Introduction to Exceptional Field Theories

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Geometry and Duality — AEI Potsdam

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Lab PHYS
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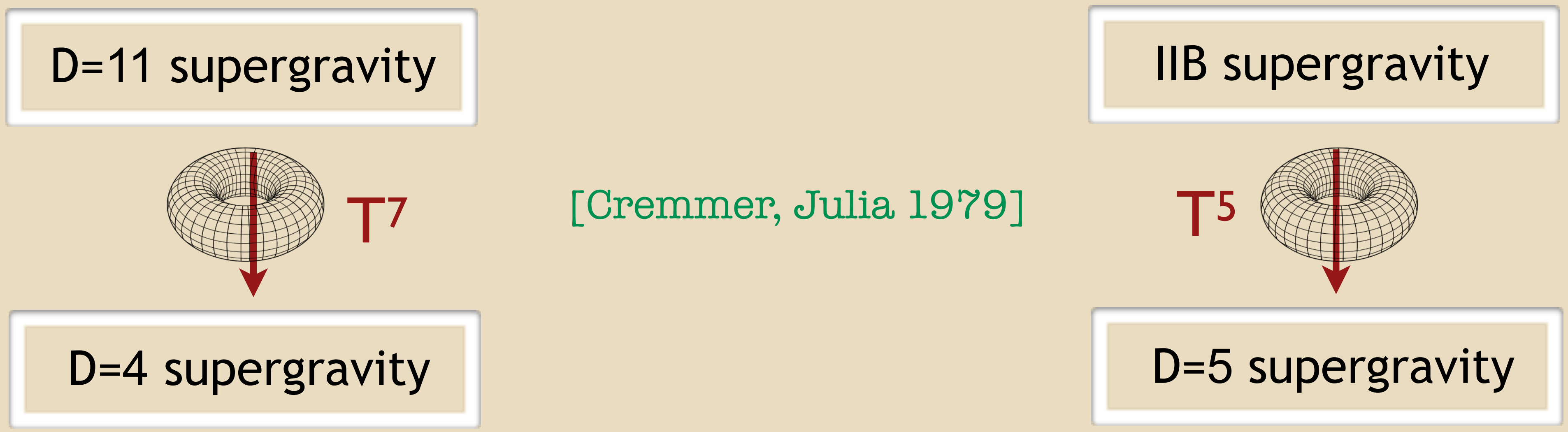
duality symmetries in supergravity

- upon toroidal reduction on T^d, eleven-dimensional supergravity exhibits the global exceptional symmetry group $E_{d(d)}$



maximal supersymmetry, global $E_{7(7)}$

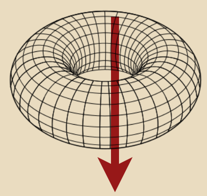
maximal supersymmetry, global $E_{6(6)}$

after proper dualization/reorganisation of the fields

duality symmetries in supergravity

- upon toroidal reduction on T^d, eleven-dimensional supergravity exhibits the global exceptional symmetry group $E_{d(d)}$

D=11 supergravity



T^7

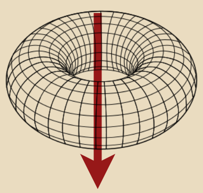
D=4 supergravity

maximal supersymmetry, global $E_{7(7)}$

[Cremmer, Julia 1979]

IIB supergravity

T^5



D=5 supergravity

maximal supersymmetry, global $E_{6(6)}$

after proper dualization/reorganisation of the fields

- the GL(d) subgroups can be explained geometrically

- the compact subgroup $SU(8) \subset E_{7(7)}$ can be made visible already in eleven dimensions [de Wit, Nicolai 1986]

$$SO(1,10) \longrightarrow SO(1,3) \times SO(7) \longrightarrow SO(1,3) \times SU(8)$$

to which extent are (remnants of) the full exceptional groups present in D=11 ?

exceptional field theory

to which extent are (remnants of) the full exceptional groups present in D=11 ?

[Hull, Tseytlin, Duff, Siegel, Hillmann, Hohm, Zwiebach, Berman, Godazgar, Godazgar, Perry, West, Musaev, Coimbra, Strickland-Constable, Waldram, Pacheco, Kwak, Jeon, Lee, Park, Suh, Blair, Malek, Cederwall, Kleinschmidt, Thompson, Edlund, Karlsson, Aldazabal, Graña, Marqués, Rosabal, Geissbühler, ... , ... , ...]

double field theory

generalized geometry

exceptional geometry

gauged supergravity

$\longrightarrow$ exceptional field theory

plan

exceptional field theory

- exceptional geometry & tensor hierarchy
- invariant action functionals

applications

- consistent truncations and AdS vacua
- Kaluza-Klein spectroscopy

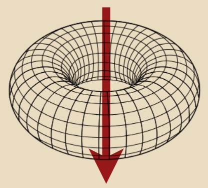
based on work with Olaf Hohm, Emanuel Malek,
Arnaud Baguet, Hadi Godazgar, Mahdi Godazgar, Hermann Nicolai,
Gianluca Inverso, Marc Magro, Edvard Musaev, Mario Trigiante,
Guillaume Bossard, Martin Cederwall, Franz Ciceri, Axel Kleinschmidt,
Jakob Palmkvist, Dan Butter, Ergin Sezgin

example: $E_{6(6)}$ exceptional field theory (ExFT)

based on the exceptional symmetry group $E_{6(6)}$
of D=5 maximal supergravity

IIB supergravity

T^5



D=5 supergravity

maximal supersymmetry, global $E_{6(6)}$

D=5 maximal supergravity

after reduction of D=11 supergravity on T^6 and proper dualization of the dof's,
the D=5 bosonic Lagrangian takes the $E_{6(6)}$ invariant form

$$\mathcal{L} \;=\; R + \frac{1}{24}\,\partial_\mu \mathcal{M}_{MN} \partial^\mu \mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}_{MN}\, F_{\mu\nu}{}^{M} F^{\mu\nu\,N} \;+ e^{-1}\,\mathcal{L}_{\rm top}$$

[Cremmer, 1980]

$g_{\mu\nu}$: 5 x 5 external metric

$\mathcal{M}_{MN}$: 27 x 27 internal metric (scalars), parametrizing the coset $E_{6(6)}/USp(8)$

$A_\mu{}^M$: 27 vector fields $\longleftrightarrow$ 27 two-form fields $B_{\mu\nu\,M}$

with $\mathcal{L}_{\rm top} \;=\; d_{KMN}\, F^M \wedge F^N \wedge A^K$

D=5 maximal supergravity

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exceptional field theory:

— same Kaluza-Klein reorganisation of the higher-dimensional fields

— keeping the dependence on all internal coordinates

$E_{6(6)}$ exceptional field theory

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$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu}\mathcal{D}_\mu \mathcal{M}^{MN}\mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4}\mathcal{M}_{MN}\mathcal{F}^{\mu\nu M}\mathcal{F}^N_{\mu\nu} + e^{-1}\mathcal{L}_{\rm top} - V(\mathcal{M}, e).$$

[Hohm, HS]

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[Hohm, HS]

- non-abelian gauge structure:
  - internal coordinate dependence gives rise to an (infinite-dimensional) non-abelian gauge structure: the internal diffeomorphisms
  - known from Kaluza-Klein theory
  - here: also p-form gauge symmetries and dualization

$E_{6(6)}$ exceptional field theory

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[Hohm, HS]

- non-abelian gauge structure: generalized diffeomorphisms

$$\mathcal{D}_\mu \;=\; \partial_\mu - \mathcal{L}_{\mathcal{A}_\mu} \qquad\qquad \mathcal{L}_\Lambda V^M \;=\; \Lambda^N \partial_N V^M - \kappa \Big[\partial_N \Lambda^M\Big]_{\rm adj} V^N$$

[Coimbra, Strickland-Constable, Waldram]

> combining into a single vector parameter $\Lambda^M \;\in\; \mathbf{27}$

$$\Lambda^M \;=\; \begin{cases} \Lambda^m & \text{internal diffeomorphisms} \\ \Lambda_{mn} & \text{internal 3-form gauge transformations} \\ \Lambda_{klmnp} & \text{internal 6-form gauge transformations} \end{cases}$$

> by construction $E_{6(6)}$ covariant: $\mathcal{M}^{-1}\mathcal{L}_\Lambda \mathcal{M} \;\in\; \mathfrak{e}_{6(6)} \qquad \mathcal{L}_\Lambda d^{KMN} \;=\; 0$

$E_{6(6)}$ exceptional field theory

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[Coimbra, Strickland-Constable, Waldram]

- embedding $\partial_m \longrightarrow \partial_M$ subject to the section constraint

$$d^{KMN}\,\partial_M \otimes \partial_N \;=\; 0 \qquad \left\{ \begin{array}{rcl} d^{KMN}\,\partial_M \partial_N f &=& 0 \\ d^{KMN}\,\partial_M f\,\partial_N g &=& 0 \end{array} \right.$$

[Berman, Perry, Cederwall, Kleinschmidt, Thompson]

covariant restriction down to 6 coordinates, breaking $E_{6(6)}$ to GL(6)

$$\text{IID:} \quad \partial_M \;\rightarrow\; \{\partial_m, \cancel{\partial^{mn}}, \cancel{\partial^{mnpqr}}\}$$

$$\mathbf{27} \longrightarrow 6 + 15 + 6$$

$E_{6(6)}$ exceptional field theory

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[Berman, Perry, Cederwall, Kleinschmidt, Thompson]

covariant restriction down to 6 (5) coordinates, breaking $E_{6(6)}$ to GL(6) (GL(5) x SL(2))

IID: $\partial_M \;\rightarrow\; \{\partial_m, \cancel{\partial^{mn}}, \cancel{\partial^{mnpqr}}\}$

IIB: $\partial_M \;\rightarrow\; \{\partial_i, \cancel{\partial^{ijk}}, \cancel{\partial^{\alpha}}, \cancel{\partial^{i\alpha}}\}$

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[Coimbra, Strickland-Constable, Waldram]

- non-associative gauge algebra $\longrightarrow$ modified YM field strengths

$$\mathcal{F}_{\mu\nu}{}^{M} \;=\; 2\,\partial_{[\mu}\mathcal{A}_{\nu]}{}^{M} - [\mathcal{A}_\mu, \mathcal{A}_\nu]_{\rm E}^{M} + 10\, d^{MNK}\,\partial_K\,\mathcal{B}_{\mu\nu\,N}$$

with $\mathbf{27}$ two-forms $\mathcal{B}_{\mu\nu\,M}$

and topological term $d\mathcal{L}_{\rm top} \;=\; d_{KMN}\,\mathcal{F}^K \wedge \mathcal{F}^M \wedge \mathcal{F}^N - 40\, d^{KMN}\,\mathcal{H}_K \wedge \partial_M \mathcal{H}_N$

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[Hohm, HS]

- “potential”

$$\begin{aligned} V(\mathcal{M},e) \;=\; & \frac{1}{24}\,\mathcal{M}^{MN}\partial_M \mathcal{M}^{KL}\,(12\,\partial_L \mathcal{M}_{NK} - \partial_N \mathcal{M}_{KL}) \\ & -\frac{1}{2}\,g^{-1}\partial_M g\,\partial_N \mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}^{MN} g^{-1}\partial_M g\,g^{-1}\partial_N g - \frac{1}{4}\mathcal{M}^{MN}\partial_M g^{\mu\nu}\,\partial_N g_{\mu\nu} \end{aligned}$$

— invariant under generalized diffeomorphisms

— generalized (internal) curvature scalar

$E_{6(6)}$ exceptional field theory

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- unique two-derivative action with generalized diffeomorphism invariance
  - modulo section constraints
  - internal Λ^M & external ξ^μ diffeomorphisms
  - uniquely fixed by bosonic symmetries (but can be supersymmetrized)

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- section constraint admits two inequivalent solutions $d^{KMN}\, \partial_M \otimes \partial_N = 0$

IID: $\partial_M \to \{\partial_m, \cancel{\partial^{mn}}, \cancel{\partial^{mnpqr}}\}$

IIB: $\partial_M \to \{\partial_i, \cancel{\partial^{ijk}}, \cancel{\partial^{\alpha}}, \cancel{\partial^{i\alpha}}\}$

together with proper dictionary of ExFT fields into IID/IIB supergravity

$$\mathscr{M}_{11\text{D}} = \begin{pmatrix} \mathcal{M}_{kn} & \mathcal{M}_k{}^{mn} & \mathcal{M}_k{}^{mnpqr} \\ \mathcal{M}^{kl}{}_n & \mathcal{M}^{kl,mn} & \mathcal{M}^{kl,mnpqr} \\ \mathcal{M}^{hijkl}{}_n & \mathcal{M}^{hijkl,mn} & \mathcal{M}^{hijkl,mnpqr} \end{pmatrix} \qquad \mathscr{M}_{\text{IIB}} = \begin{pmatrix} \mathcal{M}_{im} & \mathcal{M}_{i\alpha}{}^{mnp} & \mathcal{M}_{i\alpha}{}^{\beta} & \mathcal{M}_i{}^{m\beta} \\ \mathcal{M}^{ijk}{}_m & \mathcal{M}^{ijk,mnp} & \mathcal{M}^{ijk,\beta} & \mathcal{M}^{ijk,m\beta} \\ \mathcal{M}^{\alpha}{}_m & \mathcal{M}^{\alpha,mnp} & \mathcal{M}^{\alpha\beta} & \mathcal{M}^{\alpha,m\beta} \\ \mathcal{M}^{i}{}_m & \mathcal{M}^{i,mnp} & \mathcal{M}^{i,\beta} & \mathcal{M}^{i,m\beta} \end{pmatrix}$$

the ExFT equations of motion reproduce full IID/IIB supergravity

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$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^{N}_{\mu\nu} + e^{-1} \mathcal{L}_{\mathrm{top}} - V(\mathcal{M}, e).$$

dictionary

dictionary

D=11 sugra

IIB sugra

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Exceptional field theories

- similar construction can be done for all finite-dimensional duality groups

  group G , representation $\mathscr{R}_{\rm v} \ni \Lambda^M$

  closure of generalized diffeomorphisms $\mathcal{L}_\Lambda V^M = \Lambda^N \partial_N V^M - \kappa\,(\partial_L \Lambda^K)(t^\alpha)_K{}^L (t_\alpha)_N{}^M\, V^N$

  $\longrightarrow$ section constraints $\mathbb{Y}_{KL}{}^{MN}\, \partial_M \otimes \partial_N \equiv 0$

  - define invariant action functionals

$$V = \gamma^{1/2}\Big(\frac{1}{12}M^{MN}(\partial_M M^{KL})(\partial_N M_{KL}) - \frac{1}{2}M^{MN}(\partial_N M^{KL})(\partial_L M_{MK}) + \frac{1}{12}M^{MN}(M^{KL}\partial_M M_{KL})(M^{RS}\partial_N M_{RS}) + \frac{1}{4}M^{MN}M^{PQ}(M^{RS}\partial_P M_{RS})(\partial_M M_{NQ})\Big)$$

[Berman, Godazgar, Perry, West]
[Coimbra, Strickland-Constable, Waldram]

  - extend to external coordinates, tensor hierarchy $\longrightarrow$ ExFT (full sugra)

$$\mathcal{L} = \widehat{R} + \frac{1}{24}\,\mathcal{D}_\mu \mathcal{M}_{MN}\,\mathcal{D}^\mu \mathcal{M}^{MN} - \frac{1}{4}\,\mathcal{M}_{MN}\,\mathcal{F}_{\mu\nu}{}^M \mathcal{F}^{\mu\nu N} + \mathcal{L}_{\rm top} - V_{\rm pot}(\mathcal{M}_{MN}, g_{\mu\nu})$$

[Hohm, H.S.]
[Hohm, Wang][Abzalov, Bakhmatov, Musaev]
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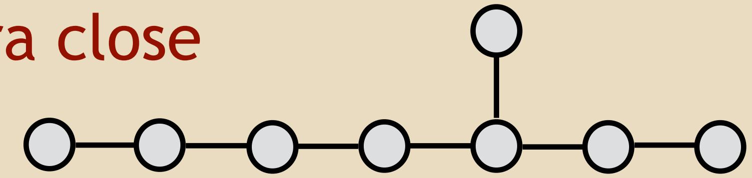
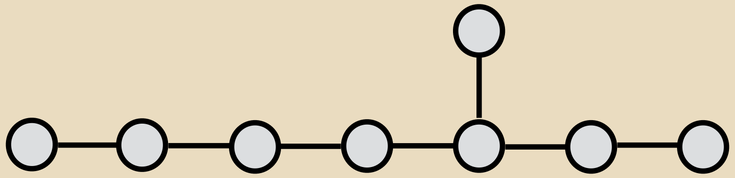
[Hohm, H.S.]
[Hohm, Wang][Abzalov, Bakhmatov, Musaev]
[Berman, Blair, Malek, Rudolph]

- works perfectly for the groups $\mathrm{GL}(2), \mathrm{SL}(2)\times\mathrm{SL}(3), \mathrm{SL}(5), \mathrm{SO}(5{,}5)$, $\mathrm{E}_{6(6)}$, $\mathrm{E}_{7(7)}$

  but closure fails for $\mathrm{E}_{8(8)}$! (the D=3 U-duality group)
  no choice of section constraints makes the algebra close



$E_{8(8)}$ exceptional field theory



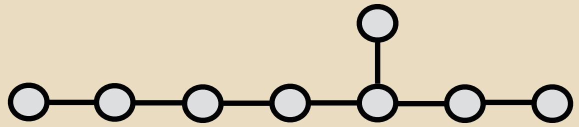
- despite non-closure of the algebra, one can construct invariant action functionals!

$$
\begin{aligned}
V(\mathcal{M},g) \;=\; & -\frac{1}{240}\mathcal{M}^{MN}\partial_M\mathcal{M}^{KL}\,\partial_N\mathcal{M}_{KL} + \frac{1}{2}\mathcal{M}^{MN}\partial_M\mathcal{M}^{KL}\partial_L\mathcal{M}_{NK} \\
& + \frac{1}{7200}\,f^{NQ}{}_P f^{MS}{}_R\,\mathcal{M}^{PK}\partial_M\mathcal{M}_{QK}\mathcal{M}^{RL}\partial_N\mathcal{M}_{SL} \\
& - \frac{1}{2}g^{-1}\partial_M g\,\partial_N\mathcal{M}^{MN} - \frac{1}{4}\mathcal{M}^{MN}g^{-1}\partial_M g\,g^{-1}\partial_N g - \frac{1}{4}\mathcal{M}^{MN}\partial_M g^{\mu\nu}\partial_N g_{\mu\nu}
\end{aligned}
$$

[Hohm, H.S.]

$\longrightarrow$ $E_{8(8)}$ generalized diffeomorphisms close into additional symmetries ..!

$E_{8(8)}$ exceptional field theory



- generalized Lie derivative for $E_{8(8)}$ $\qquad \Lambda^M \in \mathcal{R}_{\mathbf{248}}$

$$\mathcal{L}_\Lambda V^M \;=\; \Lambda^N\partial_N V^M - \kappa\,(\partial_L\Lambda^K)(\mathrm{t}^\alpha)_K{}^L(\mathrm{t}_\alpha)_N{}^M\,V^N$$

$$\mathcal{L}_\Sigma V^M \;=\; -\,\Sigma_K\,f^{KM}{}_N\,V^N$$

additional symmetries (constrained rotations)

with parameter Σ_M constrained analogous to the section constraints

$$\mathbb{Y}_{KL}{}^{MN}\,\Sigma_M\,\Sigma_N \;\equiv\; 0 \qquad\qquad \mathbb{Y}_{KL}{}^{MN}\,\Sigma_M\,\partial_N \;\equiv\; 0$$

- the full algebra closes !

$$\left[\,\delta_{(\Lambda_1,\Sigma_1)},\,\delta_{(\Lambda_2,\Sigma_2)}\,\right] \;=\; \delta_{(\Lambda_{12},\Sigma_{12})}$$

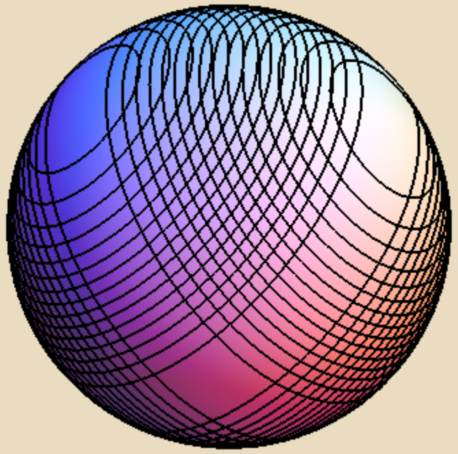
$$\Sigma_{12\,M} \;\equiv\; -\,2\,\Sigma_{[2\,M}\partial_N\Lambda_{1]}^N + 2\,\Lambda_{[2}^N\partial_N\Sigma_{1]\,M} - 2\,\Sigma_{[2}^N\partial_M\Lambda_{1]\,N} + f^N{}_{KL}\,\Lambda_{[2}^K\,\partial_M\partial_N\Lambda_{1]}^L$$

- ExFT requires additional gauge connection $B_{\mu\,M}$

$$D_\mu V^M \;=\; \partial_\mu V^M - A_\mu{}^K\partial_K V^M + 60\,\mathbb{P}^M{}_N{}^K{}_L\,\partial_K A_\mu{}^L\,V^N + \boxed{B_\mu{}^L f^M{}_{NL} V^N}$$

constrained, not present in the dimensionally reduced theory
related to the dual graviton ..

applications: consistent truncations — spheres



consistent truncations from ExFT

manifestly duality covariant formulation of maximal supergravity

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

dictionary

dictionary

D=11 sugra

IIB sugra

consistent truncations from ExFT

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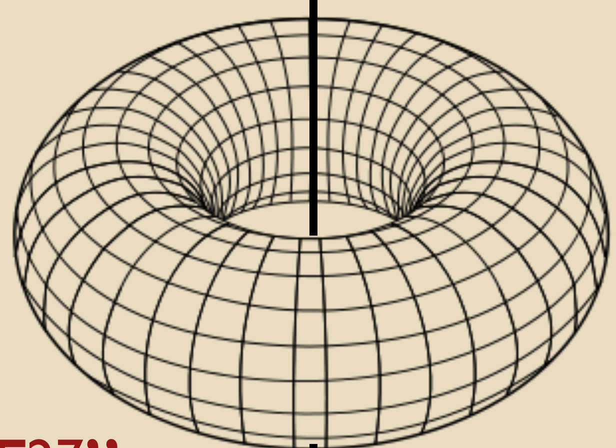
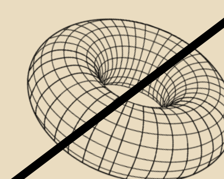


“T^{27}”

IIB sugra



T^5

D=5 maximal sugra

global $E_{6(6)}$

‘explains’ supergravity’s exceptional global symmetry groups

consistent truncations from ExFT

manifestly duality covariant formulation of maximal supergravity

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

dictionary

dictionary

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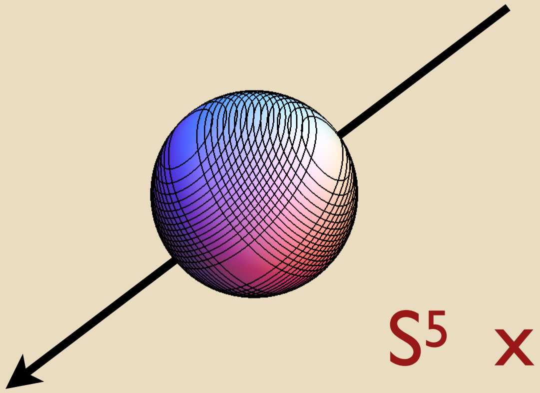


$S^5 \times AdS_5$

D=5 gauged sugra

non-abelian gauge group

also allows a compact description of non-linear reductions

consistent truncations from ExFT

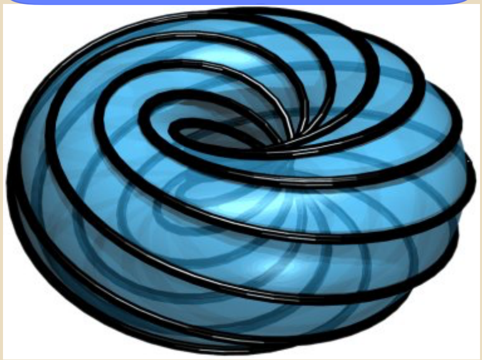
manifestly duality covariant formulation of maximal supergravity

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

dictionary

dictionary

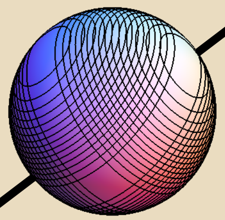
GSS



D=11 sugra

IIB sugra

captured by a twisted torus
generalized Scherk-Schwarz reduction



S^5 x AdS_5

[Kaloper, Myers, Dabholkar, Hull, Reid-Edwards, Dall'Agata, Prezas, HS, Trigiante, Hohm, Kwak, Aldazabal, Baron, Nunez, Marqués, Geissbühler, Graña, Berman, Musaev, Thompson, Rosabal, Lee, Strickland-Constable, Waldram, Dibitetto, Roest, Malek, Blumenhagen, Hassler, Lust, Cho, Fernández-Melgarejo, Jeon, Park, Guarino, Varela, Inverso, Ciceri, ...]

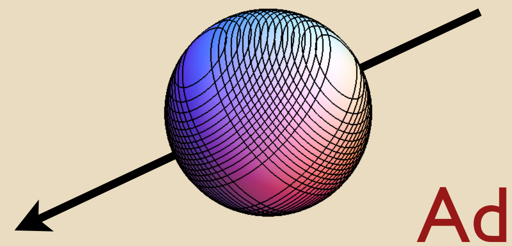
D=5 gauged sugra
non-abelian gauge group

also allows a compact description of non-linear reductions

consistent truncations from ExFT

- AdS_5 x S^5 background of IIB
  - KK towers of linearized fluctuations

IIB sugra



AdS_5 x S^5

D=5 maximal sugra
gauge group SO(6)

[Gunaydin,Romans,Warner]

- AdS_5 x S^5 : lowest KK-multiplet $\longrightarrow$ D=5 maximal supergravity
  - non-linear embedding in IIB such that any D=5 solution defines a IIB solution

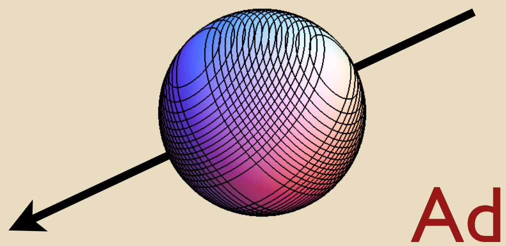
$$
\begin{aligned}
ds^2 \;&=\; \Delta^{-2/3}(x,y)\, \mathrm{g}_{\mu\nu}(x)\, dx^\mu dx^\nu + G_{mn}(x,y)\left(dy^m + \mathcal{K}_{[ab]}{}^m(y) A_\mu^{ab}(x) dx^\mu\right)\left(dy^n + \mathcal{K}_{[cd]}{}^n(y) A_\nu^{cd}(x) dx^\nu\right) \\
G^{mn}(x,y) &= \Delta^{2/3}(x,y)\, \mathcal{K}_{[ab]}{}^m(y) \mathcal{K}_{[cd]}{}^n(y)\, M^{ab,cd}(x)
\end{aligned}
$$

etc.

consistent truncations from ExFT

- AdS$_5$ x S^5 background of IIB
  - KK towers of linearized fluctuations

IIB sugra



AdS$_5$ x S^5

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$$ds^2 = \Delta^{-2/3}(x,y)\, g_{\mu\nu}(x)\, dx^\mu dx^\nu + G_{mn}(x,y)\left(dy^m + \mathcal{K}_{[ab]}{}^m(y) A_\mu^{ab}(x) dx^\mu\right)\left(dy^n + \mathcal{K}_{[cd]}{}^n(y) A_\nu^{cd}(x) dx^\nu\right)$$

$$G^{mn}(x,y) = \Delta^{2/3}(x,y)\, \mathcal{K}_{[ab]}{}^m(y) \mathcal{K}_{[cd]}{}^n(y)\, M^{ab,cd}(x) \qquad \text{etc.}$$

- construction of IIB solutions
- holography: trust D=5 supergravity calculations
- used to be scarce (only few examples until recently)

AdS$_4$ x S^7 : [de Wit, Nicolai] 1987

AdS$_7$ x S^4 : [Nastase, van Nieuwenhuizen, Vaman] 1999

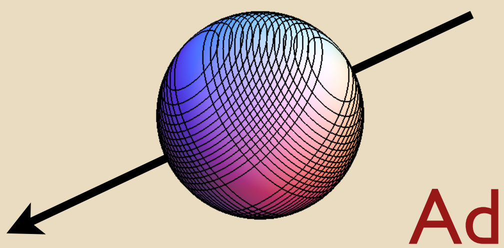
consistent truncations from ExFT

ExFT

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

dictionary

IIB sugra



$AdS_5 \times S^5$

D=5 maximal sugra

gauge group SO(6)

consistent truncations from ExFT

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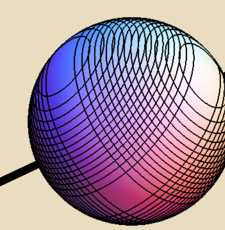
dictionary

captured by a
generalized Scherk-Schwarz
reduction of ExFT

GSS
$U(Y) \in E_{6(6)}$

IIB sugra

$$\begin{aligned}
\mathcal{M}_{MN}(x,Y) &= U_M{}^K(Y)\, M_{KL}(x)\, U_N{}^L(Y) \\
\mathcal{A}_\mu{}^M(x,Y) &= \rho^{-1}(Y)\, (U^{-1})_K{}^M(Y)\, A_\mu{}^K(x) \\
\mathcal{B}_{\mu\nu\, M}(x,Y) &= \rho^{-2}(Y)\, U_M{}^K(Y)\, B_{\mu\nu\, K}(x)
\end{aligned}$$



$AdS_5 \times S^5$

D=5 maximal sugra
gauge group SO(6)

consistent truncations from ExFT

ExFT

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

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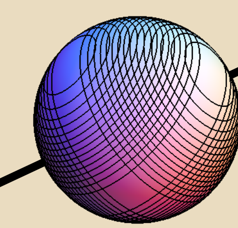
GSS
$U(Y) \in E_{6(6)}$

IIB sugra



$AdS_5 \times S^5$

D=5 maximal sugra
gauge group SO(6)

in terms of an $E_{6(6)}$–valued twist matrix $U_M{}^N(Y)$ and scale factor $\rho(Y)$

- system of consistency equations $\left[(U^{-1})_M{}^P (U^{-1})_N{}^L\, \partial_P U_L{}^K\right]_{\mathbf{351}} \stackrel{!}{=} \rho\, X_{MN}{}^K$
- generalized (Leibniz) parallelizability $\mathcal{L}_{\mathcal{U}_M} \mathcal{U}_N = X_{MN}{}^K\, \mathcal{U}_K$
- no general classification of its solutions (Lie algebras vs Leibniz algebras)

consistent truncations from ExFT

ExFT

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^{N}_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

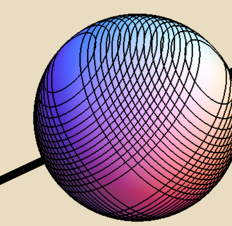
dictionary

IIB sugra

GSS

$U(Y) \in E_{6(6)}$

$$\begin{aligned} \mathcal{M}_{MN}(x,Y) &= U_M{}^K(Y)\, M_{KL}(x)\, U_N{}^L(Y) \\ \mathcal{A}_\mu{}^M(x,Y) &= \rho^{-1}(Y)\, (U^{-1})_K{}^M(Y)\, A_\mu{}^K(x) \\ \mathcal{B}_{\mu\nu\, M}(x,Y) &= \rho^{-2}(Y)\, U_M{}^K(Y)\, B_{\mu\nu\, K}(x) \end{aligned}$$



AdS_5 x S^5

D=5 maximal sugra

gauge group SO(6)

in terms of an $E_{6(6)}$—valued twist matrix $U_M{}^N(Y)$ and scale factor $\rho(Y)$

- twist matrix for AdS_5 x S^5

$$U = \begin{pmatrix} g^{-1/2} \partial_i \mathcal{Y}^A \\ \mathcal{Y}^A - 2\,\zeta^i \partial_i \mathcal{Y}^A \end{pmatrix} \in \mathrm{SL}(6) \subset \mathrm{E}_{6(6)}$$

built from sphere harmonics & volume form

AdS$_5$ x S^5

- e.g. metric (standard Kaluza-Klein form)

$$ds^2 = \Delta^{-2/3}(x,y)\, \mathrm{g}_{\mu\nu}(x)\, dx^\mu dx^\nu + G_{mn}(x,y)\left(dy^m + \mathcal{K}_{[ab]}{}^m(y) A_\mu^{ab}(x) dx^\mu\right)\left(dy^n + \mathcal{K}_{[cd]}{}^n(y) A_\nu^{cd}(x) dx^\nu\right)$$

$$G^{mn}(x,y) = \Delta^{2/3}(x,y)\, \mathcal{K}_{[ab]}{}^m(y)\mathcal{K}_{[cd]}{}^n(y)\, M^{ab,cd}(x)$$

- e.g. 4-form (after reconstructing all components, in Kaluza-Klein basis)

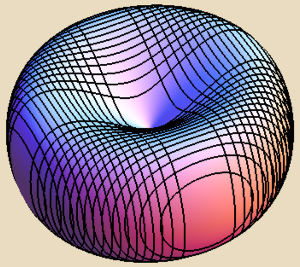
$$\begin{aligned}
C_{klmn} &= \tilde{C}_{klmn} + \frac{1}{16}\tilde{\omega}_{klmnp}\Delta^{4/3} m_{\alpha\beta}\tilde{G}^{pq}\partial_q(\Delta^{-4/3}m^{\alpha\beta}),\\
C_{\mu kmn} &= \frac{\sqrt{2}}{4}\mathcal{Z}_{[ab]kmn}A_\mu{}^{ab},\\
C_{\mu\nu mn} &= \frac{\sqrt{2}}{4}\mathcal{K}_{[ab]}{}^k\mathcal{Z}_{[cd]kmn}A_{[\mu}{}^{ab}A_{\nu]}{}^{cd},\\
C_{m\mu\nu\rho} &= -\frac{1}{32}\mathcal{K}_{[ab]m}\left(2\sqrt{|\mathrm{g}|}\varepsilon_{\mu\nu\rho\sigma\tau}M_{ab,N}F^{\sigma\tau N} + \sqrt{2}\varepsilon_{abcdef}\Omega^{cdef}_{\mu\nu\rho}\right) - \frac{1}{4}\sqrt{2}\mathcal{K}_{[ab]}{}^k\mathcal{K}_{[cd]}{}^l\mathcal{Z}_{[ef]mkl}(A_{[\mu}{}^{ab}A_\nu{}^{cd}A_{\rho]}{}^{ef}),\\
C_{\mu\nu\rho\sigma} &= -\frac{1}{16}\mathcal{Y}_a\mathcal{Y}^b\left(\sqrt{|\mathrm{g}|}\varepsilon_{\mu\nu\rho\sigma\tau}D^\tau M_{bc,N}M^{Nca} + 2\sqrt{2}\varepsilon_{cdefgb}F_{[\mu\nu}{}^{cd}A_\rho{}^{ef}A_{\sigma]}{}^{ga}\right)\\
&\quad + \frac{1}{4}\left(\sqrt{2}\mathcal{K}_{[ab]}{}^k\mathcal{K}_{[cd]}{}^l\mathcal{K}_{[ef]}{}^n\mathcal{Z}_{[gh]kln} - \mathcal{Y}_h\mathcal{Y}^j\varepsilon_{abcegj}\eta_{df}\right)A_{[\mu}{}^{ab}A_\nu{}^{cd}A_\rho{}^{ef}A_{\sigma]}{}^{gh} + \Lambda_{\mu\nu\rho\sigma}(x).
\end{aligned}$$

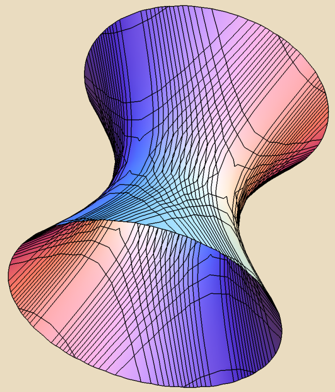
$$\begin{aligned}
D_{[\mu}\Lambda_{\nu\rho\sigma\tau]} = &-\frac{1}{80}\mathcal{Y}_a\mathcal{Y}^b\sqrt{|\mathrm{g}|}\varepsilon_{\mu\nu\rho\sigma\tau}D_\lambda(M^{Nac}D^\lambda M_{bc,N})\\
&+\frac{1}{40}\mathcal{Y}_a\mathcal{Y}^b\sqrt{|\mathrm{g}|}\varepsilon_{\mu\nu\rho\sigma\tau}F^{\kappa\lambda N}\left(M_{bc,N}F_{\kappa\lambda}{}^{ac} - \frac{1}{2}\sqrt{10}\varepsilon_{\alpha\beta}\eta_{db}M^{d\alpha}{}_N B_{\kappa\lambda}{}^{a\beta}\right)\\
&+\frac{1}{100}\sqrt{|\mathrm{g}|}\varepsilon_{\mu\nu\rho\sigma\tau}\mathcal{Y}_a\mathcal{Y}^b(10M^{ac,fd} + \mathcal{X}^{(af)ec,d}{}_e)\eta_{cd}\eta_{bf}\\
&+\frac{1}{32}\sqrt{2}\varepsilon_{abcdef}F_{[\mu\nu}{}^{ab}F_{\rho\sigma}{}^{cd}A_{\tau]}{}^{ef} + \frac{1}{16}F_{[\mu\nu}{}^{ab}A_\rho{}^{cd}A_\sigma{}^{ef}A_{\tau]}{}^{gh}\varepsilon_{abcdeh}\eta_{fh}\\
&+\frac{1}{40}\sqrt{2}A_{[\mu}{}^{ab}A_\nu{}^{cd}A_\rho{}^{ef}A_\sigma{}^{gh}A_{\tau]}{}^{ij}\varepsilon_{abcegi}\eta_{df}\eta_{hj},
\end{aligned}$$

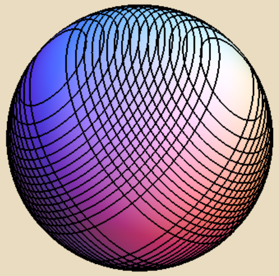
- **proves the consistent truncation of IIB on AdS$_5$ x S^5**

applications: other consistent truncations

- hyperboloids
- product manifolds







other consistent truncations: hyperboloids

ExFT

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\rm top} - V(\mathcal{M}, e).$$

GSS
$U(Y) \in E_{6(6)}$

D=5 maximal sugra
gauge group SO(6)

- similar: twist matrices $\tilde{U}, \hat{U} \;\in\; \mathrm{SL}(6)$
  associated to $\mathrm{SO}(p,q)$ and $\mathrm{CSO}(p,q,r)$

  built from sphere harmonics on $\mathrm{SO}(p,q)/\mathrm{SO}(p,q-1)$ $\quad \tilde{U} = \begin{pmatrix} g^{-1/2}\partial_i \mathcal{Y}^A \\ \mathcal{Y}^A - 2\,\zeta^i \partial_i \mathcal{Y}^A \end{pmatrix} \in \mathrm{SL}(6)$

other consistent truncations: hyperboloids

ExFT

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).$$

GSS
$\tilde{U}(Y) \in E_{6(6)}$

D=5 maximal sugra
gauge group SO(p,q)

GSS
$U(Y) \in E_{6(6)}$

D=5 maximal sugra
gauge group SO(6)

GSS
$\hat{U}(Y) \in E_{6(6)}$

D=5 maximal sugra
gauge group CSO(p,q,r)

- similar: twist matrices $\tilde{U}, \hat{U} \;\in\; \mathrm{SL}(6)$
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other consistent truncations: hyperboloids

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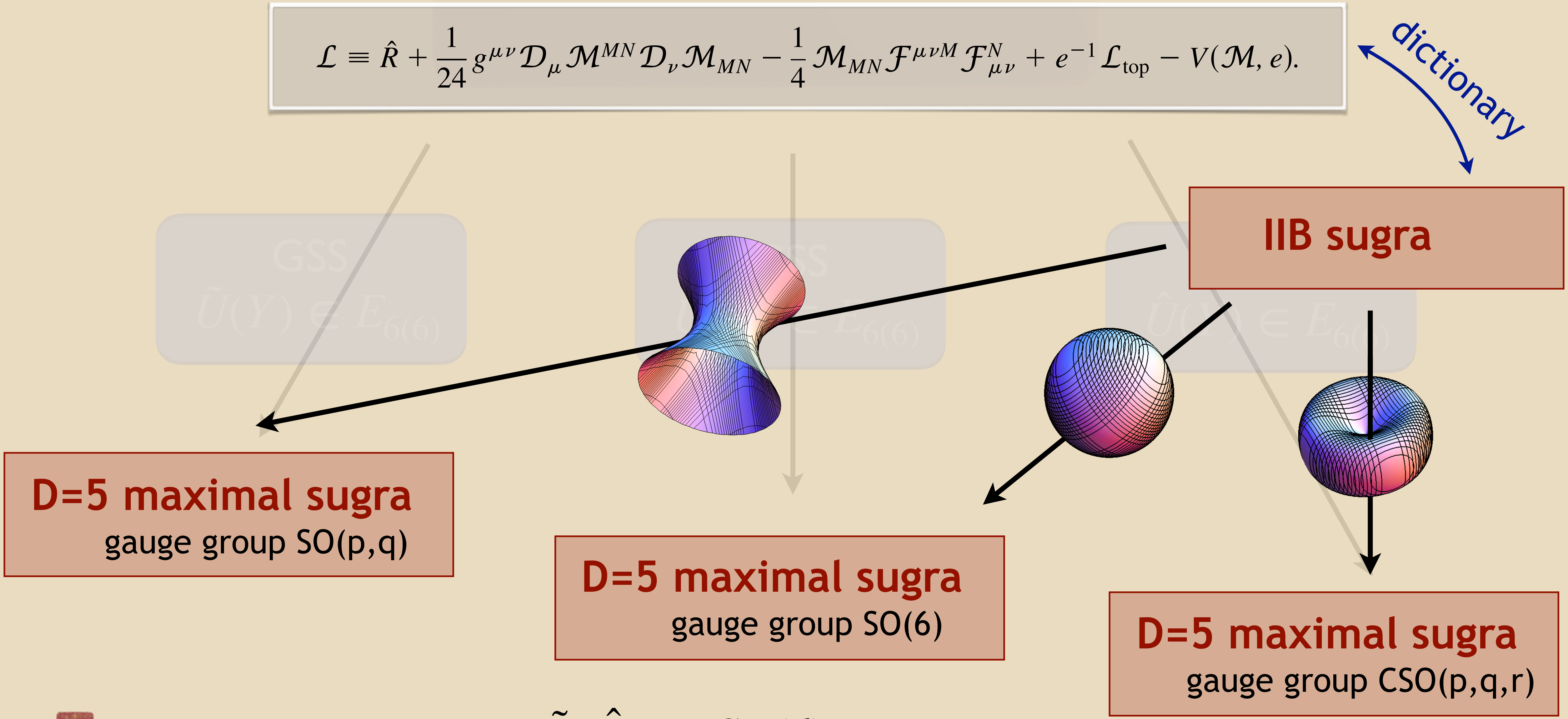

- similar: twist matrices $\tilde{U}, \hat{U} \in \mathrm{SL}(6)$ associated to $\mathrm{SO}(p,q)$ and $\mathrm{CSO}(p,q,r)$
  - background: (warped) hyperboloids [Hull, Warner] [Baron, Dall'Agata]
  - in general no IIB solutions, still consistent truncations!

other consistent truncations: product manifolds [Inverso, HS, Trigiante]

a priori generalized parallelizability is not inherited to products

$$S^p \times S^q \,, \;\; S^p \times H^q \,, \;\; \text{etc..}$$

- possible for compactifications to D=4, described within $E_{7(7)}$ ExFT

  based on electric/magnetic split of internal coordinates

  $$G = \mathrm{E}_{7(7)} \subset \mathrm{Sp}(56) \,, \quad \mathscr{R}_{\mathrm{v}} = \mathbf{56} \qquad \{Y^M\} \longrightarrow \{Y^{AB}, Y_{AB}\} \qquad A, B = 1, \ldots, 8$$

other consistent truncations: product manifolds [Inverso, HS, Trigiante]

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Scherk-Schwarz product ansatz $\boxed{U(Y) \;=\; \mathring{U}(y)\tilde{U}(\tilde{y})}$ $\qquad S^p \times S^q$

$$\mathring{U}(y) \;:\; \{y^i\} = \{Y^{i8}\} \subset \{Y^{AB}\} \qquad \tilde{U}(\tilde{y}) \;:\; \{\tilde{y}_a\} = \{Y_{a7}\} \subset \{Y_{AB}\}$$

$p+q$				
	6 :	S^6	$\{y^1, y^2, y^3, y^4, y^5, y^6\}$	IIA
	5 :	S^5 x S^1	$\{y^1, y^2, y^3, y^4, y^5, \tilde{y}_6\}$	IIB
	4 :	S^4 x S^2	$\{y^1, y^2, y^3, y^4, \tilde{y}_5, \tilde{y}_6\}$	IIA
	3 :	S^3 x S^3	$\{y^1, y^2, y^3, \tilde{y}_4, \tilde{y}_5, \tilde{y}_6\}$	IIB

induces D=4 dyonic gaugings $\quad (\mathrm{SO}(p,q) \times \mathrm{SO}(p',q')) \ltimes N \quad$ [Dall'Agata, Inverso]

other consistent truncations: product manifolds [Inverso, HS, Trigiante]

- an example: $p{+}q = 5 \quad \{y^1, y^2, y^3, y^4, y^5, \tilde{y}_6\}$

$$U(Y) = \mathring{U}(y)\tilde{U}(\tilde{y})$$

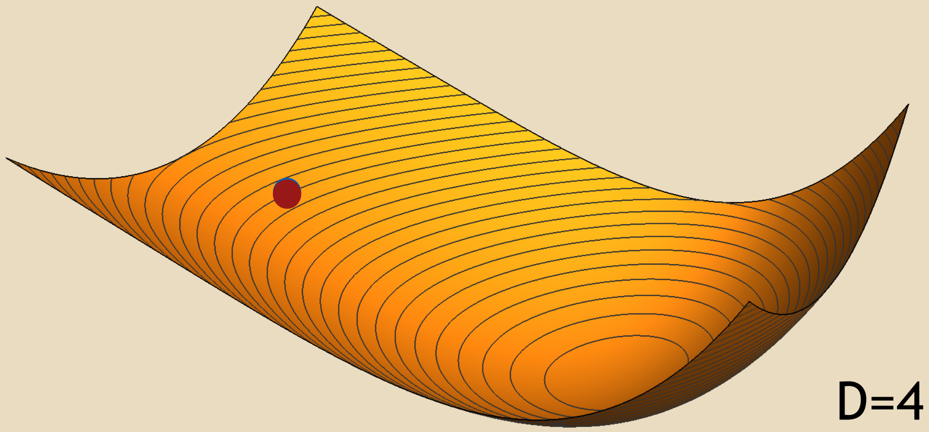
IIB sugra

$\mathrm{AdS}_4 \times S^5 \times H^1$

D=4 maximal sugra

gauge group $[\mathrm{SO}(1,1) \times \mathrm{SO}(6)] \ltimes T^{12}$

- $\mathrm{SO}(6)$: not stationary



D=4 scalar potential

other consistent truncations: product manifolds [Inverso, HS, Trigiante]

an example: $p+q=5 \quad \{y^1, y^2, y^3, y^4, y^5, \tilde{y}_6\}$

$$U(Y) = \mathring{U}(y)\tilde{U}(\tilde{y})$$

IIB sugra

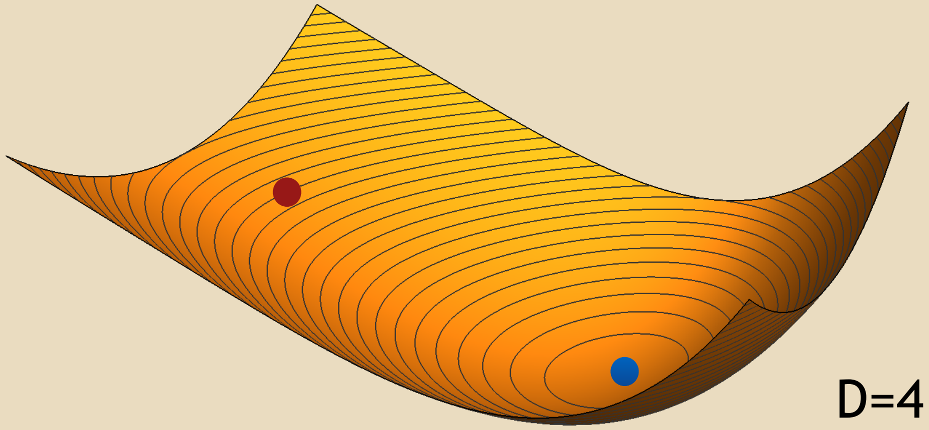
$\mathrm{AdS}_4 \times S^5 \times H^1$

D=4 maximal sugra

gauge group $[\mathrm{SO}(1,1) \times \mathrm{SO}(6)] \ltimes T^{12}$

$\mathrm{SO}(6)$: not stationary

$\mathrm{SO}(4) : \mathcal{N}=4,\ \mathrm{AdS}_4$ vacuum



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$\mathrm{AdS}_4 \times S^2 \times S^2 \times \Sigma$

D=4 maximal sugra

gauge group $[\mathrm{SO}(1,1) \times \mathrm{SO}(6)] \ltimes T^{12}$

Janus solution
[D'Hoker, Estes, Gutperle]

$$
\begin{aligned}
ds^2 \;&=\; \Delta^3 \sin^2 x\,(1+2\cos^2 x)\, d\mathcal{Y}_1^p d\mathcal{Y}_1^p \\
&\quad + \Delta^3\,(1+2\sin^2 x)\cos^2 x\, d\mathcal{Y}_2^p d\mathcal{Y}_2^p \\
&\quad + \Delta^{-1}\,(dx dx + d\eta\, d\eta) + \frac{1}{2}\,\Delta^{-1}\, ds^2_{\mathrm{AdS}_4} \\
H_3{}^+ \;&=\; 2\sqrt{2}\, 3^{-1/4}\, \omega_{S^1_1} \wedge d\left(\frac{e^{-\eta}\sin^3 x}{1+2\sin^2 x}\right)
\end{aligned}
$$

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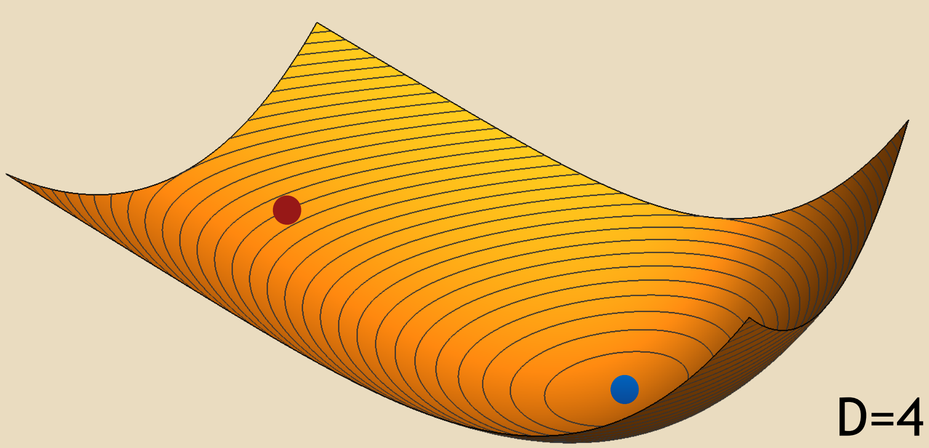
- $\mathrm{SO}(4) : \mathcal{N} = 4,\ \mathrm{AdS}_4$ vacuum



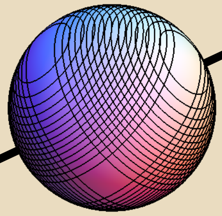
D=4 scalar potential

> with a maximally supersymmetric consistent truncation around!

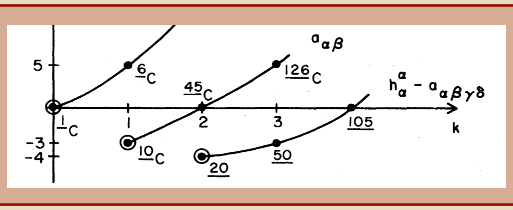
applications: Kaluza-Klein spectroscopy

[E. Malek, HS]

IIB sugra



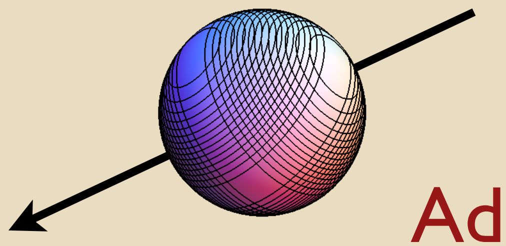
$AdS_5 \times S^5$



Kaluza-Klein spectroscopy

- $\mathrm{AdS}_5 \times S^5$ background of IIB
  - KK towers of linearized fluctuations

IIB sugra



$\mathrm{AdS}_5 \times S^5$

D=5 maximal sugra
gauge group SO(6)

[Gunaydin,Romans,Warner]

- $\mathrm{AdS}_5 \times S^5$: lowest KK-multiplet $\longrightarrow$ D=5 maximal supergravity
  - non-linear embedding in IIB such that any D=5 solution defines a IIB solution

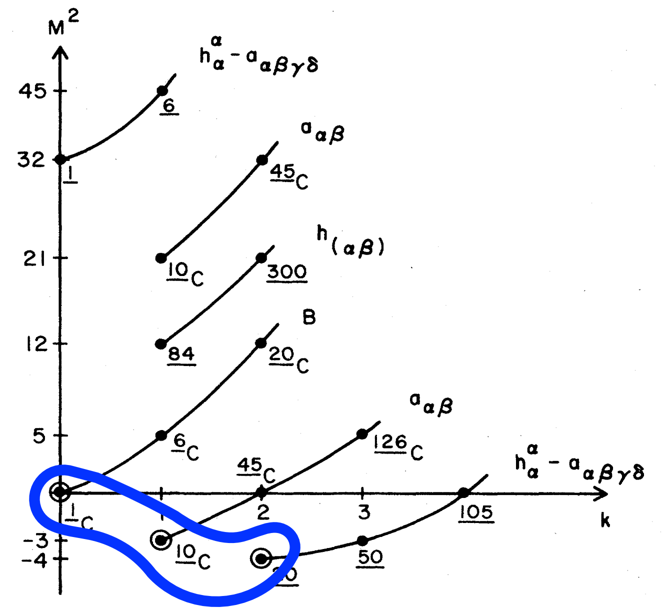


FIG. 2. Mass spectrum of scalars.

[Kim, Romans, van Nieuwenhuizen]

Kaluza-Klein spectroscopy

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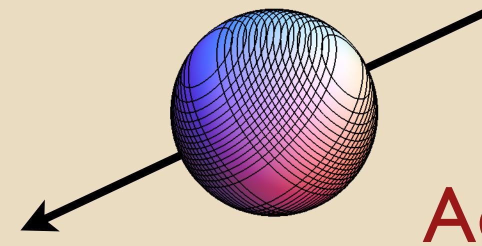

AdS₅ x S⁵

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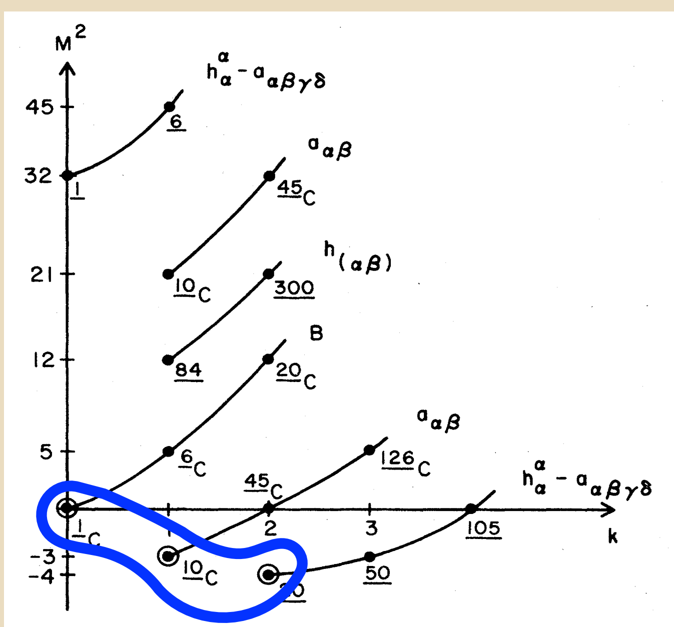


FIG. 2. Mass spectrum of scalars.

[Kim, Romans, van Nieuwenhuizen]

- in ExFT variables

$$\mathcal{M}_{MN}(x,Y) \;=\; U_M{}^K(Y)\, M_{KL}(x)\, U_N{}^L(Y)$$

$$\mathcal{A}_\mu{}^M(x,Y) \;=\; \rho^{-1}(Y)\,(U^{-1})_K{}^M(Y)\, A_\mu{}^K(x)$$

$$\mathcal{B}_{\mu\nu\,M}(x,Y) \;=\; \rho^{-2}(Y)\, U_M{}^K(Y)\, B_{\mu\nu\,K}(x)$$

- what about the higher Kaluza-Klein modes ?

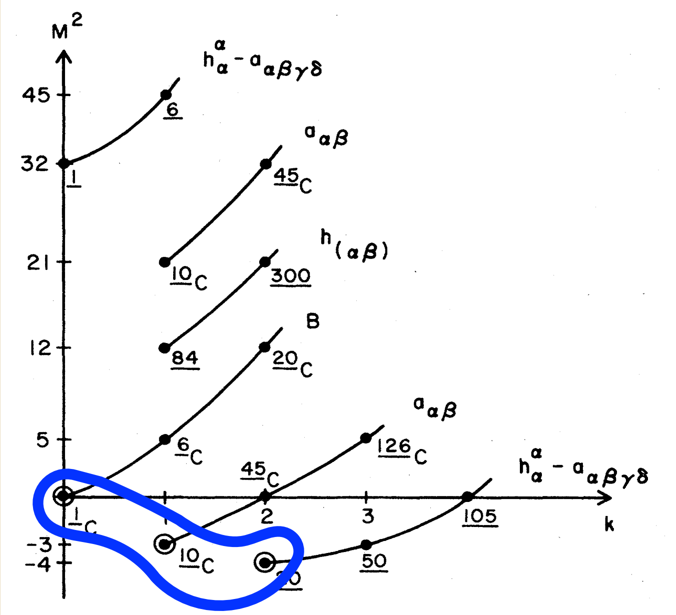
Kaluza-Klein spectroscopy

- traditionally: expand fluctuations in S^5 sphere harmonics

10D scalar: $\phi(x,y) = \sum_\Sigma \phi_\Sigma(x)\,\mathcal{Y}^\Sigma(y) = \sum_n \phi_{[n,0,0]}(x)\,\mathcal{Y}^{[n,0,0]}$ $\mathcal{Y}^{[n,0,0]} = \mathcal{Y}^{((A_1}\ldots\mathcal{Y}^{A_n))}$

10D internal metric: $g_{kl}(x,y) = \sum_n g_{[n,0,0]}(x)\,\mathcal{Y}_{kl}^{[n,0,0]} + \sum_n g_{[n,0,0]}(x)\,\mathcal{Y}_{kl}^{[n,1,1]} + \sum_n g_{[n,2,2]}(x)\,\mathcal{Y}_{kl}^{[n,2,2]}$

etc.



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etc.

- linearize & diagonalize field equations $\longrightarrow$ mass spectrum

TABLE III. Complete mass spectrum.

Spin	Field	Masses on S^5		Irred. reps.
2	$h'_{\mu\nu} = H^{I_1}_{\mu\nu} Y^{I_1}$	$M^2 = e^2 k(k+4)$	$(k \geq 0)$	1,6,20,...
1	$h_{\alpha\mu} = B^{I_5}_{\mu} Y^{I_5}_{\alpha}$	$M^2 = e^2(k-1)(k+1)$	$(k \geq 1)$	15,64,175,...
	$a_{\mu\alpha\beta\gamma} = \phi^{I_5}_{\mu} \epsilon_{\alpha\beta\gamma}{}^{\delta\epsilon} D_\delta Y^{I_5}_{\epsilon}$	$M^2 = e^2(k+3)(k+5)$	$(k \geq 1)$	15,64,175,...
0	$h^{\alpha}_{\alpha} = \pi^{I_1} Y^{I_1}$	$M^2 = e^2 k(k-4)$	$(k \geq 2)$	20,50,...
	$a_{\alpha\beta\gamma\delta} = b^{I_1} \epsilon_{\alpha\beta\gamma\delta}{}^{\epsilon} D_\epsilon Y^{I_1}$	$M^2 = e^2(k+4)(k+8)$	$(k \geq 0)$	1,6,20,...
0	$h_{(\alpha\beta)} = \phi^{I_{14}} Y^{I_{14}}_{(\alpha\beta)}$	$M^2 = e^2 k(k+4)$	$(k \geq 2)$	84,300,...
0	$B = B^{I_1} Y^{I_1}$	$M^2 = e^2 k(k+4)$	$(k \geq 0)$	$1_c, 6_c, 20_c, \ldots$
ant	$a_{\mu\nu\alpha\beta} = b^{I_{10,\pm}}_{\mu\nu} Y^{I_{10,\pm}}_{[\alpha\beta]}$	$M^2 = e^2(k+2)^2$	$(k \geq 1)$	$10_c, 45_c, \ldots$
ant	$A_{\mu\nu} = a^{I_1}_{\mu\nu} Y^{I_1}$	$M^2 = e^2 k^2$	$(k \geq 1)$	$6_c, 20_c, \ldots$
		$M^2 = e^2(k+4)^2$	$(k \geq 0)$	$1_c, 6_c, \ldots$
1	$A_{\mu\alpha} = a^{I_5}_{\mu} Y^{I_5}_{\alpha}$	$M^2 = e^2(k+1)(k+3)$	$(k \geq 1)$	$15_c, 64_c, \ldots$
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		$M^2 = e^2(k+2)(k+6)$	$(k \geq 1)$	$10_c, 45_c, \ldots$
$\frac{3}{2}$	$\psi_\mu = \psi^{I_L}_{\mu} \Xi^{I_L}$	$M = ek$	$(k \geq 0)$	4,20,...
		$M = -e(k + \frac{10}{2})$	$(k \geq 0)$	4*,20*,...
$\frac{1}{2}$	$\psi_{(\alpha)} = \psi^{I_T} \Xi^{I_T}_{\alpha}$	$M = e(k + \frac{5}{2})$	$(k \geq 0)$	36*,140*,...
		$M = -e(k + \frac{9}{2})$	$(k \geq 0)$	36,140,...
$\frac{1}{2}$	$\psi_{(\alpha)} = \psi^{I_L} D_{(\alpha)} \Xi^{I_L} + \chi \tau_\alpha \eta^{+}$	$M = e(k + \frac{11}{2})$	$(k \geq 0)$	4,20,...
		$M = -e(k - \frac{1}{2})$	$(k \geq 1)$	20*,...
$\frac{1}{2}$	$\lambda = \lambda^{I_L} \Xi^{I_L}$	$M = e(k + \frac{7}{2})$	$(k \geq 0)$	4,20,...
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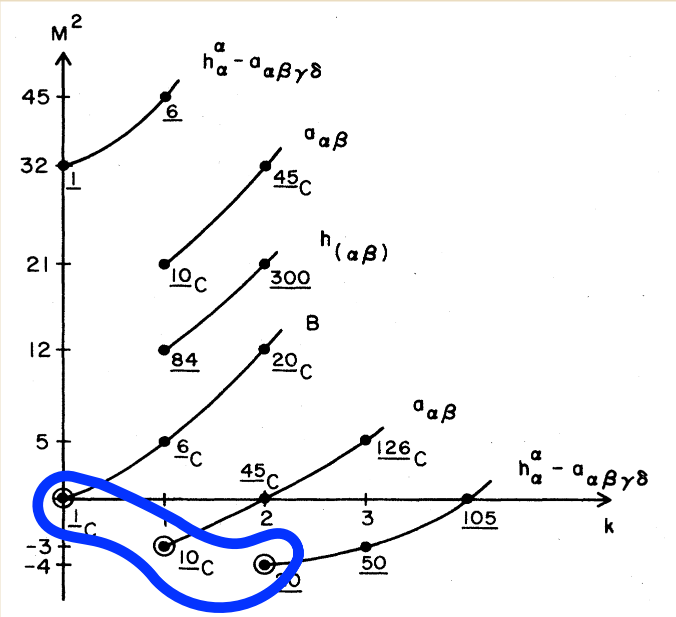

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[Kim, Romans, van Nieuwenhuizen]

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etc.

- linearize & diagonalize field equations $\longrightarrow$ mass spectrum
- combine into 1/2-BPS multiplets $\mathscr{B}_{[2,0,0](0,0)} \oplus \mathscr{B}_{[3,0,0](0,0)} \oplus \mathscr{B}_{[4,0,0](0,0)} \oplus \dots$

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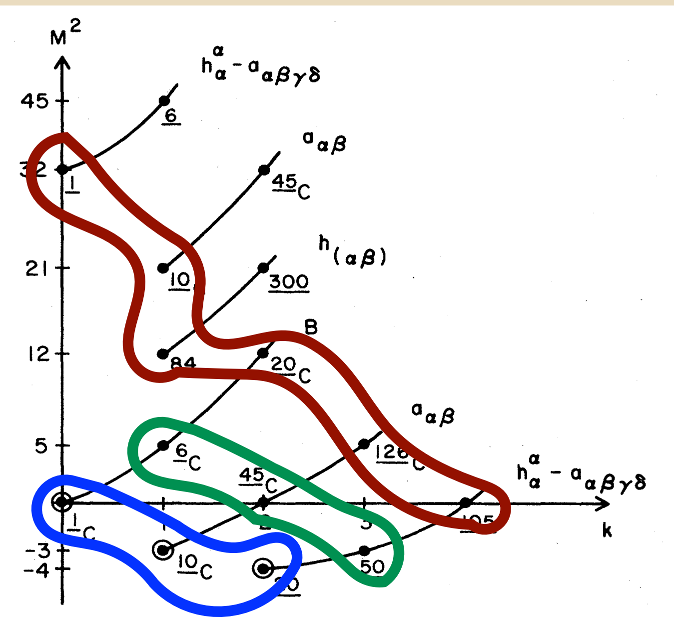

FIG. 2. Mass spectrum of scalars.

[Kim, Romans, van Nieuwenhuizen]

1/2-BPS $\mathscr{B}_{[n,0,0](0,0)}$

$[n,00](00)$
$[n-1,10](0\tfrac12)+[n-1,01](\tfrac12 0)$
$[n-2,02](00)+[n-2,20](00)+[n-1,00](01)+[n-1,00](10)+[n-2,11](\tfrac12\tfrac12)$
$[n-2,10](0\tfrac12)+[n-3,12](0\tfrac12)+[n-2,01](\tfrac12 0)+[n-3,21](\tfrac12 0)+[n-2,01](\tfrac12 1)+[n-2,10](1\tfrac12)$
$2[n-2,00](00)+[n-4,22](00)+[n-3,02](01)+[n-3,20](10)+2[n-3,11](\tfrac12\tfrac12)+[n-2,00](11)$
$[n-3,10](0\tfrac12)+[n-4,12](0\tfrac12)+[n-3,01](\tfrac12 0)+[n-4,21](\tfrac12 0)+[n-3,01](\tfrac12 1)+[n-3,10](1\tfrac12)$
$[n-4,02](00)+[n-4,20](00)+[n-3,00](01)+[n-3,00](10)+[n-4,11](\tfrac12\tfrac12)$
$[n-4,10](0\tfrac12)+[n-4,01](\tfrac12 0)$
$[n-4,00](00)$

Kaluza-Klein spectroscopy in ExFT

[Malek, HS]

- lowest KK-multiplet > in ExFT variables

$$\mathcal{A}_\mu{}^M(x,Y) \;=\; \rho^{-1}(Y)\,(U^{-1})_K{}^M(Y)\,A_\mu{}^K(x) \qquad \mathcal{B}_{\mu\nu\,M}(x,Y) \;=\; \rho^{-2}(Y)\,U_M{}^K(Y)\,B_{\mu\nu\,K}(x)$$

- extend to higher Kaluza-Klein modes > with the tower of scalar harmonics Σ

$$\mathcal{A}_\mu{}^M(x,Y) \;=\; \rho^{-1}(Y)\,(U^{-1})_K{}^M(Y)\,\sum_\Sigma A_\mu{}^{K,\Sigma}(x)\,\mathcal{Y}^\Sigma$$

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$$\mathcal{B}_{\mu\nu\,M}(x,Y) = \rho^{-2}(Y)\,U_M{}^K(Y)\sum_\Sigma B_{\mu\nu\,K,\Sigma}(x)\,\mathcal{Y}^\Sigma$$

- reproduces the full Kaluza-Klein spectrum
  - combining the lowest KK-multiplet with the tower of scalar fluctuations
- for fixed $\Sigma = [n,0,0]$, this combines all fluctuations of the multiplet $\mathscr{B}_{[n,0,0](0,0)}$
  - mixing of different states occurs from the ExFT – IIB dictionary and upon evaluating the product $U_M{}^K(Y)\,\mathcal{Y}^\Sigma(Y)$

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- for fixed $\Sigma = [n,0,0]$, this combines all fluctuations of the multiplet $\mathscr{B}_{[n,0,0](0,0)}$
  - mixing of different states occurs from the ExFT — IIB dictionary and upon evaluating the product $U_M{}^K(Y)\,\mathcal{Y}^\Sigma(Y)$
- simple and compact (re-)derivation of the supergravity spectrum on S^5
- direct identification of BPS multiplet components within IIB supergravity

Kaluza-Klein spectroscopy in ExFT

[Malek, HS]

$$
\begin{aligned}
\mathcal{A}_\mu{}^M(x,Y) &= \rho^{-1}(Y)\,(U^{-1})_K{}^M(Y)\,\sum_\Sigma A_\mu{}^{K,\Sigma}(x)\,\mathcal{Y}^\Sigma \\
\mathcal{B}_{\mu\nu\,M}(x,Y) &= \rho^{-2}(Y)\,U_M{}^K(Y)\,\sum_\Sigma B_{\mu\nu\,K,\Sigma}(x)\,\mathcal{Y}^\Sigma
\end{aligned}
$$

- linear in fluctuations, exact in the lowest Kaluza-Klein multiplet
  - allows to switch on non-vanishing background for the D=5 supergravity scalars !

Kaluza-Klein spectroscopy in ExFT

[Malek, HS]

$$\mathcal{A}_\mu{}^M(x,Y) \;=\; \rho^{-1}(Y)\,(U^{-1})_K{}^M(Y)\,\sum_\Sigma A_\mu{}^{K,\Sigma}(x)\,\mathcal{Y}^\Sigma$$

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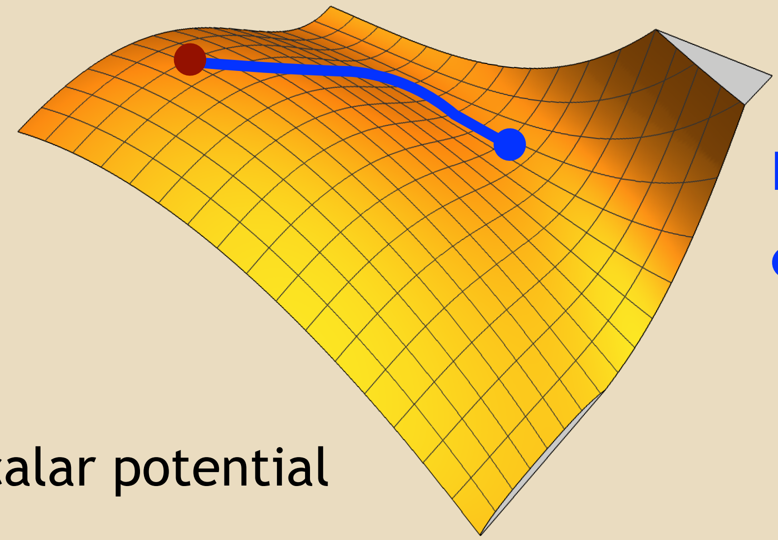
- $\mathcal{N}=8$, SO(6), round S^5



Freedman-Gubser-Pilch-Warner flow

- $\mathcal{N}=2$, U(2), deformed S^5

Kaluza-Klein spectroscopy in ExFT

[Malek, HS]

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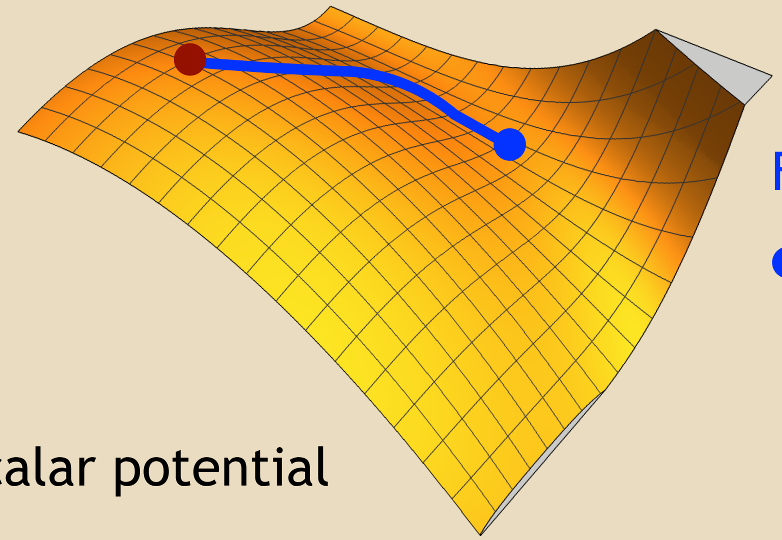

D=5 scalar potential

Freedman-Gubser-Pilch-Warner flow

- $\mathcal{N}=2$, U(2), deformed S^5

- the above ansatz allows to compute the full KK spectrum around the $\mathcal{N}=2$ point
  - so far only known for the 128 dof's from the supergravity multiplet [FGPW '99]
  - IIB analysis would require harmonic analysis on a non-symmetric space & effect of non-vanishing p-form fluxes, possible in the spin-2 sector [Bachas, Estes]

Kaluza-Klein spectroscopy in ExFT

[Malek, HS]

$$\mathcal{A}_\mu{}^M(x,Y) \;=\; \rho^{-1}(Y)\,(U^{-1})_K{}^M(Y)\,\sum_\Sigma A_\mu{}^{K,\Sigma}(x)\,\mathcal{Y}^\Sigma$$

$$\mathcal{B}_{\mu\nu\,M}(x,Y) \;=\; \rho^{-2}(Y)\,U_M{}^K(Y)\,\sum_\Sigma B_{\mu\nu\,K,\Sigma}(x)\,\mathcal{Y}^\Sigma$$

- the above ansatz allows to compute the full KK spectrum around the $\mathcal{N}=2$ point
  - plug into the ExFT action and linearize in fluctuations

$$\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\rm top} - V(\mathcal{M}, e).$$

- e.g. mass matrix for vector fluctuations $A_\mu{}^{M,\Sigma}$

$$M_{M\Sigma,N\Omega} \;\propto\; \frac{1}{3}\,X^{\rm s}_{ML}{}^K\,X^{\rm s}_{NK}{}^L\,\delta^{\Sigma\Omega} + 2\left(X^{\rm s}_{MK}{}^N - X^{\rm s}_{NM}{}^K\right)\mathcal{T}_{K,\Omega\Sigma}$$
$$-6\,\left(\mathbb{P}^K{}_M{}^L{}_N + \mathbb{P}^M{}_K{}^L{}_N\right)\mathcal{T}_{L,\Omega\Lambda}\mathcal{T}_{K,\Lambda\Sigma} + \frac{8}{3}\,\mathcal{T}_{N,\Omega\Lambda}\mathcal{T}_{M,\Lambda\Sigma}$$

in terms of
- symmetrized D=5 embedding tensor $X^{\rm s}_{MN}{}^K \equiv X_{MN}{}^K + X_{MK}{}^N$
- adjoint projector $\mathbb{P}^M{}_N{}^K{}_L \;=\; (t^\alpha)_N{}^M\,(t_\alpha)_L{}^K$
- action of dressed Killing vector field $\mathcal{K}_M{}^m\partial_m\mathcal{Y}^\Sigma = \mathcal{T}_{M,\Sigma\Omega}\,\mathcal{Y}^\Omega$

Kaluza-Klein spectroscopy in ExFT

[Malek, HS]

$$\mathcal{A}_\mu{}^M(x,Y) \;=\; \rho^{-1}(Y)\,(U^{-1})_K{}^M(Y)\,\sum_\Sigma A_\mu{}^{K,\Sigma}(x)\,\mathcal{Y}^\Sigma$$

$$\mathcal{B}_{\mu\nu\,M}(x,Y) \;=\; \rho^{-2}(Y)\,U_M{}^K(Y)\,\sum_\Sigma B_{\mu\nu\,K,\Sigma}(x)\,\mathcal{Y}^\Sigma$$

- the above ansatz allows to compute the full KK spectrum around the $\mathcal{N}=2$ point
- e.g. at level $\Sigma = [3,0,0]$ in multiplets $D(E_0, j_1, j_2; r)$ of $\mathrm{SU}(2)\times\mathrm{SU}(2,2\,|\,1)$

$$\mathbf{0}\;:\quad D(1+\tfrac12\sqrt{37},0,0;1)_{\mathbb{C}} + D(1+\tfrac12\sqrt{61},0,0;1)_{\mathbb{C}} + D_{\mathrm{S}}(\tfrac92,\tfrac12,\tfrac12;1)_{\mathbb{C}} + 2\,D_{\mathrm{S}}(\tfrac92,\tfrac12,0;-1)_{\mathbb{C}} + D(\tfrac92,\tfrac12,0;1)_{\mathbb{C}}$$

$$\mathbf{\tfrac{1}{2}}\;:\quad D(1+\tfrac14\sqrt{145},\tfrac12,\tfrac12;\tfrac12)_{\mathbb{C}} + D(1+\tfrac14\sqrt{193},0,0;\tfrac12)_{\mathbb{C}} + D(\tfrac{15}{4},\tfrac12,0;\tfrac12)_{\mathbb{C}} + D(\tfrac{17}{4},\tfrac12,0;-\tfrac12)_{\mathbb{C}}$$
$$+D_{\mathrm{S}}(\tfrac{15}{4},0,0;\tfrac52)_{\mathbb{C}} + D_{\mathrm{S}}(\tfrac{17}{4},0,0;\tfrac32)_{\mathbb{C}}$$

$$\mathbf{1}\;:\quad 2\,D(1+\sqrt7,0,0;0) + D(1+\sqrt7,\tfrac12,0;0)_{\mathbb{C}} + D_{\mathrm{S}}(\tfrac72,\tfrac12,0;1)_{\mathbb{C}} + D_{\mathrm{S}}(3,\tfrac12,0;2)_{\mathbb{C}}$$

$$\mathbf{\tfrac{3}{2}}\;:\quad D_{\mathrm{S}}(\tfrac94,0,0;\tfrac32)_{\mathbb{C}}$$

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$$\mathbf{\tfrac{1}{2}}: \quad D(1+\tfrac{1}{4}\sqrt{145},\tfrac{1}{2},\tfrac{1}{2};\tfrac{1}{2})_\mathbb{C}+D(1+\tfrac{1}{4}\sqrt{193},0,0;\tfrac{1}{2})_\mathbb{C}+D(\tfrac{15}{4},\tfrac{1}{2},0;\tfrac{1}{2})_\mathbb{C}+D(\tfrac{17}{4},\tfrac{1}{2},0;-\tfrac{1}{2})_\mathbb{C}$$
$$+D_\mathrm{S}(\tfrac{15}{4},0,0;\tfrac{5}{2})_\mathbb{C}+D_\mathrm{S}(\tfrac{17}{4},0,0;\tfrac{3}{2})_\mathbb{C}$$

$$\mathbf{1}: \quad 2\,D(1+\sqrt{7},0,0;0)+D(1+\sqrt{7},\tfrac{1}{2},0;0)_\mathbb{C}+D_\mathrm{S}(\tfrac{7}{2},\tfrac{1}{2},0;1)_\mathbb{C}+D_\mathrm{S}(3,\tfrac{1}{2},0;2)_\mathbb{C}$$

$$\mathbf{\tfrac{3}{2}}: \quad D_\mathrm{S}(\tfrac{9}{4},0,0;\tfrac{3}{2})_\mathbb{C}$$

in terms of semi-short and long multiplets

Kaluza-Klein spectroscopy in ExFT

[Malek, HS]

$$
\begin{aligned}
\mathcal{A}_\mu{}^M(x,Y) &= \rho^{-1}(Y)\,(U^{-1})_K{}^M(Y)\,\sum_\Sigma A_\mu{}^{K,\Sigma}(x)\,\mathcal{Y}^\Sigma \\
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in terms of semi-short and long multiplets

Kaluza-Klein spectroscopy in ExFT

[Malek, HS]

- ExFT offers access to the full KK spectrum around any AdS vacuum located within maximal D=5 supergravity

- the same applies to vacua in all other maximal supergravities

- some other interesting candidates:

  - $\mathcal{N}=2$ vacuum in $D=4$ maximal $\mathrm{SO}(8)$ supergravity
    [Warner][Nicolai,Warner][Klebanov, Klose, Murugan][Klebanov, Pufu, Rocha]

  - $\mathcal{N}=3$ vacuum in $D=4$ maximal $\mathrm{ISO}(7)$ supergravity, Romans' mIIA on S^6
    [Guarino, Jafferis, Varela] $\longrightarrow$ holography with Chern-Simons matter theories

  - Stable (?) $\mathcal{N}=0$ vacuum in $D=4$ maximal $\mathrm{SO}(8)$ supergravity
    [Fischbacher, Pilch, Warner][Godazgar, Godazgar, Krüger, Nicolai, Pilch]
    $\longrightarrow$ non-supersymmetric stable AdS vacua ?

  - Recent numerical scan of vacua in $D=4$ maximal $\mathrm{SO}(8)$ supergravity
    [Comsa, Firsching, Fischbacher]

  - ...

- to be generalized to vacua in half-maximal (quarter-maximal) supergravities ...

other developments — conclusions

other examples of consistent truncations

- consistent truncations with less supersymmetry via DFT
  [Baguet, Malek, Pope, HS, Sarioglu]
  - S^3 reduction of the bosonic string
  - more general: Pauli reduction of the bosonic string on group manifold G
  - example G = SO*(4), type II uplift of D=4 Minkowski vacua
  - $AdS_3 \times S^3$ reductions from 6D supergravity, N=(1,1) and N=(2,0) w tensor-multiplets

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- consistent truncations with less supersymmetry in ExFT (in type II sugra)
  [Malek] [Malek, HS, Vall Camell]

  embedding of half-maximal supergravity into ExFT

  construction and classification of supersymmetric AdS vacua

  half-maximal supersymmetric AdS vacua induce consistent truncations around

other applications / developments

- ExFT for infinite dimensional Kac-Moody algebras
  - affine $E_{9(9)}$: infinite-dimensional highest-weight representations

    [Bossard, Cederwall, Ciceri, Inverso, Kleinschmidt, Palmkvist, HS] $\longrightarrow$ Gianluca's talk
  - towards E_{11} ExFT [Bossard, Kleinschmidt, Sezgin][West]

- fermions & superspace
  - $E_{7(7)}$: super-diffeomorphisms in $(4+56\,|\,32)$

    [Butter, HS, Sezgin] $\longrightarrow$ [Howe, Lindström 1981]

- embedding of massive IIA theory
  - by deformations of ExFT
  - by Scherk-Schwarz reduction violating the section constraints

    [Ciceri, Guarino, Inverso] [Cassani, de Felice, Petrini, Strickland-Constable, Waldram]

- embedding of 'generalized IIB' theory
  - background from η-deformed $AdS_5 \times S^5$ sigma model
  - T-dual of IIA with non-isometric dilaton

    [Baguet, Magro, HS] [Sakatani , Uehara, Yoshida]

other applications / developments

- unifying framework for brane solutions
  - organisation of exotic branes

    [Berman, Rudolph, Bakhmatov, Kleinschmidt, Musaev, Otsuki, Fernandez-Melgarejo, Kimura, Sakatani] → Chris' talk

- orbifolds and orientifolds in ExFT
  - unified approach in terms of generalized orbifolds (O-folds) [Blair, Malek, Thompson]

- exceptional string sigma model
  - string sigma model with ExFT background fields [Arvanitakis, Blair]

- ExFT loop calculations
  - duality covariant graviton amplitudes [Bossard, Kleinschmidt]

- underlying mathematical structures
  - L_∞-algebras, Borchers superalgebras, tensor hierarchy algebras

    [Cederwall, Palmkvist][Hohm, Kupriyanov, Lüst, Traube]
    [Cagnacci, Codina, Marques][Arvanitakis][Hohm, Zwiebach] → Olaf's talk

conclusions

- **exceptional field theory**
  - manifestly duality covariant formulation of maximal supergravity
  - unique theory with generalized diffeomorphism invariance
  - upon an explicit solution of the section constraints
    the theory reproduces full D=11 supergravity and full D=10 IIB supergravity
- powerful tools for
  - consistent truncations
  - construction of vacua
  - Kaluza-Klein spectroscopy

conclusions

exceptional field theory

- manifestly duality covariant formulation of maximal supergravity
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challenges

- higher order corrections ⟶ Diego's talk
- decrease number of external dimensions ⟶ unifying picture
- weaken / relax section constraints

[Bossard, Kleinschmidt, Sezgin]

new variables for supergravity
— or hints towards a more fundamental structure ..?