Asymptotic structure of electromagnetism and gravity in the asymptotically flat case

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Introduction

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What is the situation in the asymptotically flat context ?

This will be the subject of this talk.

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and even infinite-dimensional !

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while the BMS group naturally emerges at null infinity,

previous analyses of asymptotically flat spaces at spatial infinity did not exhibit any sign of the BMS group.

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But if a transformation is a symmetry of a theory, it should be visible in any formulation !

So : which picture should be trusted ?

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Analyses at null infinity and at spatial infinity completely agree !

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Does the BMS symmetry depends on the existence of a sufficiently smooth null infinity ?

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where B_∞ is a boundary term at infinity and where

$$\mathcal{H}^{grav} = -\sqrt{g} R + \frac{1}{\sqrt{g}} (\pi^{ij} \pi_{ij} - \frac{1}{2} \pi^2) \approx 0, \quad \mathcal{H}_i^{grav} = -2 \nabla_j \pi^j_{\ i} \approx 0.$$

(Dirac, Arnowitt-Deser-Misner, Regge-Teitelboim)

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This puts strong and interesting restrictions.

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$$S_H[A_i,\pi^i,A_t] = \int dt \left\{ \int d^3x \pi^i \partial_t A_i - \int d^3x \left(\frac{1}{2}\pi^i\pi_i + \frac{1}{4}F^{ij}F_{ij} + A_t \mathscr{G} \right) \right\}$$

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where $\mathscr{G} = -\partial_k \pi^k \approx 0$ (Gauss' law).

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for the electric field.

These conditions imply

$$F_{ik} = \frac{f_{ik}^{(1)}(\mathbf{n})}{r^2} + \frac{f_{ik}^{(2)}(\mathbf{n})}{r^3} + o(r^{-3})$$

for the magnetic field.

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One needs to strengthen the boundary conditions.

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because the leading orders of Lorentz parameters are parity-odd (boosts and rotations).

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They are also fulfilled by the electromagnetic field of a magnetic monopole (up to a gauge transformation).

From the point of view of containing the known solutions, these boundary conditions seem therefore acceptable.

Asymptotic symmetries and charges

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The physical asymptotic symmetry algebra is the quotient of all the gauge transformations preserving the boundary conditions by the ideal of “trivial" asymptotic symmetries, which have zero charges for all configurations obeying the boundary conditions.

We must therefore both determine the asymptotic gauge transformations that preserve the boundary conditions and compute the value of their generators.

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... but is this the case ?

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The odd part ϵ_{odd} of the gauge parameter ϵ gives a zero contribution to the charges $G[\epsilon]$.

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By contrast, the gauge transformations with ϵ_{even} are improper gauge transformations that do change the physical state of the system.

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Can one devise a “more liberal” strengthening of the original boundary conditions ?

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The answer is affirmative...

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The new boundary conditions that do not suffer from the above drawback are simply the above ones...

up to a gradient !

That is, one demands that the leading order of A_k be even up to $\partial_k \Phi$ for some Φ (of order 1)

$$a_k^{(1)}(-\mathbf{n}) = a_k^{(1)}(\mathbf{n}) + (\partial_k \Phi)^{(1)}(\mathbf{n}), \;\; p^k_{(1)}(-\mathbf{n}) = -p^k_{(1)}(\mathbf{n}).$$

Idea

The answer is affirmative...

and surprisingly simple !

The new boundary conditions that do not suffer from the above drawback are simply the above ones...

up to a gradient !

That is, one demands that the leading order of A_k be even up to $\partial_k\Phi$ for some Φ (of order 1)

$$\bar{a}_k^{(1)}(-\mathbf{n}) = \bar{a}_k^{(1)}(\mathbf{n}) + (\partial_k\Phi)^{(1)}(\mathbf{n}), \;\; p^k_{(1)}(-\mathbf{n}) = -p^k_{(1)}(\mathbf{n}).$$

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and it would be wrong to take the quotient by such transformations.

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is indeed finite.

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These problems can be cured and one finds in the end complete agreement with the null infinity analysis (see MH + C. Troessaert).

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Some form of parity conditions are needed, however, in order to have a finite symplectic structure.

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Do these relaxed parity conditions involving a twist lead to a consistent description

(finite symplectic form, well-defined generators) ?

BMS group at spatial infinity

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The procedure turns out to be very similar to what was found for electromagnetism.

(1) The symplectic form is finite provided one requires that the constraints hold faster at infinity than dictated by the decrease of the canonical variables.

(2) The (asymptotic) boosts also raise difficulties. These are solved by imposing $h_{rA} = 0$ to leading order.

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$$\xi = b_i x^i + T(\mathbf{n}) + O\left(r^{-1}\right)$$
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The zero mode of T and the first spherical harmonic component of W describe translations.

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$$P_\xi[g_{ij}, \pi^{ij}] = \int d^3x \left(\xi \mathscr{H} + \xi^i \mathscr{H}_i\right) + \mathscr{B}_\xi[g_{ij}, \pi^{ij}]$$

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The algebra of the generators can be easily verified to be the BMS algebra.

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In particular, the “matching conditions” of Strominger, which involve the antipodal map, are in fact a consequence of the boundary conditions at spatial infinity

(and leading singularities are absent).

References

M. Henneaux and C. Troessaert, “Asymptotic symmetries of electromagnetism at spatial infinity,” JHEP **1805** (2018) 137 [arXiv :1803.10194 [hep-th]] ;

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Conclusions and comments

The asymptotic symmetries of electromagnetism and gravity in the asymptotically flat context are infinite-dimensional and contain the finite dimensional symmetries of the background as a subgroup.

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Once it is included, one finds a perfect match with the results obtained at null infinity.

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