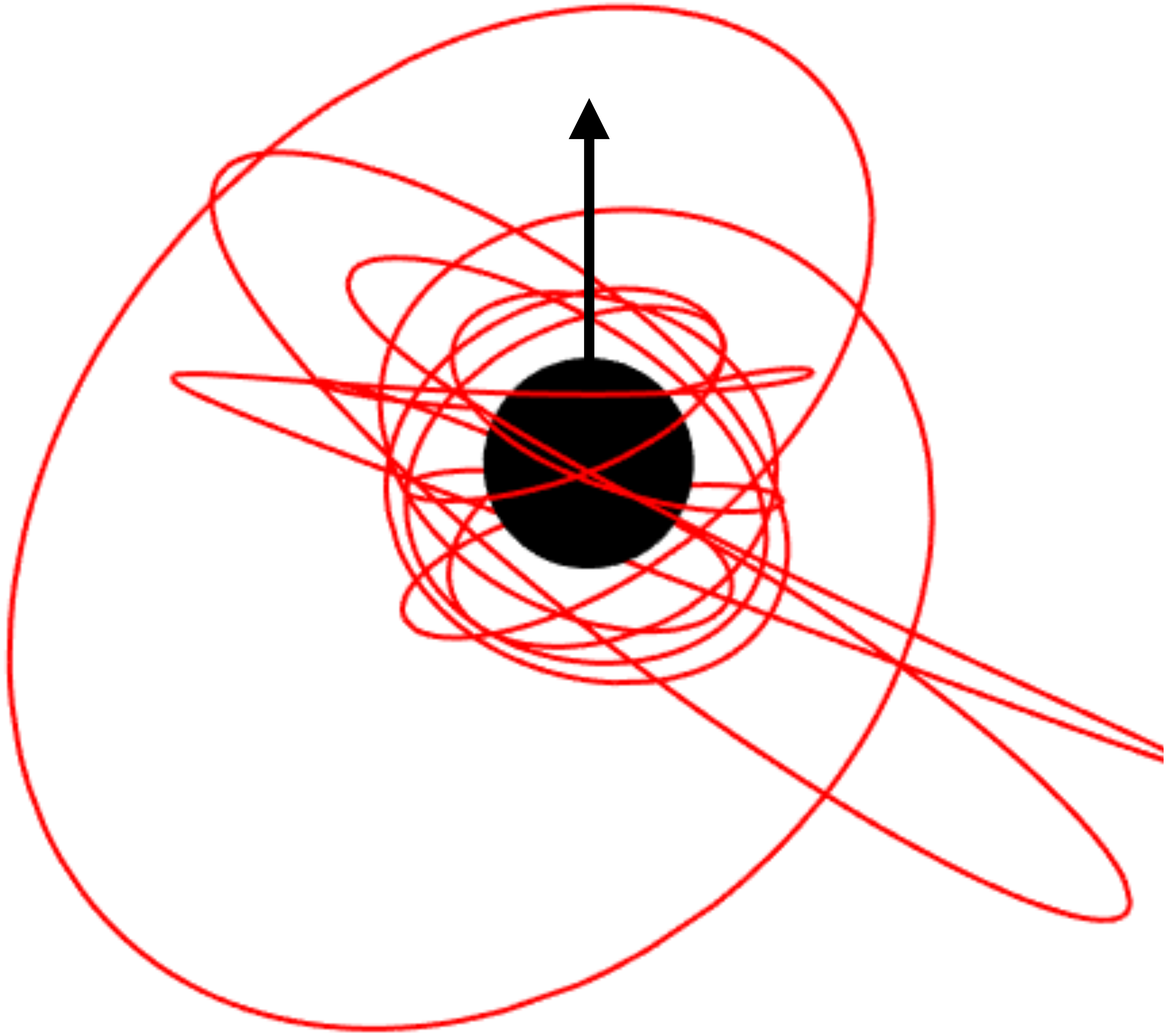
High Mass Ratios

Larry Kidder, Harald Pfeiffer, Barry Wardell

Extreme Mass Ratio Inspirals

- Mass ratio q~1:1,000,000.
- Many (>10,000) orbits.
- Generic (eccentric, inclined) orbital configurations.
- Both black holes spinning.
- LISA parameter estimation needs ~1 radian accuracy in the phase of the waveform.
- High mass ratio methods may also be useful for Intermediate Mass Ratio Inspirals.



Extreme Mass Ratio Inspiral Modelling goals

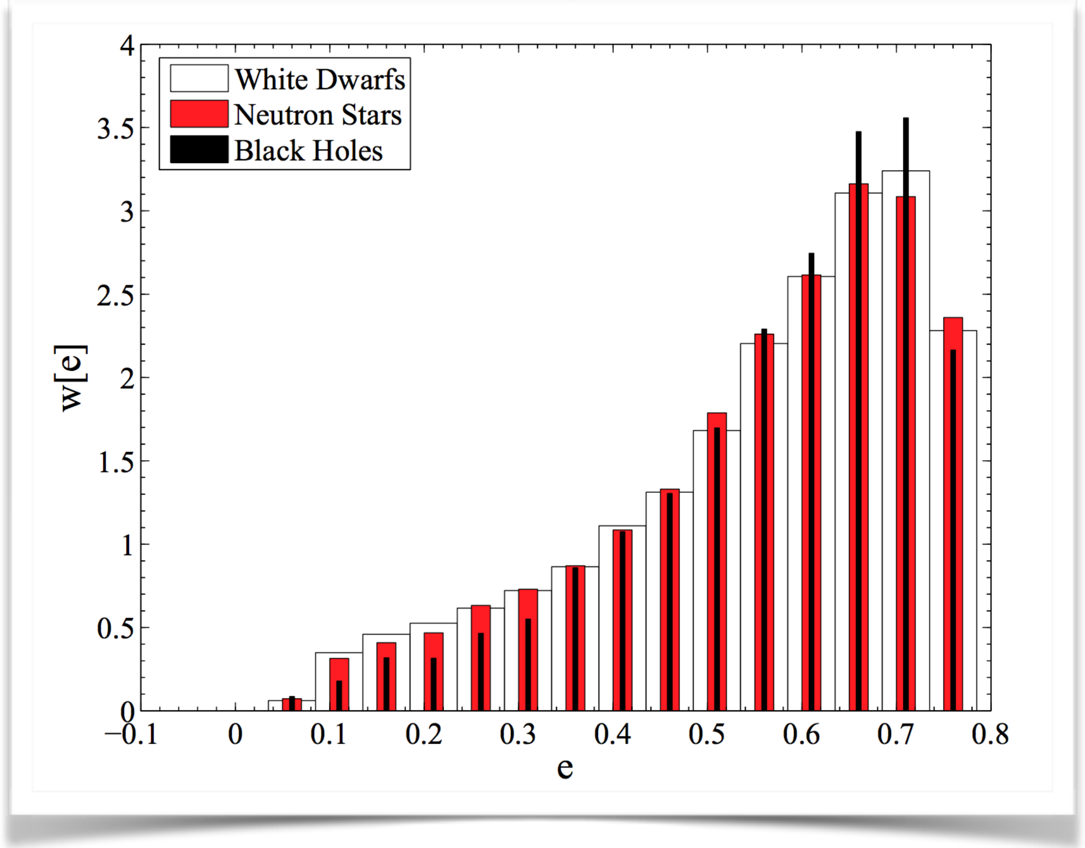
Waveform templates need to be accurate across the parameter space

and generated rapidly

- *Instantaneous SNR is very low so use matched filtering to extract signal*
- *Template must track waveform phase to better than 1 radian over 10s to 100s of thousands of cycles*

- *Need to cover 14 dimensional parameter space, so each template must be generated in a few seconds*

- *Primary and secondary spinning*
- *Motion of secondary can be highly eccentric and inclined*



Hopman & Alexander ApJ 629 (2005) 362-372

Perturbation Theory for EMRIs

Perturbation theory (Gravitational self-force): Solve Einstein equations perturbatively. For EMRIs with LISA we need to solve for:

1. First order metric perturbation (gravitational self-force)

2. Second order metric perturbation (dissipative part)

3. Evolving orbital inspiral

4. Waveform

$$g_{\alpha\beta} = g^{0}_{\alpha\beta} + \epsilon h^{(1)}_{\alpha\beta} + \epsilon^2 h^{(2)}_{ab} + \mathcal{O}(\epsilon^3)$$

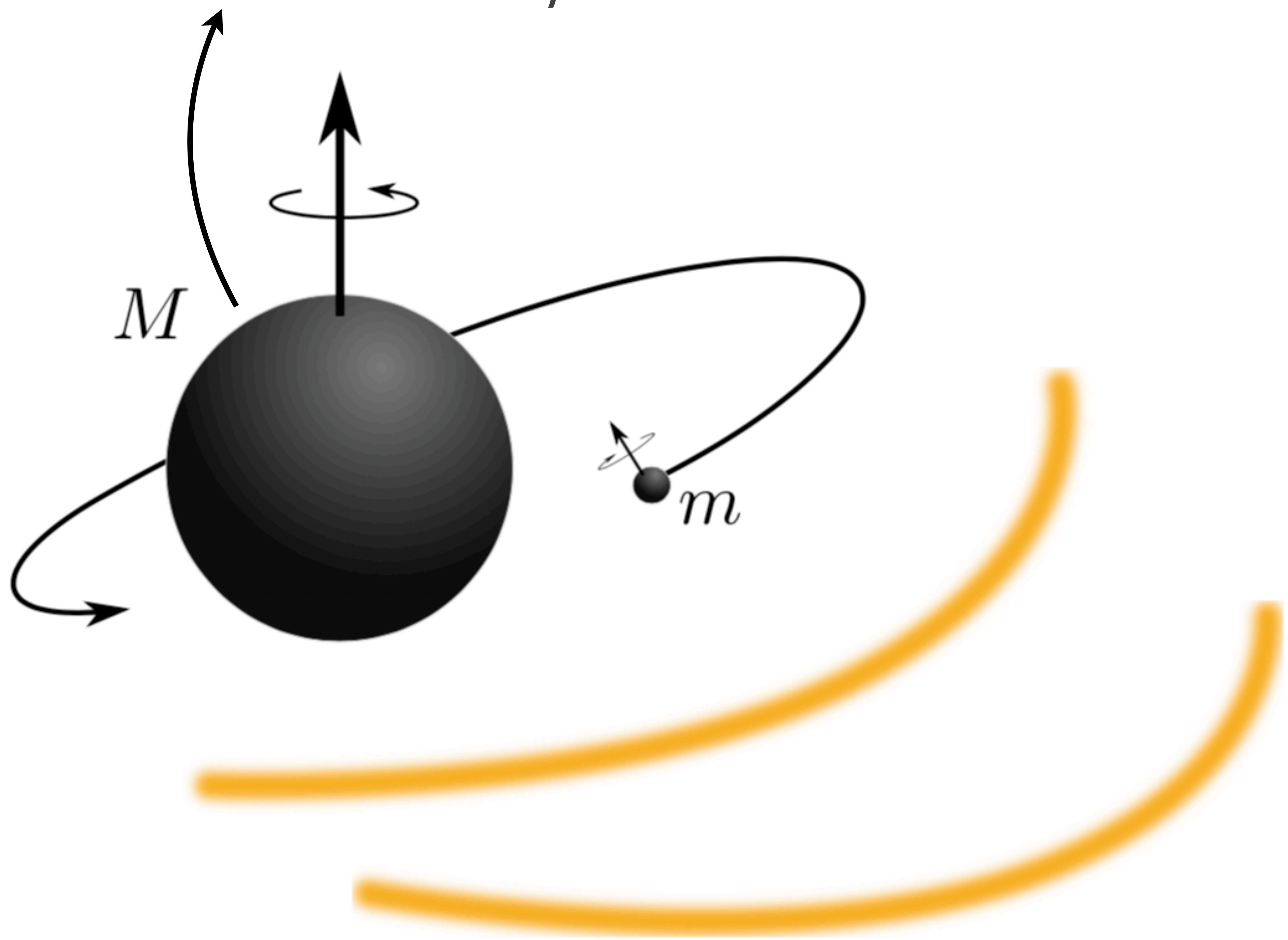


Image credit: A. Pound

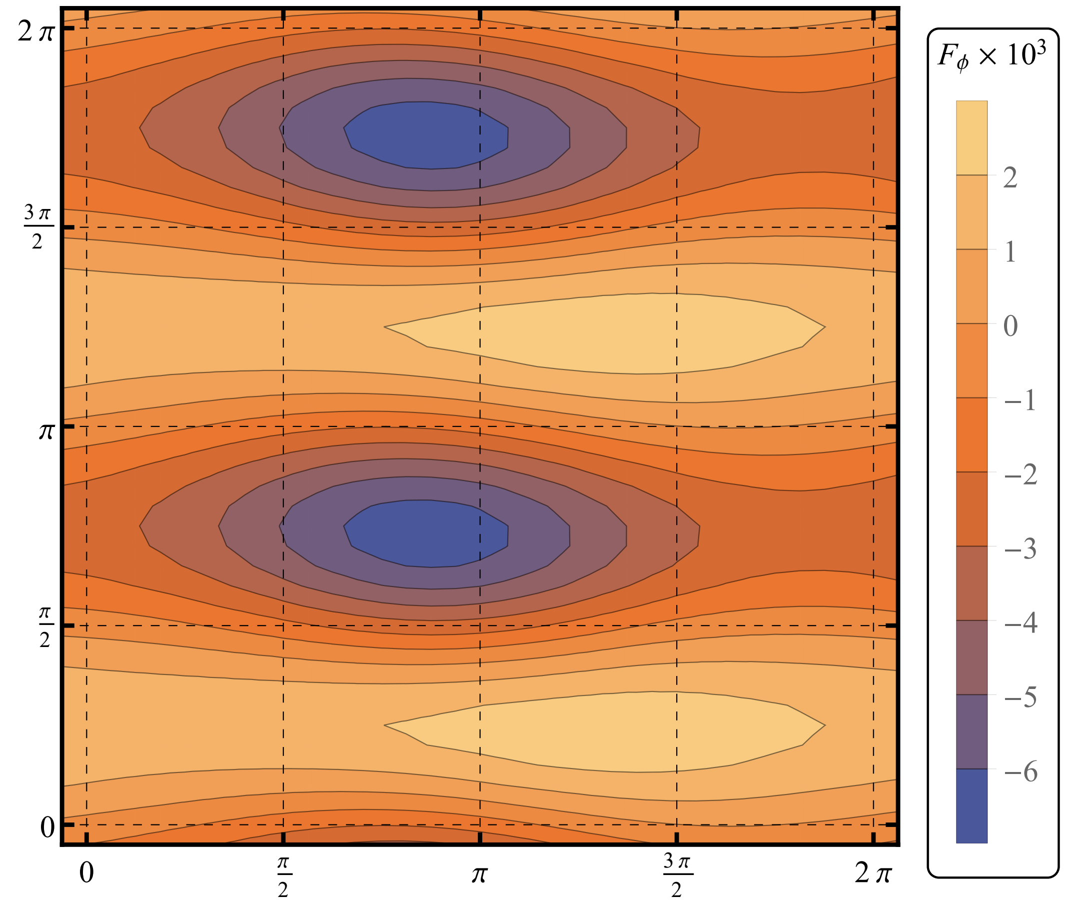
$$\Box\, \bar{h}^{(1)}_{\alpha\beta} + 2C_{\alpha\ \beta}^{\ \gamma\ \delta}\bar{h}^{(1)}_{\gamma\delta} = -16\pi T_{\alpha\beta}$$

$$\Box\, \bar{h}^{(2)}_{\alpha\beta} + 2C_{\alpha\ \beta}^{\ \gamma\ \delta}\bar{h}^{(2)}_{\gamma\delta} = S(h^{(1)})$$

$$a^{\mu} \propto \nabla_{\gamma}(h^{(1)}_{\alpha\beta} + h^{(2)}_{\alpha\beta})$$

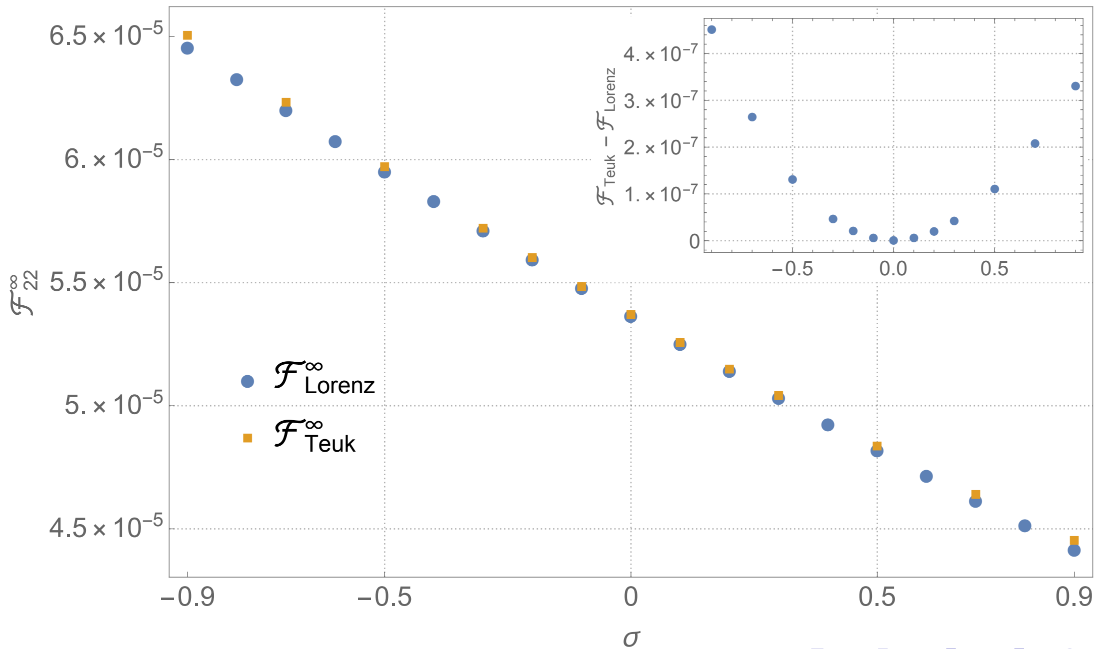
State of the art in Gravitational Self-force

a=0.9M, p=10, e=0.1, z_{max}=0.9



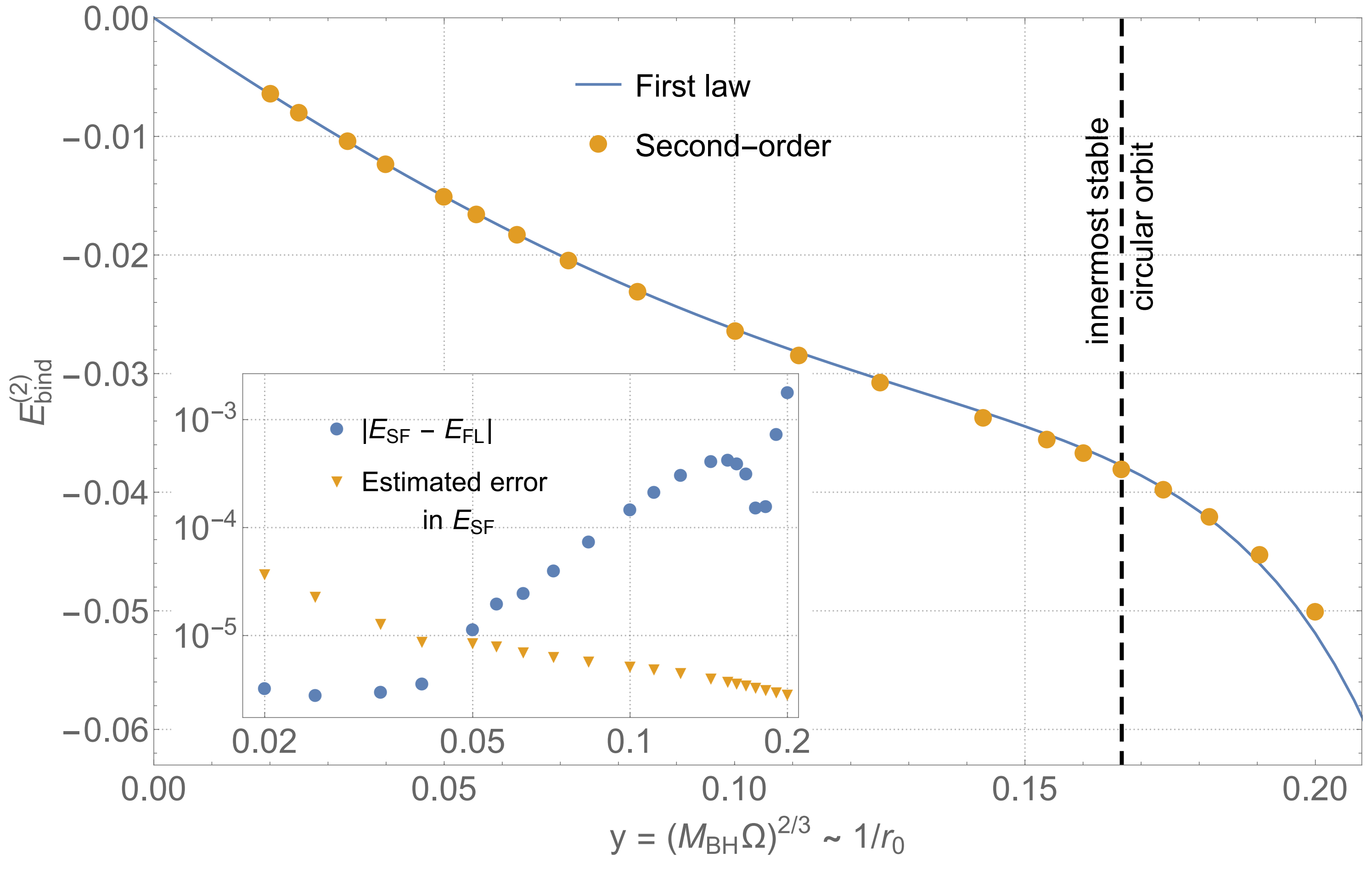
M. van de Meent, Phys. Rev. D 97, 104033 (2018)

First order self-force: generic orbits with larger BH spinning (Kerr).



S. Akcay

Spinning secondary



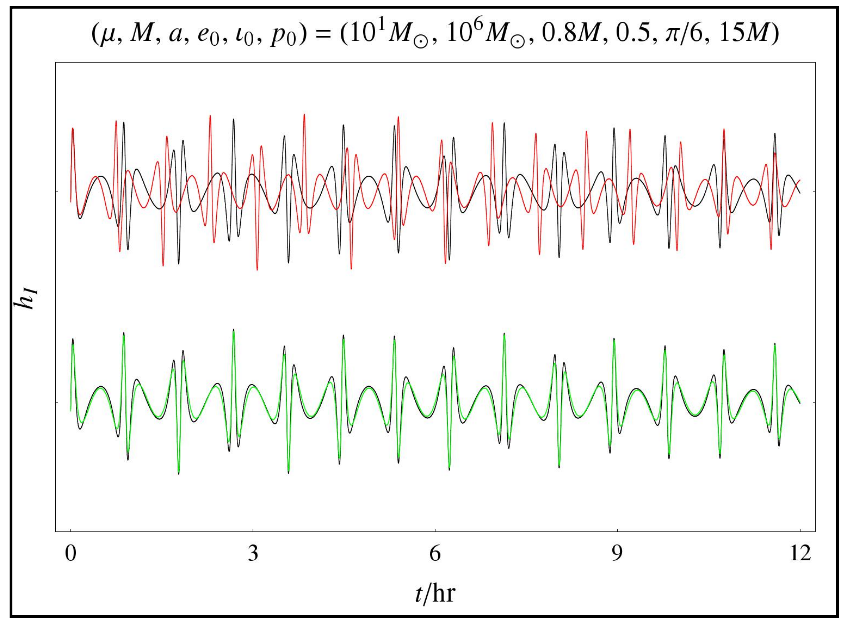
J. Miller, A. Pound, N. Warburton and B. Wardell

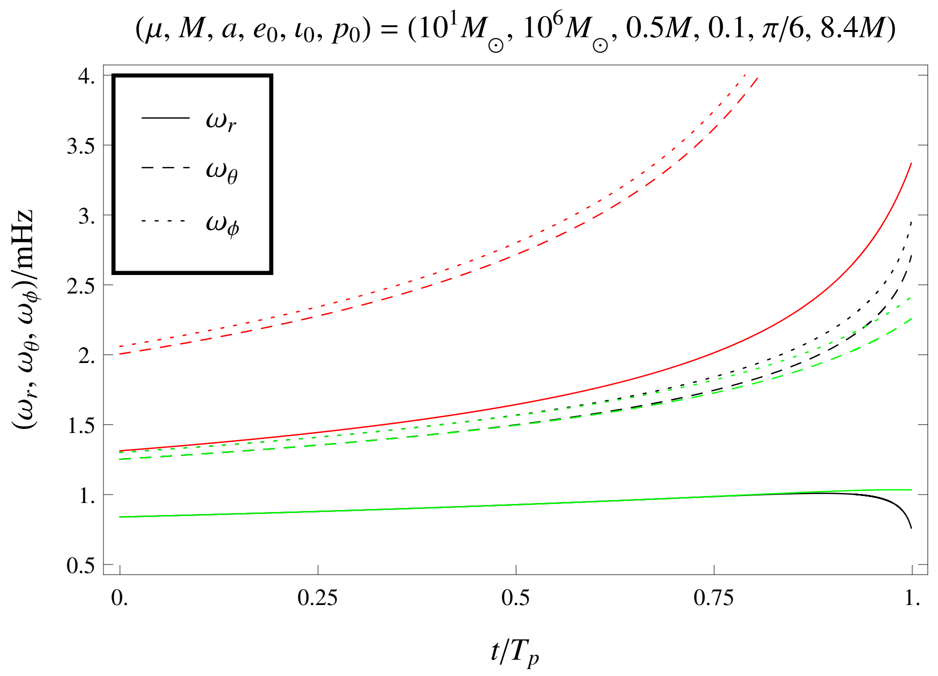
Second order self-force: circular orbits, non-spinning (Schwarzschild)

Waveforms

A. Chua, C. Moore and J. Gair

S.Hughes





A. Chua, C. Moore and J. Gair

Kludge

Time to calculate phase space inspiral

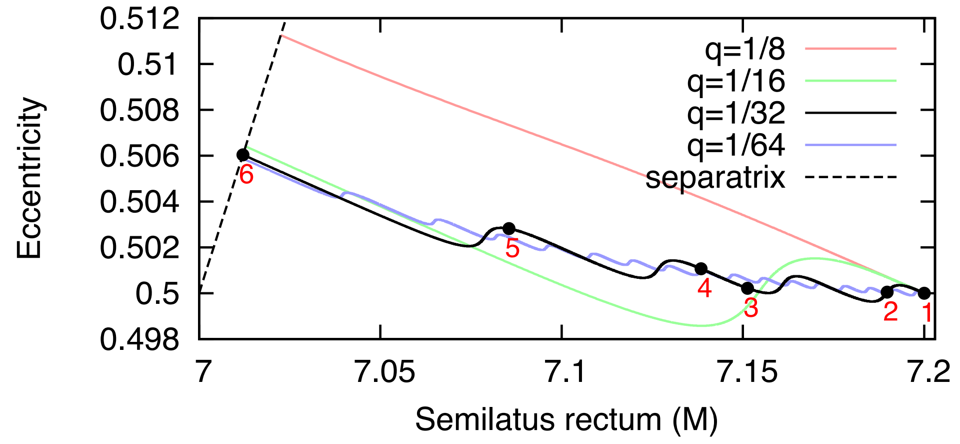
Mass Ratio	Time (Full)	Time (NIT)	Speed up
10^{-3}	6.2s	~0.008s	~700
10^{-4}	43s	~0.008s	~5,000
10^{-5}	5m40s	~0.008s	~40,000
10^{-6}	42m20s	~0.008s	~300,000
	C++	C++	

M. van de Meent and N. Warburton

Near-identity transformations

E. Flanagan, J. Moxon and A. Pound

Two-timescale expansion



S. Detweiler, P. Diener, I. Vega, B. Wardell

Self-consistent evolution

Efficient methods for evolving orbits

Open Tools and Datasets

Black Hole Perturbation Toolkit

Open tools for black hole perturbation theory

Introduction | Toolkit and Data Repository | Status and Documentation | Contributors and Users

Project List

Current **Mathematica** projects in the Toolkit include:

- SpinWeightedSpheroidalHarmonics: Tools for computing spin-weighted spheroidal harmonics and their associated eigenvalues.
- KerrGeodesics: Tools for computing bound timelike geodesics about a Kerr black hole.
- Teukolsky: A set of functions for computing solutions to the Teukolsky equation for perturbations of the spacetime of a Kerr black hole.
- QuasiNormalModes: Tools for computing quasinormal modes in Schwarzschild and Kerr spacetime
- GeneralRelativityTensors: Provides a set of functions for performing coordinate-based tensor calculations with a focus on general relativity and black holes in particular.

Current **C/C++** projects in the Toolkit include:

- Fast Self-forced Inspirals: Code to compute self-force inspirals rapidly using the near-identity transformed (NIT'd) equations of motion.
- EMRI Kludge Suite: A suite of software for computing kludge waveforms for generic extreme mass-ratio inspirals into a Kerr black hole.
- Gremlin: Code to solve the Teukolsky equation with a point-particle source

Current **Python** and **SageMath** projects in the Toolkit include:

- kerrgeodesic_gw: SageMath code to compute the gravitational waves from a particle on a circular orbit about a Kerr black hole. Also included is code to compute spin weighted spheroidal harmonics.

http://bhptoolkit.org

Fluxes.nb 100%

```
In[107]:= << Teukolsky`

In[108]:= a = 0.9; p = 10.0; e = 0; x = 1;
          orbit = KerrGeoOrbit[a, p, e, x]

Out[109]= KerrGeoOrbitFunction[0.9,10.,0,1.,<<>>]

In[110]:= s = -2; l = 2; m = 2;
          ψ4[l, m] = TeukolskyPointParticleMode[s, l, m, 0, 0, orbit]

Out[111]= TeukolskyModeObject[-2,2,2,0,0,<<>>]

In[112]:= ψ4[l, m]["Fluxes"]

Out[112]= <|FluxInf → 0.000022273,
           FluxHor → -5.98368×10^-8, FluxTotal → 0.0000222132|>
```

Numerical relativity burgeoning: 1000's of simulations

- BAM/Cardiff/Palma

  waveform modelling "Phenom"

  e.g. Husa+ 15, Hannam+ 14, Kahn+16

- Georgia Tech

  merger/ringdown properties

  e.g. Pekowsky+ 13, Jani+ 15

- RIT

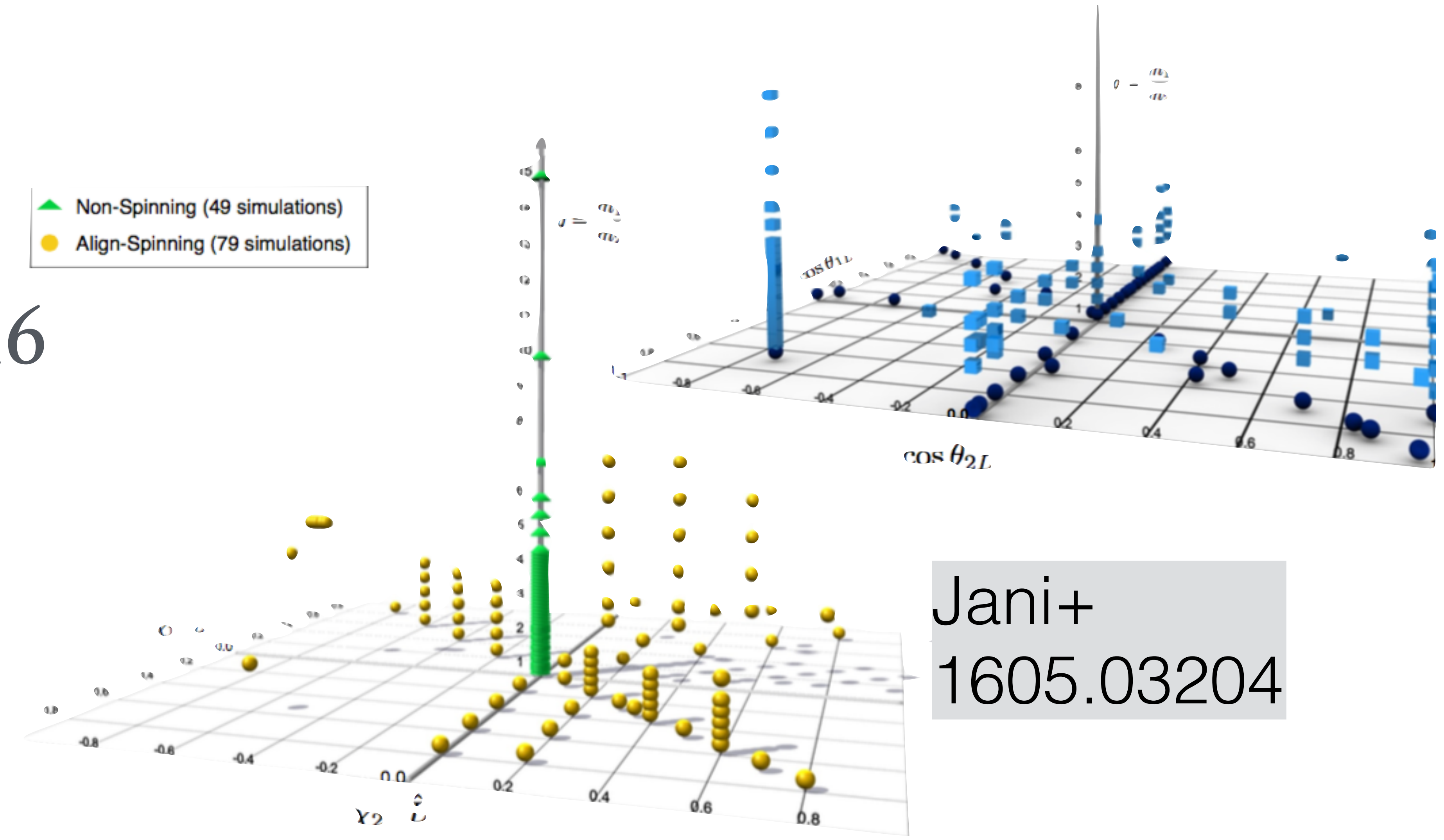
  remnant mass/spin/kick formulae

  e.g. Healy+ 14, Zlochower+ 15, Healy+ 17, Healy

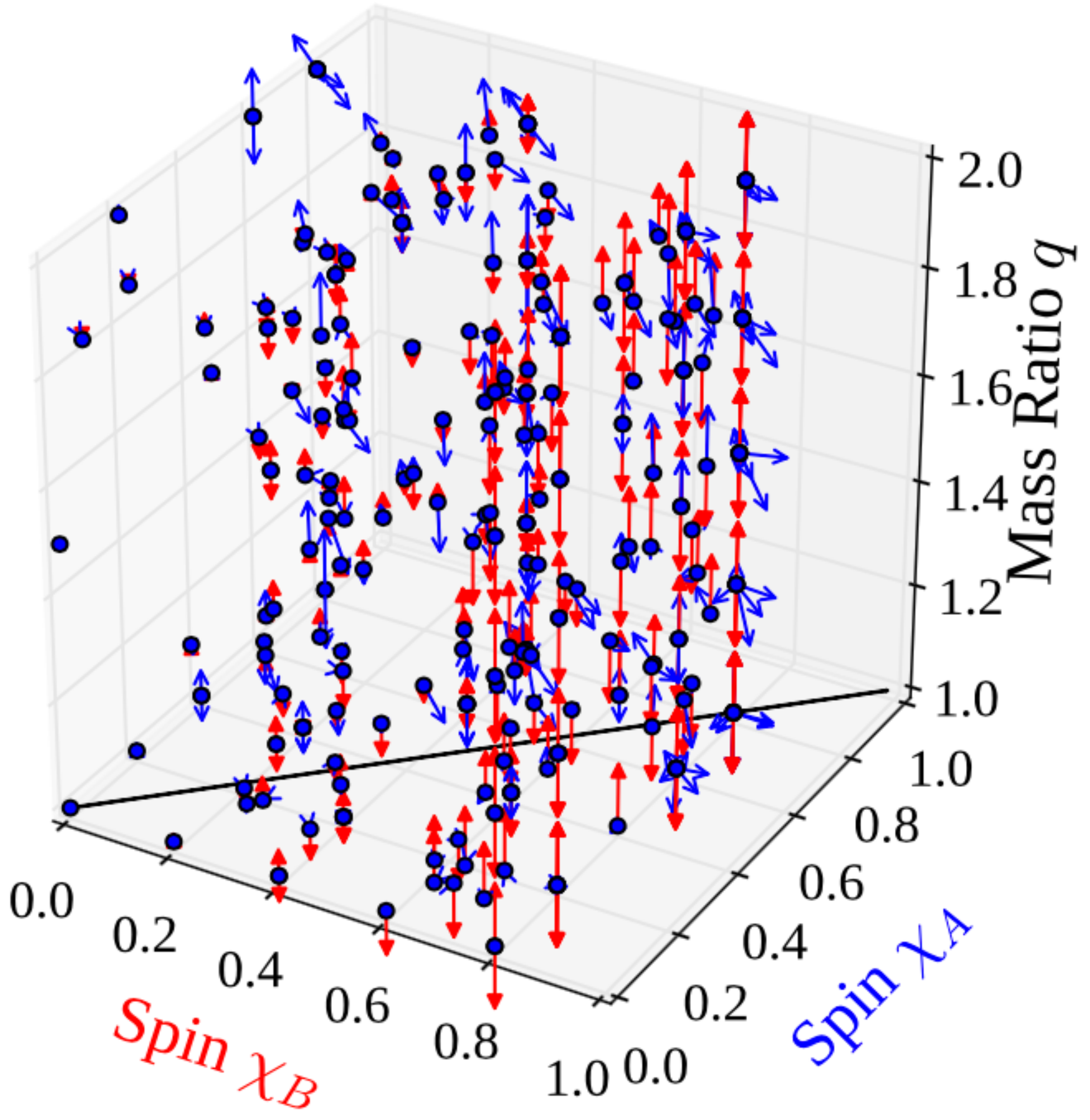
- SXS

  waveform modelling "SEOBNR"

  e.g. Mroue..HP+ 13, Chu..HP+ 15, Hemberger+ 1
  Blackman..HP+ 17, Boyle+ 2019

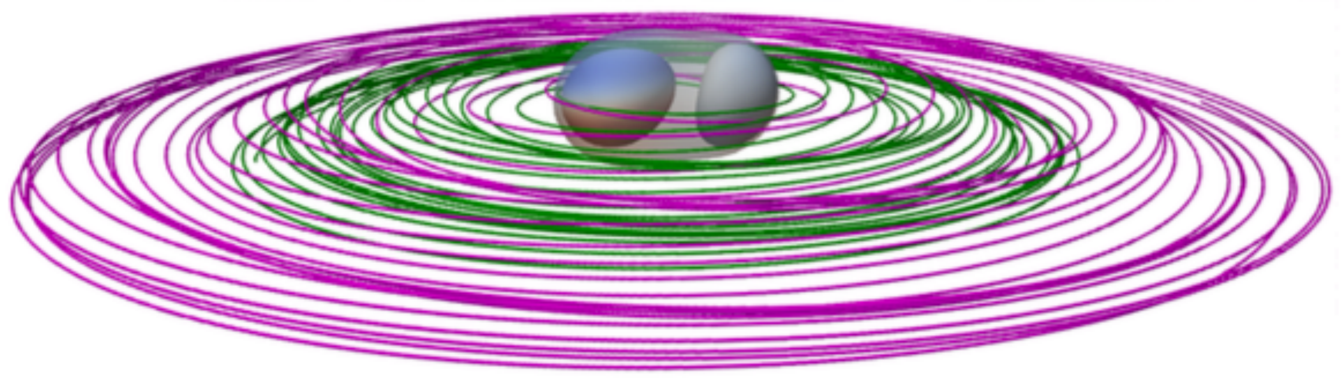


Jani+ 1605.03204



Blackman+ 16

Parameter-space coverage

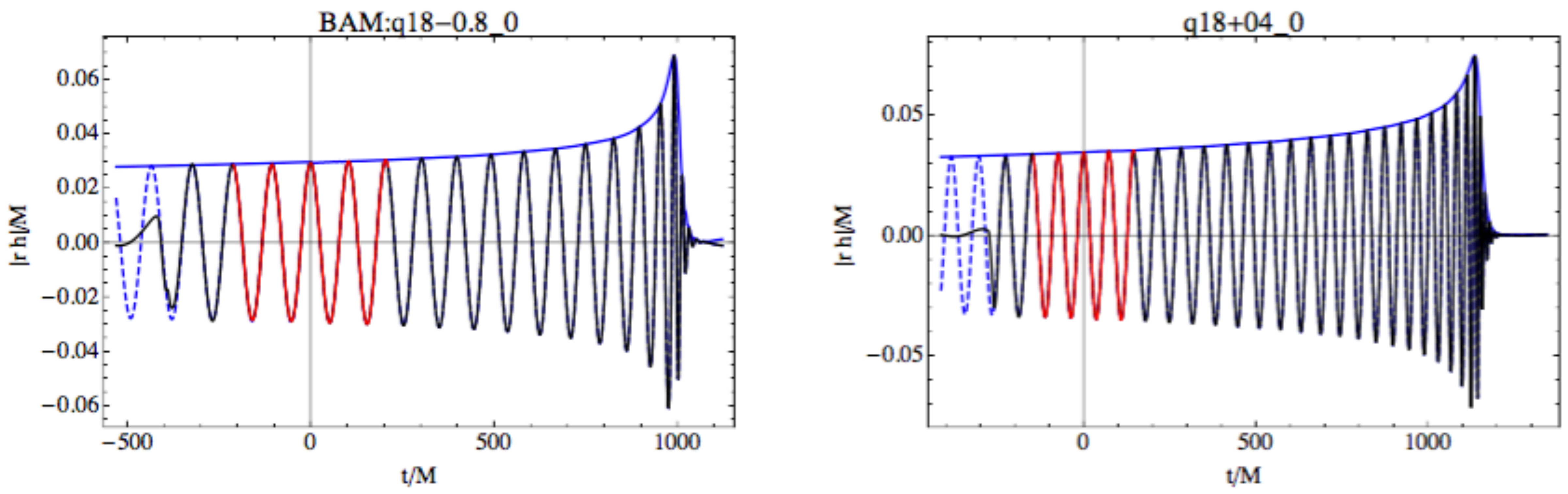


q=1: **S/M²=0.998**
q>1: S/M²=0.95

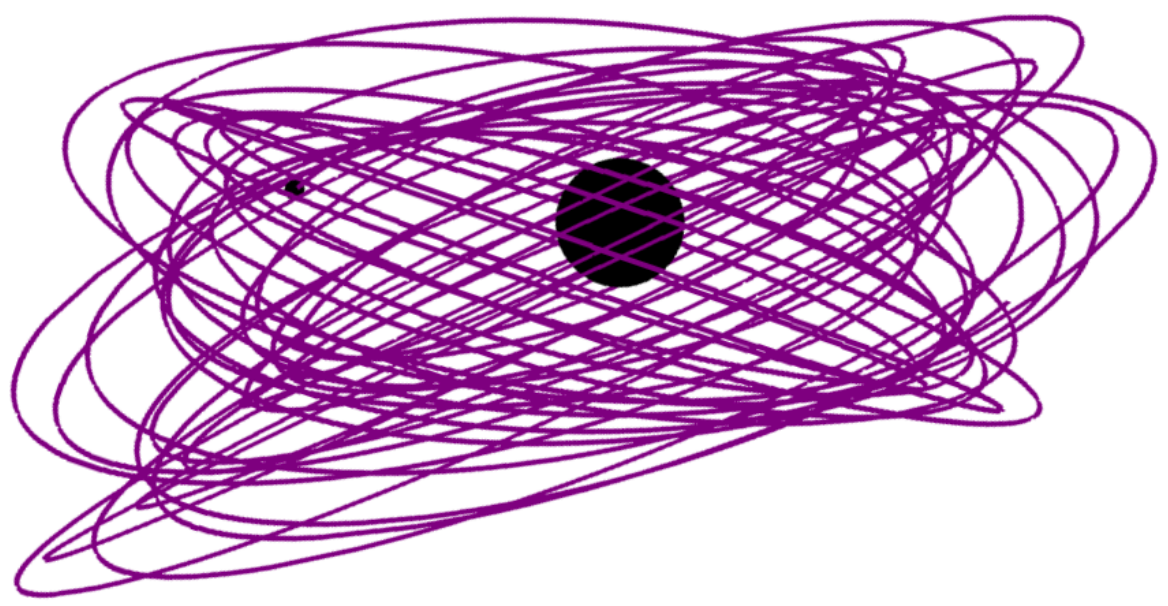
Scheel+ 14
Lovelace..HP+15
Boyle+ 19

Husa+ 15

q=18

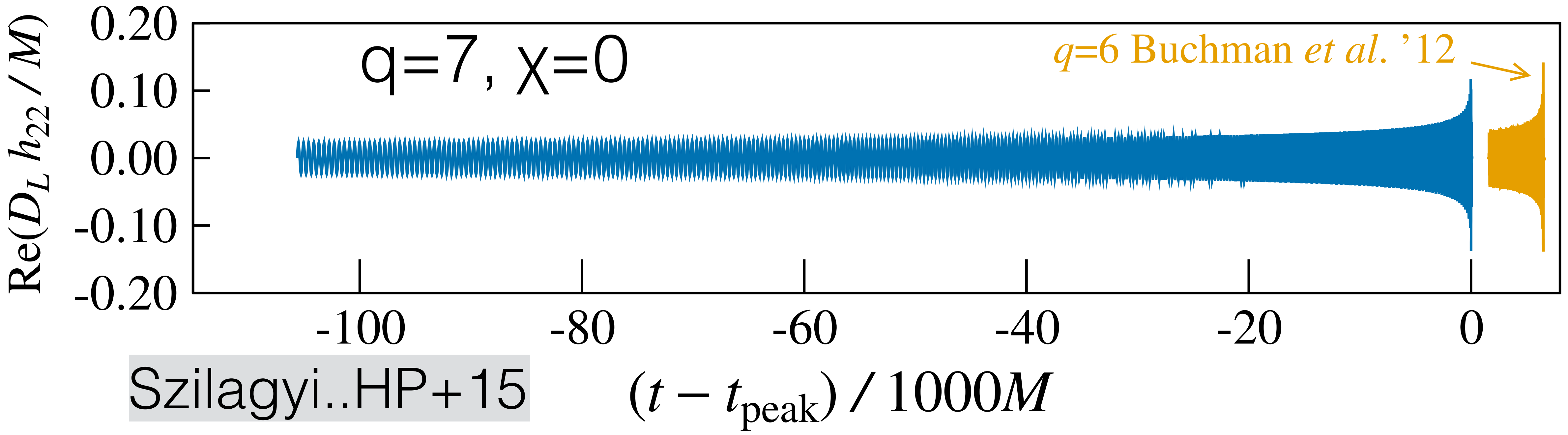


eccentric, precessing



Lewis, Zimmerman, HP 17

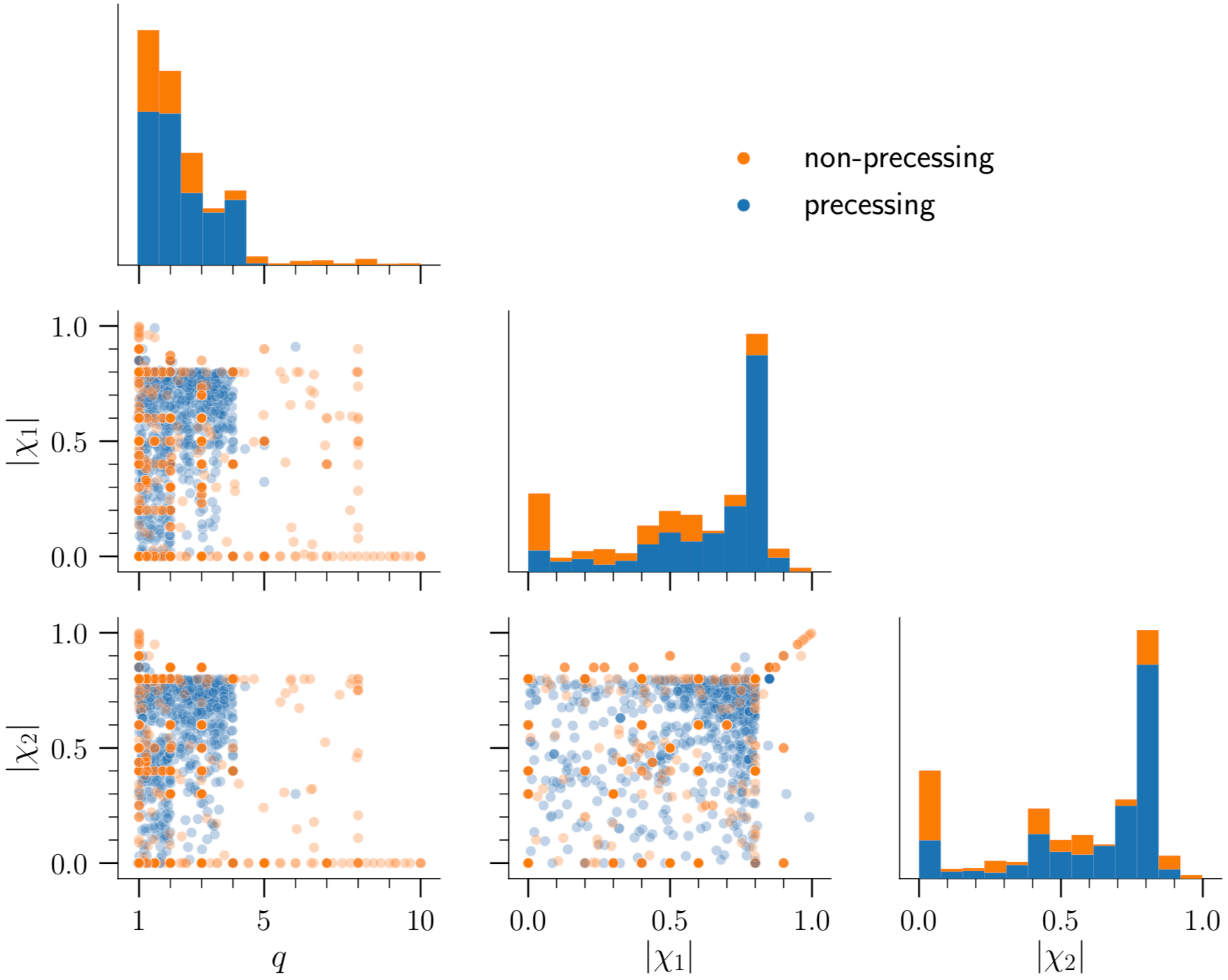
350GW cycles

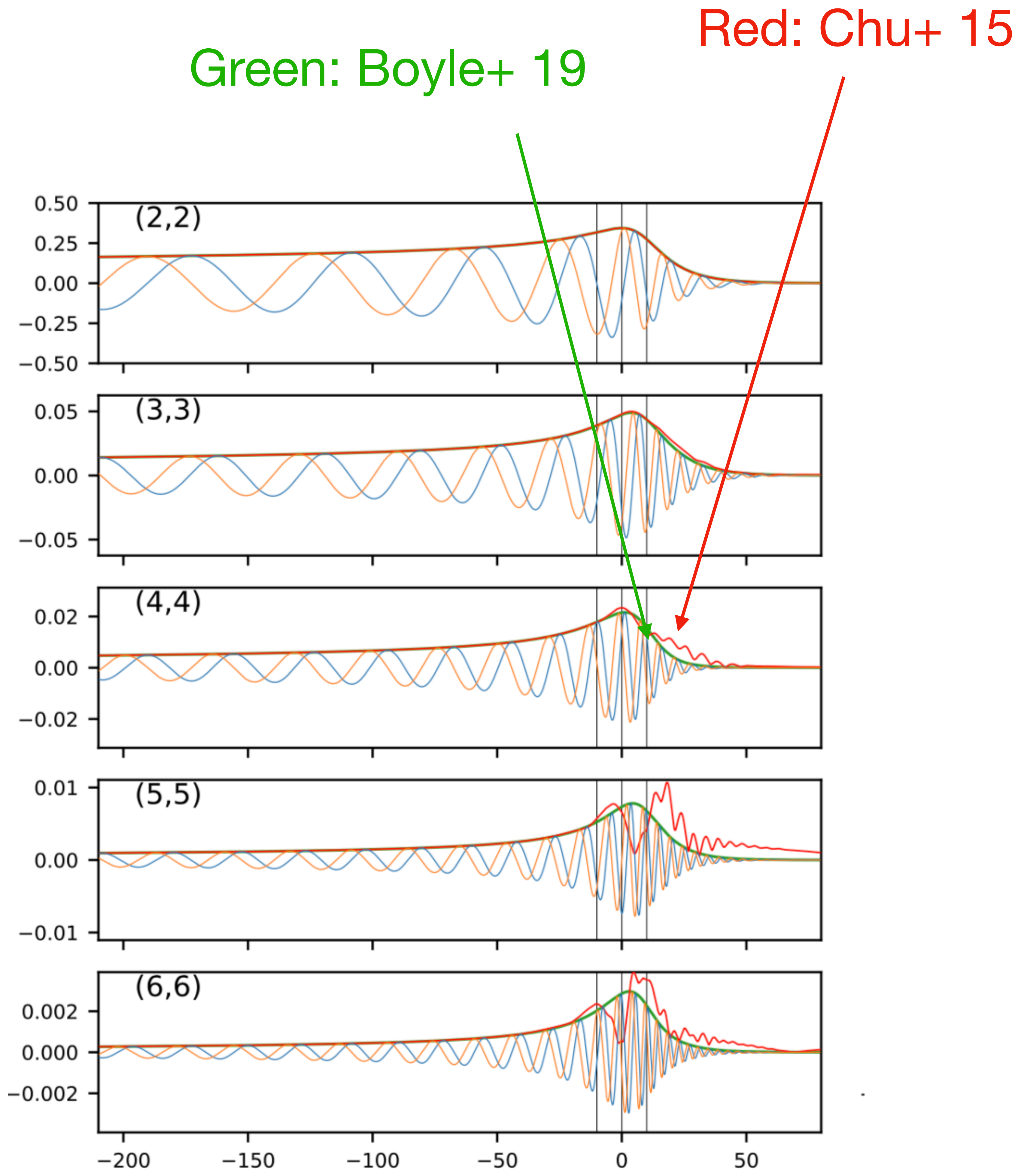


Szilagyi..HP+15

2019-SXS Catalog (~2000 sims)

- More sims, better sims.
  But only somewhat higher q.





SXS (Boyle+), 2019

Current SpEC parameter space trade-offs

- **Relevant**: spin-magnitudes, mass-ratio, length, accuracy
  - *no apparent difficulty with precession, eccentricity, higher modes*
- **"easy" region** of parameter space
  - $\lesssim$ 2 months, virtually no code issues
  - spins $\lesssim 0.8$, $q \lesssim 4$, $f_{min} \gtrsim 20\text{Hz}@50M_\odot$, aLIGO design
- **"tedious" region**
  - easily 6+ months (no upper limit), few code issues
  - **one of:** spins $\lesssim 0.9$ *or* $q \lesssim 10$ *or* $f_{min} \sim 15\text{Hz}@50M_\odot$ *or* 3G/LISA
- else: **"Frontier" region**
  - significant code issues. *If* resolvable, still 6+ months

BBH Numerical Relativity future

- Limitation **Wall-clock time.**
  Sims run O(1) month
- Scaling of number of time-steps

$$N_{\rm steps} \propto \frac{q^2}{(M\Omega_i)^{8/3}}$$

q — more steps per orbit
(Courant limit — *numerics*)
q — more orbits per inspiral
(physics)
$(M\Omega)^{8/3}$ — start frequency

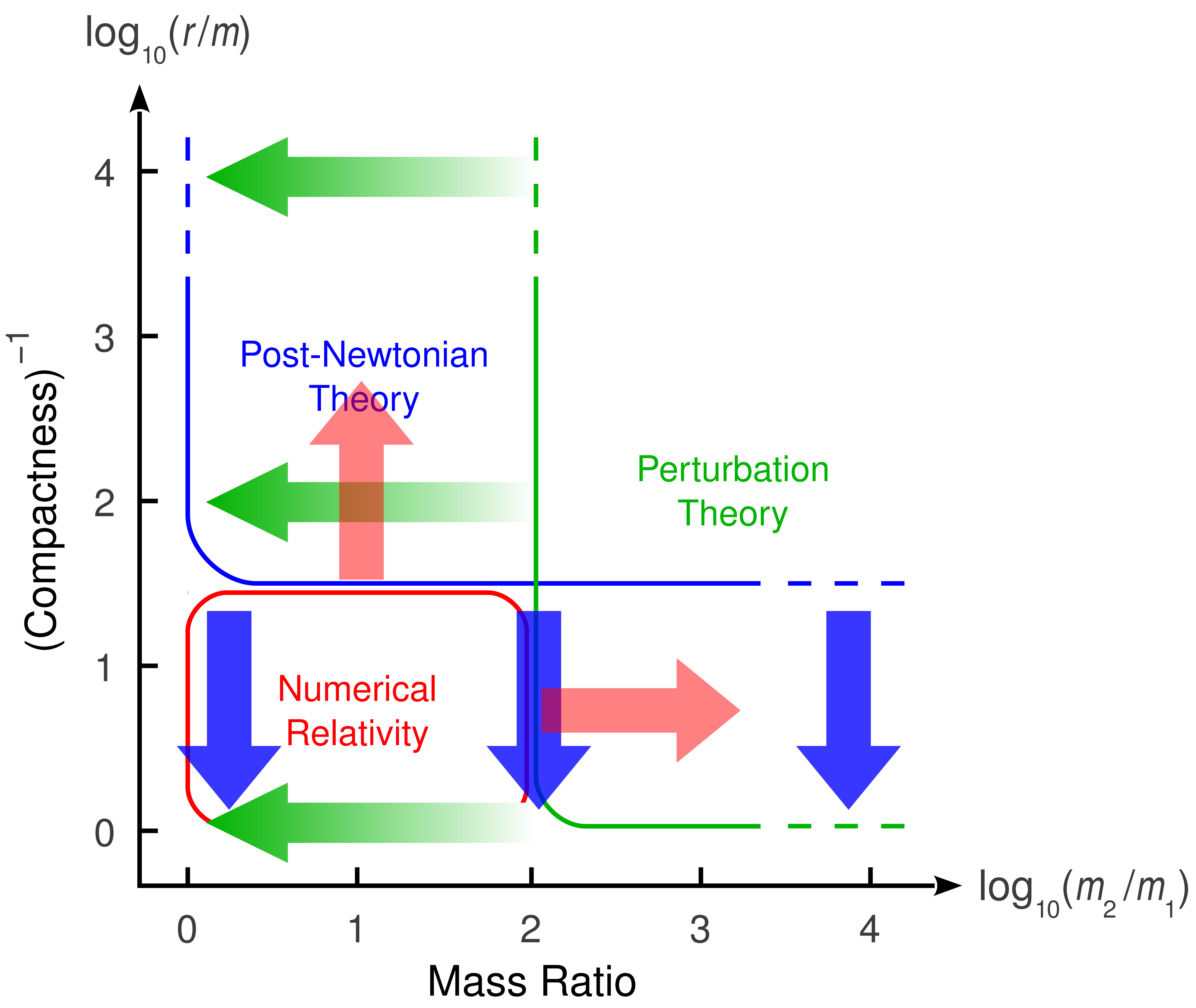
$\chi \gtrsim 0.6$: extra factor $\sim 1/(1-\chi_1)(1-\chi_2)$
χ_2 larger impact than χ_1

- Factor 2 in mass-ratio, factor 2 in low-frequency, higher accuracy
  … and assume same CPU-time per step …
  ➔ **O(100) increase in wall-time (with current codes)**
- Need:
  - Better parallel scaling ➔ reduce constant of proportionality
  - Circumvent small BH courant limit ➔ mitigate q-scaling
  - ***Either requires nearly complete re-development of NR code***

Improving NR

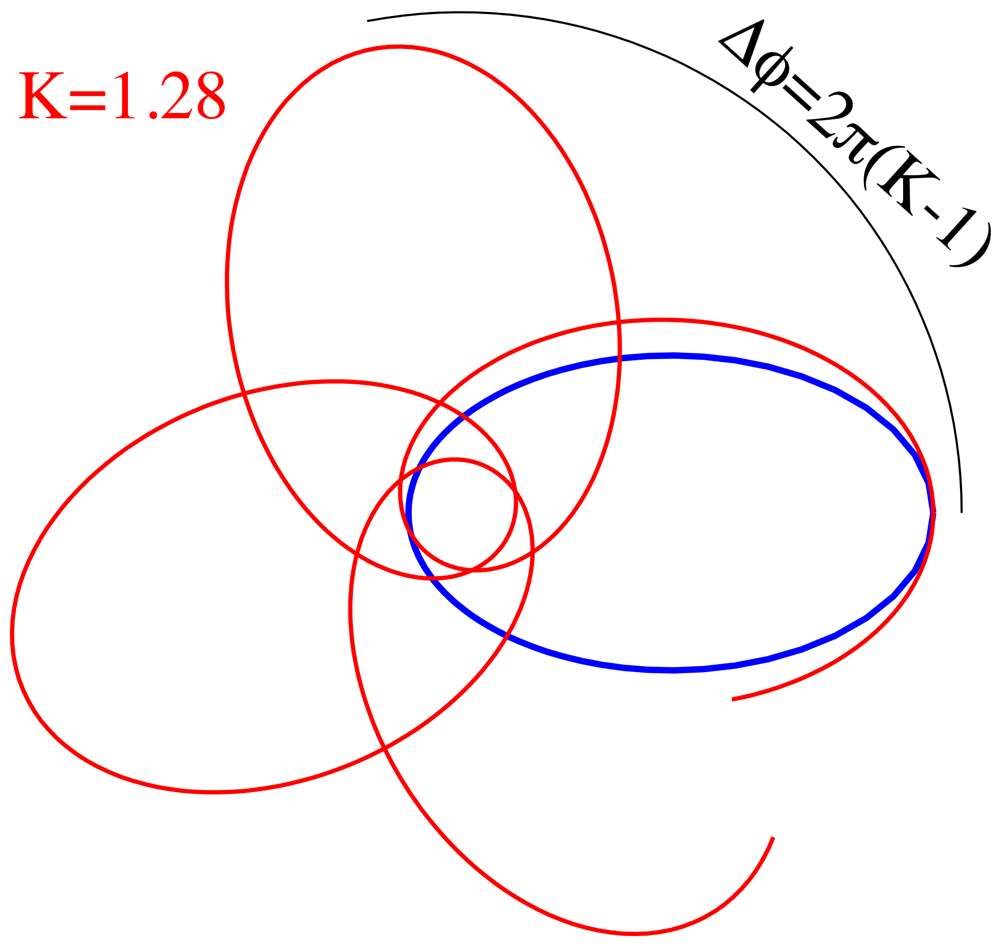
- Changing landscape of high-performance computing (more cores, accelerators)
- New codes, e.g. SpECTRE, DENDRO-GR being developed (open source on GitHub)
- Think about new numerical techniques (implicit-explicit time-stepping, two-timescale for inspired)
- Might narrow the “gap”, but not bridge it completely; may need radical new techniques

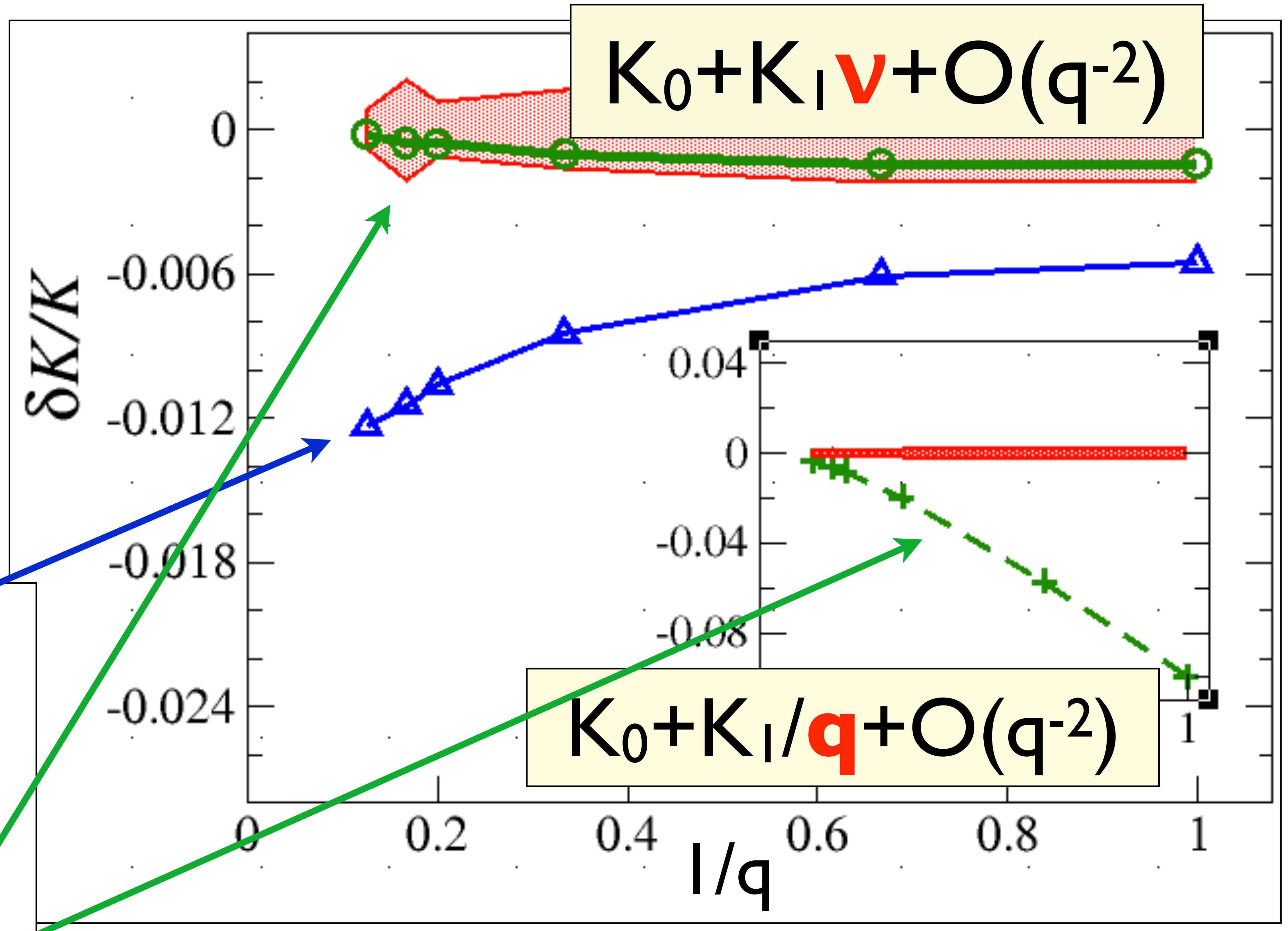
Bridging the "gap"

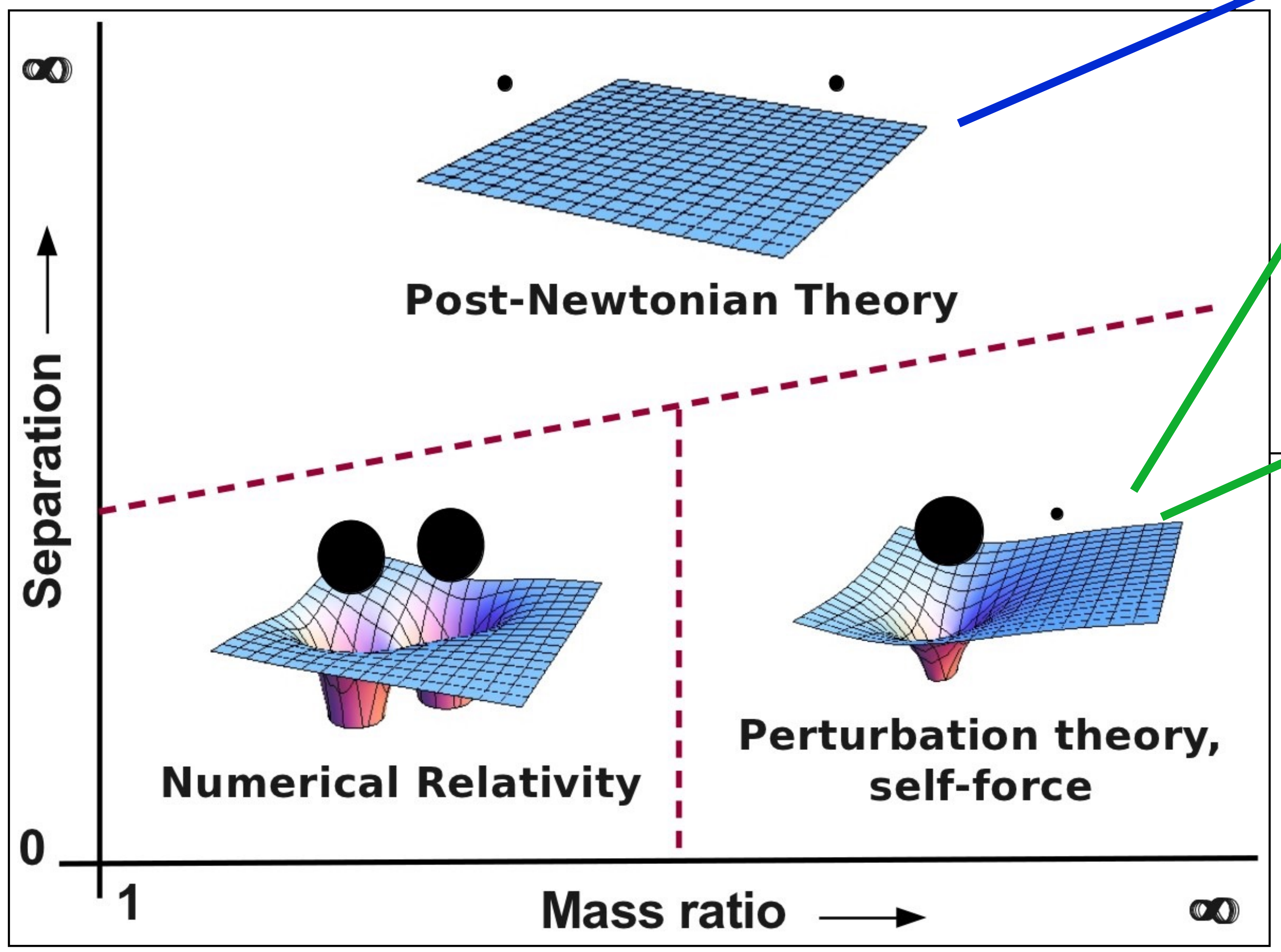


- What are the regions of validity of SF, NR, EOB/PN, PM?
- Depends on what quantity you are interested in.
- Are there comparisons we should be trying to make, that have not yet been made?
- What improvements are required to close the gap (in all areas)?

Bridging the “gap”







Expressed in <u>symmetric mass-ratio</u> $\nu = q/(1+q)^2$, perturbation theory works at equal masses!

Mroue..HP+ 09; Le Tiec..HP+ 2011

Discussion topics

- What needs to be done to **push SF to more comparable mass-ratios**?
  - More people working on 2-SF?
  - Say we have 2-SF, resummation on top?
- In EMRI regime, **what science is lost without 2-SF?**
  - LIGO PE biases smaller than originally projected.
  - Are the O(1) phase-errors after 1/v orbits compensated by a O(v) fractional change in mass-ratio?
- What will become possible in **NR in, say, 5 years?**
- What needs to be done to **push NR toward higher mass-ratios**?
  - computational techniques (optimisation, parallelisation)
  - formulations (e.g. gauge)
  - Can NR beat the Courant limit?
- What role is played by **PN, PM, EOB?**
- Is more attention needed for **BBH Resonances?**
  - At what mass-ratios do they become important?
  - What science is lost w/o models of resonances?
  - Can NR play a role in exploring resonances?

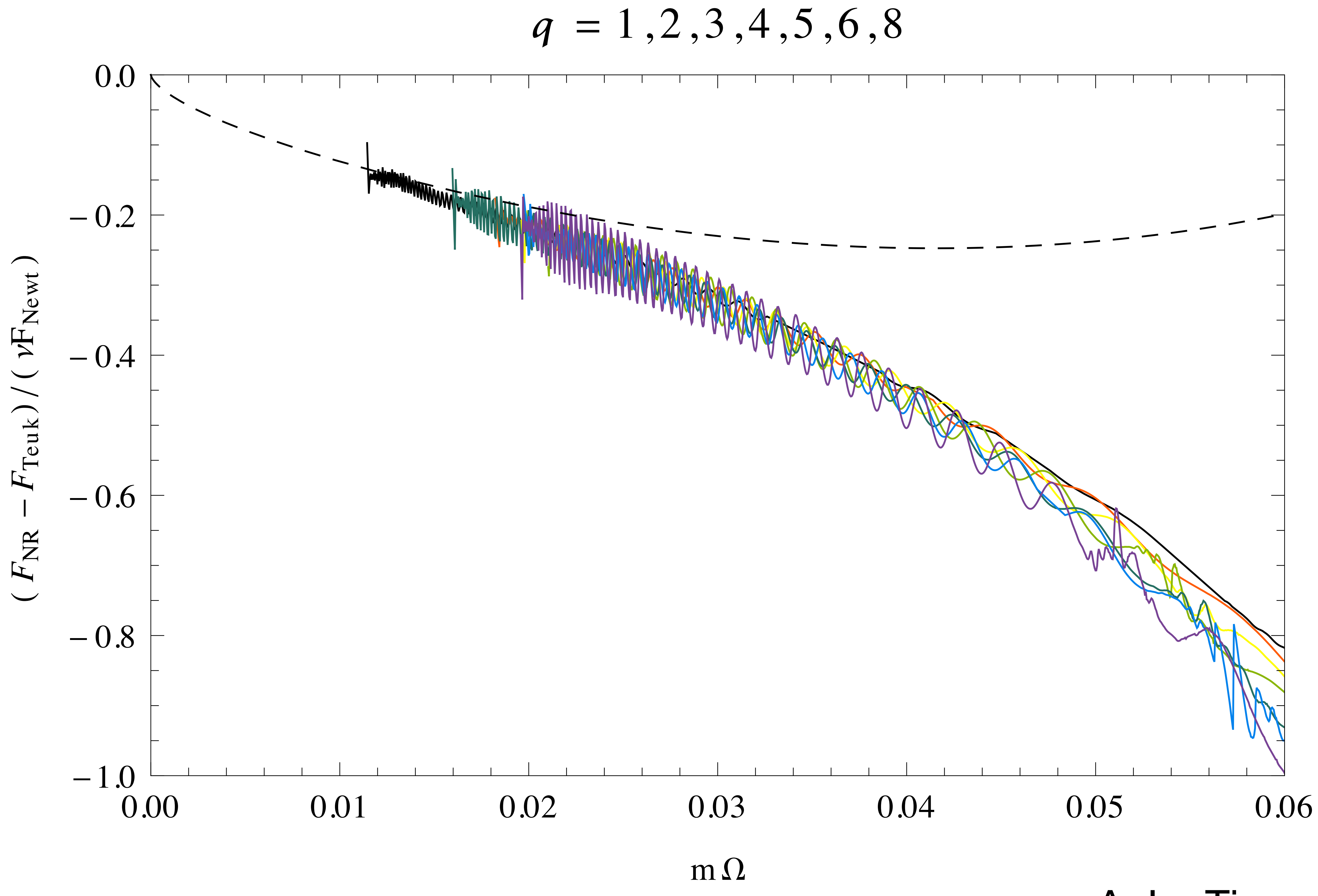
- How can we **quantify a possible IMRI-gap?**
  - intermediate quantities (periastron advance)
  - waveforms
- How can we **bridge the IMRI-gap?**
  - push NR + SF?
  - Interpolation methods like EOB? How to quantify errors?
  - Something entirely different?

- What **info is needed for waveform modeling** (from NR, SF)?
  - phase-coherent, complete inspirals
  - shorter inspiral snippets, say $\sim v^{-1/2}$ orbits
  - instantaneous info: $F_{GW}(\Omega_{GW})$, $h_{lm}(\Omega_{GW})$
  - Other characteristic information (periastron/redshift/…)

- **Templates, fast waveform models and data-analysis**
  - What is needed to interface SF/PN/EOB/NR
  - What is required by LISA data-analysis
  - Can kludge models be ‘automatically’ updated with new info

Bridging the “gap”

Gravitational wave energy flux comparisons indicate that self-force may be sufficient even for q=1!



A. Le Tiec