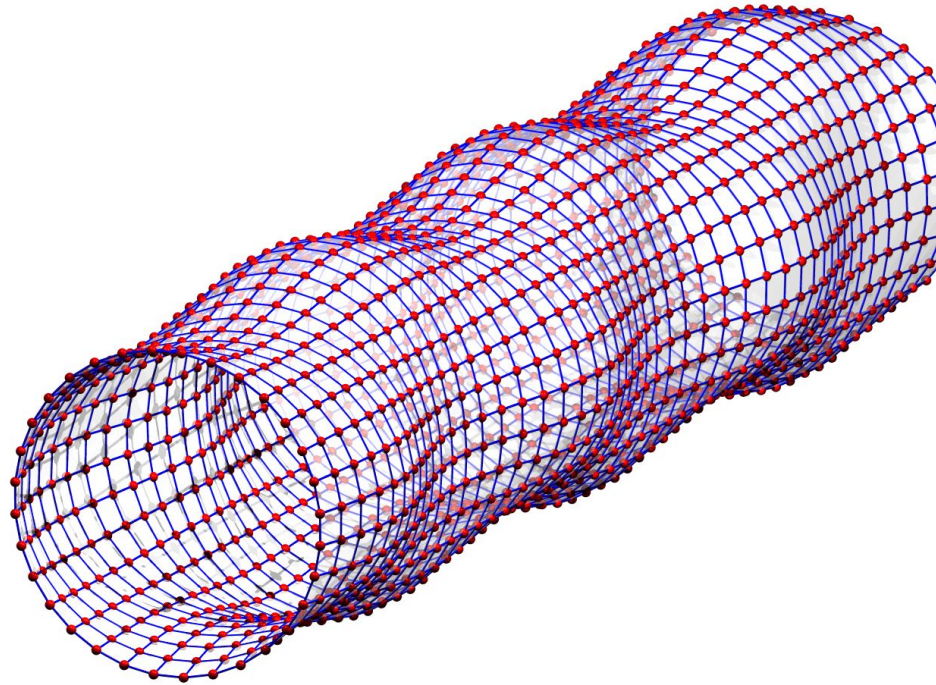


Testing GW polarizations with LISA

Maxence Corman

Coordinators:
Macarena Lagos, Alberto Mangiagli



Collaborative Project

- Motivated by discussions during CosWG workshop 2023
- In collaboration with the Fundamental Physics WG
- ~30 members: theory + data analysis
- Weekly meetings + targeted smaller meetings

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```
wf_extra_pol = {}
for pol in extra_pol_list: # Modify this for extra polarizations
    wf_extra_pol[pol] = {}
    wf_extra_pol[pol]['freq'] = extra_pol_freq.copy()
    if pol=='hp' or pol=='hb': #PAOLA - added hb (LETS ASSUME hn=hp, JUST TO TEST)
        wf_extra_pol[pol]['amp'] = np.abs(Kplus22) * wfhlm[(2,2)]['amp']
        wf_extra_pol[pol]['phase'] = np.angle(Kplus22) * wfhlm[(2,2)]['phase']
    elif pol=='hc':
        wf_extra_pol[pol]['amp'] = np.abs(Kcross22) * wfhlm[(2,2)]['amp']
        wf_extra_pol[pol]['phase'] = np.angle(Kcross22) * wfhlm[(2,2)]['phase']
    else:
        raise ValueError('Key for extra polarization not recognized.')

return wf_extra_pol
```

Organisation of the project

We split the project in two sub-groups: theory and data analysis

Theory:

- Review of the literature
- Collection of existing theories
- Deriving expressions that are currently implemented by the data analysis group

Data analysis:

- Familiarise with lisabeta code
- Run preliminary tests within GR
- First implementation of additional polarisations

Weekly meetings + separate smaller meetings to target specific topics/issues

We created a google doc with the project tasks for people to enroll in



PPE tasks had no one at the beginning so we invited few new people to cover the specific objectives

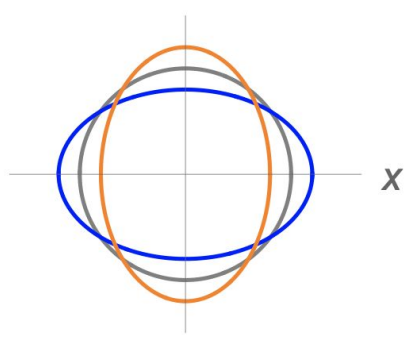
Beyond GR

Lovelock's theorem: GR propagates with two degrees of freedom

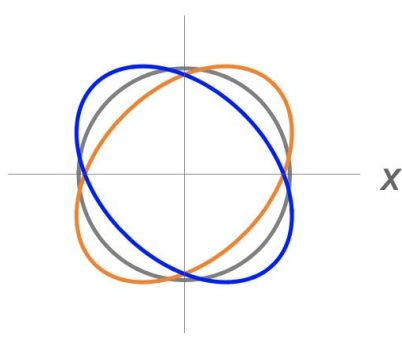


GW Polarizations

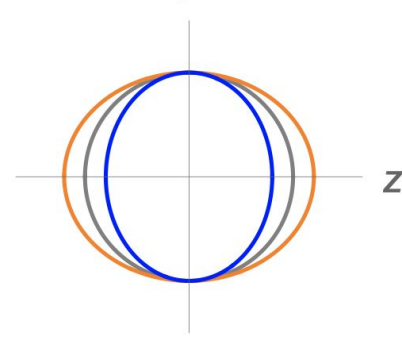
+ mode
 y



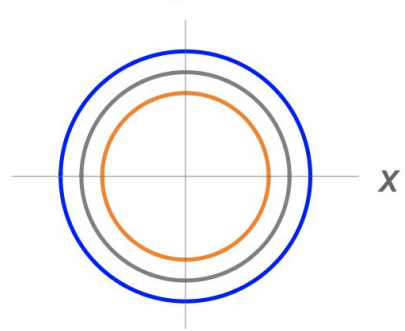
\times mode
 y



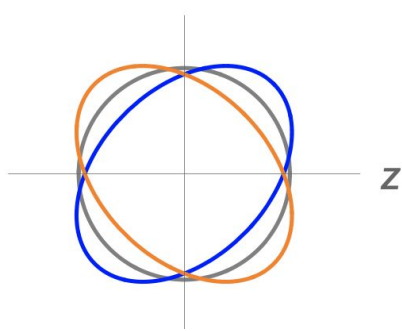
longitudinal mode
 y



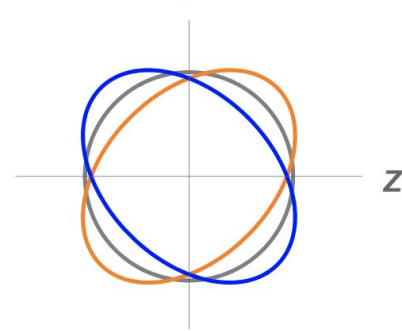
breathing mode
 y



vector- x mode
 x

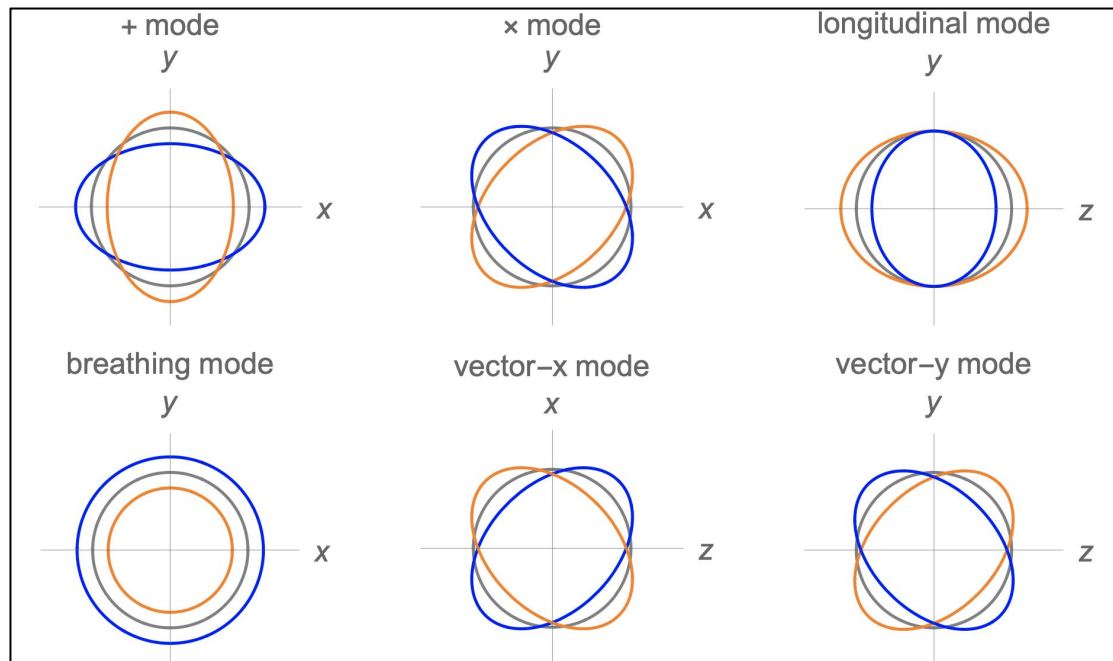


vector- y mode
 y



Six Polarizations

$$h_{ij} = \begin{pmatrix} h_b + h_+ & h_\times & h_{v1} \\ h_\times & h_b - h_+ & h_{v2} \\ h_{v1} & h_{v2} & h_l \end{pmatrix}$$



Theory-agnostic Approach

Goals:

- Develop parametrized model for extra polarizations, during inspiral
- Use specific gravity theories as inspiration:
 - Emission: modified binary evolution + extra polarizations
 - Propagation: assume all polarizations propagate at the same constant speed (EFT arguments)
- Forecast precision on extra polarizations
- Map parametrized model to known gravity theories

Theory-agnostic Approach

Multipolar Decomposition

$$h_+(t) - ih_\times(t) = \sum_{\ell,m} h_T^{(\ell,m)}(t) Y_{-2}^{(\ell,m)}(\iota, \phi_c)$$

$$h_{v_1}(t) - ih_{v_2}(t) = \sum_{\ell,m} h_V^{(\ell,m)}(t) Y_{-1}^{(\ell,m)}(\iota, \phi_c)$$

$$h_{b,l}(t) = \Re \sum_{\ell,m} h_{b,l}^{(\ell,m)}(t) Y^{(\ell,m)}(\iota, \phi_c)$$

PPE Approach

PN expansion

For $\ell=1,2$

[Chatziioannou+ 2012]

T = Tensor , p = {V, b, l}

Emission only!

$$\tilde{h}_T^{(2,2)}(f) = \tilde{h}_{GR}^{(2,2)}(f) (1 + \underline{\alpha} u^a) e^{i \underline{\beta} u^b}$$

$$\tilde{h}_p^{(1,1)}(f) = A^{(1)} \underline{\alpha}_{p1} u_1^{a_{p1}} e^{-i \Psi_{GR}^{(1)}} e^{i \underline{\beta}_1 u_1^b},$$

$$\tilde{h}_p^{(2,2)}(f) = A^{(2)} \underline{\alpha}_{p2} u_2^{a_{p1}+1} e^{-i \Psi_{GR}^{(2)}} e^{i 2 \beta_1 u_2^b},$$

(K. Schumacher, Y. Xie,+)

Connection between PPE and specific theories

For the Tensor part:

Theories	PPE Phase Parameters		Binary Type
	Magnitude (β)	Exp. (b)	
Scalar-Tensor [95, 96]	$-\frac{5}{7168}\eta^{2/5}(\alpha_1 - \alpha_2)^2$	<u>-7</u>	Any
EdGB [97]	$-\frac{5}{7168}\zeta_{\text{EdGB}}\frac{(m_1^2 s_2^{\text{EdGB}} - m_2^2 s_1^{\text{EdGB}})^2}{m^4 \eta^{18/5}}$	<u>-7</u>	Any
DCS [82, 98]	$\frac{481525}{3670016}\eta^{-14/5}\zeta_{\text{DCS}}\left[-2\delta_m\chi_a\chi_s + \left(1 - \frac{4992\eta}{19261}\right)\chi_a^2 + \left(1 - \frac{72052\eta}{19261}\right)\chi_s^2\right]$	-1	BH/BH
Einstein-Æther [99]	$-\frac{5}{3584}\eta^{2/5}\frac{(s_1^{\text{EA}} - s_2^{\text{EA}})^2}{[(1-s_1^{\text{EA}})(1-s_2^{\text{EA}})]^{4/3}}\left[\frac{(c_{14}-2)w_0^3 - w_1^3}{c_{14}w_0^3w_1^3}\right]$	<u>-7</u>	Any
Khronometric [99]	$-\frac{5}{3584}\eta^{2/5}\frac{(s_1^{\text{kh}} - s_2^{\text{kh}})^2}{[(1-s_1^{\text{kh}})(1-s_2^{\text{kh}})]^{4/3}}\sqrt{\bar{\alpha}_{\text{kh}}}\left[\frac{(\bar{\beta}_{\text{kh}}-1)(2+\bar{\beta}_{\text{kh}}+3\bar{\lambda}_{\text{kh}})}{(\bar{\alpha}_{\text{kh}}-2)(\bar{\beta}_{\text{kh}}+\bar{\lambda}_{\text{kh}})}\right]^{3/2}$	<u>-7</u>	Any
Noncommutative [100]	$-\frac{75}{256}\eta^{-4/5}(2\eta - 1)\Lambda^2$	-1	BH/BH
Varying- G [92]	$-\frac{25}{851968}\eta_0^{3/5}\dot{G}_{\text{C},0}\left[11\mathbf{m}_0 + 3(\mathbf{s}_{1,0} + \mathbf{s}_{2,0} - \delta_{\dot{G}})\mathbf{m}_0 - 41(\mathbf{m}_{1,0}\mathbf{s}_{1,0} + \mathbf{m}_{2,0}\mathbf{s}_{2,0})\right]$	-13	Any

Connection between PPE and specific theories

Work in progress for extra polarizations:

Theories	a	b	a_{b1}	a_{l1}	a_{v_11}	a_{v_22}
Scalar-Tensor [62]	-2	-7	-9/2	-21/2	-	-
Einstein-Æther [130]	-2	-7	-9/2	-9/2	-9/2	-9/2
Rosen's theory [27]	-2	-7	-9/2	-9/2	-9/2	-9/2
Lightman-Lee Theory [27]	-2	-7	-9/2	-9/2	-9/2	-9/2
Lorentz-Breaking [cite]	?	?	?	?	?	?

(M. Zhu, S. Akama, L. Perivolaropoulos, A. Nilsson,+)

We also obtain expressions for the mapping to α_{p_1,p_2} and $\beta_{1,2}$ for the extra polarizations

The mapping to PPE formalism for extra polarizations is a new result of this project

How many (extra)parameters to infer?

GR

11 parameters

- 2 masses
- 1 distance
- 2 spin magnitudes
- 2 angles (sky posit.)
- 2 angles (L angular mom.)
- Φ_c, T_c

PPE for Tensor

2 parameters

- α, β

$$\tilde{h}_T^{(2,2)}(f) = \tilde{h}_{GR}^{(2,2)}(f)(1 + \alpha u^a) e^{i\beta u^b}$$

PPE for scalar & vector

7 parameters

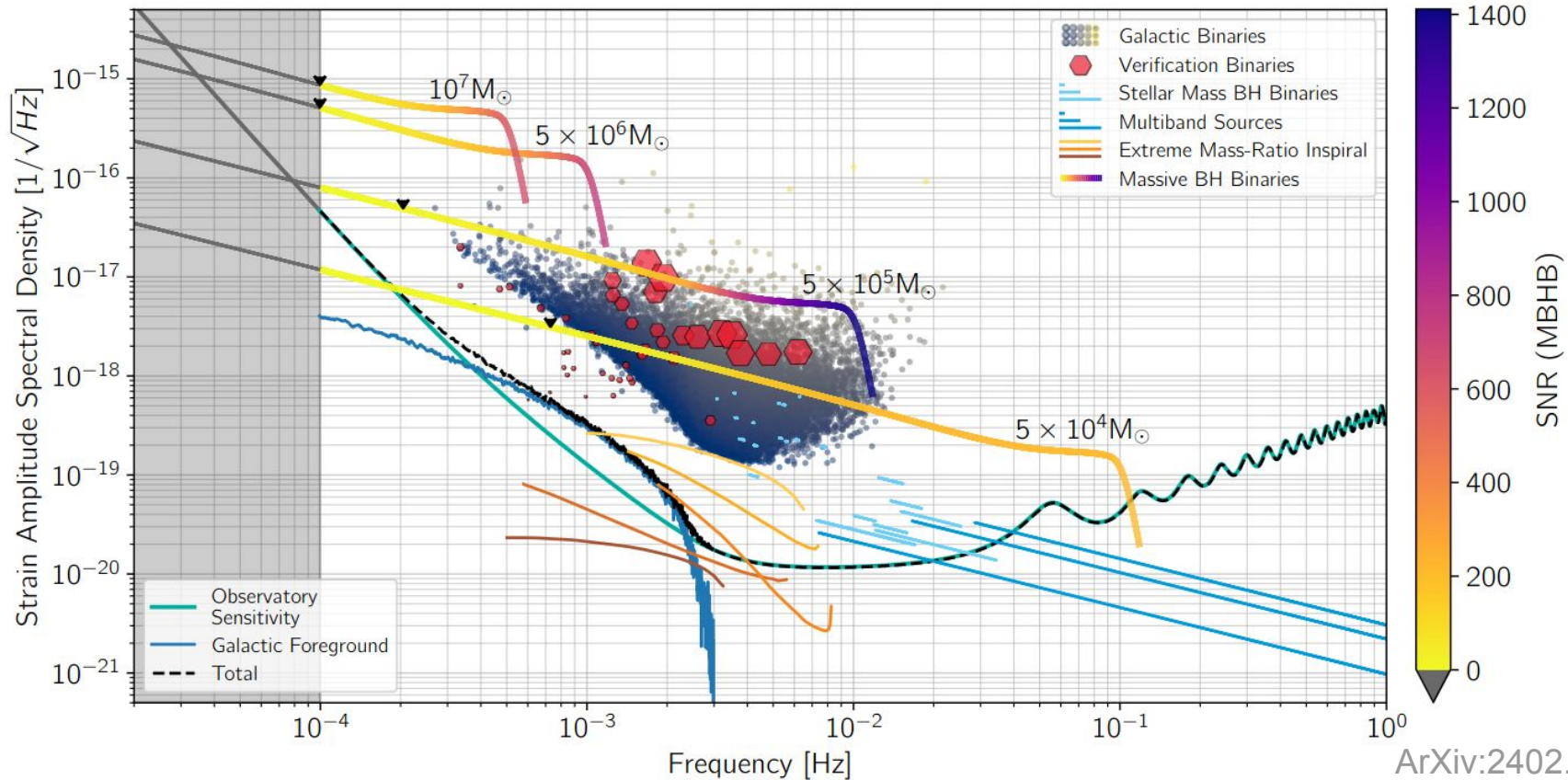
- α_p^j, β_1

$$p = \{v_1, v_2, l, b\} \quad j = \{(1,1), (2,2)\}$$

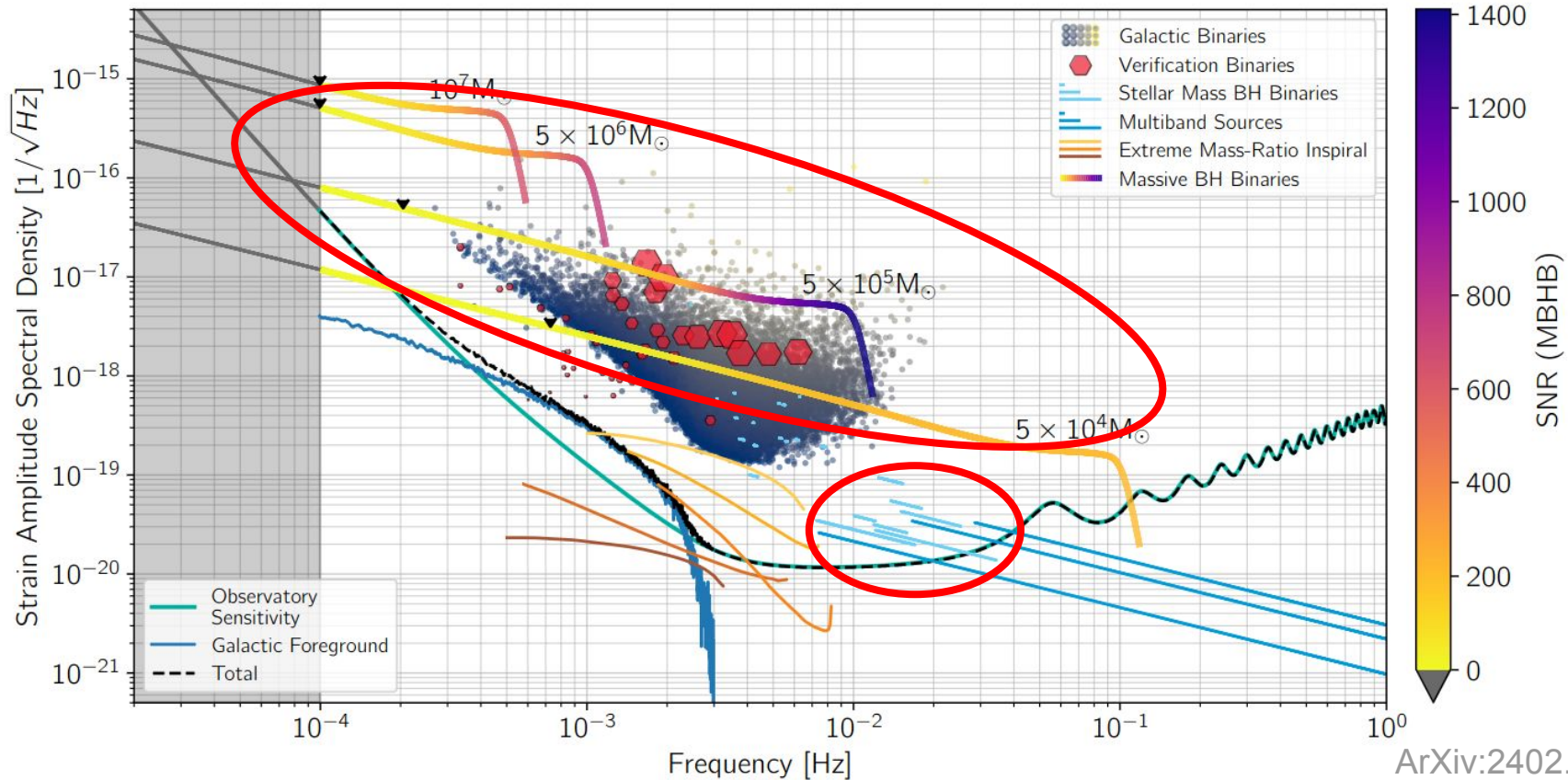
$$\tilde{h}_p^{(1,1)}(f) = A^{(1)} \alpha_{p1} u_1^{a_{p1}} e^{-i\Psi_{GR}^{(1)}} e^{i\beta_1 u_1^b},$$

$$\tilde{h}_p^{(2,2)}(f) = A^{(2)} \alpha_{p2} u_2^{a_{p1}+1} e^{-i\Psi_{GR}^{(2)}} e^{i2\beta_1 u_2^b},$$

Which sources are we targeting?



Which sources are we targeting?

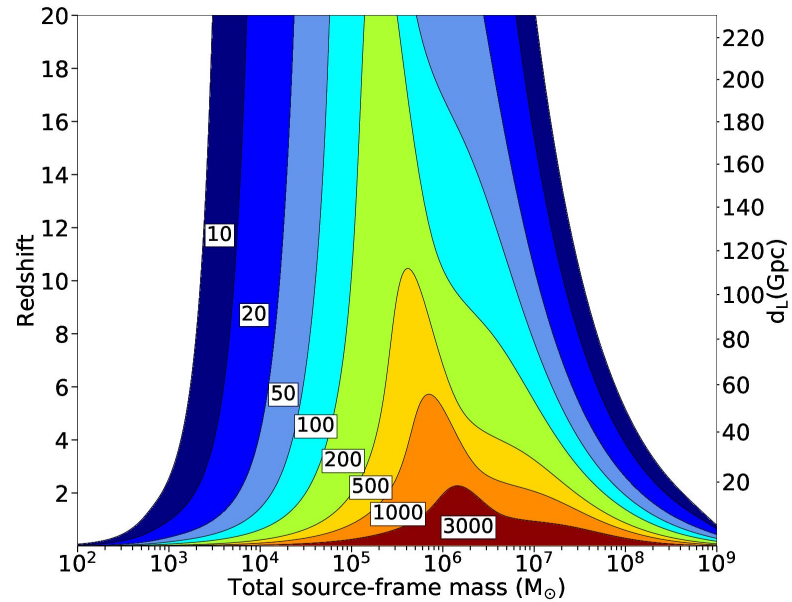


Massive BHBs and Stellar BHBs

Massive BHBs

- ✓ Strong SNR
- ✓ Inspiral-merger-ringdown
- ✗ Short signal (max ~1 month)
- ✗ Not favoured in some theories

$$h_{\text{EdGB}} \sim 1/m_{\text{tot}}^4$$



Massive BHBs and Stellar BHBs

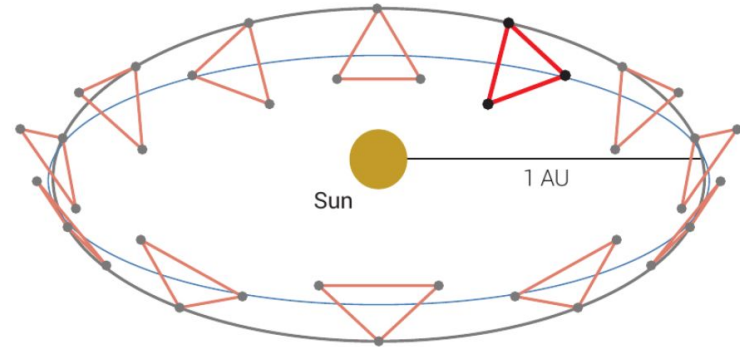
Massive BHBs

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Stellar BHBs

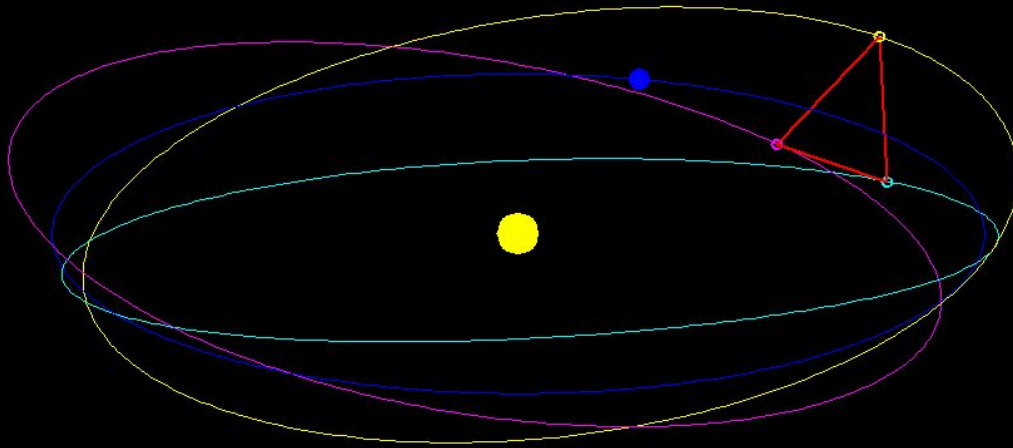
- ✓ Long inspiral (~years)
- ✓ Excellent determination of extrinsic parameters
- ✓ Wavelength comparable to arm length: optimal for scalar polarizations [Tinto+ 2010]
- ✗ Low SNR and not as many



EMRIs?

- ✓ very large number of cycles in band
- ✓ scalar test: Masseli, Barsanti + 2004.11895
- ✗ no waveform at 1PA or parametrized waveforms

Why LISA



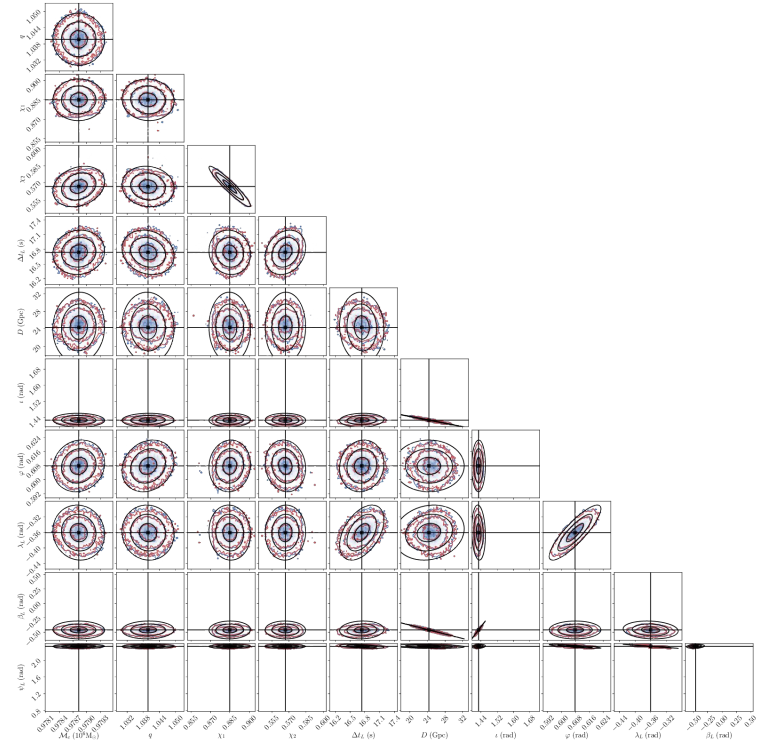
- High SNR: up to 10^3
- Long inspirals: weeks to years [cite]
- Probe 3D GW distortions
- Wavelength comparable to arm length: optimal for scalar polarizations [cite]

Simulate the GW signal

We use *lisabeta* (Marsat+20) to simulate the GW signal from SBHBs and MBHBs

https://gitlab.in2p3.fr/marsat/lisabeta_release

- ✓ IMRPhenomXHM
- ✓ Include low frequency response (motion of the detector) + high frequency response
- ✓ Fisher+Bayesian analysis
- ✓ Repository stored on gitlab.in2p3 → easily accessible to members of the project
- ✓ $h_{+/\times}$ implemented via spherical harmonics → easy extension to extra-polarisations



Analysis setup

Both for MBHBs and SBHBs we plan to assess LISA's capabilities in two ways:

	<u>Exploration of the parameter space</u>	<u>Selected sources</u>
<u>MBHBs</u>	M_{tot} in $[10^5, 10^7] M_{\odot}$ z in $[0.5, 5]$	From LDC Sangria dataset 15 MBHBs
<u>SBHBs</u>	M_{tot} in $[60, 90, 120, 150] M_{\odot}$ d_L in $[100, 200, 300, 400] \text{ Mpc}$	From LDC Yorsh dataset 8 SBHBs (though not representative of a real population)
	<u>Fisher analysis</u>	<u>MCMC runs</u>

Or catalogues: ready for SBHBs (Babak+23), need to ask permission for MBHBs

Inspiral-only or Inspiral-Merger-Ringdown?

For MBHBs, we have two further options: to focus on inspiral-only analysis or to include merger and ringdown

Inspiral-only

- ✓ Non-GR modification is cut when it's still well understood
- ✗ Worse estimates on the binary parameters

Inspiral-Merger-Ringdown

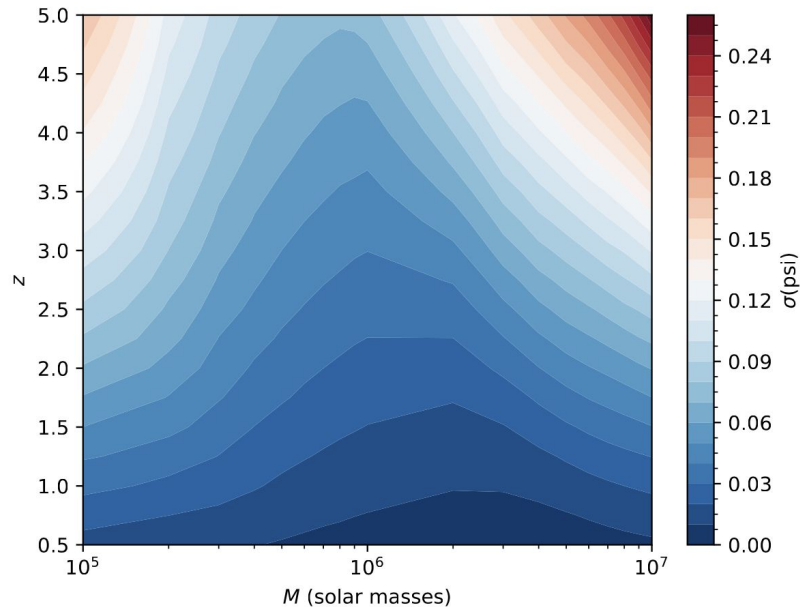
- ✓ Higher harmonics help breaking degeneracies
- ✗ Need to suppress the extra-polarisations at to merger with a window function: spurious effect might be introduced

Current plan: start with the inspiral only and expand the analysis to merger and ringdown if sufficient time is left (window function is implemented but not fully tested)

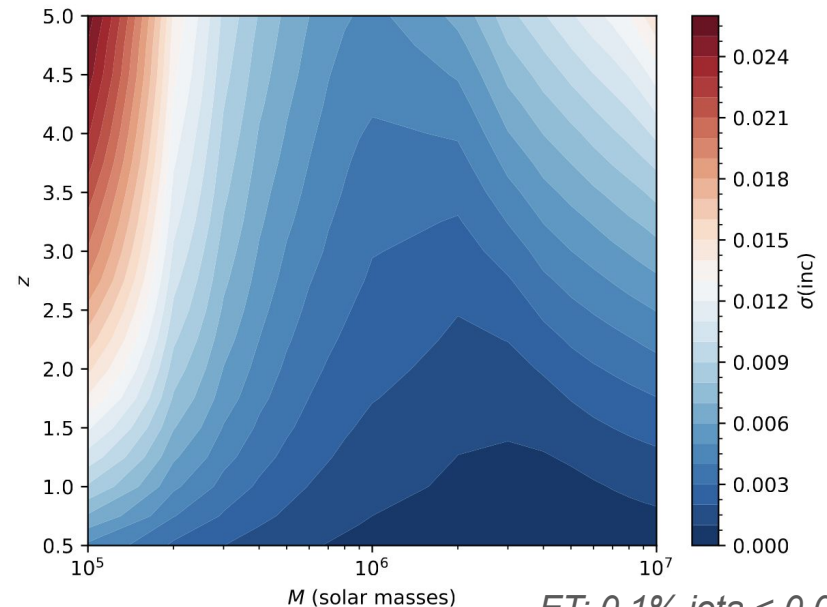
Preliminary results for MBHBs

We checked LISA ability to constrain inclination and polarisation for MBHBs in GR
(P. M. Delgado, G. Orlando, R. Theriault)

Polarization angles: inclination $\iota \in [0, \pi]$ and orientation $\psi \in [0, \pi]$



ET: 1% $\psi < 0.1$ rad

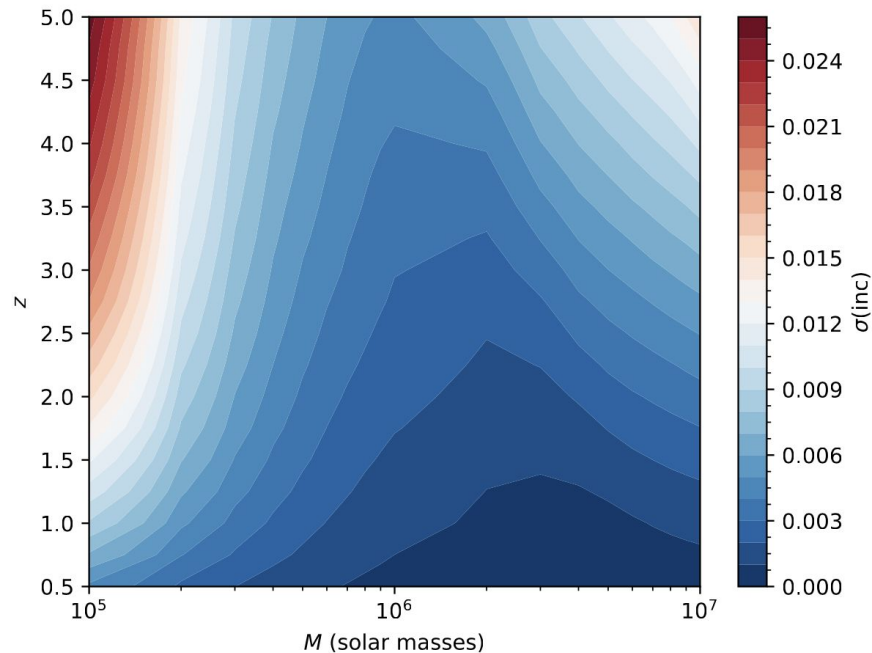
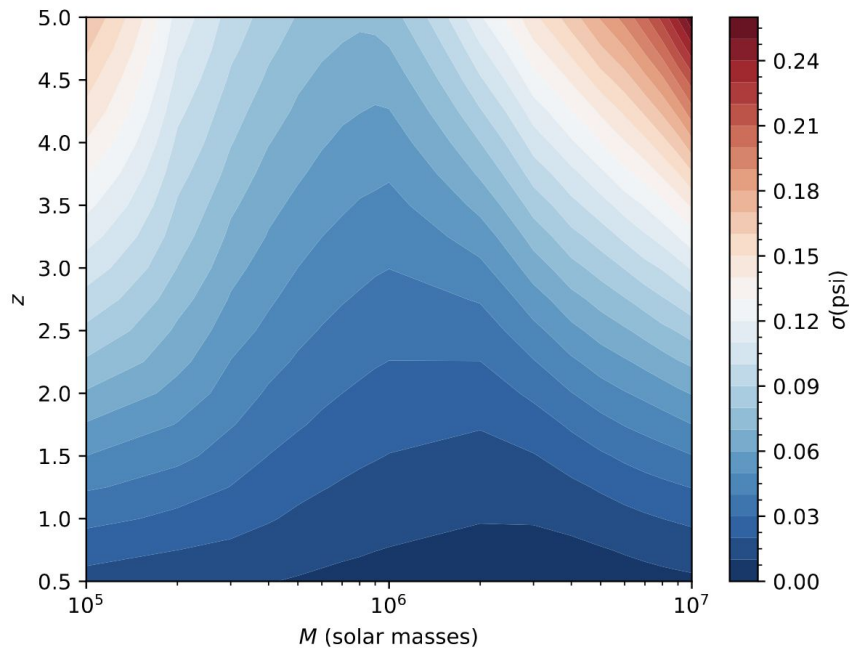


ET: 0.1% $\iota < 0.01$ rad

Results confirmed by few MCMC runs (M. Corman)

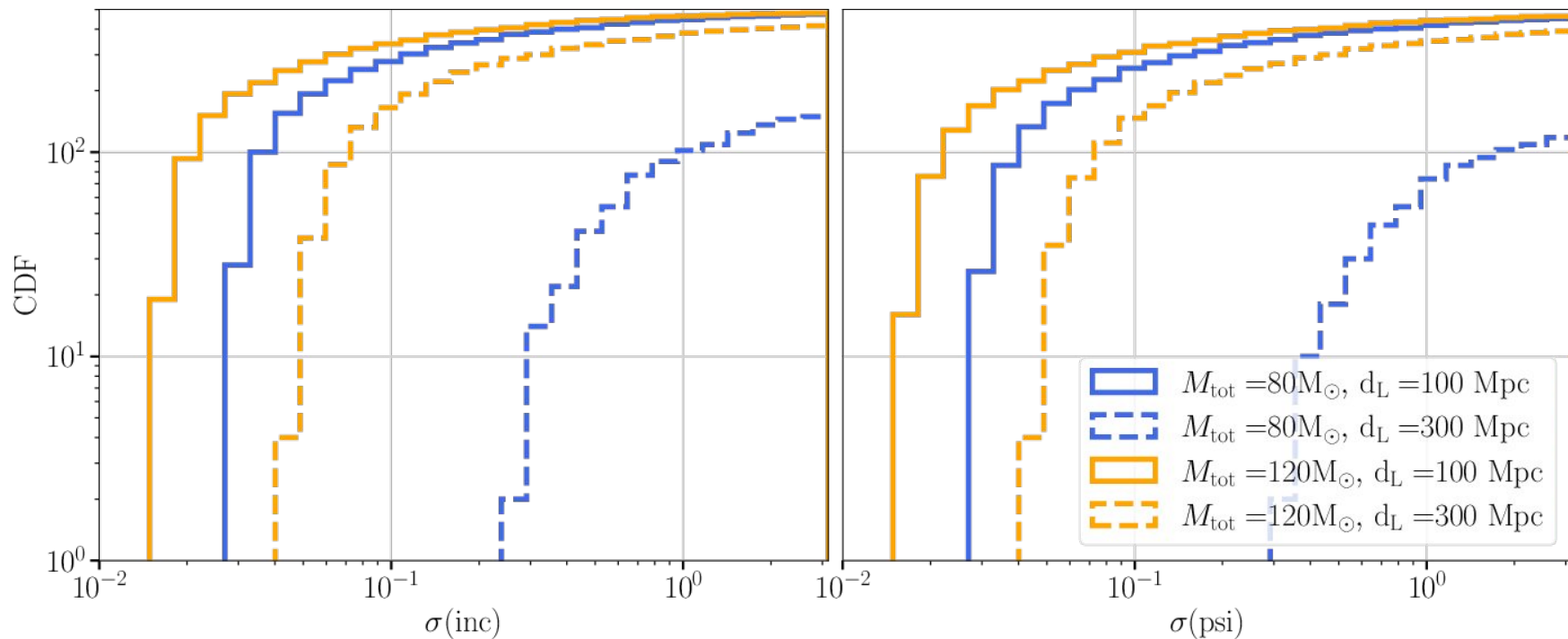
GR polarization precision: MBBHs

Polarization angles: inclination $\iota \in [0, \pi]$ and orientation $\psi \in [0, \pi]$



Preliminary results for SBHBs

As in the previous slide but for SBHBs
(B. Sutton, A. Verma)



Current status for data analysis part

Completed tasks:

- Extension of LISA response to extra-polarisations (S. Marsat)
- Implementation of the EdGB model (M. Corman)
- Implementation of scalar, vector and tensor polarizations (M. Corman)
- Implementation of window function for merger and ringdown (M. Corman, M. Piarulli)

Work in progress:

- Improvement of user interface to increase code attractiveness during and (eventually) after the project (M. Corman)
- Expansion of fisher tools for extra-polarisations parameters (P. M. Delgado)
- Preliminary tests of new implementations (P. M. Delgado, M. Corman, M. Piarulli)

Conclusion

The project is proceeding smoothly, weekly calls are well attended and we have improvements at each iterations

Not (yet) any strong feedbacks on the new rules of the CosWG for collaborative projects

Future steps:

- Finalise the preliminary steps
- EdGB for SBBHs, and comparison to MBBHs
- Tests of scalar and vector polarizations
- Comparison with EMRIs
- Comparison to related works: e.g. on null channels [arXiv:2102.03972] or using DWDs [arXiv:2208.10831]

Conclusion

Project is still ongoing so feel free to contact Maca or Alberto if you're interested in participating

Macarena Lagos: mal2346@columbia.edu

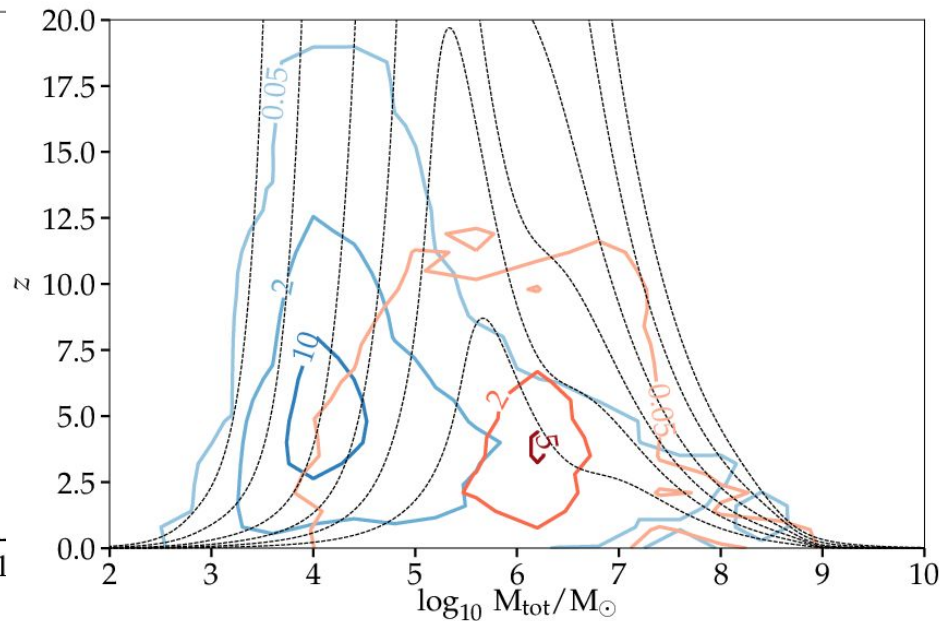
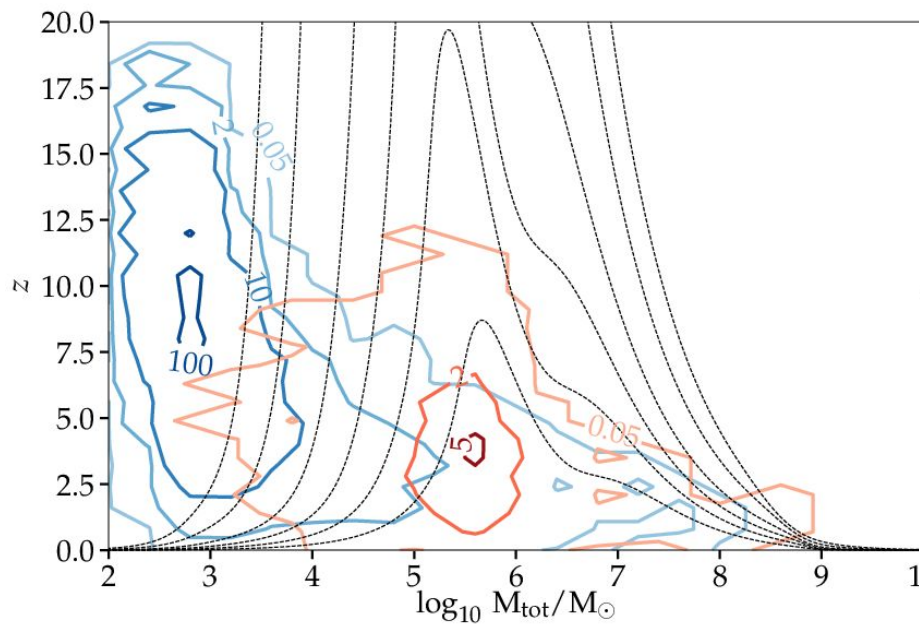
Alberto Mangiagli: mangiagli@apc.in2p3.fr

Or chat with me during one of the coffee breaks!

Thanks! Any questions?

Backup slides

MBHBs mass and redshift distributions



Blue: Light seeds
Red: Heavy seeds

Contour lines: $\frac{dN}{d \log_{10} M_{\text{tot}} dz dt} \times 4 \text{yr}$