Higher-derivative corrections from dualities

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Some goals

- Understand how duality constrains/organizes higher derivatives.
- New methods to *compute* higher derivatives.
- Construct duality covariant formulations of higher derivatives.
- Unmask the generalized geometric structure underlying higher derivatives.

Some history

- Sen 1991: $O(d,d)$ symmetry to all orders in α'.
- Meissner, Kaloper and Meissner 1997: Cosmological and circle reductions of four-derivative interactions.
- Garousi 2010, Hohm and Zwiebach 2015: Methods to constrain higher derivatives.
- Hohm, Siegel, Zwiebach 2013: First duality covariant formulation of higher derivatives.

Two derivatives

The universal NSNS sector of supergravity

$$S = \int dx \sqrt{-g} e^{-2\phi} \left(R + 4 (\partial\phi)^2 - \frac{1}{12} H^2 \right)$$

Obtained from string theory through scattering amplitudes or beta-functions.

Can we get this action from symmetry principles?

Two derivatives

We have diffeos, Lorentz and gauge symmetries

$$\begin{aligned} \delta e &= L_\xi e + e \cdot \Lambda \\ \delta B &= L_\xi B + d\lambda \\ \delta \phi &= L_\xi \phi \end{aligned}$$

which up to boundary terms give

$$S = \int dx \sqrt{-g} e^{-2\phi} \left(R + \alpha \, (\partial\phi)^2 + \beta \, H^2 \right)$$

Two derivatives

If on top we add supersymmetry the relative coefficients get fixed

$$S = \int dx \sqrt{-g} e^{-2\phi} \left(R + 4 \left(\partial\phi\right)^2 - \frac{1}{12} H^2 \right)$$

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so the action is fixed by symmetry principles.

- Supersymmetry is also successful in constraining higher derivatives.
- It might not be enough: the bosonic string is not supersymmetric.
- Can T-duality be used as a symmetry principle?

Two derivatives

After a Kaluza-Klein reduction

$$G = \begin{pmatrix} g + AGA^t & AG \\ GA^t & G \end{pmatrix} \ , \quad B = \begin{pmatrix} b + \frac{1}{2}(AV^t - VA^t) - ABA^t & V + AB \\ -V^t + BA^t & B \end{pmatrix}$$

we get an ungauged half-maximal supergravity with a global $O(d,d)$ symmetry

$$\mathcal{A} = (V \ , \ A) \ , \quad \mathcal{M} = \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix}$$

$$h \in O(d,d) \ , \quad \mathcal{A}' = h\mathcal{A} \ , \quad \mathcal{M}' = h\mathcal{M}h^t$$

Two derivatives

But this is only true for the specific values of the parameters

$$S = \int dx \sqrt{-g} e^{-2\phi} \left(R + 4 \, (\partial\phi)^2 - \frac{1}{12} \, H^2 \right)$$

because $O(d,d)$ mixes the gauge and gravitational sectors.

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Duality as a method to constrain actions: the parameters should be fixed by demanding the emergence of $O(d, d)$ symmetries after toroidal compactifications.

Four derivatives

When higher derivatives enter the game funny things happen...

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Suppose we have a two-derivative action that is fixed by some symmetries

$$S = \int dx \mathcal{L}^0[\Phi] \ , \quad \delta^0 S = 0$$

Invariant four-derivative terms might not exist

$$S = \int dx (\mathcal{L}^0[\Phi] + \mathcal{L}^1[\Phi]) \ , \quad \delta^0 S \neq 0$$

so the transformations might need corrections

$$\delta S = \delta^0 S + \delta^1 S^0 = 0$$

Four derivatives

Which can be written as

$$\delta^0 S^1 + \delta^1 S^0 = \int dx \left(\delta^0 \mathcal{L}^1 + \frac{\delta \mathcal{L}^0}{\delta \Phi} \delta^1 \Phi \right) = 0$$

The new terms need not be invariant, but their transformation must be proportional to the lowest order eom.

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Supersymmetric transformations seem to receive higher derivative corrections.

Four derivatives

Due to field redefinitions the four-derivative string actions can be written in various ways.

Universal NSNS sector by Metsaev and Tseytlin, 1987

$$S_{MT} = \int dx \sqrt{-g} e^{-2\phi} \left[R + 4(\partial\phi)^2 - \frac{1}{12} H^2 + \frac{a-b}{4} H^{\mu\nu\rho} \Omega_{\mu\nu\rho} \right.$$
$$\left. - \frac{a+b}{8} \left(\mathrm{Riem}^2 - \frac{1}{2} H\, H\, \mathrm{Riem} + \frac{1}{24} H^4 - \frac{1}{8} (H^2_{\mu\nu})^2 \right) \right]$$

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	Bosonic	Heterotic	Type II
$a+b$	$-2\alpha'$	$-\alpha'$	0
$a-b$	0	$-\alpha'$	0

Four derivatives

Bergshoeff and de Roo, 1989 (for heterotic)

$$S_{BdR} = \int dx\sqrt{-g}e^{-2\phi}\left(R + 4(\partial\phi)^2 - \frac{1}{12}\widehat{H}^2 \right.$$
$$\left. + \frac{a}{8}R^{(-)}_{\mu\nu a}{}^{b}R^{(-)\mu\nu}{}_{b}{}^{a} + \frac{b}{8}R^{(+)}_{\mu\nu a}{}^{b}R^{(+)\mu\nu}{}_{b}{}^{a}\right)$$

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Hiddenly contains higher orders...

$$\begin{aligned} \omega^{(\pm)} &= \omega \pm \tfrac{1}{2}\widehat{H} \\ \widehat{H} &= dB - \tfrac{3}{2}\, a\, \Omega^{(-)} + \tfrac{3}{2}\, b\, \Omega^{(+)} \\ \Omega^{(\pm)} &= \mathrm{tr}\left(\omega^{(\pm)} \wedge d\omega^{(\pm)} + \tfrac{2}{3}\omega^{(\pm)} \wedge \omega^{(\pm)} \wedge \omega^{(\pm)}\right) \end{aligned}$$

Four derivatives

The BdR action is invariant and

$$\widehat{H} = dB - \tfrac{3}{2}\, a\, \Omega^{(-)} + \tfrac{3}{2}\, b\, \Omega^{(+)}$$

is covariant due to the Green-Schwarz transformation of B

$$\begin{aligned}
\delta\phi &= L_\xi \phi \\
\delta e^a &= L_\xi e^a + e^b \Lambda_b{}^a \\
\delta\Omega^{(\pm)} &= L_\xi \Omega^{(\pm)} - d\ \mathrm{tr}\left(d\Lambda \wedge \omega^{(\pm)}\right) \\
\delta B &= L_\xi B + d\lambda - \frac{a}{2}\mathrm{tr}\left(d\Lambda \wedge \omega^{(-)}\right) + \frac{b}{2}\mathrm{tr}\left(d\Lambda \wedge \omega^{(+)}\right)
\end{aligned}$$

Four derivatives

Green-Schwarz transformation of the two-form

$$\delta_\Lambda B_{\mu\nu} = \frac{a-b}{2}\omega_{[\mu a}{}^{b}\partial_{\nu]}\Lambda_b{}^a - \frac{a+b}{4}H_{[\mu a}{}^{b}\partial_{\nu]}\Lambda_b{}^a$$

- $a-b$ term is not removable unless $a=b$ (bosonic string).
- $a+b$ term is removable through

$$B_{\mu\nu} \to B_{\mu\nu} + \frac{a+b}{4}H_{[\mu a}{}^{b}\omega_{\nu]b}{}^{a}$$

Four derivatives

Additional remarks:

- There is another famous scheme by Hull and Townsend 1987 where the first order gravitational sector is a quadratic Gauss-Bonnet.
- To second order the heterotic string receives no corrections in the bosonic sector, but reappear at order α'^3 through quartic Riemann interactions (more on this later).
- The bosonic string is known to contain cubic Riemann interactions, cubic Gauss-Bonnet terms, quartic Riemann interactions, etc. The full NSNS sector is not known beyond the first order.
- Type II and M-theory corrections start with eight derivatives through quartic Riemann interactions.
- On top of these corrections there are g_s corrections that we are neglecting.

Duality constraints on higher derivatives

The bosonic string is not supersymmetric

$$S_{MT} = \int dx \sqrt{-g} e^{-2\phi} \left[R + 4(\partial\phi)^2 - \frac{1}{12} H^2 + \frac{\alpha'}{4} \left(\mathrm{Riem}^2 - \frac{1}{2} H\, H\, \mathrm{Riem} + \frac{1}{24} H^4 - \frac{1}{8} H^2_{\mu\nu} H^{2\mu\nu} \right) \right]$$

but it should be constrained by duality!

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Meissner 1997 compactified this action and realized that field redefinitions were required to form duality multiplets.

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Hohm and Zwiebach 2015 converted this observation into a method to constrain higher derivatives.

Duality constraints on higher derivatives

The idea is to perform a cosmological reduction

$$G = \mathrm{diag}(-n(t), g(t)) \ , \quad B = \mathrm{diag}(0, b(t))$$

Plug the ansatz into the action and define

$$L = g^{-1}\dot{g} \ , \quad M = g^{-1}\dot{b} \ , \quad d = \phi - \tfrac{1}{2}\log\sqrt{g}$$

We end with an effective action depending only on M, L, $\dot{d}$ and time-derivatives thereof.

Duality constraints on higher derivatives

One can then use the equations of motion

$$\begin{aligned}
\dot{L} &= M^2 + \dot{d} L & \text{[eom } g\text{]} \\
\dot{M} &= ML + \dot{d} M & \text{[eom } b\text{]} \\
\ddot{d} &= \tfrac{1}{2}[\dot{d}^2 - \tfrac{1}{4}\mathrm{tr}(M^2 - L^2)] & \text{[eom } d\text{]} \\
\dot{d}^2 &= -\tfrac{1}{4}\mathrm{tr}(M^2 - L^2) & \text{[eom } n\text{]}
\end{aligned}$$

and integration by parts

$$\dot{d}^n \; f(M, L) = g_n(M, L)$$

to take the effective action to a final canonical form depending only on M and L.

Duality constraints on higher derivatives

To determine if the effective action is duality symmetric, find the minimal form of the most general invariant action using duality constraints, eoms and integration by parts.

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It turns out to be a linear combination of powers of

$$\mathrm{tr}(\dot{\mathcal{S}}^n)\ , \quad \mathrm{n\ even}\ \geq 4\ , \qquad \mathcal{S} \equiv \mathcal{M}\eta^{-1}$$

$$S = \int dt \left(a_1 \mathrm{tr}(\dot{\mathcal{S}}^4) + b_1 \mathrm{tr}(\dot{\mathcal{S}}^6) + c_1 \mathrm{tr}(\dot{\mathcal{S}}^8) + c_2 \mathrm{tr}(\dot{\mathcal{S}}^4)^2 + \dots \right)$$

and this in turn can be rewritten in terms of M and L.

Duality constraints on higher derivatives

Example: Consider the minimal form of the four-derivative terms found by Metsaev and Tseytlin

$$\mathcal{L}^1 = \gamma_1\, Riem^2 + \gamma_2\, HHRiem + \gamma_3\, H^4 + \gamma_4\, (H^2_{\mu\nu})^2 + \gamma_5\, (H^2)^2$$
$$+\gamma_6\, H^2_{\mu\nu}\partial^\mu\phi\partial^\nu\phi + \gamma_7\, H^2(\partial\phi)^2 + \gamma_8\, (\partial\phi)^4$$

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The method determines the action up to some terms

$$\begin{aligned} \mathcal{L}^1 &= Riem^2 - \tfrac{1}{2}HHRiem + \tfrac{1}{24}H^4 - \tfrac{1}{8}(H^2_{\mu\nu})^2 \\ &\quad + t\left(H^4 + \tfrac{1}{12}(H^2)^2\right) + u\left(H^2_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - \tfrac{1}{3}H^2(\partial\phi)^2\right) \end{aligned}$$

Duality constraints on higher derivatives

These approaches are predictive but...

- Need general inputs that get very messy with higher derivatives.
- Leave unfixed coefficients.
- Perturbative in α'.

If we want to understand the role of duality, it might be better to move to a duality covariant framework:
Double Field Theory, Siegel 1993; Hull, Zwiebach and Hohm 2009.

Two derivatives in Double Field Theory

Three symmetries:

- Global $G = O(D, D)$ indices $M, N, P, \ldots$
  Invariant metric η_{MN}.

- Local $H = \underline{O(D)} \times \overline{O(D)}$ indices $A, B, C, \ldots$
  Invariants $\underline{P}_{AB}$ & $\bar{P}_{AB}$ and parameter $\Lambda_{[AB]}$.

- Generalized diffeomorphisms $\widehat{\mathcal{L}}$ generated by ξ^M / $\widehat{\mathcal{L}}_\xi \eta = 0$.
  Close wrt a C-bracket.

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Two derivatives in Double Field Theory

The fields are duality covariant multiplets

$$E_M{}^A \ , \quad d$$

The generalized frame is a constrained field

$$\eta_{MN} = E_M{}^A\,(\bar{P}_{AB} + P_{AB})\,E_N{}^B$$

and defines a generalized metric (also constrained)

$$\mathcal{H}_{MN} = E_M{}^A(\bar{P}_{AB} - P_{AB})E_N{}^B$$

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Double space but strong-constrained

$$\eta^{MN} \partial_M \partial_N \cdots = 0$$

Two derivatives in Double Field Theory

Duality covariant gauge transformations

$$\begin{aligned} \delta e^{-2d} &= \partial_M \left(\xi^M e^{-2d}\right) \\ \delta E_M{}^A &= (\widehat{\mathcal{L}}_\xi E)_M{}^A + E_M{}^B \, \Lambda_B{}^A \end{aligned}$$

which imply

$$\delta \mathcal{H}_{MN} = (\widehat{\mathcal{L}}_\xi \mathcal{H})_{MN}$$

Two derivatives in Double Field Theory

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which imply

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Composite objects with nice transformation properties:

$$F_{ABC} = 3 E^M{}_{[A} \partial_M E^N{}_B E^P{}_{C]} \eta_{NP}$$
$$F_A = \partial_M E^M{}_A - 2 E^M{}_A \partial_M d$$

Two derivatives in Double Field Theory

Two equivalent ways of writing an action

$$S = \int dX\, e^{-2d}\, \mathcal{R}$$

The metric formalism

$$\begin{aligned}\mathcal{R} \;=\;& 4\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\partial_M d\,\partial_N d + 4\partial_M\mathcal{H}^{MN}\partial_N d \\ &+\frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_K\mathcal{H}_{LN}\end{aligned}$$

Two derivatives in Double Field Theory

Two equivalent ways of writing an action

$$S = \int dX \, e^{-2d} \, \mathcal{R}$$

The frame formalism

$$\begin{aligned} \mathcal{R} \;=\;& -2 D_{\underline{A}} F^{\underline{A}} + 2 D_{\overline{A}} F^{\overline{A}} + F_{\underline{A}} F^{\underline{A}} - F_{\overline{A}} F^{\overline{A}} \\ & -\frac{1}{6} F_{\underline{ABC}} F^{\underline{ABC}} + \frac{1}{6} F_{\overline{ABC}} F^{\overline{ABC}} - \frac{1}{2} F_{\underline{AB}\overline{C}} F^{\underline{AB}\overline{C}} - \frac{1}{2} F_{\overline{AB}\underline{C}} F^{\overline{AB}\underline{C}} \end{aligned}$$

Two derivatives in Double Field Theory

To make contact with supergravity... $GL(D) \in O(D,D)$ decomposition:

$$E \to e \oplus B \ , \qquad d \to \phi \oplus g$$

The gauge symmetries reduce to

$$\left.\begin{aligned} \delta E &= \widehat{\mathcal{L}}_\xi E + E \cdot \Lambda \\ \delta d &= \xi \cdot d - \tfrac{1}{2}\, \partial \cdot \xi \end{aligned}\right\} \quad \to \quad \left\{\begin{aligned} \delta e &= L_\xi e + e \cdot \Lambda \\ \delta B &= L_\xi B + d\lambda \\ \delta \phi &= L_\xi \phi \end{aligned}\right.$$

Two derivatives in Double Field Theory

To make contact with supergravity... $GL(D) \in O(D,D)$ decomposition:

$$E \to e \oplus B \ , \qquad d \to \phi \oplus g$$

and the action reduces to the standard supergravity action

$$S = \int dx \sqrt{-g} e^{-2\phi} \left(R + 4\,(\partial\phi)^2 - \frac{1}{12}\,H^2 \right)$$

with fixed coefficients.

Towards more than two derivatives in Double Field Theory

Some obstructions to naively searching for higher corrections with these symmetries:

- no-go theorem by Hohm and Zwiebach 2011: It is impossible to write quadratic Riemann interactions in terms of a Generalized Metric.
- Need non-covariant field redefinitions.
- Generalized Riemann tensor not covariant/unique/determined. Jeon et. al. 2010, Coimbra et. al. 2011, Hohm et. al. 2011.

Double α'-Geometry

Hohm, Siegel and Zwiebach, 2013, first duality covariant deformation of $\widehat{\mathcal{L}}$

$$\delta \mathcal{M}_{MN} = \widehat{\mathcal{L}}_\xi \mathcal{M}_{MN} - \partial_M \mathcal{M}^{PQ} \partial_{P[Q}\xi_{N]} - \partial_P \mathcal{M}_{QM} \partial_N{}^{[Q}\xi^{P]} - \frac{1}{2}\partial_M{}^K \mathcal{M}^{PQ} \partial_{NP[Q}\xi_{K]} + (M \leftrightarrow N)$$

- Invariant action: finite and exact.
- Double metric $\mathcal{M}$ unconstrained, duality covariant, related to $\mathcal{H}$ through eoms.
- Exact corrected C-bracket

$$[\xi_1 \, , \, \xi_2]^M_{\alpha'} = [\xi_1 \, , \, \xi_2]^M_{(C)} + \partial_P \xi^Q_{[1} \partial^M \partial_Q \xi^P_{2]}$$

Double α'-Geometry

Hohm and Zwiebach, 2014

Studying field perturbations around flat space, the correction induces

$$\delta B_{\mu\nu} = L_\xi B_{\mu\nu} + 2\partial_{[\mu}\lambda_{\nu]} + \frac{1}{2}\partial_{[\mu}(\partial_\rho \xi^\sigma)\,\Gamma^\rho{}_{\nu]\sigma}$$

which is a Green-Schwarz transformation of B wrt diffeos.

The deformation of $\widehat{\mathcal{L}}$ is a generalized Green-Schwarz transformation.

Double α'-Geometry

- The formulation in terms of the double metric is intriguing: finite expansion for infinite higher derivatives.

- The action contains:
  - Chern-Simons corrections to the three-form, Hohm, Zwiebach 2014
  - Cubic Riemann interactions, Hohm, Naseer, Zwiebach 2016
  - Cubic Gauss-Bonnet interactions, Lescano, Marques 2016

- Alternates Z_2 parity $B \to -B$ symmetry order by order. As a consequence, it is unrelated to the bosonic and heterotic strings: no quadratic Riemann interactions.

Duality in the heterotic string

There are two approaches to T-duality covariant first order corrections in heterotic supergravity:

- Extending the duality group

  Bedoya, Marques and Nunez 2014

  Coimbra, Minasian, Triendl and Waldram 2014

- Deforming the gauge symmetries

  Marques and Nunez 2015

  Hohm and Zwiebach 2014

Extending the duality group

It is based on two facts related to heterotic supergravity

$$\mathcal{L} = R + 4(\partial\phi)^2 - \frac{1}{12}\widehat{H}^2 - \frac{1}{4}F^2 + \text{fermions}$$

where

$$\widehat{H} = dB + CS(A) + \text{fermions}$$

Extending the duality group

The first observation is due to Bergshoeff and de Roo 1988

gauge fields	A	$\leftrightarrow$	$\omega^{(-)}$	spin con. w/torsion
gauginos	χ	$\leftrightarrow$	$D\psi$	gravitino curvature

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$$\begin{array}{llcll} \text{gauge fields} & A & \leftrightarrow & \omega^{(-)} & \text{spin con. w/torsion} \\ \text{gauginos} & \chi & \leftrightarrow & D\psi & \text{gravitino curvature} \end{array}$$

The Bergshoeff-de Roo identification is based on supersymmetry

$$\begin{array}{lcl} \delta A = \bar{\epsilon}\gamma\chi & \leftrightarrow & \delta\omega^{(-)} = \bar{\epsilon}\gamma D\psi \\ \delta\chi = F_{\mu\nu}\gamma^{\mu\nu}\epsilon & \leftrightarrow & \delta D\psi = R_{-\mu\nu}\gamma^{\mu\nu}\epsilon \end{array}$$

The pair $(\omega^{(-)}, D\psi)$ *effectively* behaves as a gauge multiplet.

Extending the duality group

First order corrections are obtained by including extra Lorentz multiplets and *identifying* them with $(\omega^{(-)}, D\psi)$

$$\mathcal{L} = R + 4(\partial\phi)^2 - \frac{1}{12}\widehat{H}^2 - \frac{1}{4}F^2 + \frac{1}{4}R^{(-)2} + \text{fermions}$$

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where

$$\widehat{H} = dB + CS(A) - CS(\omega^{(-)}) + \text{fermions}$$

- $CS(\omega^{(-)})$ deforms the transformation of $\omega^{(-)}$ itself, rendering the identification ill-defined to second order.
- Noether procedure for higher orders, Bergshoeff and de Roo 1989.

Extending the duality group

The second observation is due to Hohm and Kwak 2011

Gauge multiplets are incorporated into DFT through extensions of the duality group and local symmetries

$$\mathcal{G} = O(D, D + k) \ , \qquad \mathcal{H} = \underline{O(D)} \times \overline{O(D + k)}$$

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The second observation is due to Hohm and Kwak 2011

Gauge multiplets are incorporated into DFT through extensions of the duality group and local symmetries

$$\mathcal{G} = O(D, D + k)\ , \qquad \mathcal{H} = \underline{O(D)} \times \overline{O(D + k)}$$

Under a $GL(D)$ and $O(D)$ decomposition the generalized fields include the gauge multiplet components

$$\mathcal{E} \to e \oplus B \oplus A\ , \qquad \Psi \to \psi \oplus \chi$$

Generalized diffeomorphisms $\to$ $GL(D)$ diffs $\oplus$ B-shifts $\oplus$ $\mathcal{K}$

Extending the duality group

Based on these observations we make a further extension of the duality group

$$\mathcal{E} \to e \oplus B \oplus A \oplus A' \ , \quad \Psi \to \psi \oplus \chi \oplus \chi'$$

and perform a Bergshoeff-de Roo identification

$$\mathcal{K}' \quad \leftrightarrow \quad O(D) \in \overline{O(D + k + k')}$$

$$A' = \omega^{(-)} \ , \qquad \chi' = D\psi$$

Extending the duality group

Pro

- It is guaranteed to work to first order.

Cons

- It is guaranteed not to work to higher orders.
- The identification is done *after* the $GL(D)$ and $O(D)$ decomposition, so the procedure is not duality covariant.

Deforming the gauge symmetries

In the heterotic string the Green-Schwarz transformation

$$\delta B_{\mu\nu} = \frac{1}{2}\partial_{[\mu}\Lambda_a{}^b\,\omega^{(-)a}_{\nu]b}$$

fixes the higher-derivative corrections to the three-form field strength

$$\widehat{H}_{\mu\nu\rho} = 3\partial_{[\mu}B_{\nu\rho]} + \frac{3}{2}\left(\omega^{(-)b}_{[\mu a}\partial_\nu\omega^{(-)a}_{\rho]b} + \frac{2}{3}\omega^{(-)b}_{[\mu a}\omega^{(-)c}_{\nu b}\omega^{(-)a}_{\rho]c}\right)$$

Deforming the gauge symmetries

The heterotic string contains in addition the Riemann-squared term not required by the Green-Schwarz deformation.

Deforming the gauge symmetries

The heterotic string contains in addition the Riemann-squared term not required by the Green-Schwarz deformation.

$$S_{het} = \int dx \sqrt{-g}\, e^{-2\phi} \left(R + 4(\partial\phi)^2 - \frac{1}{12} \widehat{H}^{\mu\nu\rho} \widehat{H}_{\mu\nu\rho} \right.$$
$$\left. - \frac{1}{8} R^{(-)}_{\mu\nu a}{}^{b} R^{(-)\mu\nu}{}_{b}{}^{a} \right)$$

Deforming the gauge symmetries

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T-duality mixes the two-form and the gravitational sector: this is why we seek for a *duality covariant* generalization of the Green-Schwarz transformation.

Deforming the gauge symmetries

How do we turn the Green-Schwarz transformation

$$\delta B_{\mu\nu} = \frac{1}{2}\partial_{[\mu}\Lambda_a{}^b\,\omega^{(-)a}_{\nu]b}$$

into a *duality covariant* form?

Naively, just double...

$$\partial_\mu \to \partial_M \ , \quad \Lambda_a{}^b \to \Lambda_A{}^B \ , \quad \omega^{(-)b}_{\mu a} \to \omega_{MA}{}^B$$

Deforming the gauge symmetries

How do we turn the Green-Schwarz transformation

$$\delta B_{\mu\nu} = \frac{1}{2}\partial_{[\mu}\Lambda_a{}^b\,\omega^{(-)a}_{\nu]b}$$

into a *duality covariant* form?

Then, write the obvious...

$$\delta E_M{}^C = \frac{1}{2}\partial_{[M}\Lambda_A{}^B\,\omega_{N]B}{}^A\,E^{NC}$$

Deforming the gauge symmetries

How do we turn the Green-Schwarz transformation

$$\delta B_{\mu\nu} = \frac{1}{2}\partial_{[\mu}\Lambda_a{}^b\,\omega^{(-)a}_{\nu]b}$$

into a *duality covariant* form?

Select one factor of the double Lorentz group...

$$\delta E_M{}^C = \frac{1}{2}\partial_{[M}\Lambda_{\underline{A}}{}^{\underline{B}}\,\omega_{N]\underline{B}}{}^{\underline{A}}\,E^{NC}$$

Deforming the gauge symmetries

How do we turn the Green-Schwarz transformation

$$\delta B_{\mu\nu} = \frac{1}{2}\partial_{[\mu}\Lambda_a{}^b\, \omega_{\nu]b}^{(-)a}$$

into a *duality covariant* form?

Remove undetermined components of the spin connection...

$$\delta E_M{}^C = \frac{1}{2}\partial_{[M}\Lambda_{\underline{A}}{}^{\underline{B}}\, \omega_{\overline{N}]\underline{B}}{}^{\underline{A}}\, E^{NC}$$

Deforming the gauge symmetries

How do we turn the Green-Schwarz transformation

$$\delta B_{\mu\nu} = \frac{1}{2}\partial_{[\mu}\Lambda_a{}^b\,\omega^{(-)a}_{\nu]b}$$

into a *duality covariant* form?

Impose duality constraints on the generalized frame...

$$\delta E_M{}^C = \frac{1}{2}\partial_{[\underline{M}}\Lambda_{\underline{A}}{}^{\underline{B}}\,\omega_{\overline{N}]\underline{B}}{}^{\underline{A}}\,E^{NC}$$

Deforming the gauge symmetries

How do we turn the Green-Schwarz transformation

$$\delta B_{\mu\nu} = \frac{1}{2}\partial_{[\mu}\Lambda_a{}^b\,\omega^{(-)a}_{\nu]b}$$

into a *duality covariant* form?

Replace the spin connection by the generalized fluxes...

$$\delta E_M{}^C = \frac{1}{2}\partial_{[\underline{M}}\Lambda_{\underline{A}}{}^{\underline{B}}\,F_{\overline{N}]\underline{B}}{}^{\underline{A}}\,E^{NC}$$

Deforming the gauge symmetries

We could as well have chosen the opposite projections

$$\delta E_M{}^A = \widehat{\mathcal{L}}_\xi E_M{}^A + E_M{}^B \Lambda_B{}^A + \left(a\, \partial_{[\underline{M}} \Lambda_{\underline{B}}{}^{\underline{C}}\, F_{\overline{N}]\underline{C}}{}^{\underline{B}} - b\, \partial_{[\overline{M}} \Lambda_{\overline{B}}{}^{\overline{C}}\, F_{\underline{N}]\overline{C}}{}^{\overline{B}} \right) E^{NA}$$

Deforming the gauge symmetries

We could as well have chosen the opposite projections

$$\delta E_M{}^A = \widehat{\mathcal{L}}_\xi E_M{}^A + E_M{}^B \Lambda_B{}^A + \left(a\, \partial_{[\underline{M}} \Lambda_{\underline{B}}{}^{\underline{C}}\, F_{\overline{N}]\underline{C}}{}^{\underline{B}} - b\, \partial_{[\overline{M}} \Lambda_{\overline{B}}{}^{\overline{C}}\, F_{\underline{N}]\overline{C}}{}^{\overline{B}} \right) E^{NA}$$

This is the two-parameter generalized Green-Schwarz transformation, Marques and Nunez, 2015.

The parameters a and b are $\mathcal{O}(\alpha')$.

Deforming the gauge symmetries

The gauge transformations preserve the constraints and close

$$\left[\delta_{(\xi_1\,,\Lambda_1)},\delta_{(\xi_2\,,\Lambda_2)}\right]=\delta_{(\xi_{21}\,,\Lambda_{21})}$$

w.r.t. the brackets

$$\xi_{12}^M = [\xi_1\,,\,\xi_2]_{(C)}^M - \frac{a}{2}\,\Lambda_{[1\underline{A}}{}^{\underline{B}}\partial^M\Lambda_{2]\underline{B}}{}^{\underline{A}} + \frac{b}{2}\,\Lambda_{[1\overline{A}}{}^{\overline{B}}\partial^M\Lambda_{2]\overline{B}}{}^{\overline{A}}$$

$$\begin{aligned}\Lambda_{12\,AB} &= 2\xi_{[1}^P\partial_P\Lambda_{2]\,AB} - 2\Lambda_{[1\,A}{}^C\Lambda_{2]\,CB} \\ &\quad + a\,\partial_{[\overline{A}}\Lambda_1^{\underline{CD}}\partial_{\overline{B}]}\Lambda_{2\underline{DC}} + a\,\partial_{[\underline{A}}\Lambda_1^{\underline{CD}}\partial_{\underline{B}]}\Lambda_{2\underline{DC}} \\ &\quad - b\,\partial_{[\underline{A}}\Lambda_1^{\overline{CD}}\partial_{\underline{B}]}\Lambda_{2\overline{DC}} - b\,\partial_{[\overline{A}}\Lambda_1^{\overline{CD}}\partial_{\overline{B}]}\Lambda_{2\overline{DC}}\end{aligned}$$

Deforming the gauge symmetries

The first-order gauge invariant action is fixed

$$S = \int dX e^{-2d} \left(\mathcal{R} + a\,\mathcal{R}^{(-)} + b\,\mathcal{R}^{(+)} \right)$$

for some $\mathcal{R}^{(\pm)}$ that satisfy:

- $\mathcal{R}^{(\pm)}$ must transform as scalars under diffeomorphisms.
- $\widetilde{\delta}_\Lambda \mathcal{R} + \delta_\Lambda \left(a\,\mathcal{R}^{(-)} + b\,\mathcal{R}^{(+)} \right) = 0.$

Deforming the gauge symmetries

One finds

$$\begin{aligned}
\mathcal{R}^{(-)} \;=\;& -\partial_A\partial_B F_{CD\,E}\;F_{FGH}\left(P^{CF}P^{D\,G}\bar{P}^{AE}\bar{P}^{BH}+P^{CF}P^{D\,G}\bar{P}^{AH}\bar{P}^{BE}\right)\\
&+\partial_A F_{BCD}\;\partial_E F_{FGH}\left(\frac{1}{2}P^{AE}P^{BF}P^{CG}\bar{P}^{D\,H}-P^{BF}P^{CG}\bar{P}^{AD}\,\bar{P}^{EH}-\frac{1}{2}P^{BF}P^{CG}\bar{P}^{AE}\bar{P}^{D\,H}\right)\\
&+(2\,\partial_A F_B-F_AF_B)\,F_{CD\,E}F_{FGH}P^{CF}P^{D\,G}\bar{P}^{AE}\bar{P}^{BH}\\
&+2\,\partial_A F_{BCD}\;F_{FGH}F_E\left(P^{BF}P^{CG}\bar{P}^{AD}\,\bar{P}^{EH}+P^{BF}P^{CG}\bar{P}^{AH}\bar{P}^{D\,E}\right)\\
&-\partial_A F_{BCD}\;F_{EFG}F_{HIJ}\left(P^{AE}P^{BH}P^{CI}\bar{P}^{D\,F}\bar{P}^{GJ}+4\,P^{BE}P^{CH}P^{FI}\bar{P}^{AG}\bar{P}^{D\,J}-P^{BE}P^{CF}\bar{P}^{AH}\bar{P}^{D\,I}\bar{P}^{GJ}\right)\\
&+F_{ABC}F_{D\,EF}F_{GHI}F_{JKL}\left(P^{AD}P^{BG}P^{EJ}P^{HK}\bar{P}^{CL}\bar{P}^{FI}-P^{AD}P^{BG}P^{EJ}P^{HK}\bar{P}^{CF}\bar{P}^{IL}\right.\\
&\qquad\left.+P^{AD}P^{BE}P^{GJ}\bar{P}^{CH}\bar{P}^{FK}\bar{P}^{IL}+\frac{4}{3}P^{AD}P^{BG}P^{EH}\bar{P}^{CJ}\bar{P}^{FK}\bar{P}^{IL}\right)
\end{aligned}$$

and $\mathcal{R}^{(+)} = \mathcal{R}^{(-)}\left[P \leftrightarrow \bar{P}\right]$.

Deforming the gauge symmetries

After a $GL(D)$ decomposition

$$\mathcal{H} = \begin{pmatrix} \tilde{g}^{-1} & -\tilde{g}^{-1}\tilde{B} \\ \tilde{B}\tilde{g}^{-1} & \tilde{g} - \tilde{B}\tilde{g}^{-1}\tilde{B} \end{pmatrix}$$

the generalized Green-Schwarz transformation *induces* a Lorentz transformation of the components

$$\begin{aligned} \delta\tilde{g}_{\mu\nu} &= L_\xi \tilde{g}_{\mu\nu} - \frac{a}{2}\omega^{(-)}_{(\mu a}{}^{b}\partial_{\nu)}\Lambda_b{}^a - \frac{b}{2}\omega^{(+)}_{(\mu a}{}^{b}\partial_{\nu)}\Lambda_b{}^a \\ \delta\tilde{B}_{\mu\nu} &= L_\xi \tilde{B}_{\mu\nu} + 2\partial_{[\mu}\tilde{\xi}_{\nu]} + \frac{a}{2}\omega^{(-)}_{[\mu a}{}^{b}\partial_{\nu]}\Lambda_b{}^a - \frac{b}{2}\omega^{(+)}_{[\mu a}{}^{b}\partial_{\nu]}\Lambda_b{}^a \end{aligned}$$

Deforming the gauge symmetries

The two-form transforms exactly as in Bergshoeff-de Roo!

Deforming the gauge symmetries

The two-form transforms exactly as in Bergshoeff-de Roo!

The anomalous Lorentz transformation of $\widetilde{g}$

$$\delta \widetilde{g}_{\mu\nu} = L_\xi \widetilde{g}_{\mu\nu} - \frac{a}{2} \omega^{(-)}_{(\mu a}{}^{b} \partial_{\nu)} \Lambda_b{}^a - \frac{b}{2} \omega^{(+)}_{(\mu a}{}^{b} \partial_{\nu)} \Lambda_b{}^a$$

can be removed through a non-covariant field redefinition

$$\widetilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{a}{4} \omega^{(-)}_{\mu a}{}^{b} \omega^{(-)}_{\nu b}{}^{a} - \frac{b}{4} \omega^{(+)}_{\mu a}{}^{b} \omega^{(+)}_{\nu b}{}^{a}$$

such that

$$\delta g_{\mu\nu} = L_\xi g_{\mu\nu}$$

Deforming the gauge symmetries

- The generalized frame/metric are no longer Lorentz covariant in the standard sense.

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Deforming the gauge symmetries

- The generalized frame/metric are no longer Lorentz covariant in the standard sense.
- But they are still duality covariant.
- Their components $(\widetilde{g}_{\mu\nu},\ \widetilde{B}_{\mu\nu})$ transform as usual under T-duality, but their induced gauge transformations are deformed.
- Contact with the standard supergravity fields $(g_{\mu\nu},\ B_{\mu\nu})$ is achieved through non-covariant field redefinitions.

Deforming the gauge symmetries

Re-writing the first-order DFT action in terms of the gauge covariant fields

$$\mathcal{R} + a\mathcal{R}^{(-)} + b\mathcal{R}^{(+)} = R + 4(\partial\phi)^2 - \frac{1}{12}\widehat{H}_{\mu\nu\rho}\widehat{H}^{\mu\nu\rho} + \frac{a}{8}R^{(-)}_{\mu\nu a}{}^{b}R^{(-)\mu\nu}{}_{b}{}^{a} + \frac{b}{8}R^{(+)}_{\mu\nu a}{}^{b}R^{(+)\mu\nu}{}_{b}{}^{a}$$

we recover the generalized version of the action by Bergshoeff and de Roo, 1989.

Deforming the gauge symmetries

Some remarks:

- This formulation was used in Baron et. al. 2017 to compute first order corrections to half-maximal gauged supergravities.
- It is not possible to turn this frame formalism into a generalized metric formulation unless $a = -b$ (HSZ theory).
- Hohm 2016: Background independence + Duality symmetries + higher-derivatives, require new degrees of freedom with enlarged gauge symmetries.
- It is extremely hard to guess what the second-order corrections to these deformations are.

The generalized Bergshoeff-de Roo identification

To answer the question on how both approaches relate and how to extend them, we had the following idea...

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Instead of decomposing $O(D, D + k)$ w.r.t. $GL(D)$, we preserve $O(D, D)$ covariance, Hohm, Sen and Zwiebach 2014

$$\mathcal{E} \to E \oplus \mathcal{A} \ , \quad E \in O(D, D) \ , \quad E^{M\bar{A}} \mathcal{A}_M{}^{\alpha} = 0$$

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$$\mathcal{E} \to E \oplus \mathcal{A} \ , \quad E \in O(D,D) \ , \quad E^{M\bar{A}} \mathcal{A}_M{}^{\alpha} = 0$$

Only then one should look for a generalized Bergshoeff-de Roo identification

$$A_\mu{}^\alpha \ \leftrightarrow \ \omega^{(-)}_{\mu ab} \quad | \quad \mathcal{A}_{\underline{A}}{}^{\alpha} \ \leftrightarrow \ ???$$

The generalized Bergshoeff-de Roo identification

There are generalizations of everything in DFT:

$$A_\mu{}^\alpha \quad \leftrightarrow \quad \mathcal{A}_{\underline{A}}{}^\alpha$$

$$\omega^{(-)}_{\mu bc} \quad \leftrightarrow \quad \text{Generalized spin connection}$$

$$O(D) \quad \leftrightarrow \quad \underline{O(D)} \times \overline{O(D+k)}$$

The generalized Bergshoeff-de Roo identification

The correct answer turns out to be...

$$A_\mu{}^\alpha \quad \leftrightarrow \quad \mathcal{A}_{\underline{A}}{}^\alpha$$

$$\omega^{(-)}_{\mu bc} \quad \leftrightarrow \quad \mathcal{F}_{\underline{A}\overline{\mathcal{B}}\overline{\mathcal{C}}}$$

$$O(D) \quad \leftrightarrow \quad \overline{O(D+k)}$$

The generalized Bergshoeff-de Roo identification

From the $O(D, D + k)$ gen diffs we get

$$\delta \mathcal{A}_{\underline{A}\alpha} = \widehat{\mathcal{L}}_\xi \mathcal{A}_{\underline{A}\alpha} - \mathcal{D}_{\underline{A}} \lambda_\alpha + g \left[\lambda \, , \, \mathcal{A}_{\underline{A}}\right]_\alpha + \mathcal{A}_{\underline{D}\alpha} \Lambda^{\underline{D}}{}_{\underline{A}}$$

Which transforms basically as a generalized connection

$$\delta \mathcal{F}_{\underline{A}\overline{\mathcal{B}\mathcal{C}}} = \widehat{\mathcal{L}}_\xi \mathcal{F}_{\underline{A}\overline{\mathcal{B}\mathcal{C}}} - \mathcal{D}_{\underline{A}} \Lambda_{\overline{\mathcal{B}\mathcal{C}}} + \left[\Lambda \, , \, \mathcal{F}_{\underline{A}}\right]_{\overline{\mathcal{B}\mathcal{C}}} + \mathcal{F}_{\underline{D}\overline{\mathcal{B}\mathcal{C}}} \Lambda^{\underline{D}}{}_{\underline{A}}$$

The generalized Bergshoeff-de Roo identification

We can match both transformation rules by identifying

$$\mathcal{K} = \overline{O(D+k)}$$

but this is only possible if

$$k = \infty$$

The dictionary between both groups is given by the generators

$$\begin{aligned} (t^{\alpha})_{\overline{\mathcal{AB}}} (t_{\beta})^{\overline{\mathcal{AB}}} &= X_R \, \delta^{\alpha}_{\beta} \\ (t^{\alpha})_{\overline{\mathcal{AB}}} (t_{\alpha})^{\overline{\mathcal{CD}}} &= X_R \, \delta^{\overline{\mathcal{CD}}}_{\overline{\mathcal{AB}}} \end{aligned}$$

The generalized Bergshoeff-de Roo identification

The generalized Bergshoeff-de Roo identification is then
Baron, Lescano and Marques 2018

$$\begin{aligned}
\mathcal{K} &= \overline{O(D+k)} \\
-g\, \lambda_\alpha \, (t^\alpha)_{\overline{\mathcal{AB}}} &= \Lambda_{\overline{\mathcal{AB}}} \\
-g\, \mathcal{A}_{\underline{A}\alpha} \, (t^\alpha)_{\overline{\mathcal{BC}}} &= \mathcal{F}_{\underline{A}\overline{\mathcal{BC}}}[\mathcal{E}[E, \mathcal{A}]]
\end{aligned}$$

- It eliminates some dof by *identifying* them with other dof.
- It introduces two sources of infinities.
- It is exact.
- It doesn't need supersymmetry at all.
- It is *necessarily* generalized.

The generalized Bergshoeff-de Roo identification

It naturally induces a generalized Green-Schwarz transformation for the $O(D,D)$ generalized frame

$$\begin{aligned} \delta E_{M\underline{A}} &= \partial_{\overline{M}} \lambda \cdot \mathcal{A}_{\underline{A}} \\ &\downarrow \\ \delta E_{M\underline{A}} &= \partial_{\overline{M}} \Lambda \cdot \mathcal{F}_{\underline{A}} \end{aligned}$$

that holds to all orders.

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&\downarrow \\
\delta E_{M\underline{A}} &= \partial_{\overline{M}}\Lambda \cdot \mathcal{F}_{\underline{A}}
\end{aligned}
$$

that holds to all orders.
Remember that higher derivatives require deformed gauge transformations for duality covariant multiplets.

The generalized Bergshoeff-de Roo identification

To first order the result coincides with Marques and Nuñez 2015

$$\delta E_M{}^{\underline{A}} = E_M{}^{\underline{B}} \Lambda_{\underline{B}}{}^{\underline{A}} + \frac{b}{2}\, \partial_{\underline{M}} \Lambda^{\overline{BC}}\, F^{\underline{A}}{}_{\overline{BC}}$$

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The second order is a prediction of the formalism

$$\begin{aligned}
\delta E_M{}^{\underline{A}} = {} & \widehat{\mathcal{L}}_\xi E_M{}^{\underline{A}} + E_M{}^{\underline{B}}\, \Lambda_{\underline{B}}{}^{\underline{A}} + \frac{b}{2} \partial_{\overline{M}} \Lambda^{\overline{BC}}\, F^{\underline{A}}{}_{\overline{BC}} \\
& + \frac{b^2}{2}\, E_M{}^{\overline{B}} \Big[\partial_{\overline{B}} \partial^{\overline{C}} \Lambda^{\overline{EF}} \left(F_{\overline{C}\underline{D}}{}^{\underline{A}} F^{\underline{D}}{}_{\overline{EF}} + \partial_{\overline{C}} F^{\underline{A}}{}_{\overline{EF}} \right) - F^{\underline{A}}{}_{\overline{EF}} F_{\underline{C}\overline{D}}{}^{\overline{F}} \left(F^{\underline{C}\overline{HD}} \partial_{\overline{B}} \Lambda_{\overline{H}}{}^{\overline{E}} - F^{\underline{C}\overline{HE}} \partial_{\overline{B}} \Lambda_{\overline{H}}{}^{\overline{D}} \right) \\
& \qquad + F^{\underline{C}}{}_{\overline{EF}}\, \partial_{\overline{B}} \Lambda^{\overline{E}}{}_{\overline{G}} \left(F^{\underline{A}}{}_{\underline{CD}} F^{\underline{D}\overline{GF}} - \partial^{\underline{A}} F_{\underline{C}}{}^{\overline{GF}} + 2\, \partial_{\underline{C}} F^{\underline{A}\overline{GF}} \right) - F^{\underline{A}}{}_{\overline{EF}} \partial_{\overline{B}} \left(\partial^{\underline{C}} \Lambda^{\overline{ED}} F_{\underline{C}\overline{D}}{}^{\overline{F}} \right) \Big]
\end{aligned}$$

and one can in principle keep going! Work in progress...

The generalized Bergshoeff-de Roo identification

The generalized Bergshoeff-de Roo identification admits a supersymmetric extension, Baron, Lescano and Marques 2018

$$
\begin{aligned}
-g\, \mathcal{A}_{\underline{A}\alpha}\, (t^{\alpha})_{\overline{\mathcal{B}\mathcal{C}}} &= \mathcal{F}^{*}_{\underline{A}\overline{\mathcal{B}\mathcal{C}}} - \frac{1}{2} \bar{\Psi}_{\overline{\mathcal{B}}} \gamma_{\underline{A}} \Psi_{\overline{\mathcal{C}}} \\
g \Psi_{\overline{\mathcal{D}}} {\mathcal{E}_{\alpha}}^{\overline{\mathcal{D}}} (t^{\alpha})_{\overline{\mathcal{A}\mathcal{B}}} &= 2 \left[\nabla_{[\overline{\mathcal{A}}} \Psi_{\overline{\mathcal{B}}]} \right]_{\text{det.}}
\end{aligned}
$$

It is also exact and can be perturbed order by order to find fermionic corrections in the heterotic string (Work in progress by Lescano, Nuñez and Rodriguez).

The generalized Bergshoeff-de Roo identification

The generalized Green-Schwarz transformation also admits a bi-parametric extension

$$\delta E_M{}^{\underline{A}} = E_M{}^{\underline{B}} \Lambda_{\underline{B}}{}^{\underline{A}} + \frac{a}{2} \partial^{\underline{A}} \Lambda^{\underline{BC}} F_{\overline{M}\underline{BC}} + \frac{b}{2} \partial_{\overline{M}} \Lambda^{\overline{BC}} F^{\underline{A}}{}_{\overline{BC}}$$

Soon we will show how this fits into an extended version of the generalized Bergshoeff-de Roo identification Baron and Marques, in progress.

Further deformations?

The formalism seems to give a tower of symmetric gauge $\leftrightarrow$ gravitational interactions.

But the heterotic string is known to contain more than that...

$$\alpha'^3 \, \zeta(3) \, R^{(-)4}$$

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Is this a new DFT invariant or a new deformation of the Lorentz symmery?

These terms are similar to those in type II: could shed some light on the U-duality structure of higher derivatives.
Steps in this direction by Coimbra 2018.

Outlook

- More rigorous mathematical framework
- Non-perturbative (exact) treatment
- Understand the nature of the identifications better
- Predict higher order interactions
- Maximal supergravity and U-duality
- Applications to cosmology

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Many exciting results to come! Thank you for your attention!