Non-geometry and exotic branes

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Overview talk
"Geometry & Duality", Max Planck Institute for Gravitational Physics (Albert Einstein Institute), Potsdam, December 2-6, 2019

VRIJE UNIVERSITEIT BRUSSEL

Fonds Wetenschappelijk Onderzoek Vlaanderen
Opening new horizons

Applying duality to geometry leads to non-geometry

Different descriptions of same stringy physics

Geometry + Duality $\rightarrow$ Non-geometry

Background metric & form fields

Background space-time requires duality to be defined

Applying duality to branes leads to exotic branes

Different descriptions of same stringy physics

Branes + Duality → Exotic branes

Extended object, with some tension, charged under some background field

Non-standard tension and charge

These are related via the geometry sourced by the brane

Different descriptions of same stringy physics

Branes $+$ Duality $\rightarrow$ Exotic branes

Sources a *geometric* background

Sources a *non-geometric* background

There are multiple **entry points** to non-geometry

Flux compactifications
- "Nongeometric Flux Compactifications" by Shelton, Taylor, Wecht

T/U-duality as a symmetry of string & M-theory
- "U-duality and M-theory" by Obers and Pioline

Black hole microstates
- "Exotic Branes in String Theory" by de Boer and Shigemori

String theory geometry beyond SUGRA
- "A Geometry for Non-Geometric String Backgrounds" by Hull

These references are indicative starting points to go both forwards and backwards in the literature - not absolute!

Recent review: "Non-geometric backgrounds in string theory" by Plauschinn

The simplest duality to consider is T-duality

winding

momentum

coordinate

dual coordinate

radius of S^1

$R \to 1/R,\ w \leftrightarrow p,\ y \to \tilde{y}$

Geometry + T-duality

Metric g
2-form B
Dilaton ϕ

The simplest duality to consider is T-duality

winding

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radius of S^1

$R \to 1/R,\ w \leftrightarrow p,\ y \to \tilde{y}$

Geometry + T-duality

Metric g

2-form B

Dilaton ϕ

Dual background: [Buscher]

$$
\begin{aligned}
\tilde{g}_{yy} &= \frac{1}{g_{yy}} \\
\tilde{g}_{\mu y} &= \frac{B_{\mu y}}{g_{yy}} \\
\tilde{g}_{\mu\nu} &= g_{\mu\nu} - \frac{1}{g_{yy}}(g_{\mu y} g_{\nu y} - B_{\mu y} B_{\nu y}) \\
\tilde{B}_{\mu\nu} &= B_{\mu\nu} + \frac{1}{g_{yy}}(g_{\mu y} B_{\nu y} - g_{\nu y} B_{\mu y}) \\
\tilde{B}_{\mu y} &= \frac{g_{\mu y}}{g_{yy}} \\
\tilde{\phi} &= \phi - \frac{1}{2}\log g_{yy}
\end{aligned}
$$

The full T-duality group is $O(d,d)$

$i, j, \ldots = 1, \ldots, d$ (internal, torus)
$\mu, \nu, \ldots = 1, \ldots, 10-d$ (external)

$$\begin{tikzcd}[column sep=small]
 & & g, \; B, \; \phi \arrow[dll, bend right] \arrow[d] \arrow[drr, bend left] & & \\
g_{\mu\nu},\; A_\mu{}^i,\; g_{ij} & & B_{\mu\nu},\; B_{\mu i},\; B_{ij} & & \phi
\end{tikzcd}$$

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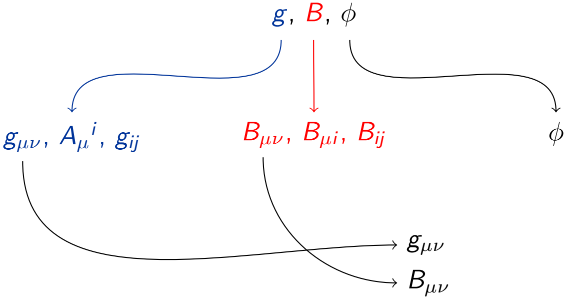

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$$g,\ B,\ \phi$$

$$g_{\mu\nu},\ A_\mu{}^i,\ g_{ij} \qquad B_{\mu\nu},\ B_{\mu i},\ B_{ij} \qquad \phi$$

$$g_{\mu\nu}$$

$$B_{\mu\nu}$$

$$\mathcal{A}_\mu{}^M = \begin{pmatrix} A_\mu{}^i \\ B_{\mu i} \end{pmatrix}$$

$M, N, \ldots = 1, \ldots, 2d$ (doubled)

The full T-duality group is $O(d,d)$

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$$\mathcal{A}_\mu{}^M = \begin{pmatrix} A_\mu{}^i \\ B_{\mu i} \end{pmatrix}$$

$$\mathcal{H}_{MN} = \begin{pmatrix} g_{ij} - B_{ik} g^{kl} B_{lj} & B_{ik} g^{kj} \\ -g^{ik} B_{kj} & g^{ij} \end{pmatrix}$$

$$\mathbf{\Phi} = \phi - \tfrac{1}{4} \log \det (g_{ij})$$

$M, N, \ldots = 1, \ldots, 2d$ (doubled)

The full T-duality group is $O(d,d)$

$i, j, \ldots = 1, \ldots, d$ (internal, torus)
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$M, N, \ldots = 1, \ldots, 2d$ (doubled)

The full T-duality group is $O(d,d)$

$$g_{\mu\nu}\,, \quad B_{\mu\nu}\,, \quad \boldsymbol{\Phi}\,, \quad \mathcal{A}_\mu{}^M = \begin{pmatrix} A_\mu{}^i \\ B_{\mu i} \end{pmatrix}\,, \quad \mathcal{H}_{MN} = \begin{pmatrix} g_{ij} - B_{ik} g^{kl} B_{lj} & B_{ik} g^{kj} \\ -g^{ik} B_{kj} & g^{ij} \end{pmatrix}$$

$O(d,d)$ transformations

$$\eta_{MN} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}\,, \quad \mathcal{P}_M{}^K \mathcal{P}_N{}^L \eta_{KL} = \eta_{MN}$$

$$\mathcal{A}_\mu{}^M \to (\mathcal{P}^{-1})^M{}_K \mathcal{A}_\mu{}^K\,, \quad \mathcal{H}_{MN} \to \mathcal{P}_M{}^K \mathcal{P}_N{}^L \mathcal{H}_{KL}$$

Geometric

$$\mathcal{P}_A = \begin{pmatrix} A & 0 \\ 0 & A^{-T} \end{pmatrix}\,, A \in \mathrm{GL}(d)$$

$$\mathcal{P}_b = \begin{pmatrix} I & b \\ 0 & I \end{pmatrix}\,, b^T = -b\,, B \to B + b$$

Non-geometric

T-duality: swap ${}^i \leftrightarrow {}_i$

$$\mathcal{P}_\beta = \begin{pmatrix} I & 0 \\ \beta & I \end{pmatrix}\,, \beta^T = -\beta$$

The reduced action leads to a (hidden) T-duality symmetry

The reduced action has lots of hidden symmetry...
Field content, field strengths, symmetries all $O(d,d)$ covariant, Lagrangian $O(d,d)$ invariant

e.g. $\delta \mathcal{A}_\mu{}^M = \partial_\mu \Lambda^M$

... but it's phenomenologically useless
A standard problem with string compactifications: lots of massless scalars ($\mathcal{H}_{MN}$, $\mathbf{\Phi}$)
(Also too many supersymmetries, too few de Sitters, etc.)

$\Rightarrow$ compactify on more complicated spaces than the flat torus

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$\Rightarrow$ compactify on more complicated spaces than the flat torus

e.g. H-flux y^i coords on T^d
$g_{ij} = \delta_{ij}$, $B_{ij} = b_{ij}(y)$
$H_{ijk} = 3\partial_{[i} b_{jk]}$ constant

e.g. Geometric flux
$g_{ij} = e_i{}^a(y) e_j{}^a(y) \delta_{ab}$, $B_{ij} = 0$
$f_{ab}{}^c = 2 e_a{}^i e_b{}^j \partial_{[i} e_{j]}{}^c$ constant

These fluxes deform (or "gauge") the reduced action

e.g. H-flux

y^i coords on T^d

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$g_{ij} = e_i{}^a(y) e_j{}^a(y) \delta_{ab}$, $B_{ij} = 0$

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The reduced action is still organised in terms of $O(d,d)$

Symmetries $\delta \mathcal{A}_\mu{}^A = \partial_\mu \Lambda^A + f_{BC}{}^A \mathcal{A}_\mu{}^B \Lambda^C$

Scalar potential $V = -\frac{1}{4} f_{DA}{}^C f_{CB}{}^D \mathcal{H}^{AB} - \frac{1}{12} f_{AC}{}^E f_{BD}{}^F \mathcal{H}^{AB} \mathcal{H}^{CD} \mathcal{H}_{EF} - \frac{1}{6} f_{ABC} f^{ABC}$

Flux components

Allowed deformations: fluxes $f_{AB}{}^C$ (A is an $O(d,d)$ index)

$$f_{abc} = H_{abc}\,, \quad f_{ab}{}^c = f_{ab}{}^c\,, \quad f_a{}^{bc} \stackrel{?}{=} Q_a{}^{bc}\,, \quad f^{abc} \stackrel{?}{=} R^{abc}$$

Flux breaks $O(d,d)$, gauges a subgroup

Here is how we will uncover non-geometry:

We will follow [Kachru, Schulz, Tripathy, Trivedi] and [Shelton, Taylor, Wecht] and look at a *toy model*

This is the T-duality chain starting with the three-torus with H-flux

$$\begin{array}{ccccccc} H_{abc} & \xrightarrow{T_c} & f_{ab}{}^{c} & \xrightarrow{T_b} & Q_a{}^{bc} & \xrightarrow{T_a} & R^{abc} \\ T^3 \text{ with flux} & & \text{Twisted torus} & & \text{T-fold} & & \text{T-fold} \\ & & & & \text{(global)} & & \text{(local)} \end{array}$$

This is sort of the **harmonic oscillator of non-geometry**

None of these are true string theory backgrounds. But they're educational.

H-flux and geometric flux are related by T-duality

$H_{abc} \longrightarrow f_{ab}{}^{c}$

T^3 with H_3

$$ds^2 = (dy^1)^2 + (dy^2)^2 + (dy^3)^2\,, \quad B_{12} = \frac{h}{2\pi} y^3$$

$$H_{123} = \frac{h}{2\pi}$$

$$\updownarrow T_1$$

Twisted torus

$$ds^2 = (dy^1 - \frac{h}{2\pi} y^3 dy^2)^2 + (dy^2)^2 + (dy^3)^2\,, \quad B_{ij} = 0$$

$$f_{23}{}^{1} = \frac{h}{2\pi}$$

$$e^1 = dy^1 - \frac{h}{2\pi} y^3 dy^2\,, \quad e^2 = dy^2\,, \quad e^3 = dy^3\,, \quad de^a = \frac{1}{2} f_{bc}{}^{a} e^b \wedge e^c$$

Another T-duality gives a T-fold

$H_{abc} \longrightarrow f_{ab}{}^{c} \longrightarrow Q_a{}^{bc}$

T^3 with H_3

$$ds^2 = (dy^1)^2 + (dy^2)^2 + (dy^3)^2\,, \quad B_{12} = \frac{h}{2\pi} y^3$$

$\updownarrow T_1$

Twisted torus

$$ds^2 = (dy^1 - \frac{h}{2\pi} y^3 dy^2)^2 + (dy^2)^2 + (dy^3)^2\,, \quad B_{ij} = 0$$

$\updownarrow T_2$

T-fold

$$ds^2 = \frac{1}{1 + \left(\frac{h}{2\pi} y^3\right)^2} \left((dy^1)^2 + (dy^2)^2\right) + (dy^3)^2$$

$$B_{12} = -\frac{\frac{h}{2\pi} y^3}{1 + \left(\frac{h}{2\pi} y^3\right)^2}$$

The first two spaces have a geometric monodromy

T^3 with H_3

$$ds^2 = (dy^1)^2 + (dy^2)^2 + (dy^3)^2\,, \quad B_{12} = \frac{h}{2\pi} y^3$$

$$y^3 \to y^3 + 2\pi\,, \quad B_{12} \to B_{12} + h$$

$\Rightarrow$ geometric $O(2,2)$ transformation (B-shift)

Twisted torus

$$ds^2 = (dy^1 - \frac{h}{2\pi} y^3 dy^2)^2 + (dy^2)^2 + (dy^3)^2\,, \quad B_{ij} = 0$$

$$y^3 \to y^3 + 2\pi\,, \quad y^1 \to y^1 + h y^2$$

$\Rightarrow$ geometric $GL(2) \subset O(2,2)$ transformation

The T-fold has a non-geometric monodromy

T-fold

$$ds^2 = \frac{1}{1+\left(\frac{h}{2\pi}y^3\right)^2}\left((dy^1)^2+(dy^2)^2\right)+(dy^3)^2$$

$$B_{12} = -\frac{\frac{h}{2\pi}y^3}{1+\left(\frac{h}{2\pi}y^3\right)^2}$$

$$y^3 \to y^3 + 2\pi\,, \quad (g,B) \to \mathcal{P}_\beta \cdot\ (g,B)$$

$\Rightarrow$ non-geometric $O(2,2)$ transformation:

$$\mathcal{P}_\beta = \begin{pmatrix} 1 & 0 \\ \beta & 1 \end{pmatrix}\,, \quad \beta = h\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Note: physical coordinates identified with duals

$$y^1 \to y^1 + h\tilde{y}^2\,, \quad y^2 \to y^2 - h\tilde{y}^1$$

We can associate to it a non-geometric flux

Definition of generalised metric:

$$\mathcal{H}_{MK}\eta^{KL}\mathcal{H}_{LN} = \eta_{MN} \qquad \mathcal{H}_{MN} = \mathcal{H}_{NM} \Rightarrow \mathcal{H}_{MN} = E_M{}^A E_N{}^B \delta_{AB}$$

$O(D,D)/O(D)\times O(D)$ coset element

***B*-field parametrisation:**

$$E_M{}^A = \begin{pmatrix} e_i{}^a & B_{ij}e_a{}^j \\ 0 & e_a{}^i \end{pmatrix}$$

$$\mathcal{H}_{MN} = \begin{pmatrix} g_{ij} - B_{ik}g^{kl}B_{lj} & B_{ik}g^{jk} \\ -g^{ik}B_{kj} & g^{ij} \end{pmatrix}$$

Bivector parametrisation:

$$E_M{}^A = \begin{pmatrix} \tilde{e}_i{}^a & 0 \\ \beta^{ij}\tilde{e}_j{}^a & \tilde{e}^a{}_i \end{pmatrix}$$

$$\mathcal{H}_{MN} = \begin{pmatrix} \tilde{g}_{ij} & -\tilde{g}_{ik}\beta^{kj} \\ \beta^{ik}\tilde{g}_{kj} & \tilde{g}^{ij} - \beta^{ik}\tilde{g}_{kl}\tilde{g}^{lj} \end{pmatrix}$$

Q-flux: $$Q_i{}^{jk} = \partial_i\beta^{jk}$$

The T-fold in this parametrisation looks geometric

T-fold

$$\widetilde{ds}^2 = (dy^1)^2 + (dy^2)^2 + (dy^3)^2$$

$$\beta^{12} = \frac{h}{2\pi} y^3$$

$$y^3 \to y^3 + 2\pi\,, \quad \beta^{12} \to \beta^{12} + h$$

Q-flux: $$Q_i{}^{jk} = \partial_i \beta^{jk}\,, \quad Q_3{}^{12} = h$$

We are tempted to do one more T-duality

$$H_{abc} \longrightarrow f_{ab}{}^{c} \longrightarrow Q_{a}{}^{bc} \xrightarrow{?} R^{abc}$$

T-fold

$$\widetilde{ds}^{2} = (dy^1)^2 + (dy^2)^2 + (dy^3)^2 \,, \quad \beta^{12} = \frac{h}{2\pi} y^3$$

$$\updownarrow T_3$$

Local T-fold

$$\widetilde{ds}^{2} = (dy^1)^2 + (dy^2)^2 + (dy^3)^2 \,, \quad \beta^{12} = \frac{h}{2\pi} \tilde{y}^3$$

R-flux:

$$R^{ijk} = 3\tilde{\partial}^{[i}\beta^{jk]}$$

$$R^{123} = h$$

We can describe such fluxes in doubled geometry

Doubled geometry: [Duff] [Tseytlin] [Siegel] [Hohm, Hull, Zwiebach]

Doubled coords: $Y^M = (y^i, \tilde{y}_i)$, $\partial_M = (\partial_i, \tilde{\partial}^i)$ $\tilde{y}_i$: winding coordinates

Generalised metric: $\mathcal{H}_{MN} = E_M{}^A E_N{}^B \delta_{AB}$

Generalised Lie derivative:

$$\mathcal{L}_\Lambda \mathcal{H}_{MN} = \Lambda^P \partial_P \mathcal{H}_{MN} + 2\partial_{(M}\Lambda^P \mathcal{H}_{N)P} - 2\eta^{PQ}\eta_{K(M|}\partial_P \mathcal{H}_{Q|N)}$$

A general Riemannian **parametrisation:**

[see Park's talk]

$$E_M{}^A = \begin{pmatrix} e_i{}^a & B_{ij} e_a{}^j \\ \beta^{ij} e_j{}^a & e_a{}^i + e_a{}^j \beta^{jk} B_{ki} \end{pmatrix}$$

Generalised fluxes:

$$f_{AB}{}^C = -E_M{}^C \mathcal{L}_{E_A} E_B{}^M$$

generalised Lie deriv

Can have: f constant, U depends on duals $\Rightarrow$ generalised Scherk-Schwarz [Geissbühler] [Aldazabal, Baron, Marqués, Núñez] [Graña, Marqués]

Here's the summary of the toy model:

Space	Flux	Features
Torus with flux	H_{abc}	Geometric
Twisted torus	$f_{ab}{}^{c}$	Geometric
T-fold	$Q_a{}^{bc}$	Locally geometric, globally non-geometric
Local T-fold	R^{abc}	Locally non-geometric (depends on dual coords)

Lesson 1: duality predicts non-geometry

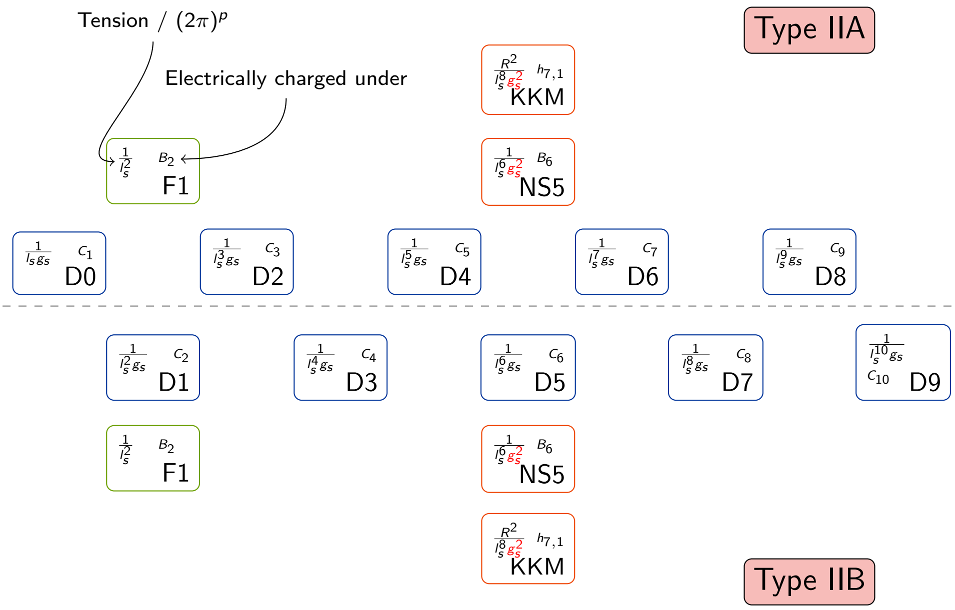
Lesson 2: non-geometry predicts additional flux compactifications

(but n.b. can get "non-geometric flux" from e.g. sphere compactifactions: imperfect diagnostic of higher dimensional non-geometry)

Remark: the U-duality (M-theory) version of the toy model is in [CB, Malek 2014]

Next: from Non-Geometry to Exotic Branes

The Standard Model of Branes looks something like this:



The Standard Model of Branes looks something like this:

Tension / $(2\pi)^p$

Electrically charged under

Type IIA

$\frac{R^2}{l_s^8 g_s^2}$ $h_{7,1}$ KKM

$\frac{1}{l_s^2}$ B_2 F1

$\frac{1}{l_s^6 g_s^2}$ B_6 NS5

$\frac{1}{l_s g_s}$ C_1 D0

$\frac{1}{l_s^3 g_s}$ C_3 D2

$\frac{1}{l_s^5 g_s}$ C_5 D4

$\frac{1}{l_s^7 g_s}$ C_7 D6

$\frac{1}{l_s^9 g_s}$ C_9 D8

$\frac{1}{l_s^2 g_s}$ C_2 D1

$\frac{1}{l_s^4 g_s}$ C_4 D3

$\frac{1}{l_s^6 g_s}$ C_6 D5

$\frac{1}{l_s^8 g_s}$ C_8 D7

$\frac{1}{l_s^{10} g_s}$ C_{10} D9

$\frac{1}{l_s^2}$ B_2 F1

$\frac{1}{l_s^6 g_s^2}$ B_6 NS5

Magnetically charged under B_2

Magnetically charged under KK vector

$\frac{R^2}{l_s^8 g_s^2}$ $h_{7,1}$ KKM

Radius of compact direction

Type IIB

This is the NS5 solution

Worldvolume

Transverse $\mathbb{R}^4$

$Q_{mag} \sim \int H_3 \sim q$

$$ds^2 = dx^2_{012345} + f dx^2_{6789}$$

$$H_{abc} = \epsilon_{abcd}\delta^{de}\partial_e f\,, \quad x^a = (x^6, x^7, x^8, x^9)$$

$$f = 1 + \frac{q}{r^2}\,, \quad r = \sqrt{(x^6)^2 + (x^7)^2 + (x^8)^2 + (x^9)^2}$$

harmonic in transverse

This is the smeared NS5 solution

Worldvolume

Transverse $\mathbb{R}^3 \times S^1$

$B_{i9} \neq 0$

$$ds^2 = dx^2_{012345} + f(dx^2_{678} + (dx^9)^2)$$

$$H_{ij9} = \epsilon_{ijk}\delta^{kl}\partial_l f\,, \quad x^i = (x^6, x^7, x^8)$$

$$f = 1 + \frac{q'}{r}\,, \quad r = \sqrt{(x^6)^2 + (x^7)^2 + (x^8)^2}$$

harmonic in transverse

Its T-dual on the 9 direction is the KKM solution

Worldvolume

Transverse $\mathbb{R}^3 \times S^1$

Special isometry direction

$$ds^2 = dx^2_{012345} + f dx^2_{678} + f^{-1}(dx^9 + A_i dx^i)^2$$

$$F_{ij} = 2\partial_{[i}A_{j]} = \epsilon_{ijk}\delta^{kl}\partial_l f\,, \quad x^i = (x^6, x^7, x^8)$$

$$f = 1 + \frac{q'}{r}\,, \quad r = \sqrt{(x^6)^2 + (x^7)^2 + (x^8)^2}$$

harmonic in transverse

And this is the smeared KKM solution

Worldvolume

Transverse $\mathbb{R}^2 \times T^2$

$$ds^2 = dx^2_{012345} + f(dr^2 + r^2 d\theta^2) + f(dx^8)^2 + f^{-1}(dx^9 + A_8 dx^8)^2$$

$$F_{i8} = 2\partial_{[i}A_{8]} = -\epsilon_{ij}\delta^{jk}\partial_l f\,, \quad x^i = (x^6, x^7)$$

$$A_8 = -q''\theta$$

$$f = -q'' \log r\,, \quad r = \sqrt{(x^6)^2 + (x^7)^2}$$

harmonic in transverse

Its T-dual on the 8 direction is an exotic brane

Worldvolume

Transverse $\mathbb{R}^2 \times T^2$

2 special isometry dirs

Not asympotically flat

$$ds^2 = dx^2_{012345} + f(dr^2 + r^2 d\theta^2) + \frac{f}{f^2 + (\theta q'')^2}((dx^8)^2 + (dx^9)^2)$$

$$f = -q'' \log r \qquad B_{89} = -\frac{\theta q''}{f^2 + (\theta q'')^2}$$

harmonic in transverse

This exotic brane has a non-geometric monodromy

Worldvolume

Transverse $\mathbb{R}^2 \times T^2$

2 special isometry dirs

Not asympotically flat

$$ds^2 = dx^2_{012345} + f(dr^2 + r^2 d\theta^2) + \frac{f}{f^2 + (\theta q'')^2}((dx^8)^2 + (dx^9)^2)$$

$$f = -q'' \log r \qquad B_{89} = -\frac{\theta q''}{f^2 + (\theta q'')^2}$$

$$\mathcal{H}_{MN} = \begin{pmatrix} f^{-1} I_2 & -f^{-1}\theta q'' \epsilon \\ f^{-1}\theta q'' \epsilon & (f + f^{-1}(\theta q'')^2) I_2 \end{pmatrix}$$

$$= \begin{pmatrix} I_2 & 0 \\ \theta q'' \epsilon & I_2 \end{pmatrix} \begin{pmatrix} f^{-1} I_2 & 0 \\ 0 & f I_2 \end{pmatrix} \begin{pmatrix} I_2 & -\theta q'' \epsilon \\ 0 & I_2 \end{pmatrix}$$

Bivector $\beta^{89} = \theta q''$

Magnetic Q-flux $Q_\theta{}^{89} = q''$

$\theta \to \theta + 2\pi$
$g, B \to$ non-geo T-duality transf. of g, B

This is the 5^2_2 brane

Wherefore art thou 5^2_2?

Brane tensions after **wrapping all spatial and special isometry directions on compact directions**

Start with NS5: wrap 5 worldvolume directions and 2 transverse

$$T_{NS5} = \frac{1}{(2\pi)^5}\frac{1}{g_s^2 l_s^6} \to \frac{R_1 \dots R_5}{g_s^2 l_s^6}$$

T-duality on x^9, $R_9 \to l_s^2/R_9$, $g_s \to l_s g_s/R_9$

$$T_{NS5} \to T_{KKM} = \frac{R_1 \dots R_5 (R_9)^2}{g_s^2 l_s^8}$$

T-duality on x^8, $R_8 \to l_s^2/R_8$, $g_s \to l_s g_s/R_8$

$$T_{KKM} \to T_{5^2_2} = \frac{R_1 \dots R_5 (R_8 R_9)^2}{g_s^2 l_s^{10}}$$

Wherefore art thou 5_2^2?

Brane tensions after **wrapping all spatial and special isometry directions on compact directions**

Start with NS5: wrap 5 worldvolume directions and 2 transverse

$$T_{NS5} = \frac{1}{(2\pi)^5}\frac{1}{g_s^2 l_s^6} \rightarrow \frac{R_1 \ldots R_5}{g_s^2 l_s^6}$$

5 radii linearly
g_s^{-2}

5_2

T-duality on x^9, $R_9 \rightarrow l_s^2/R_9$, $g_s \rightarrow l_s g_s/R_9$

$$T_{NS5} \rightarrow T_{KKM} = \frac{R_1 \ldots R_5 (R_9)^2}{g_s^2 l_s^8}$$

5 radii linearly
1 radius quadratically
g_s^{-2}

5_2^1

T-duality on x^8, $R_8 \rightarrow l_s^2/R_8$, $g_s \rightarrow l_s g_s/R_8$

$$T_{KKM} \rightarrow T_{5_2^2} = \frac{R_1 \ldots R_5 (R_8 R_9)^2}{g_s^2 l_s^{10}}$$

5 radii linearly
2 radii quadratically
g_s^{-2}

5_2^2

Acting with more dualities gives many more exotic branes

[Elitzur, Giveon, Kutasov, Rabinovici] [Blau, O'Loughlin]

[Hull] [Obers, Pioline, Rabinovici] (all 1997)

Type IIB on T^7
$T : R \to l_s^2/R,\ g_s \to l_s g_s/R$
$S : g_s \to 1/g_s,\ l_s \to g_s^{1/2} l_s$

D7 and S-dual: (p, q) 7-branes. Codim-2 in 10-dims

Key: standard name ${b_n}^c$ (number of states in 3-dims)

g_s^0	P		(7)	F1	1_0	(7)					
g_s^{-1}	D1	1_1	(7)	D3	3_1	(35)	D5	5_1	(21)	D7	7_1 (1)
g_s^{-2}	NS5	5_2	(21)	KKM	5_2^1	(42)		5_2^2	(21)		
g_s^{-3}		1_3^6	(7)		3_3^4	(35)		5_3^2	(21)		7_3 (1)
g_s^{-4}		$0_4^{(1,6)}$	(7)		1_4^6	(7)					

Depends on 6 radii quadratically, 1 cubically

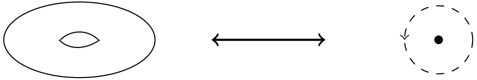
Same picture starting with IIA or M-theory branes [Obers, Pioline]

Total states $= 240 = \dim E_8 - \operatorname{rank} E_8$: U-duality missing states in Cartan, not BPS

[Bergshoeff, Riccioni, Ortin] [Kleinschmidt]

These all uplift to non-geometric backgrounds

Branes wrapping T^7: | Point particles in 3d:



Non-geometric transformations | Scalar monodromies $\in E_8$

Argument: [de Boer, Shigemori]

3d scalars from compactification: $\mathcal{M}_{MN}(g_{ij}, B_{ij}, \phi, C_0, C_{ij}, \dots) \in E_8/\mathrm{SO}(16)$ (generalisation of $\mathcal{H}_{MN}$ in T-duality)

D7 has a monodromy $\in \mathrm{SL}(2) \subset E_8$ already: $C_0 \to C_0 + 1$

Duality: D7 $\to$ other branes, monodromy $\to$ other E_8 monodromies
(Additionally, could have arbitrary E_8 monodromies)

These 3d scalar monodromies lift to non-geometric transformations of higher-dimensional metric, forms

Why should we not dismiss these branes?

Issues: [de Boer, Shigemori]
1) Non-perturbative: tension/mass $\sim g_s^{-3}$, g_s^{-4}
2) Backreaction: codim-2, metric diverges asymptotically
3) Mass is not localised at $r = 0$ but spread over spacetime
$\Rightarrow$
4) Should *not* extend solution to all of spacetime
$\to$ treat as approximate description valid near brane
$\to$ replace bad asymptotics with something else

24 7:
Multiple (24) codim-2 branes
- no overall monodromy
- compact transverse space (S^2)
- non-trivial monodromy around isolated brane

Supertubes:
Exotic brane "dipoles" along curve
- no overall monodromy
- asymptotically flat
- non-trivial monodromy through tube

~~Bird~~ ~~Brane~~ Supertube

Supertube: [Mateos, Townsend]
Spontaneous polarisation of bound state: two branes "puff up" into (higher dim.) brane "dipole"
Brane + Brane → brane′ + (angular) momentum

Curve in transverse non-compact dirs

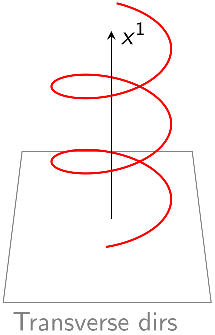
String wrapping x^1

$$\mathrm{F1}(x^1) + \mathrm{P}(x^1) \to \mathrm{f1}(\psi) + \mathrm{p}(\psi)$$

Add momentum to string
→ only transverse excitations
→ puffs up

No net charge

Angular momentum supports profile



[Dabholkar, Gauntlett, Harvey, Waldram] [Callan, Maldacena, Peet] [Lunin, Mathur]

~~Bird~~ ~~Brane~~ Supertube

Supertube: [Mateos, Townsend]
Spontaneous polarisation of bound state: two branes "puff up" into (higher dim.) brane "dipole"
Brane + Brane → brane′ + (angular) momentum

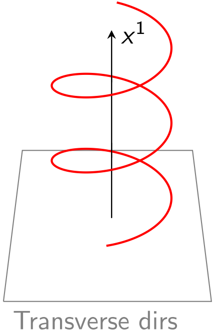
Curve in transverse non-compact dirs

S-dualise to get:

$$\mathrm{D1}(x^1) + \mathrm{P}(x^1) \to \mathrm{d1}(\psi) + \mathrm{p}(\psi)$$

No net charge

Angular momentum supports profile



~~Bird~~ ~~Brane~~ Supertube

Supertube: [Mateos, Townsend]
Spontaneous polarisation of bound state: two branes "puff up" into (higher dim.) brane "dipole"
Brane + Brane → brane′ + (angular) momentum

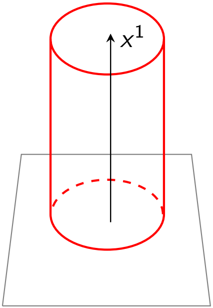
Curve in transverse non-compact dirs

T-dualise x^1 to get:

$$\mathrm{D0} + \mathrm{F1}(x^1) \to \mathrm{d2}(x^1 \psi) + \mathrm{p}(\psi)$$

No net charge:
world volume BI field F
Elec cpts: ⇒ F1 charge
Mag cpts: ⇒ D0 charge

Angular momentum
from wvol fields



Transverse dirs

[Mateos, Townsend]

An explicit asymptotically flat exotic supertube solution

[de Boer, Shigemori]:

$$\mathrm{D4}(6789) + \mathrm{D4}(4589) \to 5_2^2(4567\psi, 89) + \mathrm{p}(\psi)$$

Type IIA on $\mathbb{R}^{1,3}_{0123} \times T^6_{456789}$

Transverse non-compact: $\vec{x} = (x^1, x^2, x^3)$

$$ds^2 = -\frac{1}{\sqrt{f_1 f_2}}(dt - A_i dx^i)^2 + \sqrt{f_1 f_2}\, dx^2_{123} + \sqrt{\frac{f_1}{f_2}}\, dx^2_{45} + \sqrt{\frac{f_2}{f_1}}\, dx^2_{67} + \frac{\sqrt{f_1 f_2}}{f_1 f_2 + \gamma^2}\, dx^2_{89}$$

$$B_{89} = \frac{\gamma}{f_1 f_2 + \gamma^2} \quad e^{2\phi} = \frac{\sqrt{f_1 f_2}}{f_1 f_2 + \gamma^2} \quad C_3 \neq 0 \quad C_5 \neq 0 \quad C_1 = C_7 = 0$$

$$f_1 = 1 + \frac{Q_1}{L}\int_0^L \frac{dv}{|\vec{x} - \vec{F}(v)|}$$

$$f_2 = 1 + \frac{Q_1}{L}\int_0^L \frac{dv\,|\dot{\vec{F}}(v)|}{|\vec{x} - \vec{F}(v)|}$$

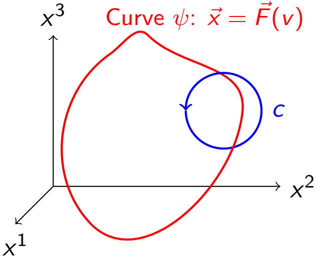
$$A_i = -\frac{Q_1}{L}\int_0^L \frac{\dot{F}_i(v)\,dv}{|\vec{x} - \vec{F}(v)|}$$

$d\gamma = \star_3 dA$

$\oint_c d\gamma = \frac{4\pi Q_1}{L}$

$\Rightarrow \gamma \to \gamma + \frac{4\pi Q_1}{L}$

$\Rightarrow$ non-geo monodromy in 89 directions



Exotic branes may therefore be relevant for BH microstates

Lesson 1: supertube effect: exotic branes inevitably appear via spontaneous polarisation of ordinary branes

Lesson 2: exotic branes may provide *non-geometric microstates* of stringy black holes

$$\mathsf{D4}(6789) + \mathsf{D4}(4589) \to 5^2_2(4567\psi, 89) + \mathsf{p}(\psi)$$

$$\updownarrow \text{T-dualise on } 678$$

$$\mathsf{D1}(9) + \mathsf{D5}(45679) \to \mathsf{kkm}(4567\psi, 9) + \mathsf{p}(\psi)$$

Two-charge black holes: quantisation of *microstate geometries* (supertubes) account for BH entropy [Lunin, Mathur] $\to$ fuzzballs [Mathur]

Three- and four-charge black holes: more branes to combine $\to$ multiple, multi-stage supertube polarisations $\Rightarrow$ need to include exotic branes? [de Boer, Shigemori]

A codim-2 configuration that makes sense 24 7

[Greene, Shapere, Vafa, Yau] [Gibbons, Green, Perry]

IIB (p,q)-branes:

$$\mathcal{H} = \frac{1}{\mathrm{Im}\,\tau}\begin{pmatrix} |\tau|^2 & \mathrm{Re}\,\tau \\ \mathrm{Re}\,\tau & 1 \end{pmatrix}, \quad \mathcal{H} \mapsto U\mathcal{H}U^T$$

$$\tau = C_0 + ie^{-\phi} \quad \tau \mapsto \frac{a\tau + b}{c\tau + d} \quad U = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2)$$

D7 monodromy: $C_0 \to C_0 + 1$

$$ds^2 = dx^2_{01234567} + \mathrm{Im}\tau\, dz d\bar{z}$$

$$z = re^{i\theta}$$

$$\tau(z) = \frac{1}{2\pi i}\log z$$

Solution near brane at origin, $\mathrm{Im}\tau \sim \log r$

A codim-2 configuration that makes sense 24 7

[Greene, Shapere, Vafa, Yau] [Gibbons, Green, Perry]

IIB (p,q)-branes:

$$\mathcal{H} = \frac{1}{\mathrm{Im}\,\tau}\begin{pmatrix} |\tau|^2 & \mathrm{Re}\,\tau \\ \mathrm{Re}\,\tau & 1 \end{pmatrix}, \quad \mathcal{H} \mapsto U\mathcal{H}U^T$$

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D7 monodromy: $C_0 \to C_0 + 1$

Dedekind $\eta \Rightarrow$ Metric

$\mathrm{SL}(2)$ invariant

$$ds^2 = dx^2_{01234567} + \mathrm{Im}\tau \frac{|\eta(\tau)|^4}{\prod_{i=1}^{N} |z - z_i|^{1/6}} dz d\bar{z}$$

$N = 24$:
transverse space is S^2

$$\tau(z) = j^{-1}\left(\frac{P(z)}{Q(z)}\right)$$

N 7-branes at $z = z_i$

Some monstrous maths

$j(\tau) = e^{-2\pi i\tau} + 744 + (196883 + 1)e^{2\pi i\tau} + \ldots$

P, Q polynomials
Zeroes of Q: locations of 7-branes
$\to$ algebraic geometry, elliptic curves
$\Rightarrow$ F-theory [Vafa]

τ complex structure of zero area T^2
$\to$ geometrises monodromy

This can be applied to $O(2,2)$

$O(2,2)$ **NSNS branes:** Type II on T^2_{89} [Hellerman, McGreevy, Williams]

$$O(2,2) \sim \mathrm{SL}(2)_\tau \times \mathrm{SL}(2)_\rho \quad \tau = \tau(T^2_{89}) \quad \rho = B_{12} + i \det g$$

$$\mathcal{H}_{MN} = \mathcal{H}_\tau \otimes \mathcal{H}_\rho$$

NS5: $\rho \to \rho + 1$
KKM: $\tau \to \tau/(-\tau+1)$
5^2_2: $\rho \to \rho/(-\rho+1)$

$$ds^2 = dx^2_{012345} + \frac{\mathrm{Im}\tau|\eta(\tau)|^4}{\prod_{i=1}^{N}|z-z_i|^{1/6}} \frac{\mathrm{Im}\rho|\eta(\rho)|^4}{\prod_{i=1}^{\tilde{N}}|z-\tilde{z}_i|^{1/6}} dz d\bar{z} + ds^2(T^2_{89})$$

$$\tau(z) = j^{-1}\left(\frac{P(z)}{Q(z)}\right) \quad \rho(z) = j^{-1}\left(\frac{\tilde{P}(z)}{\tilde{Q}(z)}\right)$$

N 5-branes at $z = z_i$, $\tilde{N}$ at $z = \tilde{z}_i$, $N + \tilde{N} = 24$

F-theory analogy: semi-auxiliary $T^2 \times T^2$?

What is the F-theory for arbitrary monodromies?

U-manifolds [Kumar, Vafa]
$T^{n>2}$ fibrations: [Liu, Minasian] [Curio, Lüst] [Lu, Roy] [Vegh, McGreevy] [Achmed-Zade, Hamilton, Lüst, Massai]
K3 fibrations → G-theory [Martucci, Morales, Pacifici] [Braun, Fucito, Morales] [Candelas, Constantin, Damian, Larfors, Morales]

General idea: associate

couple magnetically to codim-2 branes

(a subset of the) scalar moduli (transforming under monodromies)
to
geometric moduli of auxiliary space fibred over physical base

Degenerations of this fibre → (generically exotic) branes

What (if any) space realises e.g. $E_{8(8)}/\mathrm{SO}(16)$?

ExFT relationship? Some discussion in [Hohm, Wang] [Berman, CB, Malek, Rudolph] [Chabrol]

What is the geometry for non-geometry?

Doubled worldsheet [Duff] [Tseytlin] [Siegel] [Hull]

Doubled coords: $Y^M = \begin{pmatrix} y^i \\ \tilde{y}^i \end{pmatrix}$

Constraint

$$S = \int d^2\sigma \frac{1}{4} \mathcal{H}_{MN} \partial_\alpha Y^M \partial^\alpha Y^N \,, \quad \partial_\alpha Y^M = \eta^{MK} \mathcal{H}_{KN} \epsilon_\alpha{}^\beta \partial_\beta Y^N$$

Allows T-fold patching $Y^M \to \mathcal{P}^M{}_N Y^N$, $\mathcal{P} \in O(d,d)$
(Local) Polarisation: choice of y^i physical
"Section condition:" $\mathcal{H} = \mathcal{H}(y)$ or $\mathcal{H}(\tilde{y})$

Extended geometries: [See overview talks at this conference by Samtleben, Marqués]
Spacetime picture: Double Field Theory, Exceptional Field Theory
$\to$ "natural" setting for exotic branes, winding coordinate dependence (subject to section or gen. Scherk-Schwarz constraints)

Next: other topics

There are many other topics worth introducing

Classification & coupling to mixed symmetry dual fields

From E_{11}: [West] [Cook, West] [Riccioni, West] [Kleinschmidt], from "wrapping rules": [Bergshoeff, Riccioni and collaborators], from ExFT: [Fernández-Melgarejo, Kimura, Sakatani, Uehara] [Bakhmatov, Berman, Kleinschmidt, Otsuki, Musaev]

Worldvolume actions

[Eyras, Lozano] [Chatzistavrakidis, Gautason, Moutsopoulos, Zagermann] [Kimura, Sasaki, Yata]

Closed string non-commutativity and non-associativity

[Lüst] [Blumenhagen, Plauschinn] and many others

(Generalised) Orbifolds & Orientifolds

In non-geo compactifications [Dabholkar, Hull]; in ExFT [CB, Malek, Thompson]

"Mysterious duality" with del Pezzo surfaces

Branes in correspondence with curves [Iqbal, Neitzke, Vafa] [Kaidi]

Non-geometric engineering?

Branes on S-/U-folds $\rightarrow$ novel $\mathcal{N} = 3$ field theories [García-Etxebarria, Regalado]

Do exotic branes have electromagnetic duals?

Magnetic: B_{ij} $A_i^{(j)}$ $\beta^{(i)(j)}$

$$\mathrm{NS5} \xrightarrow{T_j} \mathrm{KKM} \xrightarrow{T_i} 5^2_2$$

$$\mathrm{F1} \xrightarrow{T_z} \mathrm{pp}$$

What here?

Electric: B_{tz} $A_t^{(z)}$

Do exotic branes have electromagnetic duals?

Magnetic: B_{ij} $\qquad A_i^{(j)}$ $\qquad \beta^{(i)(j)}$

$$\begin{tikzcd} \mathrm{NS5} \arrow[r, "T_j"] & \mathrm{KKM} \arrow[r, "T_i"] & 5^2_2 \end{tikzcd}$$

$$\begin{tikzcd} \mathrm{F1} \arrow[r, "T_z"] & \mathrm{pp} \arrow[r, "T_t"] & \text{Negative F1?} \end{tikzcd}$$

Electric: B_{tz} $\qquad A_t^{(z)}$ $\qquad \beta^{(t)(z)}$

[Welch]

[Sakatani]

[CB]

Electric non-geometry with non-relativistic geometry?

Negative F1

Normally $+$

$$ds^2 = \tilde{H}^{-1}(-dt^2 + dz^2) + d\vec{x}_8^{\,2}\,, \quad B_{tz} = \tilde{H}^{-1} - 1\,, \quad \tilde{H} = 1 - \frac{q}{|\vec{x}_8|^6}$$

$$\mathcal{H}_{MN} = \begin{pmatrix} \tilde{H}-2 & 0 & 0 & \tilde{H}-1 \\ 0 & 2-\tilde{H} & \tilde{H}-1 & 0 \\ 0 & \tilde{H}-1 & \tilde{H} & 0 \\ \tilde{H}-1 & 0 & 0 & \tilde{H} \end{pmatrix}$$

Negative tension branes:

[Dijkgraaf, Heidenreich, Jefferson, Vafa]

Naked singularity marks "bubble" with "exotic" string theory inside

[Hull]

Electric non-geometry with non-relativistic geometry?

Negative F1

Normally $+$

$$ds^2 = \tilde{H}^{-1}(-dt^2 + dz^2) + d\vec{x}_8{}^2\,, \quad B_{tz} = \tilde{H}^{-1} - 1\,, \quad \tilde{H} = 1 - \frac{q}{|\vec{x}_8|^6}$$

$$\mathcal{H}_{MN} = \begin{pmatrix} \tilde{H}-2 & 0 & 0 & \tilde{H}-1 \\ 0 & 2-\tilde{H} & \tilde{H}-1 & 0 \\ 0 & \tilde{H}-1 & \boxed{\begin{matrix}\tilde{H} & 0 \\ 0 & \tilde{H}\end{matrix}} \\ \tilde{H}-1 & 0 & & \end{pmatrix}$$

Degenerate at singularity $\tilde{H} = 0$
$\to$ non-relativistic limit of string

[Gomis, Ooguri] [Park & collaborators] [CB, Berman, Otsuki]

Negative tension branes:

[Dijkgraaf, Heidenreich, Jefferson, Vafa]

Naked singularity marks "bubble" with "exotic" string theory inside

[Hull]

Let's revisit the smeared NS5 on $\mathbb{R}^3 \times S^1$

Transverse $\mathbb{R}^3 \times S^1$

$$ds^2 = dx_{012345}^2 + f(dx_{678}^2 + (d\theta)^2)$$

$$f = 1 + \frac{q'}{r}\,, \quad r = \sqrt{(x^6)^2 + (x^7)^2 + (x^8)^2}$$

This is the localised NS5 solution on $\mathbb{R}^3 \times S^1$

Transverse $\mathbb{R}^3 \times S^1$

$$ds^2 = dx_{012345}^2 + f(dx_{678}^2 + (d\theta)^2)$$

$$f = 1 + \frac{q'}{r}\frac{\sinh r}{\cosh r - \cos\theta}\,, \quad r = \sqrt{(x^6)^2 + (x^7)^2 + (x^8)^2}$$

Worldsheet instanton corrections:
Conjecture: string sigma model in smeared NS5 solution is corrected to localised solution, evidence from calculation of [Tong]

The T-dual KKM solution is localised in winding space

Transverse $\mathbb{R}^3 \times S^1$

$$ds^2 = dx^2_{012345} + f dx^2_{678} + f^{-1}\left(d\tilde{\theta} + A_i dx^i\right)^2$$

$$f = 1 + \frac{q'}{r}\frac{\sinh r}{\cosh r - \cos\theta}\,, \quad r = \sqrt{(x^6)^2 + (x^7)^2 + (x^8)^2}$$

Depends on θ; physical coordinate is $\tilde{\theta}$

Worldsheet instanton corrections:
Conjecture: string sigma model in KKM solution is corrected to winding localised solution, evidence from calculation of [Harvey, Jensen] (c.f. [Gregory, Harvey, Moore])
Interpretation: doubled geometry [Jensen] [Berman, Rudolph]

Extension to 5^2_2: [Kimura, Sasaki]

Finally, final remarks

Lesson 1: Negation is not uninteresting

Lesson 2: Exotic is ordinary

Finally, final remarks

Lesson 1: Negation is not uninteresting

Lesson 2: Exotic is ordinary

Thanks for listening!