On supersymmetry breaking in supergravity

Alessandro Tomasiello

AEI, 3.12.2019

Introduction

Non-supersymmetric solutions of string theory

- have no compelling geometrical story to tell, so far (unlike susy: generalized/exceptional geometry...)
- for AdS, have been conjectured to be unstable
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Today I'll try to address the first two issues

looking at some large classes of examples.



- Breaking supersymmetry with pure spinors
- Non-BPS AdS7 solutions
 - Stability
- perturbative: part of the KK tower
- non-perturbative: bubbles

Pure spinors

'Pure spinor equations' for BPS solutions

[Graña, Minasian, Petrini, AT '05]

$$d_{H}(e^{3A-\phi}\Phi_{+}) = 0$$

$$d_{H}(e^{2A-\phi}\operatorname{Re}\Phi_{-}) = 0$$

$$RR \operatorname{flux}$$

$$d_{H}(e^{4A-\phi}\operatorname{Im}\Phi_{-}) = e^{4A} \star \lambda(F)$$
NSNS flux

 Φ_{\pm} : even/odd 'pure' forms

+ algebraic comp. condition

metric determined from Φ_\pm

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An easy class:

 $ds^2 = S^{-1/2} ds^2_{\text{Mink}_6} + K(S^{-1/2} dz^2 + S^{1/2} ds^2_{\mathbb{R}^3})$

[motivated by NS5-D6-D8]

[Imamura '01; Janssen, Meessen, Ortin '99]

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take Φ_{\pm} adapted to this metric \checkmark

fluxes are determined. Bianchi \Rightarrow

$$\Delta_3 S + \frac{1}{2} \partial_z^2 S^2 = 0 \qquad K = -\frac{4}{F_0} \partial_z S$$

rare case where single PDE

 $S(x_1, x_2, x_3, z)$ [Legramandi, AT '19]

$$\Delta_3 S + \frac{1}{2} \partial_z^2 S^2 = 0 \qquad \qquad ds^2 = S^{-1/2} ds_{\text{Mink}_6}^2 + K(S^{-1/2} dz^2 + S^{1/2} ds_{\mathbb{R}^3}^2)$$

probably rich, should be studied further. Some local solutions:

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- $S = kz + S_3(x)$ $\Delta_3 S_3 = -k^2$ gen. Green function; analytic solution

can be made **compact** with O8±

keep same metric & RR fluxes; change NSNS impose Bianchi, but not BPS

[Legramandi, AT '19]

$$\Delta_3 S + \frac{1}{2} \partial_z^2 S^2 = 0 \quad \checkmark \quad \Delta_3 S + \frac{1}{2} \partial_z^2 S^2 + c(c - 2\partial_z S) = 0$$

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$$d_{H}(e^{3A-\phi}\Phi_{+}) = 0$$

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• we have worked out several other classes

taken or generalized from classif. of $Mink_4 \times S^2$ solutions

[Macpherson, T'17; Apruzzi, Geipel, Legramandi, Macpherson, Zagermann '18; Legramandi, Macpherson '18]

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• Why does this work?

[Legramandi, AT'19]

Susy-breaking in AdS7

• All AdS₇ solutions in type II:

 $\ddot{\alpha} = F_0 \qquad \Box >$

[Apruzzi, Fazzi, Rosa, AT '13 Apruzzi, Fazzi, Passias, Rota, AT '15; Cremonesi, AT '15]

$$\frac{1}{\pi\sqrt{2}}ds^{2} = 8\sqrt{-\frac{\alpha}{\ddot{\alpha}}}ds^{2}_{\mathrm{AdS}_{7}} + \sqrt{-\frac{\ddot{\alpha}}{\alpha}}\left(dz^{2} + \frac{\alpha^{2}}{\dot{\alpha}^{2} - 2\alpha\ddot{\alpha}}ds^{2}_{S^{2}}\right)$$

interval

 α piecewise cubic

$$e^{\phi} = 2^{1/4} 162\pi^{5/2} \frac{(-\alpha/\ddot{\alpha})^{3/4}}{\sqrt{\dot{\alpha}^2 - 2\alpha\ddot{\alpha}}}$$

$$B = \pi \left(-z + \frac{\alpha \dot{\alpha}}{\dot{\alpha}^2 - 2\alpha \ddot{\alpha}} \right) \operatorname{vol}_{S^2}$$

$$F_2 = \left(\frac{\ddot{\alpha}}{162\pi^2} + \frac{\pi F_0 \alpha \dot{\alpha}}{\dot{\alpha}^2 - 2\alpha \ddot{\alpha}}\right) \operatorname{vol}_{S^2}$$



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position of a D8: z = k =its D6-charge
z ∈ [0, N] NS5 flux

 $\ddot{\alpha} = F_0 \qquad \Box >$



• Any AdS7 solution in IIA \mapsto

[Passias, Rota, AT '15; Malek, Samtleben, Vall Camell '19]

consistent truncation to 'minimal gauged 7d sugra'

fields: $g_{\mu\nu}^{(7)}, A_{\mu}^{i}, X$ [susy vacuum: X = 1]

uplift:

$$\frac{1}{\pi\sqrt{2}}ds^{2} = \frac{120}{V(X)\sqrt{X}}\sqrt{-\frac{\alpha}{\ddot{\alpha}}}ds^{2}_{AdS_{7}} + X^{5/2}\sqrt{-\frac{\ddot{\alpha}}{\alpha}}\left(dz^{2} + \frac{\alpha^{2}}{\dot{\alpha}^{2} - 2X^{5}\alpha\ddot{\alpha}}ds^{2}_{S^{2}}\right)$$
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• AdS₇ \rightarrow AdS_d \times Σ_{7-d} comp. & RG flows • non-susy AdS₇ solution: $X^5 = \frac{1}{2}$ So we get a susy-breaking sister solution:

$$\frac{1}{\pi\sqrt{2}}ds^{2} = \sqrt[3]{-\frac{\alpha}{\ddot{\alpha}}}ds^{2}_{AdS_{7}} + \sqrt{-\frac{\ddot{\alpha}}{\alpha}}\left(dz^{2} + \frac{\alpha^{2}}{\dot{\alpha}^{2} - \varkappa\alpha\ddot{\alpha}}ds^{2}_{S^{2}}\right)$$

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• Can we have a 7d theory with more than one susy vacuum?

couple minimal 7d gauged sugra to $SU(k)^2$ vector multiplets [De Luca, Gnecchi, Lo Monaco, AT '18]

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 - many nonabelian vacua \cong AdS₇ vacua with different D8 configurations
 - susy RG flows \rightarrow Nahm equations. Agree with CFT expectations



N

AdS7 (in) stability

Possible instabilities:

• 'Tachyonic' modes.

Particles with mass negative enough ('BF bound') have solutions that grow in time

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Supersymmetric vacua are protected against both. But non-supersymmetric ones?

[Breitenlohner, Freedman '82, Gibbons, Hull, Warner '83...]

• Spin-two masses: simple universal equation

[Bachas, Estes '11]

spectrum of 'warped Laplacian' $\frac{e^{-5A+2\phi}}{\sqrt{\hat{g}}}\partial_m \left(e^{7A-2\phi}\sqrt{\hat{g}}\hat{g}^{mn}\partial_n\right)$ internal metric

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Universal bound:

realized for $\psi = \alpha^{-l} Y_l$ spherical harm. on S^2

[Passias, AT'16]

[Apruzzi, De Luca, Gnecchi, Lo Monaco, AT to appear]

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internal metric
Universal bound: realized for $\psi = \alpha^{-l} Y_l$
spherical harm. on S^2
• supersymmetric case: $m^2 \ge 8l(2l+3) \ge 40$
[in units where $L_{AdS} = 1$]
• susy-breaking case: $m^2 \ge 12l(l+2) \ge 36$
 $rac{r}{r}$ no separation of scales proof for susy AdS7!

[notice: reasonable est. $M_{\rm KK} \sim \int e^{4A} R_6$ can fail for solutions with O-planes]

- Spin-zero: we can compute many masses in 7d gauged sugra
 - also: nonabelian DBI \Rightarrow same results

[Apruzzi, De Luca, Gnecchi, Lo Monaco, AT to appear]

potential: $-4\Phi^i\Phi^i - 2\epsilon_{ijk}[\Phi_i, \Phi_j]\Phi_k + [\Phi_i, \Phi_j][\Phi^i, \Phi^j]$ vacua: $[\Phi^i, \Phi^j] = \epsilon^{ijk}\Phi^k$

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[units where BF bound: $m^2 \ge -9$]

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multiplicities depend on # of D8s

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[Apruzzi, De Luca, Gnecchi, Lo Monaco, AT to appear]

multiplicities depend on # of D8s

$$m^2 = 3(k+2)(k-2) = -12, -9, 0, 15, 36, ...$$

below the BF bound!

but this -12 appears only with coincident D8s

plausible scenario: \longrightarrow

susy-breaking case:

• Another pert. instability exists for solutions with D6s

such as the non-susy version of the 'teardrop'



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[Blåbäck, Danielsson, Junghans, Van Riet, Wrase, Zagermann '11]



[Coleman, De Luccia '80] [Maldacena, Michelson, Strominger '98]



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[Ooguri, Vafa '16, Freivogel, Kleban '16]



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• this process 'shortens' the region where $F_0 = 0$



• Other possible tunnel effect:

[motivated by nonabelian potential]

$$\longrightarrow$$

2 D8s combine: one of them keeps all D6 charge, the other tunnels away

• Together these effects seem to rule everything out.

[Apruzzi, De Luca, Gnecchi, Lo Monaco, AT to appear]



• Steps towards procedure to break supersymmetry in the Minkowski case

[Legramandi, AT '19]

- Older method: consistent truncation. Are non-susy AdS7 stable?
- Tachyons make D8s repel each other

• Tunnel effects probably make remaining ones unstable