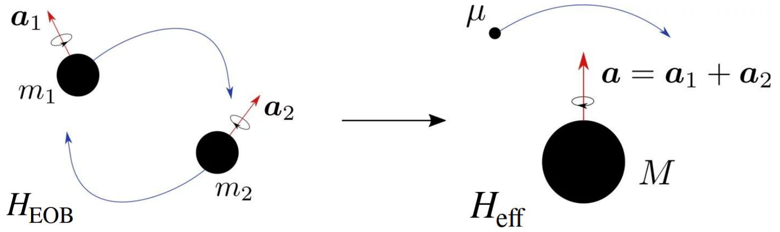

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Unknown 5PN coefficient calibrated to NR

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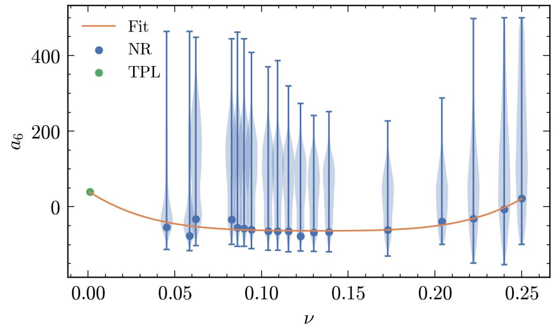

Significant freedom in the structure of the Hamiltonian, PN information, gauge and resummation choices, especially with spins → different EOB flavours: SEOBNR and TEOBResumS.

SEOBNRv5: new generation of SEOBNR models

SEOBNRv5 models: fifth generation of SEOBNR waveform models.

Developed with modular and efficient pySEOBNR Python code infrastructure: https://git.ligo.org/waveforms/software/pyseobnr [Mihaylov+ 23]

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SEOBNRv5HM: multipolar non-precessing (spins anti-/aligned with the orbital angular momentum of the system) EOB model for quasi-circular BBHs. Calibrated to NR and test-particle limit waveforms.

- Includes recent high-order PN results and improved resummations for the Hamiltonian (4 PN), the RR force and waveform modes (3.5 PN).
- Information from 2GSF fluxes in the non-spinning modes/RR force.
- Extended the NR calibration wrt SEOBNRv4 from 140 to 442 NR waveforms.
- The (2,2), (2,1),(3,3), (3,2),(4,4), (4,3),(5,5) GW modes are included.

LP+ 23
Balmelli+ 15, Khalil+ 20, Henry+ 22, Khalil+ 23
Warburton+ 21, VanDeMeent+ 23

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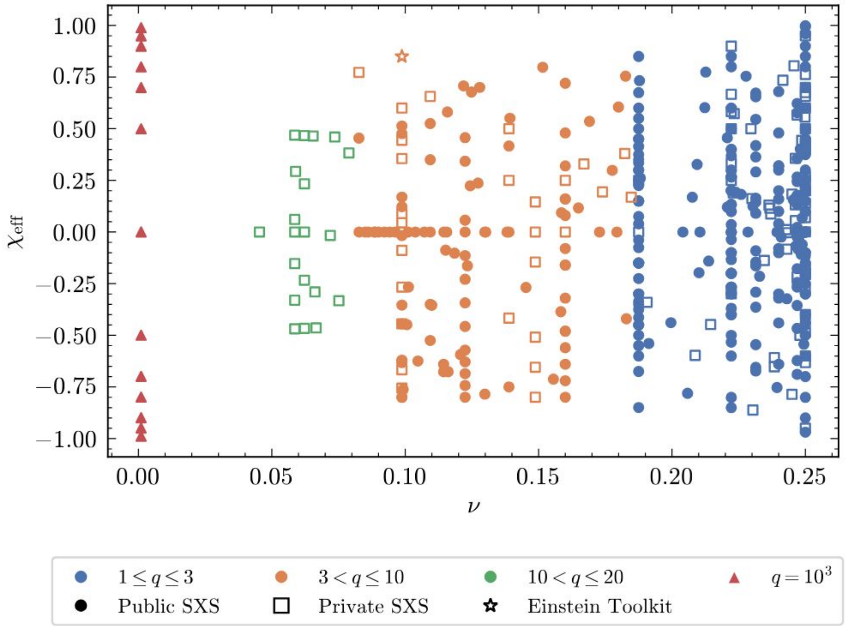
SEOBNRv5PHM: multipolar precessing-spin EOB model for quasi-circular BBHs. The model is not yet calibrated to precessing-spin NR waveforms.

Ramos-Buades+ 23
Khalil+ 23, Akcay+ 21, Estelles+ 21

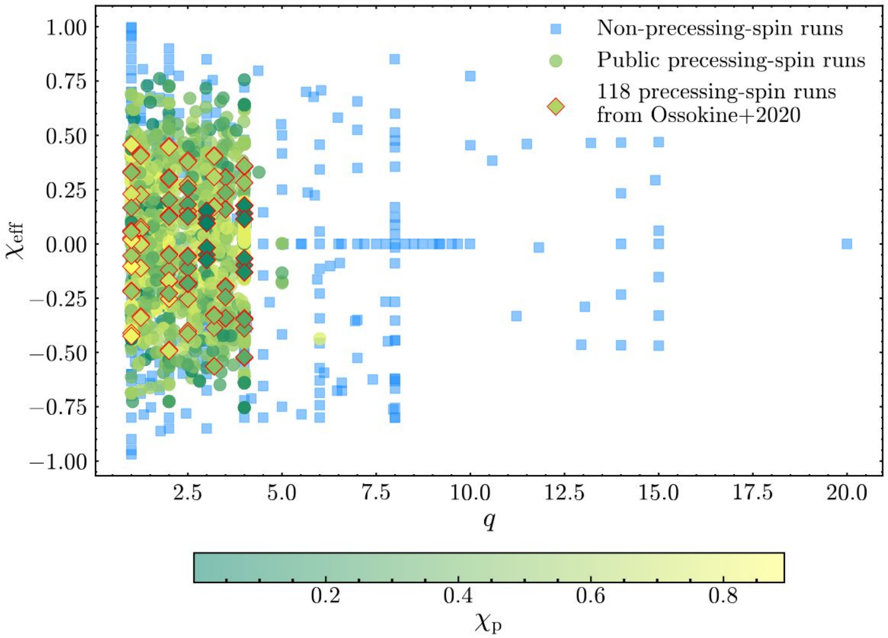
- Model built upon the accurate non-precessing SEOBNRv5HM waveforms in the "co-precessing" frame.
- Orbit-averaged in-plane spin effects via partially precessing Hamiltonian.
- Fast PN-expanded spin-precession dynamics, with consistent spin-supplementary conditions and higher-order PN information from the fully precessing SEOBNRv5 Hamiltonian.
- Highly accurate: mismatch against 1543 precessing-spin SXS NR simulations below 3% for 99.8% of cases.

SEOBNRv5 highlights: NR coverage

442 aligned-spin NR waveforms and 13 BH perturbation theory waveforms used to calibrate SEOBNRv5HM.



1543 precessing-spin SXS NR waveforms used to validate SEOBNRv5PHM.



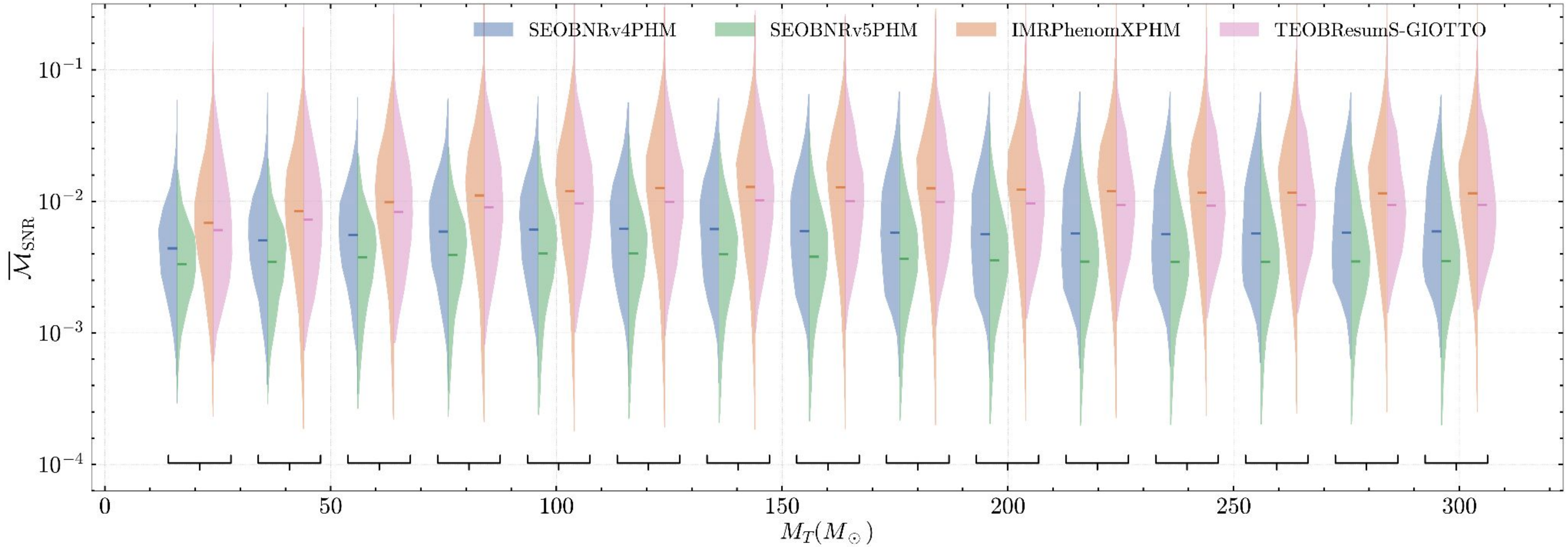
[LP+23; Ramos-Buades+ inc. LP 23]

SEOBNRv5 highlights: waveform accuracy

Agreement between models and/or NR often quantified by mismatch: $\mathcal{M} \approx 1 - \max_{t_c,\varphi_0,\psi,\phi_{\rm JL}} \left[\frac{\langle h_1, h_2 \rangle}{\sqrt{\langle h_1, h_1 \rangle \langle h_2, h_2 \rangle}} \right]$ $\langle h_1, h_2 \rangle = 4\,\mathrm{Re} \int_{f_{\rm low}}^{f_{\rm high}} df\, \frac{\tilde{h}_1(f)\, \tilde{h}_2^*(f)}{S_n(f)}$

Mismatch against 1543 precessing-spin SXS NR simulations, compare with other state-of-the-art EOB models (SEOBNRv4PHM and TEOBResumS-Giotto) and frequency-domain phenomenological model (IMRPhenomXPHM): SEOBNRv5PHM shows the lowest unfaithfulness (best accuracy) distribution for all total masses.

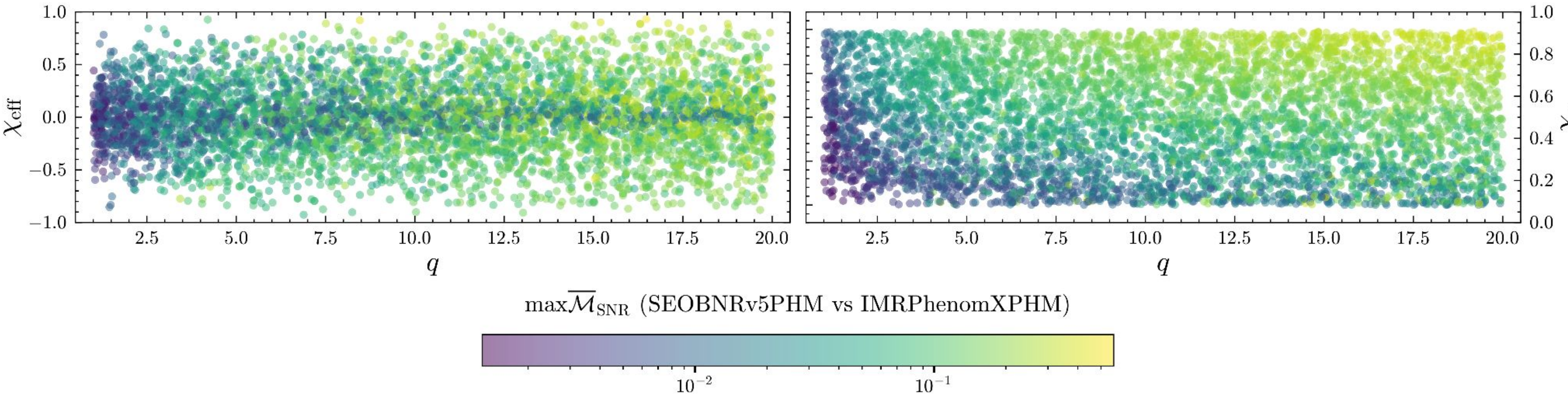
Advanced LIGO PSD and inclination π/3.



[See Hamilton+23, Ghosh+23, Thompson+23, Colleoni+23 for recent improvements to IMRPhenomXPHM and Nagar+23 for TEOBResumS-GIOTTO] [Ramos-Buades+ inc. LP 23]

SEOBNRv5 highlights: waveform systematics

Mismatch between SEOBNRv5PHM and IMRPhenomXPHM for 2000 cases with mass-ratio up to 20, spin magnitudes up to 0.99 and random orientations with a uniform distribution in the precessing effective spin.



SEOBNRv5PHM and IMRPhenomXPHM models differ significantly (up to 20-40%) in large parts of the parameter space where they are not calibrated to NR waveforms.

[Ramos-Buades+ inc. LP 23]

SEOBNRv5 highlights: NR injections

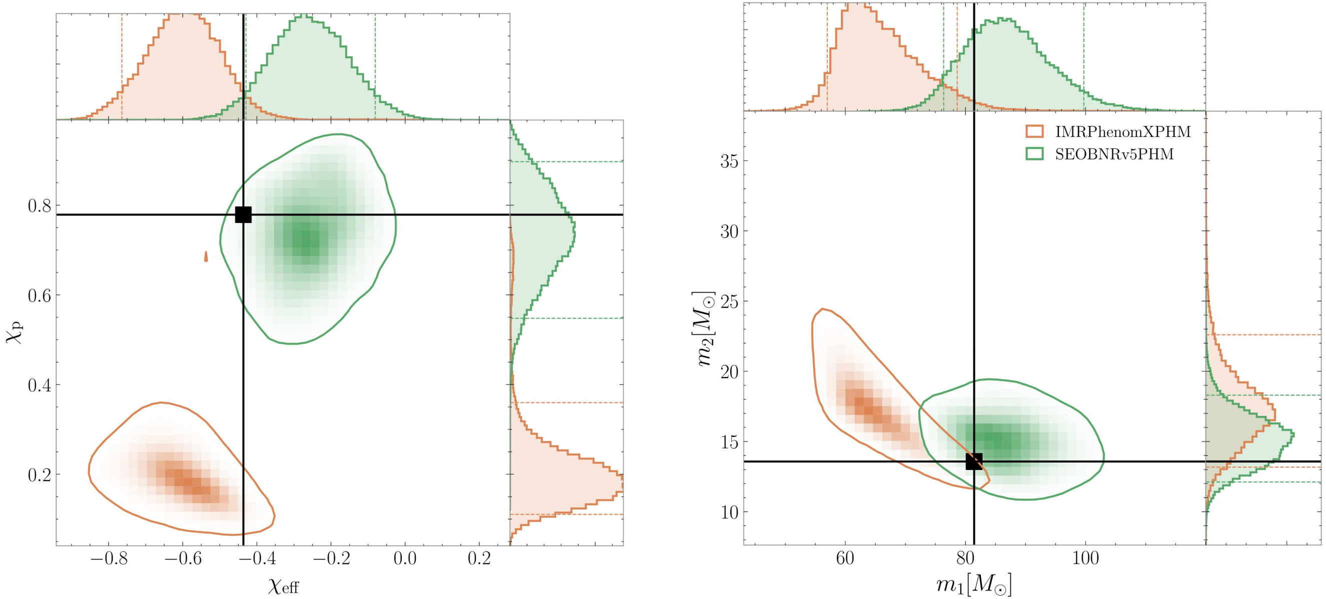
Waveform systematics can be significant even for current detectors, for challenging binary configurations.

- Injection with the NR waveform SXS:BBH:0165 for LVK at design sensitivity, SNR~20.

$$q = 6, \quad M_T = 95M_\odot, \quad \chi_1 = [-0.06, 0.78, -0.4], \quad \chi_2 = [0.08, -0.17, -0.23], \quad \iota = 0.69$$

$$\mathscr{M}^{100M_\odot}_{\rm SNR}(\text{IMRPhenomXPHM}) \sim 12\,\%$$
$$\mathscr{M}^{100M_\odot}_{\rm SNR}(\text{SEOBNRv5PHM}) \sim 2\,\%$$

- Tension in the recovery of the effective spin parameter, but other parameters are accurately recovered by SEOBNRv5PHM.



[Ramos-Buades+ inc. LP 23]

LISA waveform modelling challenges

Accuracy

Inaccurate waveform models can lead to biases in the estimated binary parameters or be misinterpreted as GR deviations.
Imperfect subtraction of high SNR MBHB signals can contaminate the analysis of overlapping signals.

Physical completeness

Waveform models need to cover the full (9D) parameter space of expected binaries:
mass ratio (1), spins (6), eccentricity (2) for BBHs in vacuum GR.
Astrophysical environments likely important for MBHBs.

Efficiency

Parameter estimation of a single event requires 10^6 - 10^9 waveform evaluations with standard methods
=> models need to generate waveforms in milliseconds.

Adapted from Maarten's LISA Symposium talk

The accuracy challenge

Significant improvement in the accuracy of current waveform models (~ two orders of magnitude) and of NR simulations is needed to avoid systematic biases in parameter estimation with high SNR signals in LISA/XG.

Pürrer+19, Ferguson+ 20, Hu+22, Toubiana+23, Dhani+24, Kapil+24
See also talks from Sylvain Marsat, Alexandre Toubiana and Jonathan Gair

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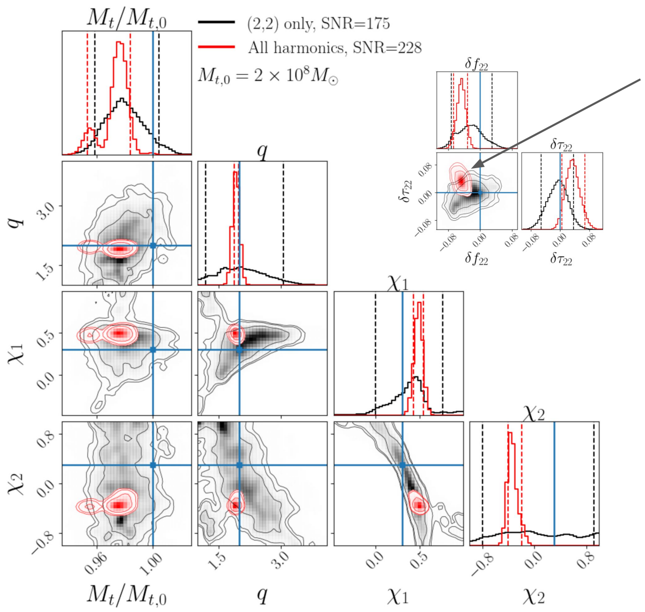

False GR deviation in the quasi-normal-mode frequency and decay time of the ringdown due to systematics.

NR injection
SEOBNRv5HM recovery

[Toubiana, LP, Buonanno, Gair & Katz 23]

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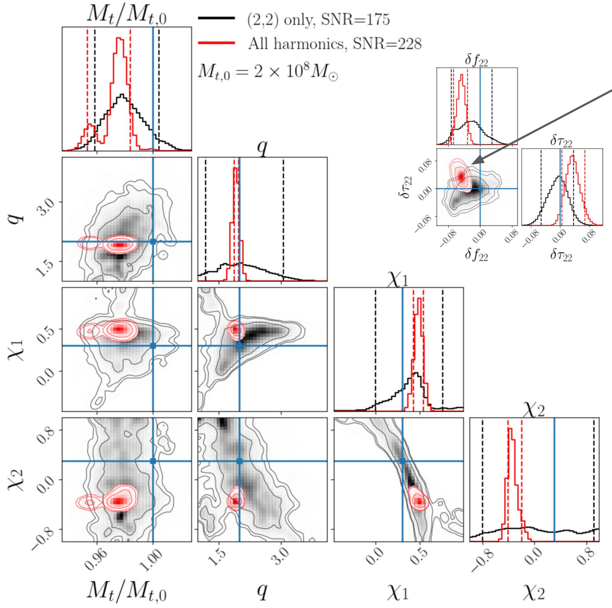

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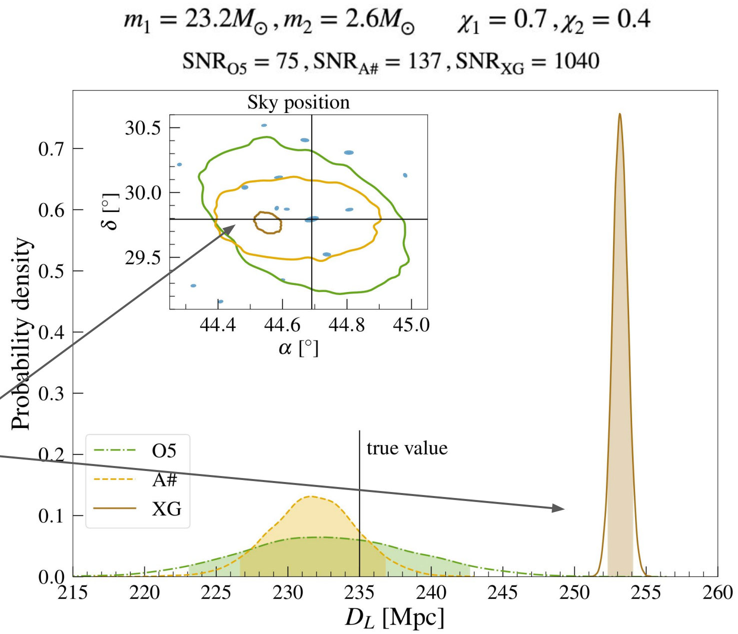
NR injection
SEOBNRv5HM recovery

Biased measurement of Hubble-Lemaitre parameter H0 due to systematics.

SEOBNRv5PHM injection
IMRPhenomXPHM recovery

[Toubiana, LP, Buonanno, Gair & Katz 23]



[Dhani, Völkel, Buonanno, Estellés, Gair, Pfeiffer LP & Toubiana 24]

The accuracy challenge

Incorporate uncertainty estimates into waveform models:

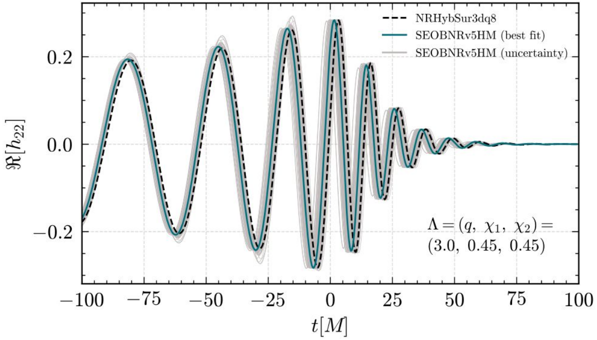
- Marginalize over NR-calibration uncertainty with priors based on NR-calibration posteriors p(θ|λ).

$$\mathcal{L}(s \mid \lambda) \propto \int d(\delta\theta(\lambda)) P[\delta\theta(\lambda)] \times \exp\left(-\frac{1}{2}\langle s - H(\lambda, \theta(\lambda) + \delta\theta(\lambda)) \mid s - H(\lambda, \theta(\lambda) + \delta\theta(\lambda))\rangle\right)$$

h: true waveform H: approximate waveform
λ: source parameters θ: NR-calibration parameters

$$h(\lambda) = H(\lambda, \theta(\lambda)) + \delta h(\lambda) \simeq H(\lambda, \theta(\lambda) + \delta\theta(\lambda))$$

best fit uncertainty



See also:
Moore & Gair 14, Doctor+ 17,
Williams+ 20, Owen+23,
Read 23, Khan 24

[LP, Buonanno & Pürrer in prep.]

The accuracy challenge

Incorporate uncertainty estimates into waveform models:

- Marginalize over NR-calibration uncertainty with priors based on NR-calibration posteriors p(θ|λ).
- Parameter estimates with slightly reduced precision (e.g. wider posteriors) but biases are significantly reduced.

$$\mathcal{L}(s \mid \lambda) \propto \int d(\delta\theta(\lambda)) P[\delta\theta(\lambda)] \times \exp\left(-\frac{1}{2}\langle s - H(\lambda, \theta(\lambda) + \delta\theta(\lambda)) \mid s - H(\lambda, \theta(\lambda) + \delta\theta(\lambda))\rangle\right)$$

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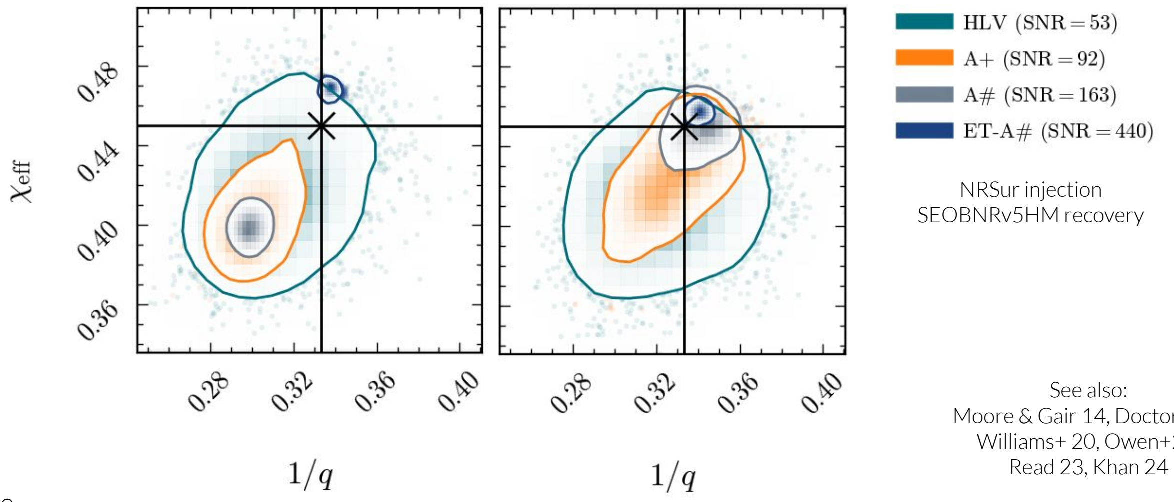
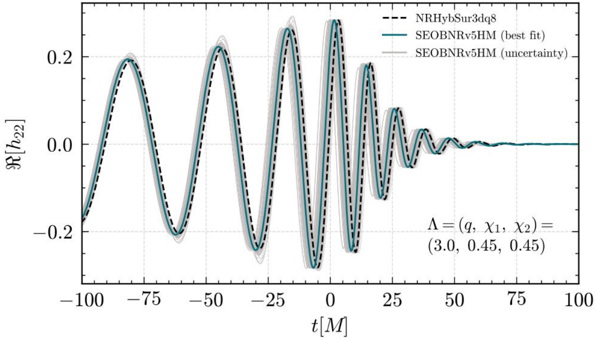
best fit   uncertainty

Without uncertainty   With uncertainty



NRSur injection
SEOBNRv5HM recovery



See also:
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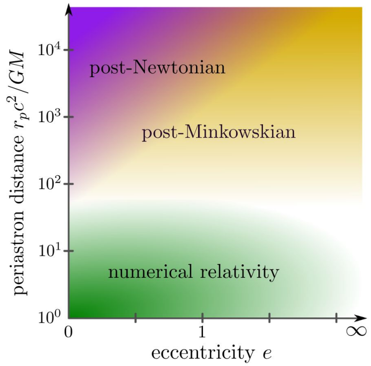
[LP, Buonanno & Pürrer in prep.]

The accuracy challenge

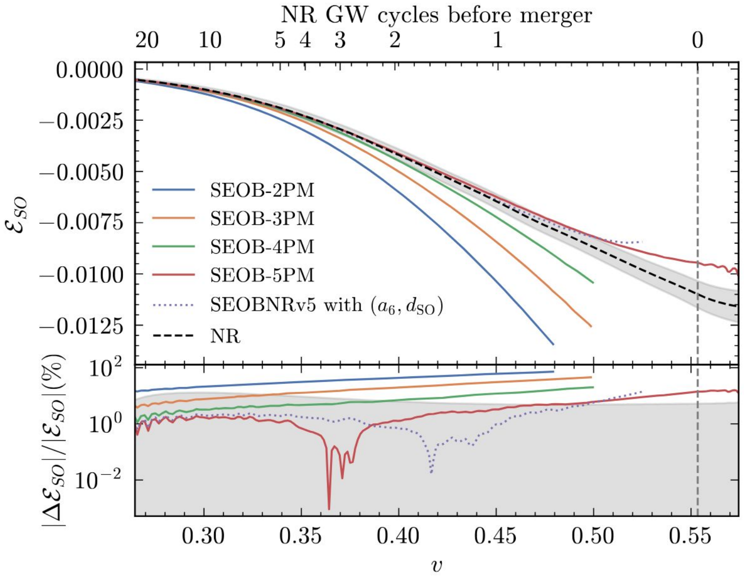
Scattering-amplitude/effective-field-theory/quantum-field-theory methods from high-energy physics have brought new tools to solve two-body problem in GR: PM approximation.

- PM approximation more accurate than PN for scattering encounters at large velocities, or large eccentricities at fixed periastron distance.
- Remarkable agreement of PM-improved EOB models with NR for scattering orbits.



Credit: Khalil+ (2022)

Damour 16, Antonelli+19, Khalil+22, Damour & Rettegno 23, Rettegno+23, Buonanno+24
See also talks tomorrow from Gregor Kälin, Oliver Long and Riccardo Gonzo



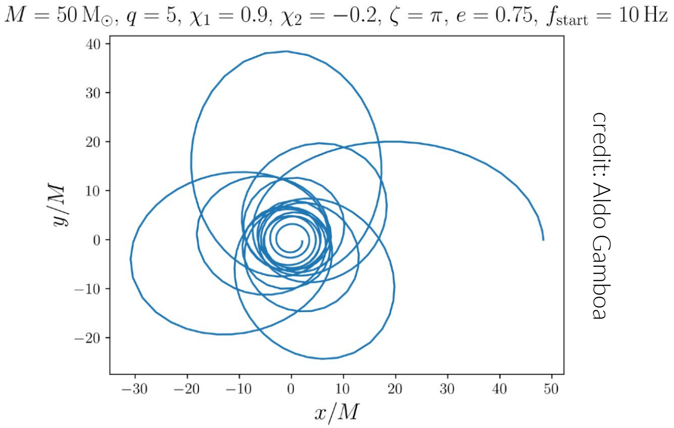
- First PM-based IMR model for spin-aligned BBHs in quasi-circular orbits: SEOBNR-PM.
- Main new feature is EOB Hamiltonian including state-of-the-art PM results, complemented by PN bound-orbit corrections.
- Excellent agreement with NR for the binding energy and the waveform, without NR calibration of the EOB Hamiltonian.

[Buonanno, Mogull, Patil & LP, 24]

Physical completeness: beyond circular orbits

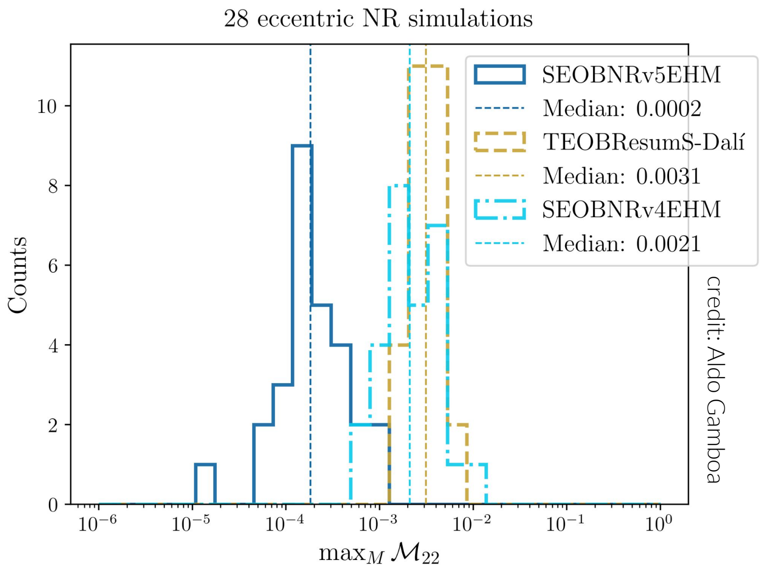
Significant progress in modeling binaries in eccentric/scattering orbits: waveform models for BBHs in generic orbits with aligned-spins and higher-modes have reached a mature stage.

- SEOBNRv5EHM: extension of SEOBNRv5HM to eccentric orbits, including 3PN eccentric corrections to waveform modes and RR force.



Gamboa+ in prep.

See also talk from Antoni Ramos-Buades



- Validated against 28 eccentric public NR simulations from the SXS collaboration with e<0.3.
- Important to extend to generic orbits and generic spins: need to model both effects to confidently distinguish eccentricity from spin precession.

Bustillo+ 20, Romero-Shaw+22, Gupte+ 24

Physical completeness: beyond vacuum-GR

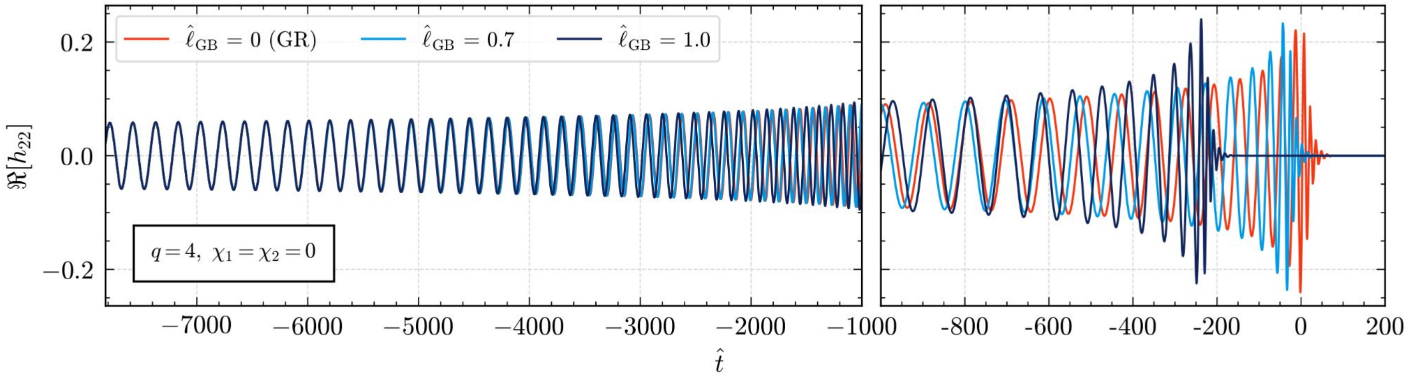
Accurate modeling beyond vacuum-GR needed to fully exploit LISA's potential:

- By modeling the binary's dynamics, EOB is a natural framework to include beyond-vacuum-GR effects.
- First beyond-GR IMR waveform, in Einstein-scalar-Gauss-Bonnet (EsGB) gravity, based on SEOBNRv5 framework:
  - Analytical PN corrections to EOB Hamiltonian, GW modes and flux, scalar flux
  - Corrections to QNM spectrum, estimate of corrections to remnant mass and spin from EOB dynamics
  - Parameterized deviations to marginalize over uncertainty in the merger (no NR calibration yet)

Julié, LP & Buonanno, 24

Julié+ 23, Jain+ 23, Sennett+ 16, Bernard+ 22, Julié+ 22, Chung+ 24

MBHB not ideal sources for testing EsGB (corrections scale with $1/M^4$), but shows the flexibility of the framework: similar strategy to include environmental effects in EOB models?



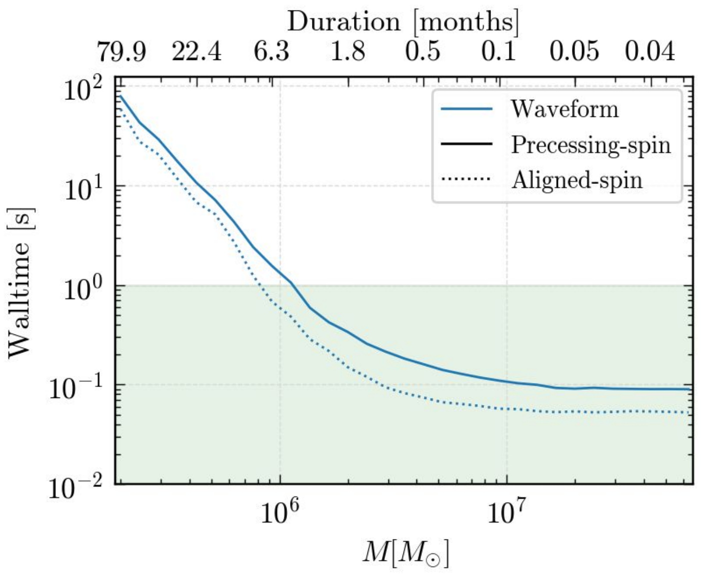
[Julié, LP & Buonanno, 24]

The efficiency challenge

Are SEOBNRv5 waveforms efficient enough to do PE with LISA?

- Benchmarks: f_{min} = 0.1 mHz, sampling rate to resolve the ringdown of (4,4) mode, including FFT.
  - Fast enough for heavy binaries ($M_{det}>10^6\ M_{\odot}$)
  - Cost prohibitive for lower masses (longer signals, higher sampling rate).

What's the bottleneck?



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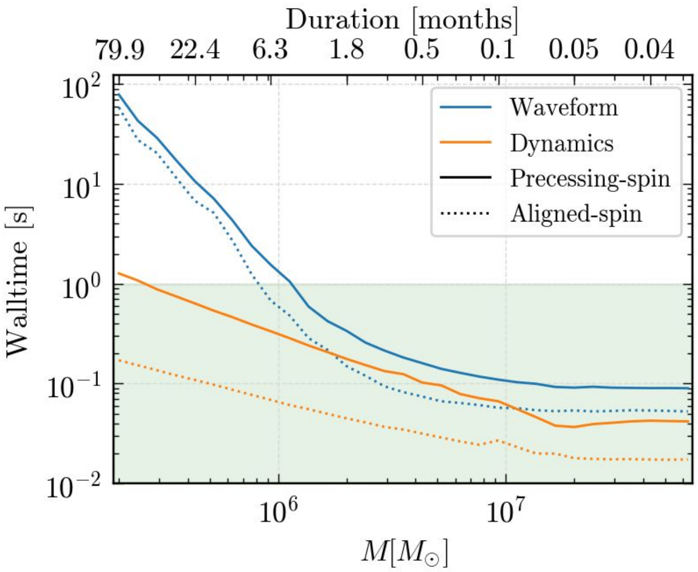


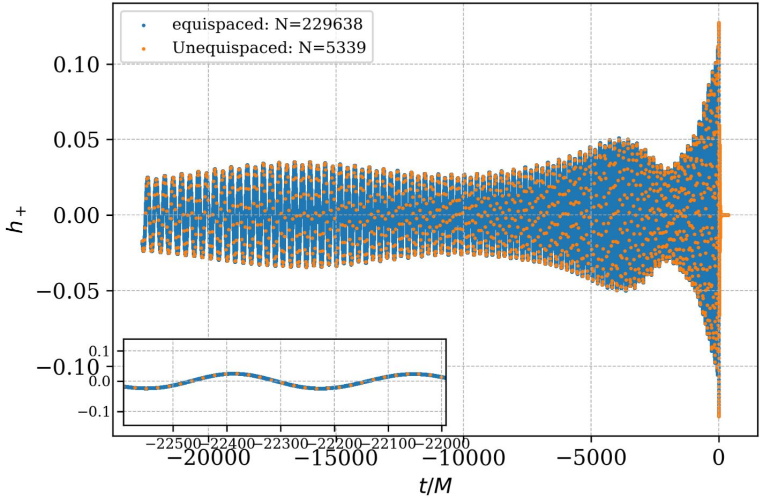
What's the bottleneck?

- Waveform vs dynamics:
  - ODE integration main cost only for high masses
  - Bottleneck for low masses is interpolation with uniform time-step, required to take FFT: issue not just for EOB, common to all time-domain models
  - Two orders of magnitude difference: lots of potential for improvement!

The efficiency challenge

- Waveform acceleration:
  - Non-uniform interpolation: no need to resolve the full waveform at the sampling rate of the ringdown. Could allow for sparse FD representation if combined with non-uniform FFT.
  - Take inspiration from EMRIs: interpolation parallelized on GPUs in Fast EMRI Waveforms. [Katz+ 21]
  - ROM/surrogates with ML: promising results, but extremely challenging to cover 9D parameter space with the required accuracy with current techniques.

    [Chua+ 19, Khan+ 20, Schmidt+ 21, Thomas+ 22, Grimbergen+ 24]



(credit: Héctor Estellés)

- PE acceleration:
  - Simulation-based inference with ML (DINGO): applied to LVK BBHs including spin-precession and eccentricity, promising results on long BNS signals. [Dax, Green+ 22, Gupte+ 24, Dax, Green+ 24]
  - Promising results on fast MCMC with JAX (automatically differentiable) waveforms. [Wong+ 23, Edwards+ 23, Wouters+ 24]

Conclusions

To fully exploit LISA's potential and avoid erroneous scientific conclusions, it's crucial to improve the accuracy of GW models in vacuum GR and incorporate all relevant physical effects.

SEOBNR is a flexible framework to produce physically complete waveforms in vacuum GR and beyond.

- Important to extend to generic orbits and generic spins. Some important physical effects still to be included for quasi-circular binaries: GW memory, multipole asymmetries, … [Estellés+ in prep.]
- Natural framework to include beyond-vacuum-GR effects, but more work is needed.
- Crucial to calibrate to spin-precessing and eccentric NR simulations, and to account for uncertainty estimates within NR calibration, even for quasi-circular, aligned-spin binaries.
- PN, PM, GSF should be pushed at higher order and combined in EOB approach in novel ways to improve analytical solutions of two-body problem.
- Important to do waveform systematic studies targeted to LISA, including the LISA response and within the LISA global fit.
- Speed remains a challenge, but many new tools and approaches are promising (ML, hardware acceleration, …). Some challenges similar to EMRIs, more interactions about waveform implementation would be very beneficial.

Backup slides

SEOBNRv5 highlights: EOB Hamiltonian

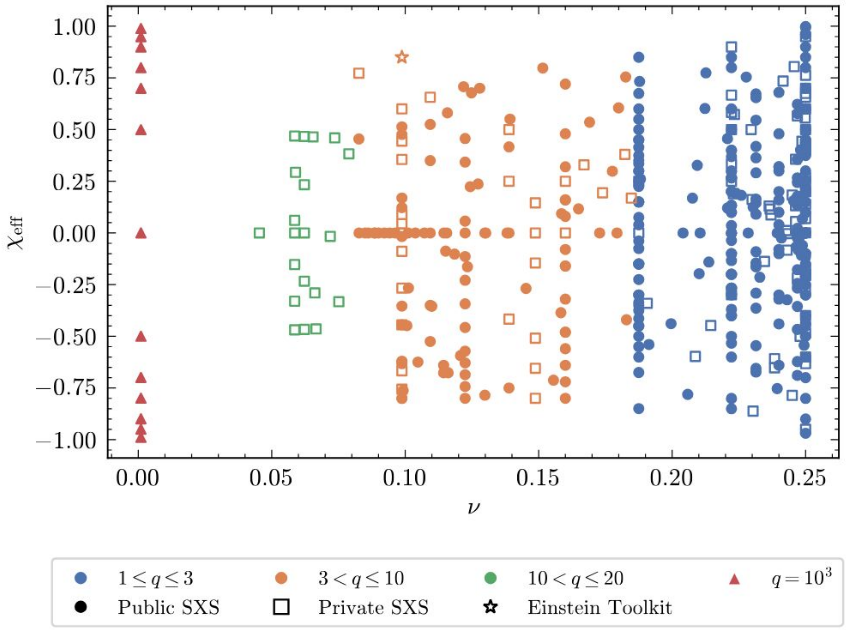
TABLE II. Summary of the main differences of the SEOBNRv5 Hamiltonian derived here, which builds on the results of Refs. [103, 104], compared to that of SEOBNRv4 and TEOBResumS.

	SEOBNRv5	SEOBNRv4 [99, 100, 107, 111]	TEOBResumS [102, 112, 113]
nonspinning part	4PN with partial 5PN in $A_{\rm noS}$ and $\bar{D}_{\rm noS}$, 5.5PN in $Q_{\rm noS}$	4PN in $A_{\rm noS}$, 3PN in $\bar{D}_{\rm noS}$ and $Q_{\rm noS}$	4PN with partial 5PN in $A_{\rm noS}$, 3PN in $\bar{D}_{\rm noS}$ and $Q_{\rm noS}$
$A_{\rm noS}$ resummation	(1,5) Padé	horizon factorization and log resummation	(1,5) Padé
$\bar{D}_{\rm noS}$ resummation	(2,3) Padé	log	Taylor expanded ($D_{\rm noS} \equiv 1/\bar{D}_{\rm noS}$ is inverse-Taylor resummed)
Hamiltonian in the $\nu \to 0$ limit	reduces to Kerr Hamiltonian for a *test mass* in a generic orbit	reduces to Kerr Hamiltonian for a *test spin*, to linear order in spin, in a generic orbit	the A potential reduces to Kerr, but not the full Hamiltonian
spin-orbit part	3.5PN, in (r, L^2) gauge, Taylor expanded	3.5PN, added in the spin map	3.5PN, in (r, p_r^2) gauge, inverse-Taylor resummed
higher-order spin information	NNLO SS (4PN), LO S^3 (3.5PN), LO S^4 (4PN)	LO SS (2PN)	NNLO SS (4PN) for circular orbits
precessing-spin Hamiltonian	yes	yes	no
spin-multipole constants included	yes	no	yes (in the SS contributions for circular orbits)

[Khalil+ inc. LP 23]

SEOBNRv5 highlights: NR calibration

442 aligned-spin NR waveforms and 13 BH perturbation theory waveforms used to calibrate SEOBNRv5HM.



High-order PN terms in EOB Hamiltonian

Merger-ringdown attachment

- Three calibration parameters: θ=(a_6, d_{SO}, Δt_{NR}).
- Bayesian calibration: define a likelihood and sample over calibration parameters for each NR simulation with parameters Λ_i=(q, χ_1, χ_2) → NR-calibration posteriors p(θ|Λ_i).

$$P(\boldsymbol{\theta}) \propto \exp\left[-\frac{1}{2}\left(\frac{\mathcal{M}_{\max}(\boldsymbol{\theta})}{\sigma_{\mathcal{M}}}\right)^2 - \frac{1}{2}\left(\frac{\delta t^{22}_{\rm peak}(\boldsymbol{\theta})}{\sigma_t}\right)^2\right]$$

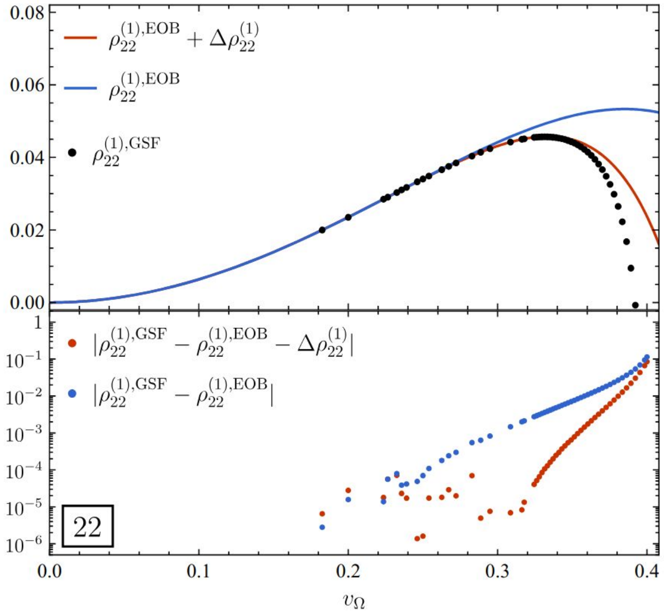
- θ(Λ): hierarchical least-square fits of point-estimate from the posteriors + test-body limit information.
- p(θ|Λ_i): NR-calibration uncertainty for each NR simulation.

[LP+ 23]

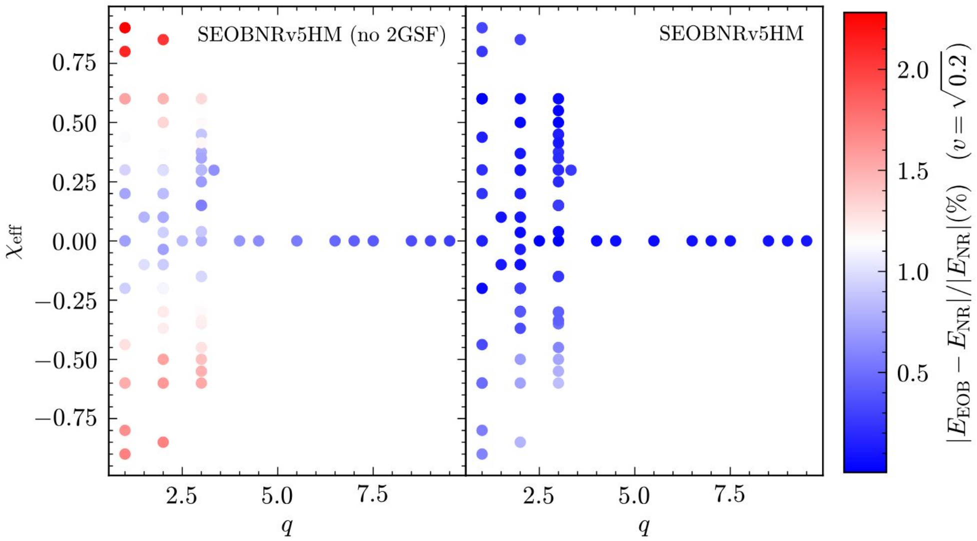
SEOBNRv5 highlights: 2GSF calibration

Improving the factorized EOB modes with 2GSF

$$h_{\ell m}^{\text{insp-plunge}} = h_{\ell m}^{\text{Newt}} e^{-im\phi} S_{\text{eff}} T_{\ell m} e^{i\delta_{\ell m}} \left(\rho_{\ell m}\right)^{\ell} h_{\ell m}^{\text{NQC}}$$



- Improved agreement with NR simulations in the energy flux/RR force after calibration to 2GSF.
- Model with 2GSF corrections also reproduces the NR binding energy much more faithfully.

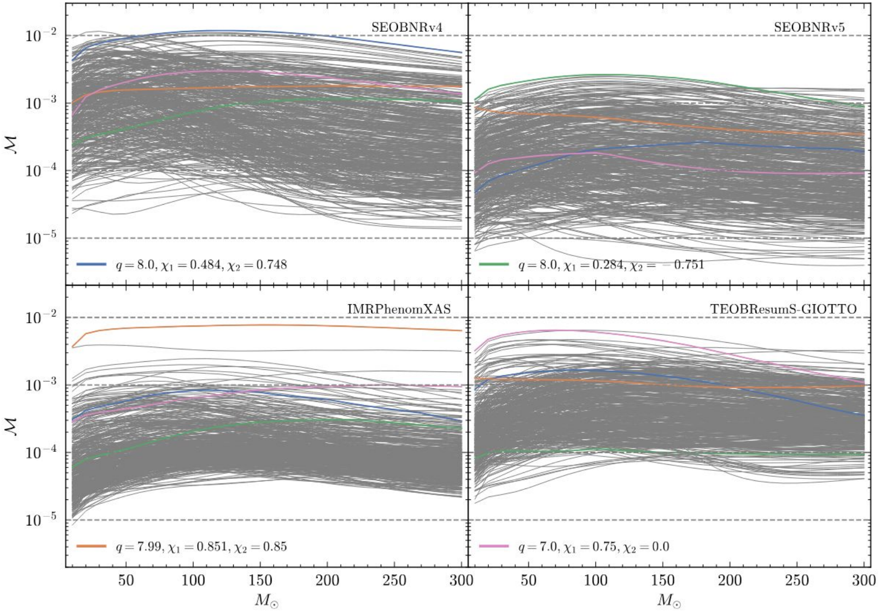


[van de Meent+ inc. LP 23]

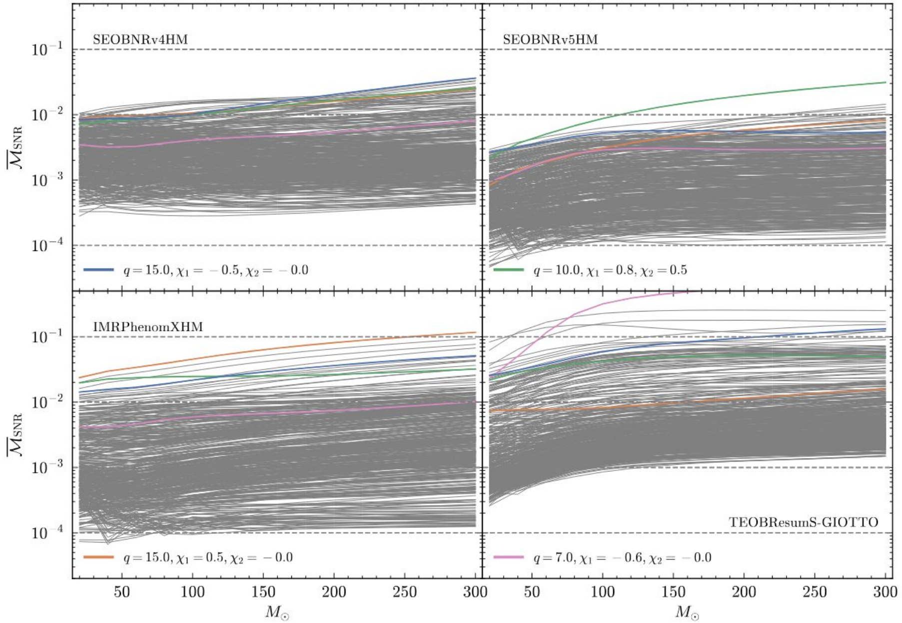
SEOBNRv5 highlights: waveform accuracy

Mismatch against 442 aligned-spin NR simulations.

(2,2) mode



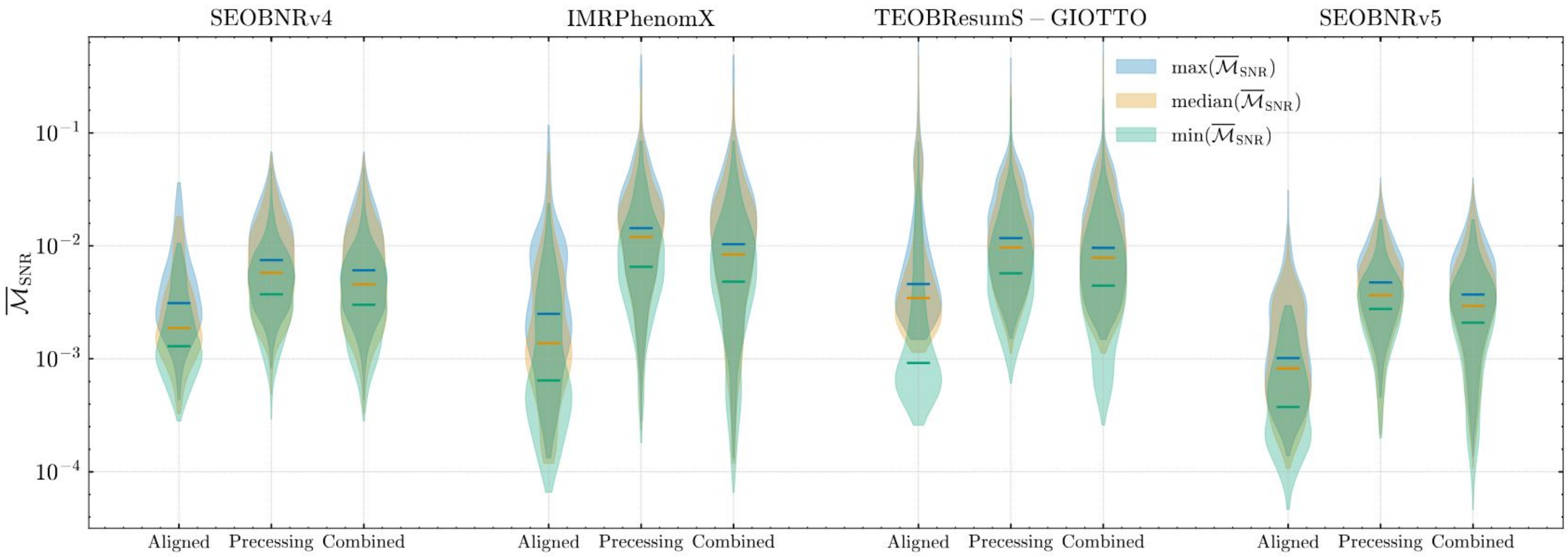
Higher-modes inclination π/3



[LP+ 23]

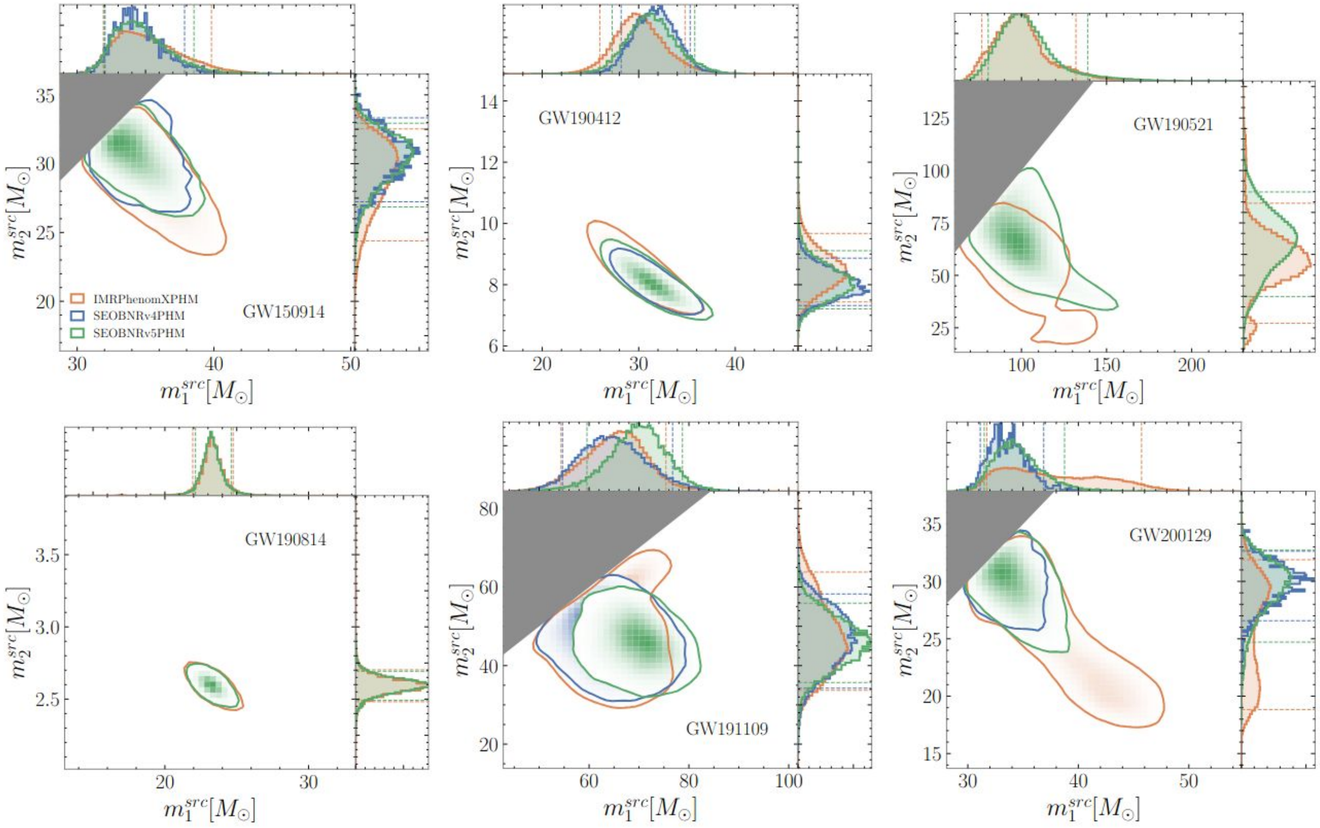
SEOBNRv5 highlights: waveform accuracy

Mismatch against 1543 precessing-spin SXS NR simulations, and 442 aligned-spin NR simulations.



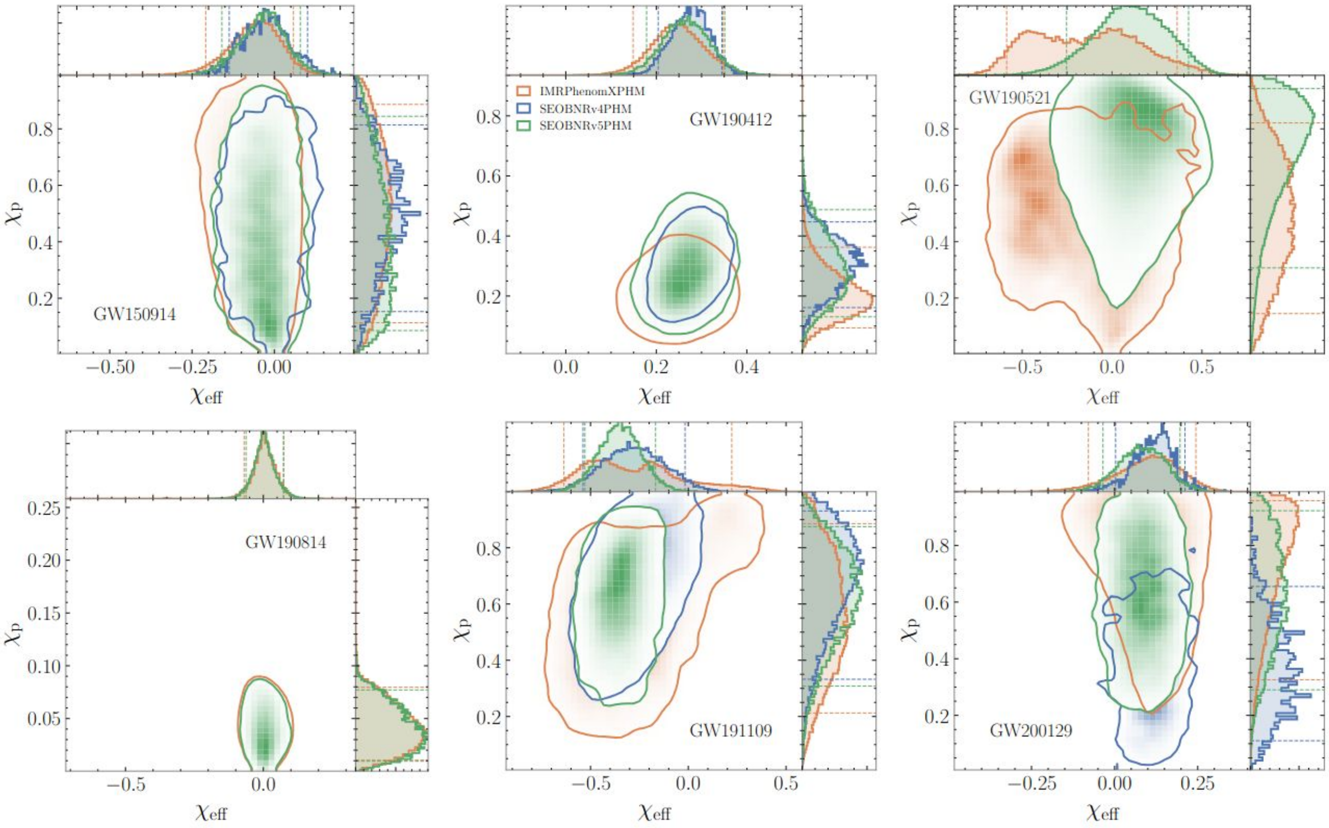
[Ramos-Buades+ inc. LP 23]

SEOBNRv5 highlights: parameter estimation



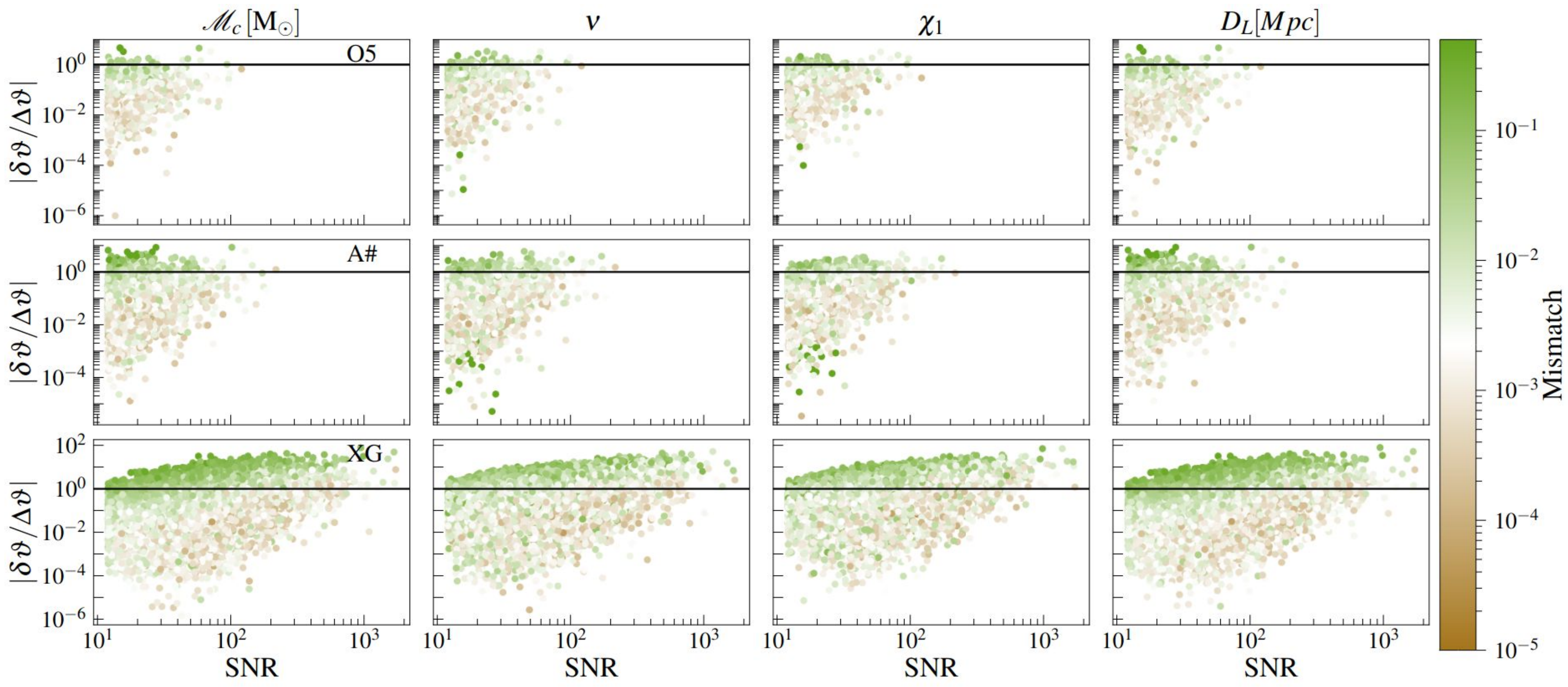
[Ramos-Buades+ inc. LP 23]

SEOBNRv5 highlights: parameter estimation



[Ramos-Buades+ inc. LP 23]

The accuracy challenge



[Dhani+ inc. LP 24]

SEOBNR-PM Hamiltonian

Effective Hamiltonian based on deformation of test-mass in Kerr

Antonelli+ 19, Khalil+ 22, Buonanno+ 24

$$H_{\mathrm{EOB}} = M\sqrt{1+2\nu\left(\frac{H_{\mathrm{eff}}}{\mu}-1\right)}$$

$$a_{\pm} = M\chi_{\pm} = a_1 \pm a_2$$

$$\delta = (m_1 - m_2)/M$$

PM-counting parametrer

$$u = GM/r$$

Odd-in-spin — Even-in-spin

$$H_{\mathrm{eff}} = \frac{M p_\phi \left(g_{a_+} a_+ + g_{a_-}\delta a_-\right)}{r^3 + a_+^2 (r+2M)} + \sqrt{A\left(\mu^2 + \frac{p_\phi^2}{r^2} + \left(1 + B_{\mathrm{np}}^{\mathrm{Kerr}}\right) p_r^2 + B_{\mathrm{npa}}^{\mathrm{Kerr}} \frac{p_\phi^2 a_+^2}{r^2}\right)}$$

$$A = \frac{1 - 2u + \chi_+^2 u^2 + \Delta A}{1 + \chi_+^2 u^2 (2u+1)}, \qquad g_{a_\pm} = \frac{\Delta g_{a_\pm}}{u^2} \qquad B_{\mathrm{np}}^{\mathrm{Kerr}} = \chi_+^2 u^2 - 2u, \quad B_{\mathrm{npa}}^{\mathrm{Kerr}} = -\frac{1+2u}{r^2 + a_+^2(1+2u)},$$

Even-in-spin PM corrections

$$\Delta A = \sum_{n=2}^{5} u^n \Delta A^{(n)} + \Delta A^{4\mathrm{PN}}$$

Odd-in-spin PM corrections

$$\Delta g_{a_\pm} = \sum_{n=2}^{5} u^n \Delta g_{a_\pm}^{(n)}$$

We lack a 5PM term only in the non-spinning case, which we correct with a 4PN tem.

	S^0	S^1	S^2	S^3	S^4	S^5
tree level	1PM	2PM	3PM	4PM	5PM	6PM
1-loop	2PM	3PM	4PM	5PM	6PM	7PM
2-loop	3PM	4PM	5PM	6PM	7PM	8PM
3-loop	4PM	5PM	6PM	7PM	8PM	9PM
4-loop	5PM	6PM	7PM	8PM	9PM	10PM

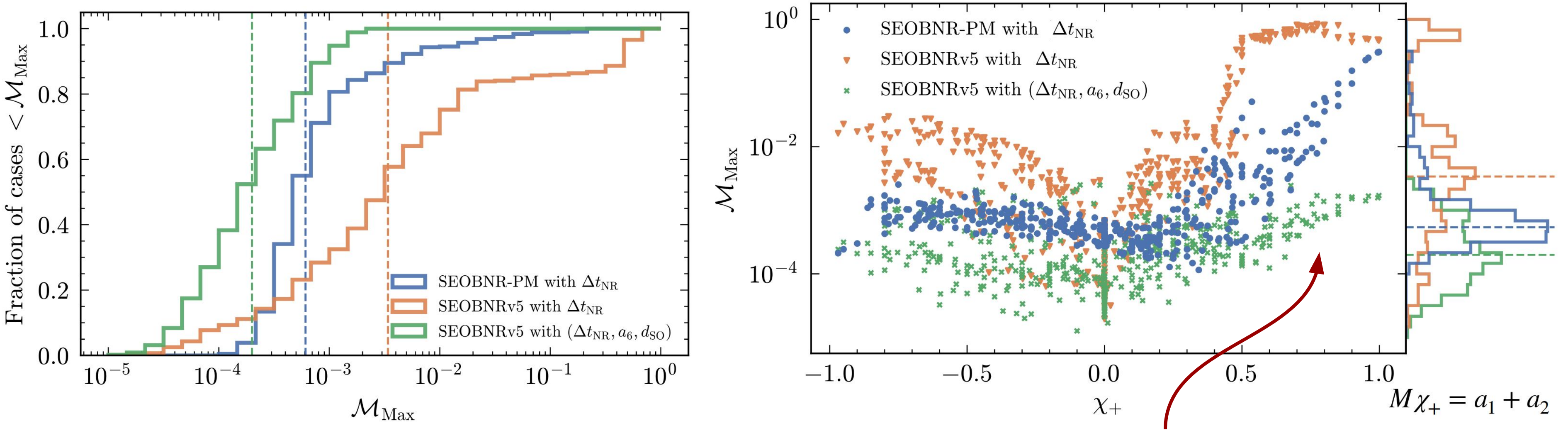
SEOBNR-PM Hamiltonian

Mismatch against 441 aligned-spin NR simulations.

Calibrating only the time to merger Δt_{NR}.

$$h_{\ell m} = \begin{cases} h_{\ell m}^{\mathrm{insp-plunge}}, & t < t_{\mathrm{peak}}^{22}, \\ h_{\ell m}^{\mathrm{merg-RD}}, & t > t_{\mathrm{peak}}^{22}. \end{cases}$$

$$t_{\mathrm{peak}}^{22} = t_{\mathrm{ISCO}} + \Delta t_{\mathrm{NR}}$$



When calibrating only the time to merger Δt_{NR} the accuracy of both SEOBNR-PM and SEOBNRv5 degrades for high positive spins.