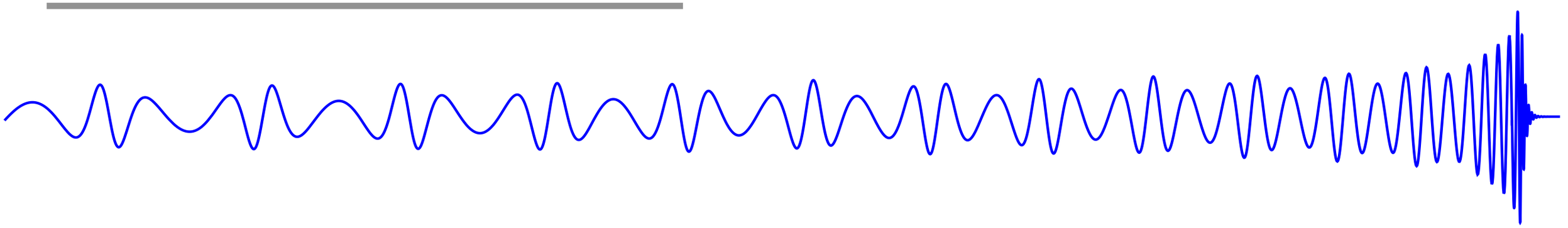
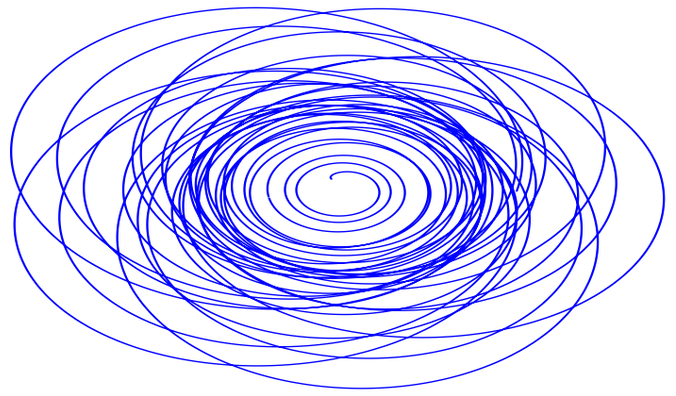
Eccentricity



- *a measure of the extent of deviation from circularity*



Many different definitions:

Analytical modeling: e.g. PN, EOB, GSF

In NR: eccentricity estimates are averaged quantities

(examples e.g. in Mroue+ arXiv:1004.4697)

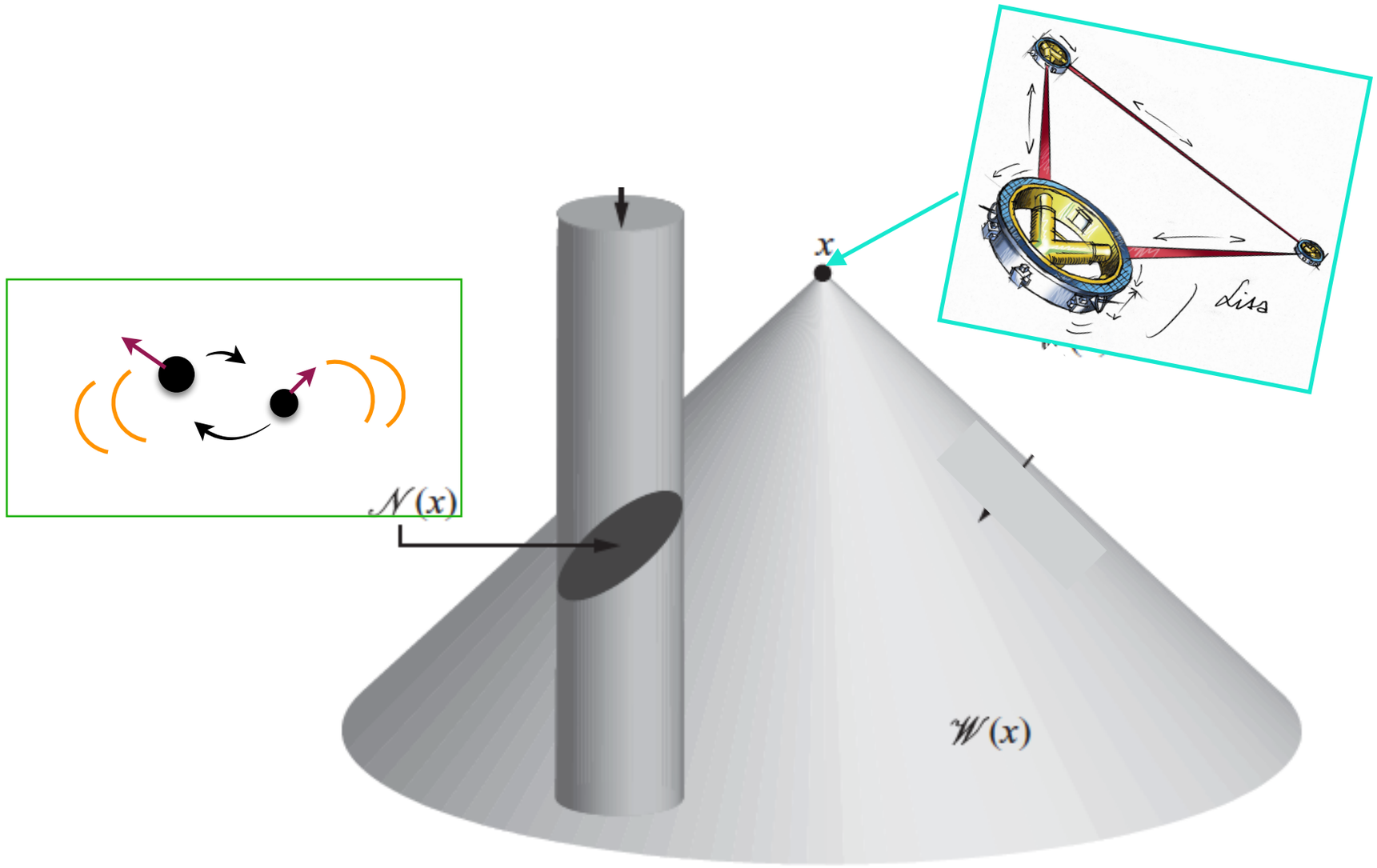
Optimization algorithm to map NR ↔ AR quantities

Habib & Huerta arXiv:1904.09295

Challenges for modeling (PN, EOB)

- Richer features (multi-frequency, higher modes important …)

- Hereditary effects less straightforward to compute explicitly



Examples of recent progress in PN

*not comprehensive

Nonspinning waveforms

- Amplitudes incl tails, post-adiabatic: Boetzel+ arXiv:1904.11814

- 3PN Frequency-domain: Moore & Yunes arXiv:1903.05203

Spins+eccentricity

- Fourier-domain waveform model Klein+ arXiv:1801.08542

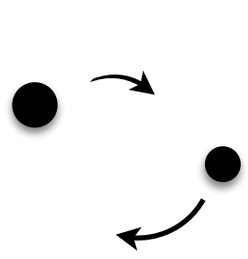
PN+NR for moderate eccentricities

- Huerta+ “ENIGMA”, Hinder+arXiv:1709.02007

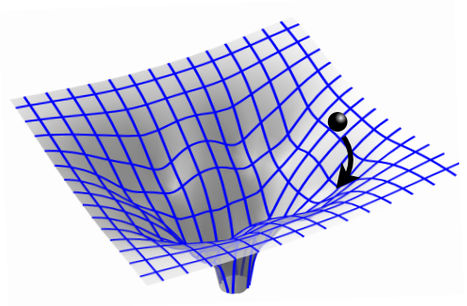
Eccentricity in EOB

Dynamics

EOB Hamiltonian for generic orbits



Map of phase space dynamics



Calibration to NR mainly for circular orbits

Radiative sector

$$h_{22}^{\rm EOB}(t) = h^{\rm Newt} e^{-2i\Phi} \mathcal{S}_{\rm eff}\, \rho^2\, T e^{i\delta}\, h^{\rm NQC}$$

Factorized waveform from PN + GSF + NR specialized to quasi-circular

multi-phase

generic orbits

Generalized light-cone trick

Other hereditary effects in EOB gauge

New NR info

Eccentricity: Self-force (SF) for EMRIs/IMRIs

- Eccentricity? Hopman & Alexander (2005) (much added work since)

  EMRIs may enter LISA passband with $e \simeq 0.7 - 0.8$ max

- Why SF? Get cumulative phase accurate to $< 0.01 - 0.1$ radian

- Status of SF for eccentric EMRIs:
  - Schwarzschild primary

    All 1st-order in mass ratio $\varepsilon = 1/q$ effects (most post-1 adiabatic effects)
    Time domain and frequency domain codes

    Barack & Sago 2007,10,11, Warburton et al. 2012, Akcay et al. 2013, Osburn et al. 2014

    FD SF data "tile" (p, e) space
    Long-term evolutions (including with secondary spin)

    Osburn et al. 2016, Warburton et al. 2017

    Fast SF inspirals van de Meent & Warburton 2018
  - Kerr primary

    Generic orbit 1st-order in ε FD code van de Meent 2018
    New, written in Mathematica. Not ported to C yet
    No tiling of (p, e, i) space yet; no full inspirals yet

Eccentricity: Roadblocks to SF for EMRIs/IMRIs

- Need at least 2nd-order SF fluxes to complete post-1 adiabatic

  See recent by Miller, Wardell, Pound 2016 and earlier by Pound, Gralla

  Cumulative radial libration phase

$$\Phi_r = \frac{\kappa_0}{\varepsilon} + \frac{\kappa_{1/2}}{\sqrt{\varepsilon}} + \kappa_1 \varepsilon^0 + \kappa_{3/2}\sqrt{\varepsilon} + \kappa_2 \varepsilon^1 + \cdots$$

  $\kappa_0 =$ 1st-order fluxes

  $\kappa_1 =$ 1st-order oscillatory diss and conserv SF + 2nd-order fluxes

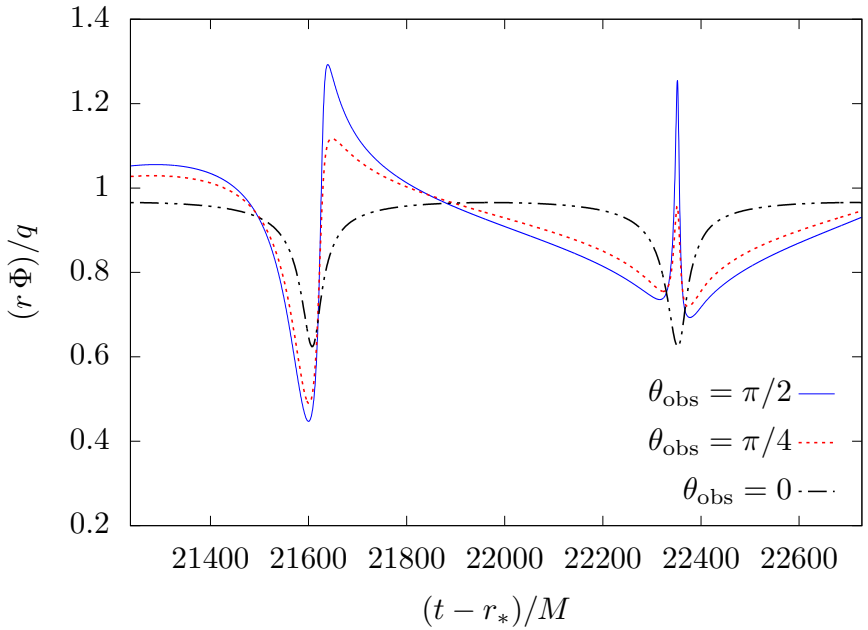
- Must cope with (on Kerr) transient resonances
  - Important Ω_r, Ω_θ resonances–1:2, 2:3, etc
  - Enhance or decrease flux during passage
  - Cause jump in orbital parameters vs adiabatic evolution

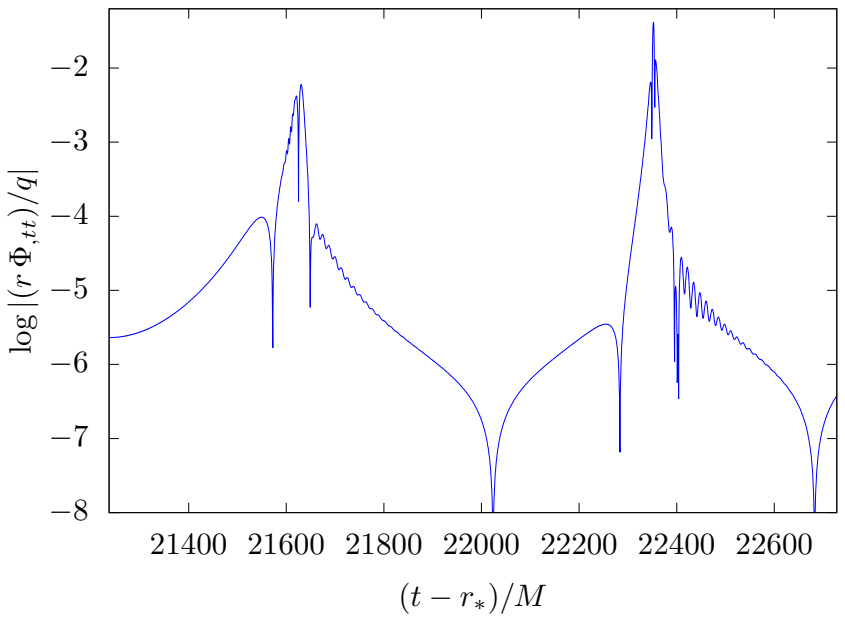
  Hinderer & Flanagan 2008, Flanagan & Hinderer 2012, Flanagan et al. 2015, Berry et al. 2016, 17

Eccentric EMRI Waveforms: Faint Quasi-Normal Bursts

- Thornburg $\sim$ 2016 found QNM excitation in TD generic Kerr scalar SF

- Nasipak et al. (soon 2019) confirmed QNM in a FD Kerr SSF code
  Examined waveform and found repeated faint QNM bursts
  (Orbit: $p = 8.0$, $e = 0.8$, $i = 0$, $a/M = 0.99$)





- In principle detectable–requires high e, high a, and (very) high SNR

Where are we now in NR?

- Several groups performed first systematic studies of eccentric BH binaries:
  - SXS (SpEC), Hinder+ (23 non-spinning), Huerta+ (89 non-spinning, $1 \leq q \leq 10$, $e \leq 0.19$), Ramos+ (poster, 63 with $e \leq 0.5$, non-precessing),
  - Key result: circularisation for moderate initial eccentricity.
- **Good news:** does not require fine-tuned initial data (like quasi-circular), evolution is not much more complicated than quasi-circular case.
- **Problems:**
  - Complete waveforms: connecting to PN regime (constructing hybrids) much more expensive: binary has to be in PN regime @ periastron
    => very far @ apastron for larger eccentricity. => long evolutions => expensive
  - With spin -> with precession. Large parameter space!
  - Higher harmonics tend to be very noisy.
- Need WFs for high mass ratio, to connect to EMRI limit. Teukolsky, self-force, etc.

Where are we now in phenomenological WF modelling?

- Personal belief: to understand/model complicated phenomenology, it is useful to utilise complementarity between time and Fourier domains.

  - Fourier domain eccentric waveforms show complex structure.

  - Time domain phenom model ~ PhenomD accuracy (poster H. Estellés+)

- Accelerated evaluation with “multi-banding” (poster García+) -> will need to be modified for eccentricity

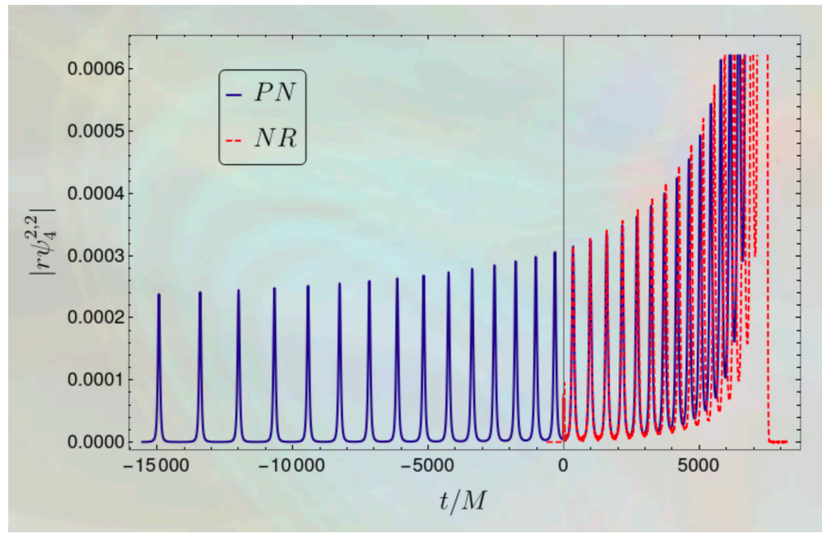
- “Standalone” time domain models: Hinder+, Huerta+.

- 0th approximation: glue eccentric PN to phenomenological model for last orbits up to ringdown (rapid circularisation for last orbits)

- Ongoing work to incorporate eccentricity into “IMRPhenom” family.

Problems, Questions, Challenges

- Need collaboration of NR and PN+ to match solutions of full GR to perturbative results.
    - Measure eccentricity in NR simulations for large eccentricity, with precession?
    - Build eccentric hybrids, add spin precession, high mass ratio => injections
        - Subdominant spherical harmonics.
        - Accuracy assessment!



- Need to “get a foot in the door” with modelling high eccentricity.
- Many problems in NR not related to eccentricity:
    - Room for optimising codes and procedures.
    - Need “cleaner” NR waveforms + solid error bars.