Introduction to E_9 exceptional field theory

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Geometry and Duality workshop

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with G. Bossard, F. Ciceri, A. Kleinschmidt, H. Samtleben – 1811.04088 + wip

Exceptional field theory

11d: $\mathbb{R}^+$

IIA: $\mathbb{R}^+ \times \mathbb{R}^+$

IIB: $SL(2) \times \mathbb{R}^+$ — $SL(2) \times \mathbb{R}^+ \times \mathbb{R}^+$

$SL(2) \times SL(3) \times \mathbb{R}^+$

$SL(5) \times \mathbb{R}^+$

$SO(5,5) \times \mathbb{R}^+$

$E_6 \times \mathbb{R}^+$

$E_7 \times \mathbb{R}^+$

$E_8 \times \mathbb{R}^+$

E_9

E_{10}

E_{11}

- $\mathrm{E}_{d(d)}$ reformulation of 11d / IIB sugra
- unify 11d / IIB gauge symmetries (on 'internal' space)
- no truncation!

why E_9 ?

- ExFTs suggest deeper structures (possibly) beyond supergravity, e.g.
  - Locally geometric solutions & exotic branes
    Bakhmatov Berman Kleinschmidt Musaev Otsuki
    Fernandez-Melgarejo Kimura Sakatani
    Berman Musaev Otsuki ....
  - generalised Scherk–Schwarz backgrounds violating section constraint
    Hull, Dabholkar, Aldazabal Baron Marques nunez, Rosabal,
    Dibitetto Melgarejo Roest, ...
  - $E_{d(d)}$ covariant amplitude computations
    Bossard Kleinschmidt

  The new structures appearing in lower D/larger $E_{d(d)}$ might teach us something

- $E_9 \quad \longleftrightarrow \quad$ classical $\int$bility of 2d (su)gra (Through Inverse scattering / Riemann-Hilbert problem)
  Geroch; Belinski Zakharov; Breitenlohner Maison; Nicolai

  How are 11d/IIB d.o.f. organised by ∞-dimensional group?

- solution generation via generalised Scherk–Schwarz

- natural framework for cod $=2,1$ exotic objects

- warm up for E_{10}, E_{11} , E_9 more tractable
  Nicolai West

Plan

- Review of ExFT
- Hidden symmetries in 2d (super)gravity
- E_9 exceptional geometry
- The dynamics

ExFT basics (up to $E_{7(7)}$)

Hull; Pires-Pacheco, Waldram; Hillman; Berman Perry; Coimbra, Str. Constable Waldram; Berman Cederwall Kleinschmidt Thompson; Nicolai Godazgar Godazgar; Aldazabal, Grana, Marques, Rosabal; Hohm, Samtleben; Ciceri Guarino GI; Cassani De Felice Petrini Str.Constable Waldram; Cederwall Palmkwist; ......

Start from (e.g.) 11d supergravity

Fields:

ds^2_{11d}	metric
$C_{(3)}$	three-form
$C_{(6)}$	dual six-form
$\vdots$	

Gauge syms:

$\xi^{(1)}$	diffeomorphisms
$\lambda_{(2)}$	three-form gauge sym
$\lambda_{(5)}$	six-form gauge sym
$\vdots$	

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Start from (e.g.) 11d **and break Lorentz:** $\mathrm{SO}(1,10) \to \underbrace{\mathrm{SO}(d)}_{y} \times \underbrace{\mathrm{SO}(1,10-d)}_{x}$

Fields:

$$\left.\begin{matrix} ds^2_{11d} \\ C_{(3)} \\ C_{(6)} \\ \vdots \end{matrix}\right\} \quad \text{dictionary*} \quad \left\{\begin{matrix} g_{\mu\nu}(x,y) \\ \mathcal{M}_{MN}(x,y) \\ A_\mu^M(x,y) \\ B^I_{\mu\nu}(x,y) \\ \vdots \end{matrix}\right.$$

$g_{\mu\nu}(x,y)$: (11-d) dimensional metric

$\mathcal{M}_{MN}(x,y)$: scalars on $\dfrac{E_{d(d)}}{K(E_{d(d)})}$

$A_\mu^M(x,y)$, $B^I_{\mu\nu}(x,y)$, …: p-forms **in** $E_{d(d)}$ **irreps**

$\boxed{\text{NO REDUCTION!}}$

Gauge syms:

$\xi^{(1)}$ diffeomorphisms

$\lambda_{(2)}$ three-form gauge sym

$\lambda_{(5)}$ six-form gauge sym

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Start from (e.g.) 11d **and break Lorentz:** $SO(1,10) \to \underbrace{SO(d)}_{y} \times \underbrace{SO(1,10-d)}_{x}$

Fields:

ds^2_{11d}, $C_{(3)}$, $C_{(6)}$, $\vdots$

dictionary*

$g_{\mu\nu}(x, y)$ (11-d) dimensional metric

$\mathcal{M}_{MN}(x, y)$ scalars on $\dfrac{E_{d(d)}}{K(E_{d(d)})}$

$A_\mu^M(x, y)$

$B^I_{\mu\nu}(x, y)$

$\vdots$

p-forms **in** $E_{d(d)}$ **irreps**

NO REDUCTION!

Gauge syms:

$\xi^{(1)}$, $\lambda_{(2)}$, $\lambda_{(5)}$, $\vdots$

along internal manifold $\longrightarrow$ $\Lambda^M(x, y)$

Generalised Diffeomorphisms

(+ external diffeo & p-form gauge trfs)

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Fields:

$$\left.\begin{array}{l} g_{\mu\nu}(x,y) \\ \mathcal{M}_{MN}(x,y) \\ A_\mu^M(x,y) \\ B_{\mu\nu}^I(x,y) \end{array}\right\} \phi$$

Gauge syms:

$\Lambda^M(x,y)$

Generalised Lie derivative:

$$\mathcal{L}_\Lambda \phi = \Lambda^M \partial_M \phi + \underbrace{\mathbb{P}^{\alpha\,M}{}_N}_{\mathfrak{e}_{d(d)}+\mathbb{R}\ \text{projector}} \partial_M \Lambda^N \overbrace{\delta_\alpha}^{\text{rigid}\ \mathfrak{e}_{d(d)}+\mathbb{R}\ \text{variation}} \phi$$

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Fields:

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Gauge syms:

$$\Lambda^M(x,y)$$

Generalised Lie derivative:

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$$[\mathcal{L}_{\Lambda_1}\,,\ \mathcal{L}_{\Lambda_2}] = \mathcal{L}_{\Lambda_{12}} \quad \Leftrightarrow \quad Y^{MN}{}_{PQ}\ \partial_M(\ldots)\,\partial_N(\ldots) = 0$$

section constraint

$$\partial_M = \left(\frac{\partial}{\partial y^m}\,,\ 0, \ldots, 0\right)$$

two (maximal) solutions: 11d & IIB

(massive IIA via deformation or non-geom. generalised Scherk–Schwarz)

(we'll get to the dynamics directly for E_9)

Hidden symmetries in 2d (super)gravity

Hidden symmetries in GR

$$4d \xrightarrow{KK} 3d$$

$$g^{4d}_{\hat{\mu}\hat{\nu}} = \left(\begin{array}{c|c} e^{-2\phi} g^{3d}_{\mu\nu} & e^{\phi} A_{\mu} \\ \hline e^{\phi} A_{\nu} & e^{2\phi} \end{array}\right)$$

Hidden symmetries in GR

$4d$

KK

$3d$

$$g^{4d}_{\hat{\mu}\hat{\nu}} = \left(\begin{array}{c|c} e^{-2\phi} g^{3d}_{\mu\nu} & e^{\phi} A_{\mu} \\ \hline e^{\phi} A_{\nu} & e^{2\phi} \end{array}\right) \xrightarrow{\text{in 3d}} d * dA = 0 \quad \Rightarrow \quad *dA = d\xi$$

$$\underbrace{\xi + \mathrm{i}\, e^{\phi}}$$

local action of $\mathbf{SL(2)}_E$ Ehlers group

Hidden symmetries in GR

$4d$

$\Big\downarrow$ *KK*

$$g^{4d}_{\hat{\mu}\hat{\nu}} = \left(\begin{array}{c|c} e^{-2\phi}\, g^{3d}_{\mu\nu} & e^{\phi} A_{\mu} \\ \hline e^{\phi} A_{\nu} & e^{2\phi} \end{array}\right) \xrightarrow{\text{in 3d}} d * dA = 0 \quad \Rightarrow \quad *dA = d\xi$$

$$\underbrace{\xi + \mathrm{i}\, e^{\phi}}_{\text{local action of } \mathbf{SL(2)}_E \text{ Ehlers group}}$$

$3d$

$\Big\downarrow$ *KK*

$$g^{4d}_{\hat{\mu}\hat{\nu}} = \left(\begin{array}{c|c|c} e^{2\sigma}\, \eta_{\mu\nu} & \multicolumn{2}{c}{\left(\text{2d vectors are non-dynamical}\right)} \\ \hline & e^{2\tilde{\phi}} & e^{\phi} A_3 \\ \hline & e^{\phi} A_3 & e^{2\phi} \end{array}\right)$$

$2d$

Hidden symmetries in GR

$4d$

KK

$$g^{4d}_{\hat\mu\hat\nu} = \left(\begin{array}{c|c} e^{-2\phi} g^{3d}_{\mu\nu} & e^{\phi} A_\mu \\ \hline e^{\phi} A_\nu & e^{2\phi} \end{array}\right) \xrightarrow{\text{in 3d}} d * dA = 0 \quad \Rightarrow \quad *dA = d\xi$$

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local action of $\mathbf{SL(2)}_E$ Ehlers group

$3d$

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$$g^{4d}_{\hat\mu\hat\nu} = \left(\begin{array}{c|c|c} e^{2\sigma}\eta_{\mu\nu} & \multicolumn{2}{c}{\left(\begin{array}{c}\text{2d vectors are} \\ \text{non-dynamical}\end{array}\right)} \\ \hline & e^{2\tilde\phi} & e^{\phi} A_3 \\ \hline & e^{\phi} A_3 & e^{2\phi} \end{array}\right) \xrightarrow{\text{in 2d}} d\xi = *dA_3$$

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local $\mathbf{SL(2)}_E$

$2d$

Hidden symmetries in GR

$4d$

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$$g^{4d}_{\hat\mu\hat\nu} = \left(\begin{array}{c|c} e^{-2\phi} g^{3d}_{\mu\nu} & e^{\phi} A_\mu \\ \hline e^{\phi} A_\nu & e^{2\phi} \end{array}\right) \xrightarrow{\text{in 3d}} d * dA = 0 \quad \Rightarrow \quad *dA = d\xi$$

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local action of $\mathbf{SL(2)}_E$ Ehlers group

$3d$

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$$\underbrace{\xi + \mathrm{i}\, e^{\phi}}$$

local $\mathbf{SL(2)}_E$

local action of $\mathbf{SL(2)}_T$ T^2 structure

$2d$

$$\boxed{\mathbf{SL(2)}_E \ '\times' \ \mathbf{SL(2)}_T = \widehat{\mathbf{SL(2)}} \quad \text{loop group} \quad \infty\text{-dim.}}$$

2d maximal sugra & the linear system

- $\widehat{\mathrm{SL}(2)}$ realised on **∞ly many dual scalar fields** extending ξ and A_3

- Maximal sugra: $\mathrm{SL}(2)_E$ becomes $E_{8(8)}$, $\widehat{\mathrm{SL}(2)}$ becomes (roughly) $\boldsymbol{E_9}$

- Symmetries & (most of) dynamics captured by **first order relations**

2d maximal sugra & the linear system

Breitenlohner Maison
Nicolai

Field content:

$$g_{\mu\nu} = e^{2\sigma}\,\eta_{\mu\nu} \qquad \text{(conformal gauge)}$$

$$\rho \quad \in \mathrm{GL}^{+}(1) \qquad \text{(internal volume)}$$

$$V_{e_8} \;\in\; E_{8(8)}/\mathrm{SO}(16) \qquad\qquad V_{e_8}(x) \longrightarrow \overset{\mathrm{SO}(16)}{h}(x)\; V_{e_8}(x)\; \overset{E_{8(8)}}{g}$$

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$$J_{e_8} \equiv M_{e_8}^{-1} d\, M_{e_8} \qquad \left(M_{e_8} \equiv V_{e_8}^{\dagger} V_{e_8}\right)$$

EOM: $\quad d{*}d\,\rho = 0 \qquad d{*}(\rho\, J_{e_8}) = 0 \qquad$ (σ eom not displayed)

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EOM:

$$d{*}d\,\rho = 0 \qquad\qquad d{*}(\rho\, J_{e_8}) = 0 \qquad\qquad (\sigma \text{ eom not displayed})$$

$$\Downarrow \qquad\qquad\qquad \Downarrow$$

$$d\tilde{\rho} = {*}d\rho \qquad\qquad dY_1 = \tfrac{1}{2}{*}(\rho\, J_{e_8})$$

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$$\Downarrow \qquad\qquad \Downarrow$$

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$$\left.\begin{aligned} dY_1 &= \tfrac{1}{2}{*}(\rho\, J_{e_8}) \\ dY_2 &= \tfrac{1}{2}\rho^2 J_{e_8} + \rho\tilde{\rho}\; {*}J_{e_8} + \tfrac{1}{2}[dY_1,\, Y_1] \\ &\vdots \end{aligned}\right\}$$

∞-ly many dual scalar fields

2d maximal sugra & the linear system

Breitenlohner Maison
Nicolai

EOM: $d{*}d\,\rho = 0$ $\qquad$ $d{*}(\rho\, J_{e_8}) = 0$

$$\Downarrow \qquad\qquad \Downarrow$$

$$d\tilde{\rho} = {*}d\rho$$

$$dY_1 = \tfrac{1}{2}{*}(\rho\, J_{e_8})$$

$$dY_2 = \tfrac{1}{2}\rho^2 J_{e_8} + \rho\tilde{\rho}\ {*}J_{e_8} + \tfrac{1}{2}[dY_1,\ Y_1]$$

$$\vdots$$

∞-ly many dual scalar fields

$$V(w) \in \frac{\hat{E}_{8(8)}}{K(\hat{E}_{8(8)})} \qquad V(w) = V_{e_8} + w^{-1}V_1 + w^{-2}V_2 + \ldots \qquad w \in \mathbb{C}$$

$$\overset{w\sim+\infty}{=} V_{e_8}\ e^{w^{-1}Y_1}\ e^{w^{-2}Y_2}\ \ldots$$

2d maximal sugra & the linear system

Breitenlohner Maison
Nicolai

EOM: $d{*}d\rho = 0$ $\quad\Downarrow\quad$ $d\tilde{\rho} = {*}d\rho$

$d{*}(\rho J_{e_8}) = 0$ $\quad\Downarrow$

$$dY_1 = \tfrac{1}{2}{*}(\rho J_{e_8})$$

$$dY_2 = \tfrac{1}{2}\rho^2 J_{e_8} + \rho\tilde{\rho}\,{*}J_{e_8} + \tfrac{1}{2}[dY_1,\ Y_1]$$

$$\vdots$$

$\int$bility

∞-ly many
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$$\overset{w\sim+\infty}{=} V_{e_8}\ e^{w^{-1}Y_1}\ e^{w^{-2}Y_2}\ \ldots$$

$$(dV\,V^{-1})(w) = c_1 dV_{e_8}V_{e_8}^{-1} + c_2(dV_{e_8}V_{e_8}^{-1})^{\top} + c_3\,{*}(dV_{e_8}V_{e_8}^{-1}) + c_4\,{*}(dV_{e_8}V_{e_8}^{-1})^{\top}$$

2d maximal sugra & the linear system

Breitenlohner Maison Nicolai

EOM: $d{*}d\,\rho = 0$ $\qquad$ $d{*}(\rho\, J_{e_8}) = 0$

$\Downarrow$ $\qquad$ $\Downarrow$

$\int$bility

$$d\tilde{\rho} = {*}d\rho$$

$$\left.\begin{aligned} dY_1 &= \tfrac{1}{2}{*}(\rho J_{e_8}) \\ dY_2 &= \tfrac{1}{2}\rho^2 J_{e_8} + \rho\tilde{\rho}\,{*}J_{e_8} + \tfrac{1}{2}[dY_1,\, Y_1] \\ &\vdots \end{aligned}\right\}$$

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$$\overset{w\sim+\infty}{=} V_{e_8}\; e^{w^{-1}Y_1}\; e^{w^{-2}Y_2}\;\ldots$$

$$(dV\,V^{-1})(w) = \frac{1}{1-\gamma^2}\,dV_{e_8}V_{e_8}^{-1} + \frac{\gamma^2}{1-\gamma^2}\,(dV_{e_8}V_{e_8}^{-1})^{\top} + \frac{\gamma}{1-\gamma^2}\Big[{*}(dV_{e_8}V_{e_8}^{-1}) + {*}(dV_{e_8}V_{e_8}^{-1})^{\top}\Big]$$

2d maximal sugra & the linear system

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$$\frac{d\gamma}{\gamma}(w) = \frac{1+\gamma^2}{1-\gamma^2}\,d\rho + \frac{2\gamma}{1-\gamma^2}\,*d\rho$$

2d maximal sugra & the linear system

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$$\left.\begin{aligned} V(x,w) &\xrightarrow{\hat{E}_{8(8)}} h(x,w)\,V(x,w)\,g(w) \\ \rho(x) &\xrightarrow{\mathrm{d}} \lambda_0\,\rho(x) \quad \text{(also acts on } V\text{)} \\ e^{\sigma(x)} &\xrightarrow{\mathrm{K}} \lambda_K e^{\sigma}(x) \end{aligned}\right\} E_9$$

2d maximal sugra & the linear system

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$$V(w) \in \frac{\hat{E}_{8(8)}}{K(\hat{E}_{8(8)})} \qquad V(w) = V_{e_8} + w^{-1} V_1 + w^{-2} V_2 + \dots \qquad w \in \mathbb{C}$$

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$$\frac{d\gamma}{\gamma}(w) = \frac{1+\gamma^2}{1-\gamma^2}\, d\rho + \frac{2\gamma}{1-\gamma^2}\, \underbrace{* d\rho}_{d\tilde{\rho}}$$

$$\left.\begin{aligned} V(x,w) &\overset{\hat{E}_{8(8)}}{\longrightarrow} h(x,w)\,V(x,w)\,g(w) \\ \rho(x) &\overset{\mathrm{d}}{\longrightarrow} \lambda_0\, \rho(x) \quad \text{(also acts on } V\text{)} \\ e^{\sigma(x)} &\overset{\mathrm{K}}{\longrightarrow} \lambda_{\mathrm{K}} e^{\sigma}(x) \end{aligned}\right\} E_9 \qquad \tilde{\rho} \overset{L_{-1}}{\longrightarrow} \tilde{\rho} + \lambda_{-1}$$

Generalised diffeomorphisms for E_9

Generalised diffeomorphisms for E_9

- $\mathfrak{e}_9$ is an affine Kac–Moody algebra over $\mathfrak{e}_{8(8)}$:

$$T^A_m \sim w^m T^A_{\mathfrak{e}_8} \qquad [T^A_m, T^B_n] = f_{\mathfrak{e}_8}{}^{AB}{}_C T^C_{m+n} + m\,\eta^{AB}_{\mathfrak{e}_8}\delta_{m+n}\mathrm{K}$$

- classical algebra is extended by central charge K

  and derivation

$$\mathrm{d} \simeq L_0 \sim -w\frac{\partial}{\partial w} \qquad [L_0, T^A_m] = -mT^A_m$$

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- classical algebra is extended by central charge K

  and derivation

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- associated Virasoro algebra $\quad L_m \sim -w^{m+1} \dfrac{\partial}{\partial w}$

- (Basic) highest weight reps $\quad V^M \, |e_M\rangle : \quad T^A_{m\geq 0}|0\rangle = 0, \quad \mathrm{K}|0\rangle = |0\rangle$

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- (Basic) highest weight reps $\quad V^M\, |e_M\rangle : \qquad T^A_{m\geq 0}|0\rangle = 0, \qquad \mathsf{K}|0\rangle = |0\rangle$

  ExFT generalised vector

Generalised diffeomorphisms for E_9

- generalised vectors V^M & derivatives ∂_M

$$\mathcal{L}_\Lambda \, V^M = \Lambda^N \partial_N V^M - \mathbb{P}_0{}^M{}_N{}^P{}_Q \, \partial_P \Lambda^Q \, V^N - (\partial \cdot \Lambda) V^M$$

- $\mathbb{P}_0$ projects onto $T^A_m \,,\ \mathrm{K}\,,\ L_0 \ \simeq \ \mathfrak{e}_9$

- section: $$\mathbb{P}_0{}^M{}_P{}^N{}_Q \, \partial_M \otimes \partial_N - \partial_M \otimes \partial_N + \partial_N \otimes \partial_M = 0$$

  solved by only **finitely many ∂_M : 11d or IIB supegravities**

Generalised diffeomorphisms for E_9

Bossard Cederwall Kleinschmidt Palmkwist Samtleben

- generalised vectors V^M & derivatives ∂_M

$$\mathcal{L}_\Lambda V^M = \Lambda^N \partial_N V^M - \mathbb{P}_0{}^M{}_N{}^P{}_Q \, \partial_P \Lambda^Q \, V^N - (\partial \cdot \Lambda) V^M$$

- $\mathbb{P}_0$ projects onto T^A_m , K , L_0 $\simeq$ $\mathfrak{e}_9$

- section: $$\mathbb{P}_0{}^M{}_P{}^N{}_Q \, \partial_M \otimes \partial_N - \partial_M \otimes \partial_N + \partial_N \otimes \partial_M = 0$$

solved by only **finitely many ∂_M : 11d or IIB supegravities**

$$[\mathcal{L}_{\Lambda_1} , \mathcal{L}_{\Lambda_2}] \neq \mathcal{L}_{\Lambda_{12}}$$

Generalised diffeomorphisms for E_9

Bossard Cederwall Kleinschmidt Palmkwist Samtleben

- generalised vectors V^M & derivatives ∂_M & constrained $\Sigma^M{}_{\underbracket{N}}$

$$\mathcal{L}_{\Lambda\,\Sigma} V^M = \Lambda^N \partial_N V^M - \mathbb{P}_0{}^M{}_N{}^P{}_Q\, \partial_{\underbracket{P}} \Lambda^Q\, V^N - (\partial\cdot\Lambda) V^M - \mathbb{P}_{-1}{}^M{}_N{}^P{}_Q\, \underset{\text{on section}}{\underset{\uparrow}{\Sigma^Q{}_{\underbracket{P}}}}\, V^N$$

- $\mathbb{P}_0$ projects onto T^A_m , K , L_0 $\simeq$ $\mathfrak{e}_9$
- $\mathbb{P}_{-1}$ projects onto T^A_m , K , L_{-1}

- section: $$\mathbb{P}_0{}^M{}_P{}^N{}_Q\, \partial_{\underbracket{M}} \otimes \partial_{\underbracket{N}} - \partial_{\underbracket{M}} \otimes \partial_{\underbracket{N}} + \partial_{\underbracket{N}} \otimes \partial_{\underbracket{M}} = 0$$

  solved by only **finitely many ∂_M : 11d or IIB supegravities**

- $\Sigma^M{}_{\underbracket{N}}$ restores closure: $$[\mathcal{L}_{\Lambda_1\,\Sigma_1}, \mathcal{L}_{\Lambda_2\,\Sigma_2}] = \mathcal{L}_{\Lambda_{12}\,\Sigma_{12}}$$

Generalised diffeomorphisms for E_9

Interpretation in $D=3$ E_8 ExFT

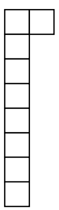
Starting from 11d sugra

ds^2_{11d}

$C_{(3)}$

$C_{(6)}$

$h_{(8|1)}$ dual graviton:



$$R^{\text{linear}}_{(2|2)} = \vec{d}\ h_{(1|1)}\ \overleftarrow{d} \qquad \vec{d} * R^{\text{linear}}_{(2|2)} = 0$$

$$\Rightarrow \qquad *\vec{d}\ h_{(1|1)}\ \overleftarrow{d} = \vec{d}\ h_{(8|1)}\ \overleftarrow{d}$$

Generalised diffeomorphisms for E_9

Interpretation in $D=3$ E_8 ExFT   Starting from 11d sugra

Hohm Samtleben

$$\left.\begin{array}{l} ds^2_{11d} \\ C_{(3)} \\ C_{(6)} \\ h_{(8|1)} \end{array}\right\} \quad \mathcal{M}_{MN}(x,y) \in \frac{E_8}{SO(16)}$$

$\psi_{m=1,\dots,8}$ dual to Kaluza–Klein A_μ^m

- ψ_m necessary for E_8 covariance
- A_μ^m necessary for gauge covariance $\mathcal{D}_\mu = \partial_\mu - \mathcal{L}_{A_\mu}$

$$A_\mu^m \frac{\partial}{\partial y^m}$$

Generalised diffeomorphisms for E_9

Interpretation in $D=3$ E_8 ExFT   Starting from 11d sugra

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$$\left.\begin{array}{l} ds^2_{11d} \\ C_{(3)} \\ C_{(6)} \end{array}\right\} \mathcal{M}_{MN}(x,y) \in \frac{E_8}{SO(16)}$$

$$h_{(8|1)} \quad \psi_{m=1,\ldots,8} \text{ dual to Kaluza–Klein } A_\mu^m$$

- ψ_m necessary for E_8 covariance
- A_μ^m necessary for gauge covariance $\mathcal{D}_\mu = \partial_\mu - \mathcal{L}_{A_\mu, B_\mu}$

$$\mathcal{L}_{A_\mu, B_\mu} \to A_\mu^m \frac{\partial}{\partial y^m}$$

need extra gauge symmetry: $\psi_m \to \psi_m + \Sigma_m$

Generalised diffeomorphisms for E_9

Interpretation in $D=3$ E_8 ExFT   Starting from 11d sugra

ds^2_{11d}

$C_{(3)}$ $\mathcal{M}_{MN}(x,y) \in \dfrac{E_8}{SO(16)}$

$C_{(6)}$

$h_{(8|1)}$ $\psi_{m=1,\dots,8}$ dual to Kaluza–Klein A^m_μ

- ψ_m necessary for E_8 covariance
- A^m_μ necessary for gauge covariance $\mathcal{D}_\mu = \partial_\mu - \mathcal{L}_{A_\mu, B_\mu}$

$$A^m_\mu \frac{\partial}{\partial y^m} = A^M_\mu \partial_{\underline{M}}$$

need extra gauge symmetry: $\psi_m \rightarrow \psi_m + \Sigma_m \longrightarrow \Sigma_{\underline{M}}$

Generalised diffeomorphisms for E_9

Interpretation in $D=2$ E_9 ExFT

$$V \equiv V_{e_8} \underbrace{e^{Y_{1A}T^A_{-1}} e^{Y_{2A}T^A_{-2}} \cdots}_{\text{All dual to } V_{e_8}}$$

E_9 covariance requires all Y^A_m on equal footing

Those without standard higher-dim. origin are gauged away by $\Sigma^M{}_{\underline{N}}$

Field content of E_9 ExFT

- **scalar fields:** $$\mathcal{V} \equiv e^{-\sigma K} \rho^{-L_0} e^{-\tilde{\rho} L_{-1}} V_{e_8} \, e^{Y_{1A} T^A_{-1}} e^{Y_{2A} T^A_{-2}} \cdots$$

  generalised metric $$\mathcal{M} \equiv \mathcal{V}^\dagger \mathcal{V}$$

  & currents $$\mathcal{J}_\mu \equiv \mathcal{M}^{-1} \mathcal{D}_\mu \mathcal{M} \qquad \mathcal{J}_{\underbracket{M}} \equiv \mathcal{M}^{-1} \partial_{\underbracket{M}} \mathcal{M}$$

- **vector fields:** $$A_\mu^M \qquad B_\mu{}^M{}_{\underbracket{N}}$$

  give gauge-connection for generalised diffeomorphisms: $$\mathcal{D}_\mu = \partial_\mu - \mathcal{L}_{A\,B}$$

- some **extra objects** will appear later

E_9 ExFT dynamics I – The potential

capturing 11d & IIB dynamics for $(x^0\,,\ x^1)$–independent configurations

The potential

Hohm Samtleben
Berman Musaev Thompson
Musaev
Cederwall Palmkwist

Standard expression for $V(\mathcal{M},\partial_M)$ $\quad (\det\mathcal{M}=1,\ g_{\mu\nu}$ in Einstein frame)

Killing–Cartan metric

$$V(\mathcal{M},\,\partial_M) = \frac{1}{4}\mathcal{M}^{MN}\eta^{\alpha\beta}\mathcal{J}_{M\alpha}\mathcal{J}_{M\beta} - \frac{1}{2}\mathcal{M}^{MN}\mathcal{J}_{P\alpha}\mathcal{J}_{Q\beta}T^{\alpha Q}{}_{M}T^{\beta P}{}_{N}$$

+ ∂_M external metric

The potential

Hohm Samtleben
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Standard expression for $V(\mathcal{M}, \partial_M)$ $\quad$ ($\det\mathcal{M} = 1$, $g_{\mu\nu}$ in Einstein frame)

Killing–Cartan metric

$$V(\mathcal{M},\, \partial_M) = \frac{1}{4}\mathcal{M}^{MN}\eta^{\alpha\beta}\mathcal{J}_{M\alpha}\mathcal{J}_{M\beta} - \frac{1}{2}\mathcal{M}^{MN}\mathcal{J}_{P\alpha}\mathcal{J}_{Q\beta}T^{\alpha Q}{}_M T^{\beta P}{}_N$$

+ ∂_M external metric

- Σ invariance requires extra non-generic term (as done in $D=3$) $\quad$ Hohm Samtleben
- $\nexists$ invariant $\eta^{\alpha\beta}$ for $\mathcal{J}_M \in \mathfrak{e}_9 \oplus L_{\pm 1}$ $\longleftarrow$ associated with $\tilde{\rho}$

for brevity, set $\tilde{\rho} = 0$ $\quad$ ($\mathcal{J} \in \mathfrak{e}_9$) $\longleftarrow$ partial gauge-fixing of Σ trfs

The potential

- Σ invariance of potential requires one more ingredient
- recall $\quad T_m^A \sim w^m \, T_{e_8}^A \;, \qquad L_m \sim -w^{m+1} \, \partial_w$
- multiplication by w:

$$\mathcal{J}_M \to w^p \, \mathcal{J}_M \equiv S_p(\mathcal{J}_M)$$

$$S_p(T_m^A) \equiv T_{m+p}^A \qquad S_p(L_m) \equiv L_{m+p}$$

The potential

- Σ invariance of potential requires one more ingredient
- recall $\quad T^A_m \sim w^m \, T^A_{e_8} \;, \qquad L_m \sim -w^{m+1} \, \partial_w$
- multiplication by w:

$$\mathcal{J}_M \to w^p \, \mathcal{J}_M \equiv S_p(\mathcal{J}_M)$$

$$S_p(T^A_m) \equiv T^A_{m+p} \qquad S_p(L_m) \equiv L_{m+p}$$

- S_p is not $\mathfrak{e}_9$ invariant: $\quad [T^\alpha, S_p] \propto \mathsf{K} \longleftarrow$ not visible in w presentation ($\mathsf{K} \sim 0$)
- define a *shifted current*, reinstate K component:

$$\mathbb{J}^{-1}_M \equiv S_{-1}(\mathcal{J}_M) + \underbrace{\chi_M}_{\text{on section}} \mathsf{K} \qquad \delta^\alpha_{e_9} \chi = \ldots - [T^\alpha, S_{-1}](\mathcal{J})$$

χ_M expected from ExFT tensor hierarchy!

The potential

$$\rho V(\mathcal{M},\, \partial_M) = \frac{1}{4}\mathcal{M}^{MN}\eta^{\alpha\beta}\mathcal{J}_{M\alpha}\mathcal{J}_{M\beta} - \frac{1}{2}\mathcal{M}^{MN}\mathcal{J}_{P\alpha}\mathcal{J}_{Q\beta}T^{\alpha Q}{}_{M}T^{\beta P}{}_{N} + \rho^{-1}\partial_M\rho\,\partial_N\mathcal{M}^{MN}$$

$$+ \frac{1}{2}\,\rho^2\mathcal{M}^{MN}\,\mathbb{J}^{-1}_{P\alpha}\,\mathbb{J}^{-1}_{Q\beta}T^{\alpha Q}{}_{M}T^{\beta P}{}_{N}$$

- E_9 & gendiffeo invariant
- Reproduces **11d** & **IIB dynamics** for (x^0, x^1) – independent configurations
- Completed to $\tilde{\rho} \neq 0$: lack of invariant $\eta^{\alpha\beta}$ compensated by χ_M

E_9 ExFT dynamics II – Twisted selfduality & full dynamics

Completing the dynamics

Full dynamics encoded in E_9 invariant pseudoaction

$$S \;=\; S^{\text{pseudo}} \;+\; \int V(\mathcal{M},\, \partial_M)$$

$\delta S = 0 \quad \cup \quad$ twisted s.d. $\quad \rightarrow \quad$ ExFT EOM

2d dynamics & twisted self-duality $\partial_M = 0$ for now

based on Julia Nicolai; Paulot

Recall the linear system $(dV_{e_8} V_{e_8}^{-1} \equiv Q_{so(16)} + P_{e_8})$

$$(dVV^{-1})(w) = Q_{so(16)} + \frac{1+\gamma^2}{1-\gamma^2} P_{e_8} + \frac{2\gamma}{1-\gamma^2} * P_{e_8}$$

$$\frac{d\gamma}{\gamma}(w) = \frac{1+\gamma^2}{1-\gamma^2} d\rho + \frac{2\gamma}{1-\gamma^2} * d\rho$$

- how to write it E_9 covariantly?
- how to write it in highest weight rep?

2d dynamics & twisted self-duality $\partial_M = 0$ for now

based on Julia Nicolai; Paulot

$$\frac{d\gamma}{\gamma}(w) = \frac{1+\gamma^2}{1-\gamma^2}\, d\rho + \frac{2\gamma}{1-\gamma^2} \; * d\rho$$

geometric series

2d dynamics & twisted self-duality $\partial_M = 0$ for now

based on Julia Nicolai; Paulot

$$\frac{d\gamma}{\gamma}(w) \overset{\gamma \sim +\infty}{=} (1 + 2\gamma^{-2} + 2\gamma^{-4} + \dots)\, d\rho \; + \; (2\gamma^{-1} + 2\gamma^{-3} + \dots) * d\rho$$

$$\gamma(w) \quad \xrightarrow{w \to +\infty} \quad c_1 w + c_0 + c_{-1} w^{-1} + c_{-2} w^{-2} + \dots$$

asymptotics

2d dynamics & twisted self-duality $\partial_M = 0$ for now

based on Julia Nicolai; Paulot

$$\frac{d\gamma}{\gamma}(w) \overset{\gamma\sim+\infty}{=} (1+2\gamma^{-2}+2\gamma^{-4}+\dots)\,d\rho \;+\; (2\gamma^{-1}+2\gamma^{-3}+\dots)*d\rho$$

$$\gamma(w) \quad \xrightarrow{w\to+\infty} \quad c_1 w + c_0 + c_{-1}w^{-1} + c_{-2}w^{-2} + \dots$$

$$\simeq \quad \underbrace{\dots\; e^{\varphi_3 L_{-3}}\; e^{\varphi_2 L_{-2}}\; e^{\varphi_1 L_{-1}}\; \rho^{L_0}}_{\Gamma^{-1}}\; w \qquad\qquad L_m = -w^{m+1}\partial_w$$

2d dynamics & twisted self-duality $\partial_M = 0$ for now

based on Julia Nicolai; Paulot

$$d\Gamma\,\Gamma^{-1} = (L_0 + 2L_{-2} + 2L_{-4} + \ldots)\; d\rho + (2L_{-1} + 2L_{-3} + \ldots)\; *d\rho$$

$$\gamma(w) \quad \xrightarrow{w\to+\infty} \quad c_1 w + c_0 + c_{-1}w^{-1} + c_{-2}w^{-2} + \ldots$$

$$\simeq \quad \underbrace{\ldots\; e^{\varphi_3 L_{-3}}\; e^{\varphi_2 L_{-2}}\; e^{\varphi_1 L_{-1}}\; \rho^{L_0}}_{\Gamma^{-1}}\; w \qquad\qquad L_m = -w^{m+1}\partial_w$$

2d dynamics & twisted self-duality $\partial_M = 0$ for now

based on Julia Nicolai; Paulot

writable in h.w.r.

$$*p \; = \; \frac{1}{w}\, p \; \equiv \; S_{-1}(p) \qquad\qquad p \equiv d\Gamma\,\Gamma^{-1} + (d\Gamma\,\Gamma^{-1})^{\dagger}$$

$$\Updownarrow$$

$$d\Gamma\,\Gamma^{-1} \; = \; (L_0 + 2L_{-2} + 2L_{-4} + \ldots)\; d\rho + \; (2L_{-1} + 2L_{-3} + \ldots)\; *d\rho$$

$$\gamma(w) \; \xrightarrow{w \to +\infty} \; c_1 w + c_0 + c_{-1} w^{-1} + c_{-2} w^{-2} + \ldots$$

$$\simeq \; \underbrace{\ldots\; e^{\varphi_3 L_{-3}}\; e^{\varphi_2 L_{-2}}\; e^{\varphi_1 L_{-1}}\; \rho^{L_0}}_{\Gamma^{-1}}\; w \qquad\qquad L_m = -w^{m+1}\partial_w$$

2d dynamics & twisted self-duality $\partial_M = 0$ for now

based on Julia Nicolai; Paulot

the dilaton sector:

$$\Gamma \;=\; \rho^{-L_0}\, e^{-\varphi_1 L_{-1}}\, e^{-\varphi_2 L_{-2}} \cdots \qquad \longrightarrow \qquad p \equiv d\Gamma\,\Gamma^{-1} + (d\Gamma\,\Gamma^{-1})^\dagger$$

$$*p \;=\; S_{-1}(p)$$

the full system:

$$\mathcal{V} \;=\; e^{-\sigma K}\, \underbrace{\rho^{-L_0}\, e^{-\varphi_1 L_{-1}}\, e^{-\varphi_2 L_{-2}} \cdots}_{\Gamma}\; \underbrace{V_{e_8}\, e^{Y_{1A} T^A_{-1}}\, e^{Y_{2A} T^A_{-2}} \cdots}_{V} \qquad \longrightarrow \qquad \mathcal{J} \;\in\; \mathfrak{e}_9 \oplus \mathfrak{vir}$$

$$*\mathcal{J} \;=\; S^{\gamma}_{-1}(\mathcal{J}) + \hat{\chi}\,\mathrm{K}$$

multiplication by γ

2d dynamics & twisted self-duality $\partial_M \neq 0$

$$\mathcal{V} = e^{-\sigma K}\rho^{-L_0}\,\underbrace{e^{-\varphi_1 L_{-1}}\,e^{-\varphi_2 L_{-2}}\cdots}\,V_{e_8}\,e^{Y_{1A}T^A_{-1}}\,e^{Y_{2A}T^A_{-2}}\cdots$$

In ExFT ($\partial_M \neq 0$), φ_m must be gauged away

2d dynamics & twisted self-duality $\partial_M \neq 0$

$$\mathcal{V} \;=\; e^{-\sigma \mathrm{K}}\, \rho^{-L_0}\, \underbrace{e^{-\varphi_1 L_{-1}}\, e^{-\varphi_2 L_{-2}} \cdots}_{\text{In ExFT } (\partial_M \neq 0),\ \varphi_m \text{ must be gauged away}}\; V_{e_8}\, e^{Y_{1A} T^A_{-1}}\, e^{Y_{2A} T^A_{-2}} \cdots$$

Generalised diffeomorphisms can be extended exactly in the right way:

$$\mathcal{L}_{\Lambda\Sigma} V^M \;=\; \Lambda^N \partial_N V^M - \mathbb{P}_0{}^M{}_N{}^P{}_Q\, \partial_P \Lambda^Q\; V^N - (\partial\cdot\Lambda) V^M \boxed{- \mathbb{P}_{-1}{}^M{}_N{}^P{}_Q\; \Sigma{}^Q{}_P\; V^N} \quad \text{gauges } L_{-1}$$

2d dynamics & twisted self-duality $\partial_M \neq 0$

$$\mathcal{V} = e^{-\sigma K} \rho^{-L_0} \underbrace{e^{-\varphi_1 L_{-1}} e^{-\varphi_2 L_{-2}} \cdots}_{} V_{e_8} e^{Y_{1A} T^A_{-1}} e^{Y_{2A} T^A_{-2}} \cdots$$

In ExFT ($\partial_M \neq 0$), φ_m must be gauged away

Generalised diffeomorphisms can be extended exactly in the right way:

$$\mathcal{L}_{\Lambda\Sigma} V^M = \Lambda^N \partial_N V^M - \mathbb{P}_0{}^M{}_N{}^P{}_Q \partial_P \Lambda^Q \, V^N - (\partial \cdot \Lambda) V^M \boxed{- \mathbb{P}_{-1}{}^M{}_N{}^P{}_Q \, \Sigma_{(1)}{}^Q{}_P \, V^N} \quad \text{gauges } L_{-1}$$

$$\boxed{- \mathbb{P}_{-2}{}^M{}_N{}^P{}_Q \, \Sigma_{(2)}{}^Q{}_P \, V^N} \quad \text{gauges } L_{-2}$$

$$\boxed{- \mathbb{P}_{-3}{}^M{}_N{}^P{}_Q \, \Sigma_{(3)}{}^Q{}_P \, V^N} \quad \text{gauges } L_{-3}$$

$$\vdots$$

vector fields $A_\mu^M \quad B_\mu^{(r)M}{}_N$ give gauge-connection: $\mathcal{J}_\mu = \mathcal{M}^{-1}(\partial_\mu - \mathcal{L}_{AB})\mathcal{M}$

2d dynamics & twisted self-duality $\partial_M \neq 0$

vector fields A_μ^M $B_\mu^{(r)M}{}_N$ give gauge-connection: $\mathcal{J}_\mu = \mathcal{M}^{-1}(\partial_\mu - \mathcal{L}_{AB})\mathcal{M}$

$$*\mathcal{J} = S_{-1}^{\gamma}(\mathcal{J}) + \hat{\chi}\,\mathsf{K}$$

2d dynamics & twisted self-duality $\partial_M \neq 0$

vector fields A_μ^M $B_\mu^{(r)M}{}_N$ give gauge-connection: $\mathcal{J}_\mu = \mathcal{M}^{-1}(\partial_\mu - \mathcal{L}_{A\,B})\mathcal{M}$

$$*\mathcal{J} = S^{\gamma}_{-1}(\mathcal{J}) + \hat{\chi}\,\mathrm{K}$$

gauge-fixing $\varphi_m = 0$, there is a more economical formulation

stripped of B_μ fields

$\mathfrak{e}_9$ valued only

$\downarrow$

$$*J - *\mathcal{B} = S_{-1}\left(J - \mathcal{M}^{-1}\mathcal{B}\mathcal{M}\right) + \tilde{\chi}\,\mathrm{K}$$

$\uparrow$

$\hat{\mathfrak{e}}_8 \oplus L_{-1}$ valued *only* (really $\equiv \mathbb{P}_{-1}\tilde{B}$)

related to $B^{(r)}$ by projection + non-covariant redefinition

Makes contact with E_{11} ExFT & **tensor hierarchy algebra**

Bossard Kleinschmidt Sezgin

Palmkwist
Bossard Kleinschmidt Palmkwist Pope Sezgin

Completing the dynamics

Full dynamics encoded in E_9 invariant pseudoaction

$$S \;=\; S^{\text{pseudo}}_{\text{topol. (+kinetic)}} \;+\; \int V(\mathcal{M}, \partial_M)$$

$\delta S = 0 \;\;\cup\;\;$ twisted s.d. $\;\;\rightarrow\;\;$ ExFT EOM

Most of S^{pseudo} found applying $S_{+1}(\,\cdot\,)$ to $\mathcal{J}$'s Bianchi id.

$$\mathcal{D}\mathcal{J} + \mathcal{J} \wedge \mathcal{J} \;=\; \text{vector field strengths}$$

and completing to an $\mathfrak{e}_9$ scalar density

$$S^{\text{pseudo}} \;\approx\; \int \mathcal{D}\hat{\chi} + \mathcal{J} \wedge \mathcal{J} + p \wedge \hat{\chi} + \chi_M \mathcal{F}^M + \ldots$$

Details depend on covariant vs. E_{11}-inspired approach

Concluding

We have discussed the basics of E_9 ExFT

- several new structures appear compared to finite dimensional duality groups
- the full dynamics are mostly in place

And then…?

- perform generalised Scherk–Schwarz

  → covariant formulation of 2d maximal gauged supegravity

  → find its most general gaugings & its scalar potential

- solution generation (AdS$_2$, exotic branes, affine U-folds)

- What does the 11d / IIB embedding into E_9 ExFT teach us?