Heterotic string theory and G2 structure manifolds

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X. de la Ossa, ML, E. Svanes (1607.03473, 1704.08717, 1709.06974)
X. de la Ossa, ML, M. Magill, E. Svanes (1904.01027)

Motivation and summary

Compactifications of heterotic string on 7-dim manifolds with G_2 structure.

Physics motivation

Assuming minimal supersymmetry, what is the 3D effective field theory?

In particular: topology/geometry $\rightsquigarrow$ spectrum, couplings, ...
can we control corrections? (problem in 3D $\mathcal{N} = 1$)

Math motivation

New perspective on geometry, deformations, invariants,...

In particular: understand coupled moduli space of vector bundle and geometry.

Motivation and summary

Heterotic string to $\mathcal{O}(\alpha')$

Green, Schwarz:84, Gross *et.al.*:85, Bergshoeff, deRoo:89

Supergravity fields (10D)

- Bosonic fields: Metric G, B-field B, dilaton ϕ, gauge field A for $G \subset E_8 \times E_8$
- Fermionic fields: Gravitino, dilatino, gaugino

Look for compactifications $\mathcal{M}_{10} = \mathcal{M}_3 \times Y$ that preserve $\mathcal{N} = 1$ SUSY

Motivation and summary

Heterotic compactifications $\mathcal{M}_{10} = \mathcal{M}_3 \times Y$

We have the following mathematical objects

- 7-dim Riemannian manifold Y with metric g_{mn}
- Vector bundle $V \to Y$ with connection A and structure group $G \subset E_8 \times E_8$
- Scalar ϕ
- 3-form flux defined by $H = \mathrm{d}B + \frac{\alpha'}{4}(CS(A) - CS(\Theta))$
  where Θ is a connection of TY

SUSY constrains the geometry:

- Y has G_2 structure φ with torsion $T = H$
- the connections A and Θ are G_2 instantons

Goal: geometry and moduli of heterotic G_2 system $[(Y, \varphi), (V, A), (TY, \Theta), H]$

Motivation and summary

Goal: geometry and moduli of heterotic G_2 system $[(Y,\varphi),(V,A),(TY,\Theta),H]$

Key ideas:

- Reformulate heterotic system as nilpotent differential $\check{\mathcal{D}}$

  $\check{\mathcal{D}}$ acts on $\mathcal{Q}$-valued forms, where topologically $\mathcal{Q} = T^*Y \oplus \mathrm{End}\, TY \oplus \mathrm{End}\, V$.
- Infinitesimal moduli counted by $\mathcal{Q}$-valued canonical G_2 cohomology $H^1_{\check{\mathcal{D}}}(\mathcal{Q})$.
- Comparison with 4D $\mathcal{N}=1$ Strominger–Hull system.
- Superpotential.
- Conclusions and outlook.

G_2 structures

Bonan:66, Fernandez–Gray:82, Bryant:87,03, Hitchin:00, Joyce:00, Chiossi–Salamon:02

Goal: geometry and moduli of heterotic G_2 system $[(Y, \varphi), (V, A), (TY, \Theta), H]$

- (Y, φ) has G_2 *structure* φ

  Comment: true whenever Y is orientable and spin.

- $\varphi \to$ Riemannian metric g on Y, and a coassociative 4-form $\psi = *\varphi$

- Heterotic compactifications:

  SUSY constrains $\mathrm{d}\varphi$ and $\mathrm{d}\psi$, i.e. the torsion of the G_2 structure

G_2 structures

Fernandez–Ugarte:98, Friedrich–Ivanov:01, Gauntlett et.al.:01, ...

- (Y, φ) has G_2 *structure* φ
- Torsion classes $\sim$ irreps of G_2:

$$\mathrm{d}\varphi = \tau_0\,\psi + 3\,\tau_1 \wedge \varphi + *\tau_3\ , \qquad \Lambda^4 = \Lambda^4_1 \oplus \Lambda^4_7 \oplus \Lambda^4_{27}\ ,$$

$$\mathrm{d}\psi = 4\,\tau_1 \wedge \psi + *\tau_2\ , \qquad \Lambda^5 = \Lambda^5_7 \oplus \Lambda^5_{14}\ .$$

- Heterotic compactifications:

SUSY $\iff$ τ_0 constant, $2\tau_1 = \mathrm{d}\phi$, $\tau_2 = 0$

$$\mathrm{d}\varphi = i_H(\varphi)\ ,\ \mathrm{d}\psi = i_H(\psi) \qquad \text{where } H = \tfrac{1}{6}\,\tau_0\,\varphi - \tau_1 \lrcorner_7\, \psi - \tau_3\ .$$

- No H $\iff$ Y has G_2 holonomy

G_2 structures

Canonical G_2 cohomology

Reyes-Carrion:93,Fernandez–Ugarte:98, see also Salamon:89

Analogue of Dolbeault operator: project d onto G_2 irreps.

- Define differential operator $\check{\mathrm{d}}$ by

$$\check{\mathrm{d}}_0 = \mathrm{d} \ , \quad \check{\mathrm{d}}_1 = \pi_7 \circ \mathrm{d} \ , \quad \check{\mathrm{d}}_2 = \pi_1 \circ \mathrm{d} \ .$$

- Can show $\boxed{\tau_2 = 0 \iff \check{\mathrm{d}}^2 = 0}$ *cf.* Dolbeault differential $\bar{\partial}$

$\rightsquigarrow$ "integrable G_2 structure"

- Differential, elliptic complex

$$0 \to \Lambda^0(Y) \xrightarrow{\check{\mathrm{d}}} \Lambda^1(Y) \xrightarrow{\check{\mathrm{d}}} \Lambda^2_7(Y) \xrightarrow{\check{\mathrm{d}}} \Lambda^3_1(Y) \to 0$$

$\to$ canonical G_2-cohomology $\boxed{H^*_{\check{\mathrm{d}}}(Y)}$ *cf.* Dolbeault cohomology $H^*_{\bar{\partial}}(X)$

G_2 instanton bundle

Goal: geometry and moduli of heterotic G_2 system $[(Y,\varphi),(V,A),(TY,\Theta),H]$

SUSY: $\boxed{F(A)\wedge\psi=0} \iff A$ is G_2 instanton

Reyes-Carrion:93, 98,Fernandez–Ugarte:98

Encode instanton constraint in differential $\mathrm{d}_A=\mathrm{d}+A\wedge$:

- Recall: $\tau_2=0 \iff \check{\mathrm{d}}^2=0$
- Generalizes to vector bundles V with instanton connection:

$$\boxed{\tau_2=0 \text{ and } F(A)\wedge\psi=0 \iff \check{\mathrm{d}}_A^2=0}$$

- $\mathrm{End}V$- valued canonical G_2-cohomology $\boxed{H^*_{\check{\mathrm{d}}_A}(Y,\mathrm{End}V)}$

Anomaly cancellation condition

Goal: geometry and moduli of heterotic G_2 system $[(Y,\varphi),(V,A),(TY,\Theta),H]$

Anomaly cancellation condition

$$\mathrm{d}B + \frac{\alpha'}{4}(CS(A) - CS(\Theta)) = H = \frac{1}{6}\tau_0\,\varphi - \tau_1\lrcorner_7\,\psi - \tau_3\ .$$

$\rightarrow$ Bianchi identity

$$\frac{\alpha'}{4}(\mathrm{tr}F(A)\wedge F(A) - \mathrm{tr}R(\Theta)\wedge R(\Theta)) = \mathrm{d}H = \mathrm{d}\left(\frac{1}{6}\tau_0\,\varphi - \tau_1\lrcorner_7\,\psi - \tau_3\right)\ .$$

Other conditions?

Hull:86, Ivanov:10, Martelli–Sparks:10

SUSY+anomaly cancellation $\Longrightarrow$ EOM $\Longleftrightarrow$ Θ is a G_2 instanton

$$R(\Theta)\wedge\psi = 0$$

Geometry as differential

Goal: geometry and moduli of heterotic G_2 system $[(Y,\varphi),(V,A),(TY,\Theta),H]$:

1. (Y,φ) 7-manifold with integrable G_2 structure
2. $(V,A),(TY,\Theta)$ G_2 instanton bundles
3. Anomaly cancellation $\mathrm{d}B + \frac{\alpha'}{4}(CS(A) - CS(\Theta)) = H = \frac{1}{6}\,\tau_0\,\varphi - \tau_1 \lrcorner_7\, \psi - \tau_3$

Encode constraints as nilpotency $\check{\mathcal{D}}^2 = 0$ of suitable differential $\check{\mathcal{D}}$

Geometry as differential

Want: Heterotic G_2 system $[(Y,\varphi),(V,A),(TY,\Theta),H] \iff \check{\mathcal{D}}^2 = 0$

- Start with

$$\mathcal{D} = \begin{pmatrix} \mathrm{d}_\zeta & \mathcal{R} & -\mathcal{F} \\ \mathcal{R} & \mathrm{d}_\Theta & 0 \\ \mathcal{F} & 0 & \mathrm{d}_A \end{pmatrix}$$

- $\mathcal{D}$ acts on $\mathcal{Q}$-valued forms where topologically $\mathcal{Q} = T^*Y \oplus \mathrm{End}\, TY \oplus \mathrm{End}\, V$
- $\mathrm{d}_A = \mathrm{d} + A\wedge$ etc.
- d_ζ has torsion $-H$ (*cf.* G_2 connection ∇_H)
- $\mathcal{F}$, $\mathcal{R}$ linear maps

$$\mathcal{F}: \quad \Omega^p(Y, T^*Y) \oplus \Omega^p(Y, \mathrm{End}(V)) \longrightarrow \Omega^{p+1}(Y, \mathrm{End}(V)) \oplus \Omega^{p+1}(Y, T^*Y)$$

$$\mathcal{F}(M) = (-1)^p\, g^{ab}\, M_a \wedge F_{bc}\, \mathrm{d}x^c \ ,$$

$$\mathcal{F}(\alpha)_a = (-1)^p\, \frac{\alpha'}{4}\, \mathrm{tr}(\alpha \wedge F_{ab}\, \mathrm{d}x^b) \ .$$

Geometry as differential

Theorem: Heterotic G_2 system $[(Y,\varphi),(V,A),(TY,\Theta),H] \iff \check{\mathcal{D}}^2 = 0$

Sketch of proof: Compute the square

$$\mathcal{D}^2 = \begin{pmatrix} \mathrm{d}_\zeta^2 + \mathcal{R}^2 - \mathcal{F}^2 & \mathrm{d}_\zeta\mathcal{R} + \mathcal{R}\mathrm{d}_\Theta & -(\mathrm{d}_\zeta\mathcal{F} + \mathcal{F}\mathrm{d}_A) \\ \mathcal{R}\mathrm{d}_\zeta + \mathrm{d}_\Theta\mathcal{R} & \mathcal{R}^2 + \mathrm{d}_\Theta^2 & -\mathcal{R}\mathcal{F} \\ \mathcal{F}\mathrm{d}_\zeta + \mathrm{d}_A\mathcal{F} & \mathcal{F}\mathcal{R} & -\mathcal{F}^2 + \mathrm{d}_A^2 \end{pmatrix} ,$$

Next, project to get $\check{\mathcal{D}}$

$$\check{\mathcal{D}}_0 = \mathcal{D} \ , \quad \check{\mathcal{D}}_1 = \pi_7 \circ \mathcal{D} \ , \quad \check{\mathcal{D}}_2 = \pi_1 \circ \mathcal{D} \ .$$

Find: components of $\mathcal{D}^2$ are (combinations of) heterotic KSE and BI, q.e.d.

Infinitesimal moduli

Goal: geometry and moduli of heterotic G_2 system $[(Y, \varphi), (V, A), (TY, \Theta), H]$

Key ideas:

- Reformulate heterotic system as nilpotent differential $\check{\mathcal{D}}$ ✓
- Infinitesimal moduli counted by $\mathcal{Q}$-valued canonical G_2 cohomology $H^1_{\check{\mathcal{D}}}(\mathcal{Q})$

Infinitesimal moduli

Consider a family

$$[(Y, \varphi(t)), (V, A(t)), (TY, \Theta(t)), H(t)]$$

which for $t = 0$ gives a heterotic G_2 system.

First guess for infinitesimal moduli

- $\delta_t \psi$ (determines $\delta_t \varphi$): geometric moduli: $H^3(Y)$ if $H = 0$
  $H = 0$: *Joyce:96, Hitchin:00, Dai, Wang, Wei:03, de Boer, Naqvi, Shomer:05,...*
- $\delta_t A$: Vector bundle moduli: $H^1(Y, \mathrm{End}(V))$
  $H = 0$: *Sa-Earp:09,...*
- $\delta_t B$: deformations of B-field, $H = \mathrm{d}B + \alpha'(...)$

SUSY and anomaly cancellation couples these objects; need more details.

Infinitesimal moduli

Geometric moduli: deformations of $[(Y, \varphi)]$

Encode deformations as 1-forms:

- Let $\delta_t \psi = i_M(\psi) = \frac{1}{3!} M_t^a \wedge \psi_{bcda} \, \mathrm{d}x^{bcd}$, where $M_t \in \Lambda^1(Y, T^*Y)$.

- Preserve $\tau_2 = 0$: $\boxed{i_{\check{\mathrm{d}}_\zeta M_t}(\psi) = 0}$
- Diffeomorphisms: $\boxed{\mathcal{L}_V \psi = i_{\check{\mathrm{d}}_\zeta V}(\psi)}$ where $V \in \Lambda^0(Y, T^*Y)$

$$\Longrightarrow \boxed{\mathcal{TM}_Y = \{M_t : i_{\check{\mathrm{d}}_\zeta M_t}(\psi) = 0\} / \{M_t : M_t = \check{\mathrm{d}}_\zeta V\}}$$

Infinitesimal moduli

Geometric moduli: deformations of $[(Y, \varphi)]$

$$\boxed{\mathcal{TM}_Y = \{M_t : i_{\check{\mathrm{d}}_\zeta M_t}(\psi) = 0\}/\{M_t : M_t = \check{\mathrm{d}}_\zeta V\}}$$

- But $\check{\mathrm{d}}_\zeta^2 \neq 0$: ζ is not an instanton

  Dimension of infinitesimal geometric moduli space is not finite, in general.
- Exception: G_2 holonomy $\quad \mathcal{TM}_Y \cong H^3_{\mathrm{d}}(Y) \cong H^1_{\check{\mathrm{d}}_\zeta}(Y, TY)$

Infinitesimal moduli

Deformations of $[(Y, \varphi), (V, A)]$

Want deformations that preserve $F \wedge \psi = 0$

$$\implies \check{\mathrm{d}}_A \alpha_t = \mathrm{d}_A \alpha_t \wedge \psi = -F \wedge i_{M_t}(\psi) = \check{\mathcal{F}}(M_t)$$

- Vector bundle moduli $\mathrm{d}_A \alpha_t = 0 \rightsquigarrow H^1(Y, \mathrm{End}(V))$
- Geometric moduli must lie in kernel of map $\check{\mathcal{F}}$

$$\implies \boxed{\mathcal{TM}_{[(Y,\varphi),(V,A)]} = H^1(Y, \mathrm{End}(V)) \oplus \ker \check{\mathcal{F}} \ , \ \ker \check{\mathcal{F}} \subset \mathcal{TM}_Y}$$

- Not enough to prove finiteness when $H \neq 0$

Infinitesimal moduli

Infinitesimal moduli of $[(Y,\varphi),(V,A),(TY,\Theta),H]$

Consider $\mathcal{Z}_t = (M_t, \kappa_t, \alpha_t)^T \in \Lambda^1(Y,\mathcal{Q})$ and compute

$$\mathcal{D}\mathcal{Z}_t = \begin{pmatrix} \mathrm{d}_\zeta M_t + \mathcal{R}(\kappa_t) - \mathcal{F}(\alpha_t) \\ \mathrm{d}_\Theta \kappa_t + \mathcal{R}(M_t) \\ \mathrm{d}_A \alpha_t + \mathcal{F}(M_t) \end{pmatrix} \qquad \text{with}\, \mathcal{D} = \begin{pmatrix} \mathrm{d}_\zeta & \mathcal{R} & -\mathcal{F} \\ \mathcal{R} & \mathrm{d}_\Theta & 0 \\ \mathcal{F} & 0 & \mathrm{d}_A \end{pmatrix}$$

- Heretotic constraints * $\implies$ $\boxed{\check{\mathcal{D}}\mathcal{Z}_t = 0}$

  * B-field variations do not decouple! $\rightsquigarrow$ antisymmetric part of matrix M_t

- Diffeomorphisms and gauge symmetry $\implies$ $\boxed{\mathcal{Z}_{triv} = \check{\mathcal{D}}\mathcal{V}}$

$$\implies \quad \boxed{\mathcal{TM}_{[(Y,\varphi),(V,A),(TY,\Theta),H]} \cong H^1_{\check{\mathcal{D}}}(\mathcal{Q})}$$

Infinitesimal moduli

Goal: geometry and moduli of heterotic G_2 system $[(Y, \varphi), (V, A), (TY, \Theta), H]$

Key ideas:

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  $\check{\mathcal{D}}$ acts on $\mathcal{Q}$-valued forms where topologically $\mathcal{Q} = T^*Y \oplus \mathrm{End}\, TY \oplus \mathrm{End}\, V$
- Infinitesimal moduli $\sim$ $\mathcal{Q}$-valued canonical G_2 cohomology $H^1_{\check{\mathcal{D}}}(\mathcal{Q})$ ✓

Comparison of 3D and 4D heterotic $\mathcal{N} = 1$ systems

Strominger–Hull system

$[(X, \Omega, \omega), (V, A), (TY, \Theta), H]$

- $SU(3)$ structure

  $\mathrm{d}(e^{-2\phi} * \omega) = 0 = \mathrm{d}(e^{-2\phi}\Omega)$

- $SU(3)$ instanton bundles (HYM eq)
- Nilpotent differential $\bar{D}$

  $Q = \mathcal{T}^* X \oplus \mathrm{End}(TX) \oplus \mathrm{End}(V) \oplus \mathcal{T} X$

Heterotic G_2 system

$[(Y, \varphi), (V, A), (TY, \Theta), H]$

- G_2 structure

  $\mathrm{d}(e^{-2\phi} * \varphi) = 0$

- G_2 instanton bundles
- Nilpotent differential $\check{\mathcal{D}}$

  $\mathcal{Q} = T^* Y \oplus \mathrm{End}(TY) \oplus \mathrm{End}(V)$

6D deformation theory *Atiyah:57, Kodaira, Spencer:58,60, Candelas, de la Ossa:91, Becker, Tseng, Yau:06, Anderson,et.al:10,11,13, Fu, Yau:11, Baraglia, Hekmati:13, Anderson, Gray, Sharpe:14, de la Ossa, Svanes:14, Garcia-Fernandez,et.al:13,15, Ashmore,et.al:19...* Also cf with talks by Dan Waldram and Charlie Strickland–Constable

Comparison of 3D and 4D heterotic $\mathcal{N} = 1$ systems

Strominger–Hull system

- Nilpotent differential $\bar{D}$

  $Q = \mathcal{T}^*X \oplus \mathrm{End}(TX) \oplus \mathrm{End}(V) \oplus \mathcal{T}X$

- SUSY $\rightarrow$ F-term/D-term
- $\bar{D}$ upper triangular
- Holomorphic Courant algebroid $\rightsquigarrow$ Hitchin's generalised geometry

Heterotic G_2 system

- Nilpotent differential $\check{\mathcal{D}}$

  $\mathcal{Q} = T^*Y \oplus \mathrm{End}(TY) \oplus \mathrm{End}(V)$

- SUSY $\rightarrow$ F-term
- $\check{\mathcal{D}}$ not upper triangular
- ——

  but, see Clarke *et.al.*:16

6D deformation theory *Atiyah:57, Kodaira, Spencer:58,60, Candelas, de la Ossa:91, Becker, Tseng, Yau:06, Anderson,et.al:10,11,13, Fu, Yau:11, Baraglia, Hekmati:13, Anderson, Gray, Sharpe:14, de la Ossa, Svanes:14, Garcia-Fernandez,et.al:13,15, Ashmore,et.al:19...* Also cf with talks by Dan Waldram and Charlie Strickland–Constable

Superpotential

Alternative perspective:

Moduli space as critical locus of a superpotential on off-shell parameter space

Strategy

- Dimensional reduction $\implies$ 3D gravitino mass $\boxed{M_{3/2} = e^K W}$
  Remark: need Hessian K
- Check $\delta W = 0 \quad \iff \mathcal{N} = 1$ heterotic G_2 system
- Follows that $\delta^2 W \quad \iff$ equation for moduli

Caveat:
3D $\mathcal{N} = 1$ supergravity lacks non-renormalisation theorems

Superpotential: Dimensional reduction

Fermionic part of 10D heterotic supergravity action

Bergshoeff, de Roo:89, Gurrieri, Lukas, Micu:07

$\rightarrow$ 3D kinetic and mass terms for gravitino

$$S_{0,f} = -\frac{1}{2\kappa_{10}^2}\int_{M^{10}} d^{10}x\sqrt{-g}\, e^{-2\phi} \left(\overline{\Psi}_M \Gamma^{MNP} D_N \Psi_P - \frac{1}{24}\left(\overline{\Psi}_M \Gamma^{MNPQR}\Psi_R + 6\,\overline{\Psi}^N \Gamma^P \Psi^Q\right) H_{NPQ}\right)$$

$$S_{3D} \supset -\frac{1}{2\kappa_3^2}\int d^3x\sqrt{-g}\left(\overline{\psi}_\mu \Gamma^{\mu\nu\kappa} D_\nu \psi_\kappa + m\overline{\psi}_\mu \Gamma^{\mu\kappa}\psi_\kappa\right)$$

Straightforward, but

- field normalisation (correct EH term)
- conventions for gravitino mass in AdS (want SUSY $\leftrightarrow 0 = M_{3/2} \sim W$).

Superpotential: Dimensional reduction

To match 3D EH term decompose

$$
\begin{aligned}
g_{10} &= e^n\, g_3 \oplus g_7 \\
\Gamma^\mu_{(10)} &= e^{-n/2}\, \Gamma^\mu_{(3)} \otimes \mathrm{Id} \otimes \sigma_2 \\
\Gamma^i_{(10)} &= \mathrm{Id} \otimes \Gamma^i_{(7)} \otimes \sigma_1 \\
\Psi_\mu &= e^{n/2}(\rho_\mu \otimes \lambda \otimes \theta) \\
\bar{\Psi}_\mu &= (\bar{\rho}_\mu \otimes \lambda^\dagger \otimes \theta^\dagger \sigma_2)\,,
\end{aligned}
$$

Then, e.g.

$$
\begin{aligned}
&\int d^{10}X\sqrt{-g_{10}}\, e^{-2\phi}\left(-\frac{1}{24}\overline{\Psi}_\mu \Gamma^{\nu\mu}\Gamma^{ijk}\Psi_\nu H_{ijk}\right) \\
&= \int d^3x\sqrt{-g_3}\,(\bar{\rho}_\mu \Gamma^{\mu\nu}\rho_\nu)\left[\frac{1}{24}\int d^7y\sqrt{g_7}\, e^{-2\phi+n}(-i\lambda^\dagger\Gamma^{ijk}\lambda)H_{ijk}\right] \\
\implies\ & m \sim \frac{1}{4}\int *_7 H \wedge \varphi \cdot e^{-2\phi+n}\,.
\end{aligned}
$$

Superpotential: Dimensional reduction

Result:

Three mass contributions

$$\tilde{M}_{3/2} = -\frac{1}{8}\int_7 e^{-2\phi+n}\left(d\varphi\wedge\varphi - 2 *_7 H\wedge\varphi + 2 *_7 f\right) , \tag{1}$$

Fixing conventions so that SUSY $\implies$ $M_{3/2} = 0$ (with $h = -\frac{2}{7}f$)

$$M_{3/2} = \frac{1}{4}\int_7 e^{-2\phi+n}\left(-\frac{1}{2}d\varphi\wedge\varphi + (H + h\varphi)\wedge\psi\right) .$$

Analysing the Einstein–Hilbert term, identify $K \simeq n$ $\implies$

$$W = \frac{1}{4}\int_Y e^{-2\phi}\ \left((H + h\,\varphi)\wedge\psi - \frac{1}{2}\,\mathrm{d}\varphi\wedge\varphi\right)$$

Superpotential: Variations and SUSY

Want to show $\delta W = 0 \iff \mathcal{N} = 1$ heterotic G_2 system

$$\delta W = \int_Y e^{-2\phi} \Big\{ -2\,\delta\phi\Big((H + h\,\varphi) \wedge \psi - \frac{1}{2}\,\mathrm{d}\varphi \wedge \varphi\Big)$$

$$- \mathcal{B} \wedge e^{2\phi}\mathrm{d}(e^{-2\phi}\,\psi) + \frac{\alpha'}{2}\,[\mathrm{tr}(\delta A\, F \wedge \psi) - \mathrm{tr}(\delta\Theta\, R(\Theta) \wedge \psi)] +$$

$$+ (H + h\varphi) \wedge \delta\psi + \delta\varphi \wedge \Big(h\,\psi - \mathrm{d}\varphi + \mathrm{d}\phi \wedge \varphi\Big)\Big\}$$

with $\delta H = \mathrm{d}\mathcal{B} + \frac{\alpha'}{2}(\mathrm{tr}(F\delta A) - \mathrm{tr}(R(\Theta)\,\delta\Theta))$.

Result: critical points of $W \iff$ heterotic G_2 system with $\tau_1 = \frac{1}{2}\,\mathrm{d}\phi$,
$H = T, h = \frac{1}{3}\,\tau_0$, and $W = 0$ ✓

Conclusions and outlook

Conclusions

Geometry and moduli of heterotic G_2 system $[(Y, \varphi), (V, A), (TY, \Theta), H]$

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  $\check{\mathcal{D}}$ acts on $\mathcal{Q}$-valued forms where topologically $\mathcal{Q} = T^*Y \oplus \mathrm{End}\, TY \oplus \mathrm{End}\, V$

- Infinitesimal moduli $\sim$ $\mathcal{Q}$-valued canonical G_2 cohomology $H^1_{\check{\mathcal{D}}}(\mathcal{Q})$
- Superpotential W s.t. $\delta W = W = 0 \iff \mathcal{N} = 1$ heterotic G_2 system

Conclusions and outlook

Outlook

- Examples and computing cohomologies
  Instantons on G2 holonomy: Walpuski:13, Sá-Earp, Menet, Nordström, Sá-Earp:15

  Heterotic G2 systems (with flux):

  Font:10, Fernandez, Ivanov, Ugarte, Villacampa:11, F, I, U, Vassiliev:15, Hinoue, Yasui:14

- Geometric invariants

- Quantum corrections

  Some control if we demand sensible deformation theory order by order in α'

- Relation to $\mathcal{N} = 2$ ($SU(3)$ structure) moduli spaces

- Structure of moduli space: metric, Hessian potential, singularities, ...

- Higher order deformations: obstructions, integrability, ...

Conclusions and outlook

Outlook

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Thank you for listening!