# High-accuracy binary modelling in scalar-tensor theories of gravity using the self-force approach

Andrew Spiers (he/him)

Collaborators: Thomas Sotiriou and Andrea Maselli

University of Nottingham

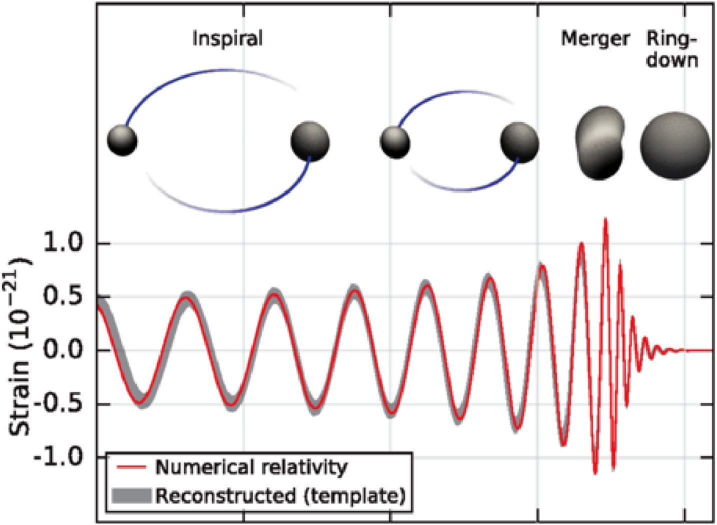
Wednesday 4$^{\text{th}}$ September, 2024

# Overview

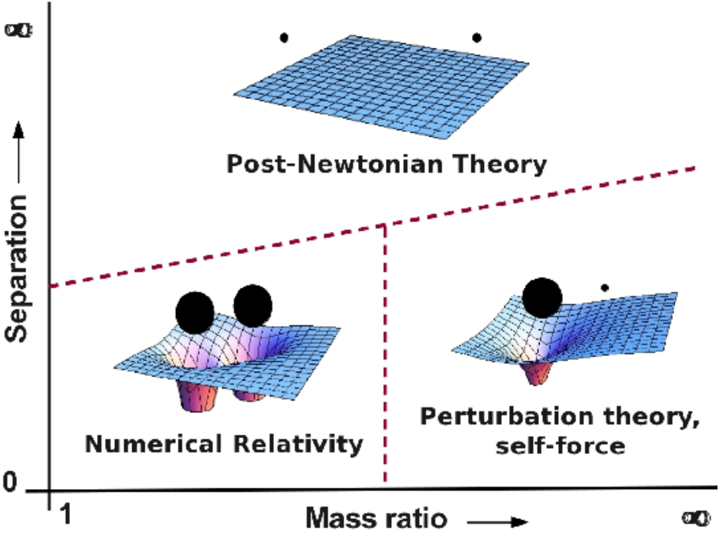
1. Modelling binaries with perturbation theory using self-force

# Overview

1. Modelling binaries with perturbation theory using self-force
2. Generalise to scalar-tensor theories of gravity

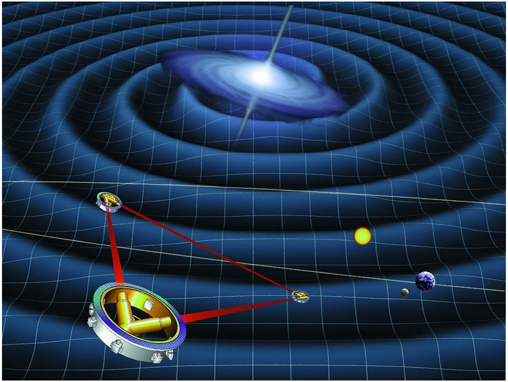



[B.P. Abbott et al. Phys. Rev. Lett., 119(16):161101, 2017]




[L. Barack, DOI:10.1007/978-3-319-06349-2 Chap. 6 (2014).]

# Extreme-Mass-Ratio Inspirals

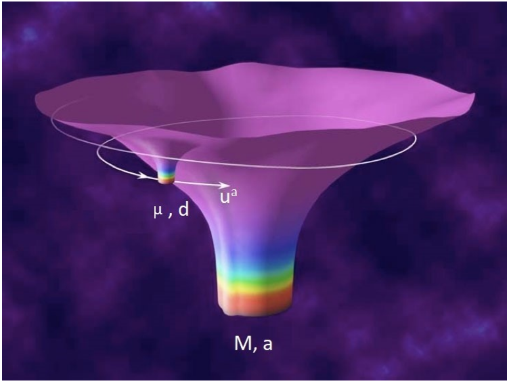


[Source: NASA, http://lisa.jpl.nasa.gov/gallery/lisa-waves.html.]

Multi-year long measurements with space-filling orbits:

- **Precise** measurements
- Potential world-leading tests of General Relativity

# Approximating the spacetime

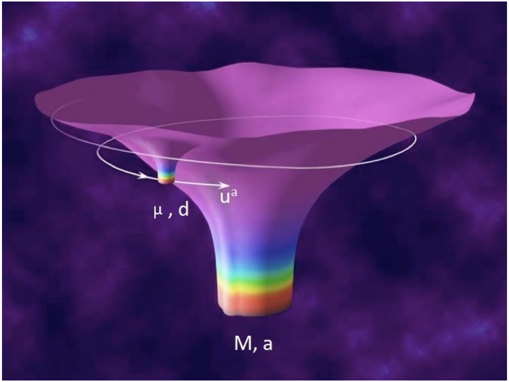



[NASA website]

$$\mathbf{g}_{ab} = g^{(0)}_{ab} + \varepsilon h^{(1)}_{ab} + \varepsilon^2 h^{(2)}_{ab} + \mathcal{O}(\varepsilon^3)$$

where $\varepsilon = \frac{\mu}{M} \ll 1$

# Approximating the spacetime



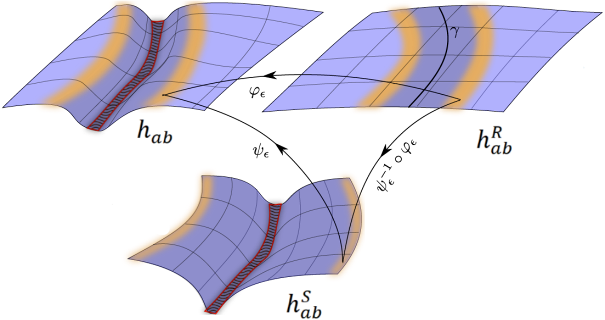

[NASA website]

$$\mathbf{g}_{ab} = g^{(0)}_{ab} + \varepsilon h^{(1)}_{ab} + \varepsilon^2 h^{(2)}_{ab} + \mathcal{O}(\varepsilon^3)$$

where $\varepsilon = \frac{\mu}{M} \ll 1$

$$ma^\alpha = \varepsilon F^\mu_{(1)}[h^{(1)}_{ab}] + \varepsilon^2 F^\mu_{(2)}[h^{(2)}_{ab}] + \mathcal{O}(\varepsilon^3)$$

# Regular and Singular fields

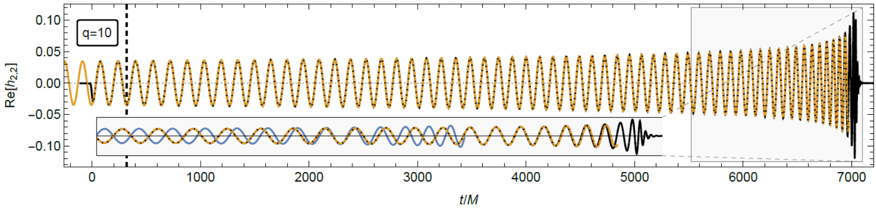



[image: Adam Pound]

$$ma^\alpha = \varepsilon F^\mu_{(1)}[h^{R(1)}_{ab}] + \varepsilon^2 F^\mu_{(2)}[h^{R(2)}_{ab}] + \mathcal{O}(\varepsilon^3)$$

[Detweiler & Whiting, PRD 67, 024025 (2003)]

# Why include second-order self-force?




[Wardell et al. PRL. 130.24 (2023): 241402]

# Self-force from the action in GR

$$S[\mathbf{g}_{ab}, \Psi] = S_0[\mathbf{g}_{ab}] + S_{\mathrm{m}}[\mathbf{g}_{ab}, \Psi],$$

[AS, Maselli & Sotiriou, Phys.Rev.D 109 (2024) 6, 064022],
[Detweiler, PRD 85, 044048 (2012)],
[Upton & Pound, PRD 103, 124016 (2021)]

# Self-force from the action in GR

$$S[\mathbf{g}_{ab}, \Psi] = S_0[\mathbf{g}_{ab}] + S_{\mathrm{m}}[\mathbf{g}_{ab}, \Psi],$$

$$S_0[\mathbf{g}_{ab}] = \int \frac{\sqrt{-\mathbf{g}}}{16\pi} R \; d^4x$$

[AS, Maselli & Sotiriou, Phys.Rev.D 109 (2024) 6, 064022],
[Detweiler, PRD 85, 044048 (2012)],
[Upton & Pound, PRD 103, 124016 (2021)]

# Self-force from the action in GR

$$S[\mathbf{g}_{ab}, \Psi] = S_0[\mathbf{g}_{ab}] + S_{\rm m}[\mathbf{g}_{ab}, \Psi],$$

$$S_0[\mathbf{g}_{ab}] = \int \frac{\sqrt{-\mathbf{g}}}{16\pi} R \, d^4x$$

$$S_{\rm m} = -\int_\gamma \mu ds$$

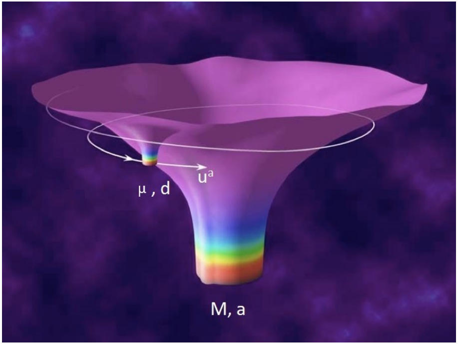
[AS, Maselli & Sotiriou, Phys.Rev.D 109 (2024) 6, 064022],
[Detweiler, PRD 85, 044048 (2012)],
[Upton & Pound, PRD 103, 124016 (2021)]

# Self-force from the action in GR

$$S[\mathbf{g}_{ab}, \Psi] = S_0[\mathbf{g}_{ab}] + S_{\rm m}[\mathbf{g}_{ab}, \Psi],$$

$$S_0[\mathbf{g}_{ab}] = \int \frac{\sqrt{-\mathbf{g}}}{16\pi} R \, d^4x$$

$$S_{\rm m} = -\int_\gamma \mu ds$$



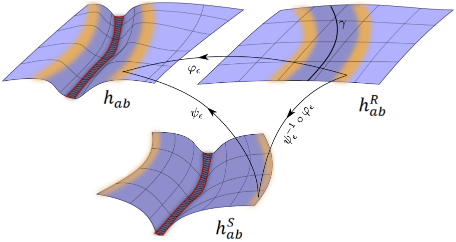

[AS, Maselli & Sotiriou, Phys.Rev.D 109 (2024) 6, 064022],
[Detweiler, PRD 85, 044048 (2012)],
[Upton & Pound, PRD 103, 124016 (2021)]

# Self-force from the action in GR

$$S[\mathbf{g}_{ab}, \Psi] = S_0[\mathbf{g}_{ab}] + S_{\mathrm{m}}[\mathbf{g}_{ab}, \Psi],$$

$$S_0[\mathbf{g}_{ab}] = \int \frac{\sqrt{-\mathbf{g}}}{16\pi} R \, d^4x$$

$$h_{ab}^{(n)} = h_{ab}^{(n)S} + h_{ab}^{(n)R}$$




[AS, Maselli & Sotiriou, Phys.Rev.D 109 (2024) 6, 064022],
[Detweiler, PRD 85, 044048 (2012)],
[Upton & Pound, PRD 103, 124016 (2021)]

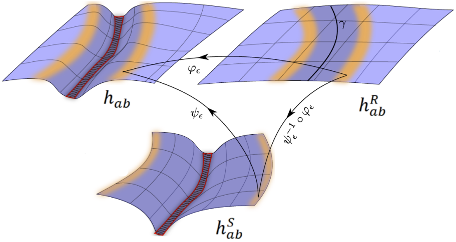
# Self-force from the action in GR

$$S[\mathbf{g}_{ab}, \Psi] = S_0[\mathbf{g}_{ab}] + S_{\rm m}[\mathbf{g}_{ab}, \Psi],$$

$$S_0[\mathbf{g}_{ab}] = \int \frac{\sqrt{-\mathbf{g}}}{16\pi} R \, d^4x$$

$$h^{(n)}_{ab} = h^{(n)S}_{ab} + h^{(n)R}_{ab}$$

$$\tilde{g}_{ab} := g^{(0)}_{ab} + h^{(1)R}_{ab} + h^{(2)R}_{ab} \dots$$




[AS, Maselli & Sotiriou, Phys.Rev.D 109 (2024) 6, 064022],
[Detweiler, PRD 85, 044048 (2012)],
[Upton & Pound, PRD 103, 124016 (2021)]

# Self-force from the action in GR

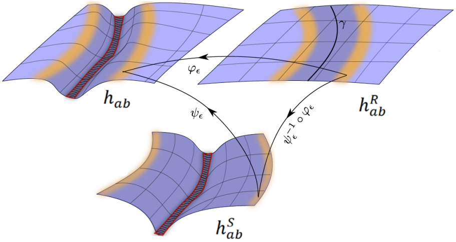
$$S[\mathbf{g}_{ab}, \Psi] = S_0[\mathbf{g}_{ab}] + S_{\mathrm{m}}[\mathbf{g}_{ab}, \Psi],$$

$$S_0[\mathbf{g}_{ab}] = \int \frac{\sqrt{-\mathbf{g}}}{16\pi} R \; d^4x$$

$$h_{ab}^{(n)} = h_{ab}^{(n)S} + h_{ab}^{(n)R}$$

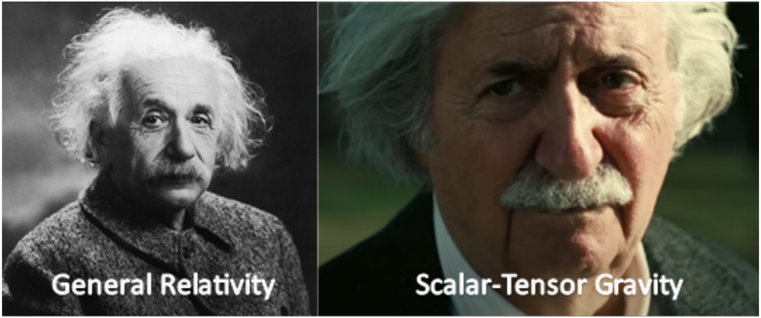
$$\tilde{g}_{ab} := g_{ab}^{(0)} + h_{ab}^{(1)R} + h_{ab}^{(2)R} \dots$$

$$S_{\mathrm{m}} = -\int_\gamma \mu d\tilde{s}$$




[AS, Maselli & Sotiriou, Phys.Rev.D 109 (2024) 6, 064022],
[Detweiler, PRD 85, 044048 (2012)],
[Upton & Pound, PRD 103, 124016 (2021)]

# Scalar-tensor theories of gravity



# Adding a scalar field

$$
\begin{aligned}
g_{ab} =& g^{(0)}_{ab} + \varepsilon^1 h^{(1)}_{ab} + \varepsilon^2 h^{(2)}_{ab} + \mathcal{O}(\varepsilon^3), \\
\varphi =& \qquad \varepsilon^1 \varphi^{(1)} + \varepsilon^2 \varphi^{(2)} + \mathcal{O}(\varepsilon^3),
\end{aligned}
$$

with $\varphi^{(0)} = 0$

# Adding a scalar field

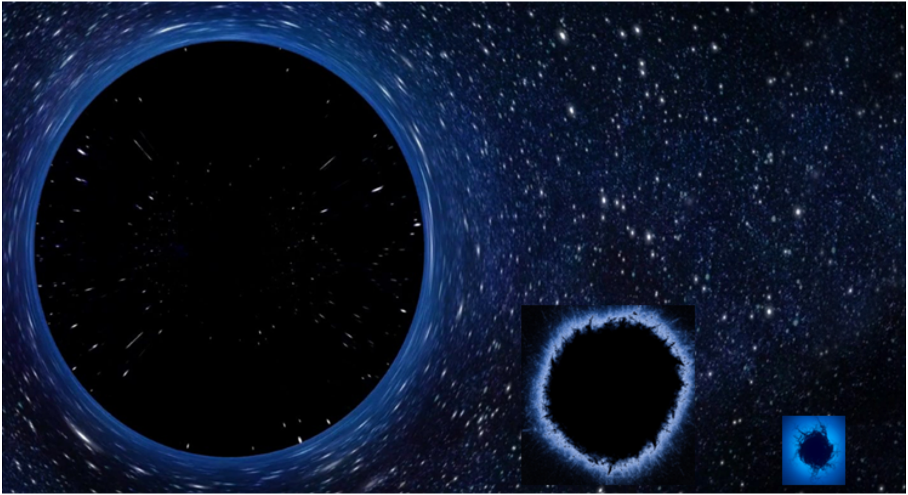
$$
\begin{aligned}
g_{ab} =& g^{(0)}_{ab} + \varepsilon^1 h^{(1)}_{ab} + \varepsilon^2 h^{(2)}_{ab} + \mathcal{O}(\varepsilon^3), \\
\varphi =& \qquad\quad \varepsilon^1 \varphi^{(1)} + \varepsilon^2 \varphi^{(2)} + \mathcal{O}(\varepsilon^3),
\end{aligned}
$$

with $\varphi^{(0)} = 0$

Self-force:

$$
ma^\alpha = \varepsilon F^a_{(1)}[h^{(1)}_{ab}, \varphi^{(1)}] + \varepsilon^2 F^a_{(2)}[h^{(2)}_{ab}, \varphi^{(2)}] + \mathcal{O}(\varepsilon^3)
$$

# The Bigger the Bolder



[Keck, Caltech. Getty Images. Andriy_A / Shutterstock. ]

# Scalar-tensor action

$$S[\mathbf{g}_{ab}, \varphi, \Psi] = S_0[\mathbf{g}_{ab}, \varphi] + \alpha S_{\mathrm{c}}[\mathbf{g}_{ab}, \varphi] + S_{\mathrm{m}}[\mathbf{g}_{ab}, \varphi, \Psi],$$

# Scalar-tensor action

$$S[\mathbf{g}_{ab}, \varphi, \Psi] = S_0[\mathbf{g}_{ab}, \varphi] + \alpha S_{\mathrm{c}}[\mathbf{g}_{ab}, \varphi] + S_{\mathrm{m}}[\mathbf{g}_{ab}, \varphi, \Psi],$$

$$S_0[\mathbf{g}_{ab}, \varphi] = \int \frac{\sqrt{-\mathbf{g}}}{16\pi}\left(R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi\right) d^4x,$$

# Scalar-tensor action

$$S[\mathbf{g}_{ab}, \varphi, \Psi] = S_0[\mathbf{g}_{ab}, \varphi] + \alpha S_{\mathrm{c}}[\mathbf{g}_{ab}, \varphi] + S_{\mathrm{m}}[\mathbf{g}_{ab}, \varphi, \Psi],$$

$$S_0[\mathbf{g}_{ab}, \varphi] = \int \frac{\sqrt{-\mathbf{g}}}{16\pi}\left(R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi\right) d^4x,$$

$$S_{\mathrm{m}} = -\int_\gamma m[\tilde{\varphi}] d\tilde{s}$$

# Scalar-tensor action

$$S[\mathbf{g}_{ab}, \varphi, \Psi] = S_0[\mathbf{g}_{ab}, \varphi] + \alpha S_{\mathrm{c}}[\mathbf{g}_{ab}, \varphi] + S_{\mathrm{m}}[\mathbf{g}_{ab}, \varphi, \Psi],$$

$$S_0[\mathbf{g}_{ab}, \varphi] = \int \frac{\sqrt{-\mathbf{g}}}{16\pi}\left(R - \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi\right) d^4x,$$

$$S_{\mathrm{m}} = -\int_\gamma m[\tilde{\varphi}] d\tilde{s}$$

$$\alpha S_{\mathrm{c}}[\mathbf{g}_{ab}, \varphi] \approx \mathcal{O}\left(\varepsilon^3\right)$$


Every Barbie Career
Beauty Inside A Box


Every Theory Model
Beauty Inside A Box

# Summary

1. We need waveforms in alternative theories of gravity to test GR

# Summary

1. We need waveforms in alternative theories of gravity to test GR
2. We have derived field equations and equations of motion up to second order (up to our assumptions)

# Summary

1. We need waveforms in alternative theories of gravity to test GR
2. We have derived field equations and equations of motion up to second order (up to our assumptions)
3. Adding 1 (or 2) new parameters: d, ($m_{[2]}$)

# Summary

1. We need waveforms in alternative theories of gravity to test GR
2. We have derived field equations and equations of motion up to second order (up to our assumptions)
3. Adding 1 (or 2) new parameters: d, $(m_{[2]})$
4. Implementation builds on GR calculation (and *no more* difficult)

# Extra slide: Non trivial $\varphi$ and $\mathbf{g}_{ab}$ coupling: $\alpha S_{\mathrm{c}}[\mathbf{g}_{ab}, \varphi]$

Dimensionless non-trivial coupling perturbation

$$\zeta := \frac{\alpha}{M^n} = \varepsilon^n \frac{\alpha}{\mu^n}.$$

[Maselli, Franchini, Gualtieri, & Sotiriou, (2020). PRL, 125(14), 141101.]

# Extra slide: Non trivial $\varphi$ and $\mathbf{g}_{ab}$ coupling: $\alpha S_{\mathrm{c}}[\mathbf{g}_{ab}, \varphi]$

Dimensionless non-trivial coupling perturbation

$$\zeta := \frac{\alpha}{M^n} = \varepsilon^n \frac{\alpha}{\mu^n}.$$

Assumptions: $\frac{\alpha}{\mu^n} = \mathcal{O}(1),\ n \geq 2$

[Maselli, Franchini, Gualtieri, & Sotiriou, (2020). PRL, 125(14), 141101.]

# Extra slide: Non trivial $\varphi$ and $\mathbf{g}_{ab}$ coupling: $\alpha S_{\mathrm{c}}[\mathbf{g}_{ab}, \varphi]$

Dimensionless non-trivial coupling perturbation

$$\zeta := \frac{\alpha}{M^n} = \varepsilon^n \frac{\alpha}{\mu^n}.$$

Assumptions: $\frac{\alpha}{\mu^n} = \mathcal{O}(1),\ n \geq 2$

Hence, $\zeta = \mathcal{O}(\varepsilon^2)$

[Maselli, Franchini, Gualtieri, & Sotiriou, (2020). PRL, 125(14), 141101.]

# Extra slide: Non trivial $\varphi$ and $\mathbf{g}_{ab}$ coupling: $\alpha S_{\rm c}[\mathbf{g}_{ab}, \varphi]$

Dimensionless non-trivial coupling perturbation

$$\zeta := \frac{\alpha}{M^n} = \varepsilon^n \frac{\alpha}{\mu^n}.$$

Assumptions: $\frac{\alpha}{\mu^n} = \mathcal{O}(1),\ n \geq 2$

Hence, $\zeta = \mathcal{O}(\varepsilon^2)$

$$\Rightarrow \alpha S_{\rm c}[\mathbf{g}_{ab}, \varphi] \approx \mathcal{O}\left(\varepsilon^3\right)$$

[Maselli, Franchini, Gualtieri, & Sotiriou, (2020). PRL, 125(14), 141101.]