Exceptional Geometries and the AdS/CFT Correspondence

Daniel Waldram, Imperial College London

5 December 2019

Geometry and Duality, AEI

Introduction

Overview

- some basic open/difficult problems in AdS/cft
- why exceptional geometry is well adapted to tackle them
- summarise some new results . . .

Builds on long history of using G-structures and generalised complex geometry to analyse supersymmetric flux backgrounds

Natural stringy extensions of standard geometrical constructions

with Anthony Ashmore, André Coimbra, Mariana Graña, Michela Petrini, Charlie Strickland-Constable, Ed Tasker, David Tennyson

$\mathcal{N} = 1$ description of $\mathcal{N} = 4$

Superpotential and Kähler potential for chiral fields (Φ^1, Φ^2, Φ^3)

$$\mathcal{W} = \tfrac{1}{6}\epsilon_{ijk}\,\mathrm{tr}\,\Phi^i\Phi^j\Phi^k \qquad\qquad \mathcal{K} = \tfrac{1}{2}\delta_{i\bar{j}}\,\mathrm{tr}\,\Phi^i\bar{\Phi}^{\bar{j}}$$

$\frac{3}{2}R$-symmetry

$$\Phi^i \to \mathrm{e}^{\mathrm{i}\alpha}\Phi^i \qquad\qquad \mathcal{W} \to \mathrm{e}^{3\mathrm{i}\alpha}\mathcal{W}$$

and SU(3) flavour symmetry $\Phi^i \to M^i{}_j\Phi^j$

F-term conditions mean Φ^i commute

$$\partial\mathcal{W}/\partial\Phi^1 = \tfrac{1}{2}[\Phi^2, \Phi^3] = 0 \quad \text{etc}$$

Chiral single-trace operators

$$\mathcal{O}_f = \sum_n \tfrac{1}{n!} f_{(i_1 \dots i_n)} \operatorname{tr} \Phi^{i_1} \dots \Phi^{i_n}. \quad \leftrightarrow \quad f(z^1, z^2, z^3)$$

chiral ring $\leftrightarrow$ ring of holomorphic functions on $\mathbb{C}^3$

Hilbert series counts number operators with charge n

$$H(t) = 1 + 3t + 6t^2 + 10t^3 + \cdots = \frac{1}{(1-t)^3}$$

Gravity dual: S^5 with F_5

- Calabi–Yau cone $C(S^5) = \mathbb{C}^3$
- chiral operators dual to particular Kaluza–Klein modes on S^5

$\mathcal{N} = 1$ marginal deformations for $\mathcal{N} = 4$

Superpotential deformation (charge 3) [*Leigh & Strassler*]

$$\mathcal{W} = \tfrac{1}{6}\epsilon_{ijk}\,\mathrm{tr}\,\Phi^i\Phi^j\Phi^k + \tfrac{1}{6}f_{ijk}\,\mathrm{tr}\,\Phi^i\Phi^j\Phi^k$$

- ten complex marginal deformations f_{ijk}
- but beta-function constrains moment map for SU(3) symmetry

$$f_{ikl}\bar{f}^{jkl} - \tfrac{1}{3}\delta_i^j f_{klm}\bar{f}^{klm} = 0$$

- exactly marginal deformation as symplectic quotient [*Kol; Green et al*]

$$\widetilde{\mathcal{M}} = \{f_{ijk}\}/\!\!/\mathrm{SU}(3)$$

Can choose

$$\Delta\mathcal{W} = \lambda_1 \operatorname{tr} Z^1 Z^2 Z^3 + \tfrac{1}{3}\lambda_2 \operatorname{tr}\left[(Z^1)^3 + (Z^2)^3 + (Z^3)^3\right]$$

F-terms now give non-commutative "Sklyanin" algebra

$$[\Phi^1, \Phi^2] + \lambda_1(\Phi^1\Phi^2 + \Phi^2\Phi^1) + \lambda_2(\Phi^3)^2 = 0 \qquad \text{etc}$$

Chiral operators for generic λ_i [*Van den Burgh*]

$$H(t) = 1 + 3t + 3t^2 + 2t^3 + 3t^4 + 3t^5 + 2t^6 + \cdots = \frac{(1+t)^3}{1-t^3}$$

Gravity dual?

Moduli space of AdS solutions, deforming S^5 and turning on more fluxes

- $\lambda_2 = 0$: "β-deformation", $U(1)^3$ isometry, exact dual solution via solution generating transformation [*Lunin & Maldacena*]
- generic?: no isometries, not a perturbation in consistent truncation

As hard as finding explicit Calabi–Yau metrics, but perturbative tour de force to 3rd order by [*Aharony, Kol & Yankielowicz*]

Kaluza–Klein expansion for duals of chiral operators on β-deformation already looks exhausting.

But ... much of field theory quite simple, since only depends on holomorphic structure. Is there some supergravity analogue?

More generally ...

Field theory characterises "non-commutative alegbraic geometry"

- quiver diagram for fields + superpotential W
- setting $\mathrm{d}\mathcal{W} = 0$ defines "Calabi–Yau algebra" [*Ginzburg*]
- chiral operators counted by "cyclic homology" [*Eager, Schmude, Tachikawa*]

Special case $\mathcal{W} = \mathcal{W}_{\mathsf{SE}} + \Delta\mathcal{W}$

- $\mathcal{W}_{\mathsf{SE}}$ defines commutative space of holomorphic functions of Calabi–Yau cone $C(M)$
- $\Delta\mathcal{W}$ is marginal deformation characterised by holomorphic function f of charge 3

What can we understand about the dual geometries?

Differential geometry description

- can we find dual of superpotential?
- calculate chiral spectrum?

tool will be exceptional geometry ...

Introduction

Generalised geometry and supersymmetric spacetimes

Exceptional Sasaki–Einstein geometries

Dual of the superpotential, new solutions and chiral operators

Generalised geometry and supersymmetric spacetimes

Supersymmetric backgrounds: Killing spinor equations

Important class of supergravity solutions for phenomenology or AdS/cft

$$\mathcal{M}_D = \begin{cases} \mathrm{Mink}_{D-d} \times M_d \\ \mathrm{AdS}_{D-d} \times M_d \end{cases} \qquad D = 10, 11$$

preserving supersymmetry, heuristically

$$\begin{aligned} \nabla_m \epsilon + (\mathrm{flux})_m \, \epsilon &= 0, \\ \gamma^m \nabla_m \epsilon + (\mathrm{flux}) \, \epsilon &= 0. \end{aligned}$$

where ϵ is spinor, extra terms include

$$\begin{aligned} F &= \mathrm{d}A \in \Gamma(\Lambda^p T^* M) && \text{flux}, \\ \mathrm{d}s^2(M_D) &= \mathrm{e}^{2\Delta} \mathrm{d}s^2(\mathrm{Mink}/\mathrm{AdS}) + \mathrm{d}s^2(M_d) && \text{warp factor} \end{aligned}$$

Minkowksi background with no flux

$$\nabla_m \epsilon = 0 \qquad \Longrightarrow \qquad \text{special holonomy}$$

Classic case: Calabi–Yau – SU(3) holonomy

Geometry encoded in pair of integrable objects

$$\mathrm{d}\omega = 0 \qquad \text{symplectic structure}$$

$$\mathrm{d}\Omega = 0 \qquad \text{complex structure}$$

$\omega \in \Gamma(\Lambda^2 T^* M)$ and $\Omega \in \Gamma(\Lambda^3 T^*_{\mathbb{C}} M)$.

G structures for Calabi–Yau

By considering stabiliser groups in $\mathrm{GL}(6,\mathbb{R})$

ω	$\mathrm{Sp}(6,\mathbb{R})$ structure
Ω	$\mathrm{SL}(3,\mathbb{C})$ structure

together (ω,Ω) is $\mathrm{SU}(3)$ structure if compatible

$$\omega\wedge\Omega=0, \qquad \tfrac{1}{6}\omega^3=\tfrac{1}{8}\mathrm{i}\Omega\wedge\bar{\Omega}$$

Torsion-free structure

$$\exists \text{ a torsion-free connection } \nabla \text{ such that } \nabla\omega=\nabla\Omega=0$$
$$\Updownarrow$$
$$\mathrm{d}\omega=\mathrm{d}\Omega=0$$

Sasaki–Einstein – SU(2) structure

Structure defined by $\sigma \in \Gamma(T^*M)$, $\omega \in \Gamma(\Lambda^2 T^*M)$ and $\Omega \in \Gamma(\Lambda^2 T^*_{\mathbb{C}}M)$

$$\mathrm{d}\sigma = 2\omega, \qquad \mathrm{d}\Omega = 3\mathrm{i}\sigma \wedge \Omega.$$

with metric $\mathrm{d}s^2(M) = \sigma \otimes \sigma + \mathrm{d}s_4^2(M)$ and $\sigma = \mathrm{d}\psi + a$ (R-symmetry)

singlet intrinsic torsion

What is the geometry of background with flux?

- special holonomy? analogues of ω and Ω? integrability?
- natural generalisation of Calabi–Yau, Sasaki–Einstein, . . . ?
- deformations? moduli spaces? . . .

(n.b. no-go means non-compact/boundary for Minkowski)

Answer

For Minkowski backgrounds

supersymmetry $\Leftrightarrow$ *torsion-free* G*-structure in exceptional geometry*

or singlet instrinsic torsion for AdS [*Coimbra, Strickland-Constable, DW*]

$E_{6(6)} \times \mathbb{R}^+$ exceptional geometry

[*Hull; Pacheco & DW; Coimbra, Strickland-Constable & DW*][*Berman et al.*]

Focus on M_5 in $D = 10$ type IIB supergravity

$$\text{bosonic fields: } g,\ \phi,\ \chi,\ B_2^i,\ C_4,\ \Delta$$

define generalised tangent space

$$E \simeq TM \oplus 2T^*M \oplus \Lambda^3 T^*M \oplus 2\Lambda^5 T^*M$$

$$V^M = (v^m, \lambda^i_m, \rho_{mnp}, \sigma^i_{m_1 \dots m_5})$$

- transforms as **27** under $E_{6(6)}$
- parametrises diffeomorphism and gauge symmetry

Generalised tensors

For example, adjoint **78** includes potentials

$$\mathrm{ad}\,\tilde{F} \simeq 3\mathbb{R} \oplus (TM \otimes T^*M) \oplus 2\Lambda^2 T^*M \oplus 2\Lambda^2 TM \oplus \Lambda^4 T^*M \oplus \Lambda^4 TM$$

$$A^M{}_N = (\ldots, B^i_{mn}, \ldots, C_{mnpq})$$

"Twisting" of generalised tensors by gauge potentials acting in adjoint

$$V = \mathrm{e}^{B^i + C} \cdot \tilde{V} \qquad A = \mathrm{e}^{B^i + C} \cdot \tilde{A}$$

Generalised Lie derivative

"Generalised diffeomorphism, GDiff" symmetries

$$
\begin{aligned}
L_V &= \text{diffeo} + \text{gauge transf} \\
&= \mathcal{L}_v - (\mathrm{d}\lambda^i + \mathrm{d}\rho)\,\cdot
\end{aligned}
$$

where forms act via adjoint (n.b. $L_V W \neq -L_W V$)

$E_{6(6)}$ cubic invariant

$$
c(V,V,V) = \epsilon_{ij}(i_v\lambda^i)\sigma^j - \tfrac{1}{2} i_v\rho\wedge\rho - \tfrac{1}{2}\epsilon_{ij}\rho\wedge\lambda^i\wedge\lambda^j \in \Gamma(\Lambda^5 T^* M)
$$

Properties

- $L_V W$ defines "Leibniz algebroid" structure on E
- only ∂_M breaks the symmetry : "section condition" (same formalism encodes IIA/B and M-theory)
- L_V is not the conventional Lie derivative on a 27-dim space

Generalised connection

$$D_M V^N = \partial_M V^N + \Gamma_M{}^N{}_P V^P, \qquad D_M c_{NPQ} = 0$$

in components not just $E_{6(6)} \times \mathbb{R}^+$ connection

$$(\partial_m V^N + \Gamma_m{}^N{}_P V^P,\ \Gamma_i^{mN}{}_P V^P,\ \Gamma^{mnpN}{}_P V^P,\ \Gamma_i^{m_1\dots m_5\,N}{}_P V^P)$$

Generalised Levi–Civita connection and supergravity

- Generalised metric encodes $\{g, \phi, \chi, B^i, C, \Delta\}$ as USp(8) structure

$$G_{MN} \in E_{6(6)} \times \mathbb{R}^+/\mathrm{USp}(8) \quad \text{at } p \in M$$

- Given notion of generalised torsion, get non-unique connection

$$D_M G_{NP} = 0, \qquad D_M c_{NPQ} = 0, \qquad \text{torsion} = 0.$$

- Spinors $\epsilon = (\epsilon_1, \epsilon_2)$ are sections of USp(8) vector bundle

$$D_M \epsilon = (\not{D}\epsilon,\ D \curlywedge \epsilon,\ D^0_M \epsilon) \qquad \not{D} \text{ and } D\curlywedge \text{ operators}$$

$$\mathbf{27} \times \mathbf{8} = \mathbf{8} + \mathbf{48} + \mathbf{160} \qquad \text{are unique}$$

[*Coimbra, Strickland–Constable, Waldram*]

Killing spinor equations

$$\delta(\text{gravitino}) = D \curlywedge \epsilon, \qquad \delta(\text{dilatino}) = \not{D}\epsilon$$

For minimal supersymmetry, single spinor ϵ defines structure

$$\text{Stab}(\epsilon) = \text{USp}(6) \subset \text{USp}(8) \subset \text{E}_{6(6)} \times \mathbb{R}^+$$

then one can show

$$\text{Mink} \iff \text{exists torsion-free } D_M \text{ with } D_M\epsilon = 0$$

$$\text{AdS} \iff \text{exists singlet-torsion } D_M \text{ with } D_M\epsilon = 0$$

"torsion-free/singlet torsion USp(6) structure"

Exceptional Sasaki–Einstein geometries

Invariant tensors

How can we define the USp(6) structure in terms of generalised tensors satisfying differential conditions? [*Ashmore, Petrini, DW*]

V structure

Generalised vector defining $F_{4(4)} \subset E_{6(6)}$ structure

$$K \in \Gamma(E) \qquad \text{such that } c(K, K, K) > 0$$

V special real geometry on ∞-dim. space of structures (vector multiplet)

H structure

Weighted, complex, adjoint tensor defining $\mathrm{SU}^*(6) \subset \mathrm{E}_{6(6)}$ structure

$$X \in \Gamma(\mathrm{ad}\, \tilde{F}_{\mathbb{C}} \otimes \det T^*M)$$

required to be a highest root in $\mathfrak{e}_6$ such that

$$0 \neq \operatorname{tr} X\bar{X} := -2\kappa^4 \in \Gamma((\det T^*M)^2)$$

equivalently if $X = \kappa(J_1 + \mathrm{i}J_2)$ get $\mathfrak{su}_2$ triplet

$$[J_\alpha, J_\beta] = 2\kappa\epsilon_{\alpha\beta\gamma}J_\gamma, \qquad \operatorname{tr} J_\alpha J_\beta = -\kappa^2\delta_{\alpha\beta}$$

Hyper-Kähler geometry on ∞-dim. space of structures (hypermultiplet)

Compatible structures

The H and V structures are compatible if

$$X \cdot K = 0 \qquad c(K, K, K) = \kappa^2$$

(analogues of $\omega \wedge \Omega = 0$ and $\frac{1}{6}\omega^3 = \frac{1}{8}\mathrm{i}\Omega \wedge \bar{\Omega}$ on cone)

the compatible pair (X, K) define an USp(6) structure

X and K come from generalised metric and Killing spinor bilinears.

Example: Sasaki–Einstein

Writing Killing vector $\xi = \sigma^\sharp = \partial/\partial\psi$ and $\tau = \chi + \mathrm{i}e^{-2\phi}$

$$X = \tfrac{1}{2}\kappa^2 u^i \left(\Omega - \mathrm{i}\Omega^\sharp\right), \qquad \text{“Cauchy–Riemann structure”}$$

$$\operatorname{ad}\tilde{F} \simeq 3\mathbb{R} \oplus (TM \otimes T^*M) \oplus \Lambda^2 T^*M \oplus \Lambda^2 TM \oplus \Lambda^4 T^*M \oplus \Lambda^4 TM,$$

where $u^i = \tau_2^{-2}(\tau, 1)^i$ and $\kappa^2 = \mathrm{vol}_5$ and

$$K = \xi - \sigma \wedge \omega, \qquad \text{“contact structure”}$$

$$E \simeq TM \oplus 2T^*M \oplus \Lambda^3 T^*M \oplus 2\Lambda^5 T^*M,$$

and L_K generates the R-symmetry

Differential conditions

Supersymmetric AdS background is equivalent to

$$L_K X = 3\mathrm{i} X, \qquad L_K K = 0.$$

so that X is charge 3, and

$$\mu_+(V) = 0, \qquad \mu_3(V) = \int_M c(K, K, V), \qquad \forall V \in \Gamma(E)$$

and are equivalent to the conditions derived in [*GMSW*] by writing (X, K) in terms of bilinears.

But what are μ_α??

Triplet of moment maps

Infinitesimally GDiff parametrised by $V \in \Gamma(E) \simeq \mathfrak{gdiff}$ and acts by

$$\delta J_\alpha = L_V J_\alpha$$

preserves HK structure on space of J_α giving triplet of moment maps

$$\mu_\alpha(V) = -\tfrac{1}{2}\epsilon_{\alpha\beta\gamma} \int_M \operatorname{tr} J_\beta (L_V J_\gamma)$$

Universal contact structure

In all cases [*c.f. Gabella, Gauntlett, Sparks, DW*]

$$K = \xi - \sigma \wedge \omega$$

with $i_\xi \sigma = 1$, $i_\xi \omega = 0$ and $\mathrm{d}\sigma = 2\omega$ and (c.f. a-max ...)

$$\frac{1}{\text{central charge}} \propto \int_M c(K, K, K)$$

is universal result

Summary

Supersymmetry with general fluxes is torsion-free/singlet torsion G structures in generalised geometry

- many cases $E_{7(7)}$, $E_{6(6)}$, $E_{5(5)} = \mathrm{Spin}(5,5)$ in M-theory or IIB generalising CY, Sasaki–Einstein, hyper-Kähler, G_2 ...
- structures for 8 supercharges [*Ashmore & DW; Graña & Ntokos*]; for 16 supercharges [*Malek; Malek, Samtleben & Vall Camell; Cassani, Josse, Petrini DW*]; for 4 supercharges [*see Charlie's talk*]
- extend to heterotic
- new solutions and new consistent truncations ...

Dual of the superpotential, new solutions and chiral operators

Applications to AdS/cft

Nice formalism but how is it useful? SCFT result of [*Kol, Green et al.*]

> *all marginal deformations are in the superpotential and are all exactly marginal unless there is a global symmetry*

In gravity dual follows directly from moment map structure [*Ashmore, Gabella, Graña, Petrini, DW*].

What about the missing deformed solutions or counting chiral operators? [*Ashmore, Petrini, Tennyson, Waldram to appear*]

Idea: analogue of Calabi–Yau

Given explicit Kähler SU(3) structure (Ω, ω) with $\frac{1}{8}\mathrm{i}\Omega \wedge \bar{\Omega} = \frac{1}{6}\omega^3$

$$\mathrm{d}\Omega = 3\mathrm{i}\alpha \wedge \Omega, \qquad \mathrm{d}\omega = 0$$

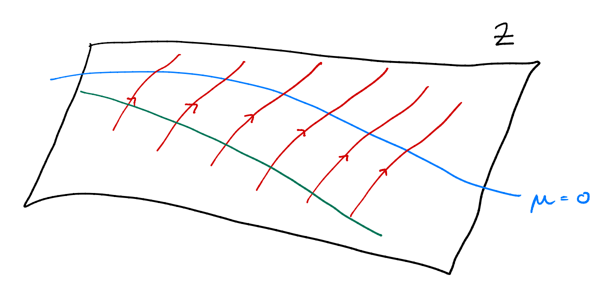
then vary Kähler form within cohomology class

$$\omega' = \omega + \mathrm{i}\partial\bar{\partial}h, \qquad \Omega' = \lambda\Omega$$

there exists Calabi–Yau solution (Ω_*, ω_*)

Symplectic quotient/extremal metric *[Yau; Donaldson, ...]*

Typical of supersymmetry conditions: first solve F-terms (holomorphic)



- Z is Kähler (infinite-dimensional) with group action G
- orbits for $G_{\mathbb{C}}$ intersect $\mu = 0$ (if "stable" – algebraic condition)

Kähler–Einstein, Sasaski–Einstein, Hermitian Yang-Mills, ...

“Exceptional Sasaki” geometry

We would like to relax one condition (analogue of Kähler on cone)

$$\mu_+(V) = 0, \qquad \cancel{\mu_3(V) = \int_M c(K, K, V)},$$

$$L_K J_+ = 3\mathrm{i} J_+, \qquad L_K K = 0.$$

and $\kappa^2 \neq c(K, K, K)$.

(Note that if only fiveform flux, then Exceptional Sasaki = Sasaki)

Final condition: GDiff moment map

- ∞-dim. space $\mathcal{Z}$ of structures X is Kähler
- condition is a moment map for GDiff
- complex $\mathrm{GDiff}_{\mathbb{C}}$ orbit generated by

$$\delta X = L_V X \qquad V \in \Gamma(E_{\mathbb{C}}) \simeq \mathfrak{gdiff}_{\mathbb{C}}$$

- intersects moment map condition on susy ESE background (X_*, K_*)

Physical interpretation

From supersymmetric multiplet structure

- fixing orbit, $[X] = \mathrm{GDiff}_{\mathbb{C}} \cdot X$ fixes superpotential $\mathcal{W}$
- equivalence class is simply ambiguity of rewriting SCFT

$$\int \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}\mathcal{K} = \int \mathrm{d}^2\theta \bar{D}^2\mathcal{K}$$

- the relation $L_K X = 3\mathrm{i}X$ fixes marginal condition $\Delta = 3$
- motion in orbit is renormalisation flow of Kähler potential, can check class $[X]$ does not change – non-renormalization theorem for $\mathcal{W}$

Existence of solutions is implied by SCFT result of [*Kol,Green et al.*]

Aside: c.f. GMPT

Translating into pure spinors on the cone [*GMPT*, *Tomasiello*]

$$\mathrm{d}\Phi_- = 0,$$

$$\mathrm{d}^{\mathcal{J}_-}(\mathrm{e}^{-3A}\,\mathrm{Im}\,\Phi_+) = F_{\mathrm{RR}}$$

$$\mathrm{d}(\mathrm{e}^{-A}\,\mathrm{Re}\,\Phi_+) = 0 \qquad \text{moment map}$$

(Already hard to solve first two equations...)

$[X]$ for Sasaki–Einstein

We have, up to $\mathrm{GDiff}_{\mathbb{C}}$,

$$X = \tfrac{1}{2}\,\mathrm{vol}_5\, u^i \left(\Omega - \mathrm{i}\Omega^\sharp\right) = \mathrm{e}^{\frac{1}{4}\mathrm{id}(\sigma\wedge\omega)} \cdot \left(-\tfrac{1}{2}\mathrm{i}u^i\sigma\wedge\Omega\right) \sim -\tfrac{1}{2}\mathrm{i}u^i\sigma\wedge\Omega$$

$$\in \Gamma(\mathrm{ad}\,\tilde{F}_{\mathbb{C}} \otimes \det T^*M) = \Gamma(T^*_{\mathbb{C}}M \oplus \Lambda^3 T^*_{\mathbb{C}}M \oplus \dots)$$

How can we deform this using the information in the function f?

- marginal $\Leftrightarrow$ $\mathcal{L}_\xi f = 3\mathrm{i}f$
- for example on S^5: $f = \frac{1}{6} f_{ijk} z^i z^j z^k$

Solution for deformed background

We find new family of Exceptional Sasaki solutions

$$\boxed{\begin{aligned} K &= \xi - \sigma \wedge \omega \\ X &= \mathrm{e}^{b^i(\tau,f)} \left(\mathrm{d}f + v^i(\tau,f)\,\sigma \wedge \Omega\right) \end{aligned}}$$

with $b^i \in \Gamma(\Lambda^2 T^*_{\mathbb{C}} M)$ linear in $\mathrm{d}f$ and v^i quadratic

- very complicated deformed metric g, axion-dilaton and fluxes
- valid for deformation of any Sasaki–Einstein
- for S^5 matches leading parts of Aharony et al. and $\mathrm{GDiff}_{\mathbb{C}}$ action gives $\{f_{ijk}\} /\!\!/ \mathrm{SU}(3)$
- can also check if $f = z^1 z^2 z^3$ then is $\mathrm{GDiff}_{\mathbb{C}}$ of LM solution

Can we calculate the chiral spectrum?

Space of integrable structures $\mathcal{Z}$ so $\delta X \in T\mathcal{Z}$

$$E_{\mathbb{C}} \xrightarrow{L.X} T\mathcal{Z} \xrightarrow{\delta\mu_+} E^*_{\mathbb{C}}$$

cohomology counts operators

$$\text{space of chiral ops, } \mathcal{C} = \frac{\{\delta X : \delta\mu_+ = 0\}}{\{\delta X = L_V X\}}$$

independent of choice of X in orbit

The integrable X defines an involutive subbundle $L_+ \subset E_{\mathbb{C}}$

$$L_V W \in \Gamma(L_+) \qquad \forall V, W \in \Gamma(L_+)$$

defines dual complex

$$\cdots \xrightarrow{\mathrm{d}_L} \Lambda^p L_+^* \xrightarrow{\mathrm{d}_L} \Lambda^{p+1} L_+^* \xrightarrow{\mathrm{d}_L} \cdots$$

and $\mathcal{C}$ is given in terms of $H^p(\Lambda^* L_+, \mathrm{d}_L)$

differential geometry dual of cyclic homology

Calculating the cohomology

If $\mathrm{d}f$ nowhere vanishing (generic) then can write

$$X = \mathrm{e}^{\tilde{b}^i(f,\tau)+\tilde{c}(f,\tau)}\,\mathrm{d}f$$

and cohomology reduces to calculating

$$\cdots \xrightarrow{\mathrm{d}} \mathrm{d}f \wedge \Lambda^p T^*_{\mathbb{C}} M \xrightarrow{\mathrm{d}} \mathrm{d}f \wedge \Lambda^{p+1} T^*_{\mathbb{C}} M \xrightarrow{\mathrm{d}} \cdots$$

which can be calculated using "Kohn–Rossi cohomology" of SE giving

$$H(t)_{S^5} = \frac{(1+t)^3}{1-t^3}$$

+ new predictions for other SE!

Summary

- exceptional geometry has allowed us to implicitly solve for a large class of new supergravity duals
- the "holomorphic" structure $[X]$ directly encodes the superpotential and determines the spectrum of short-multiplet

Questions/Extensions

- same formalism for M-theory AdS_5 – what is $[X]$?
- similar formalism for M-theory AdS_4 – solve for deformations of $d = 7$ Sasaki–Einstein
- cohomology gives supersymmetric index
- full solution? ...