Nicolai-fest

Golm, September 6-8

Most has been said already about Hermann's unique personality, and contributions to physics

To paraphrase Murat and Francois:

Hermann is the best living approximation to our image of a physicist in the (romantic) pre-war era

His long views, tolerant and demanding attitude
to physics, care for his students and younger colleagues,
have earned him *(in addition to a higher state of bliss)*
the friendship and respect of us all.

We heard a lot about Hermann's major contributions to
N=8 supergravity, and its exceptional symmetries.

Much less about some of his other important inputs,
like the **Nicolai map**, or the dW-H-N matrix model

of the **supermembrane.**

[one of most tantalizing hints about M theory]

One other aspect of Hermann's personality, as befits the Renaissance man that he is, is his **appreciation of fine wine.**

This field of human activity has its own sophisticated theories, its **landscape** and its **swampland**, and it is subject to more direct experimental verification.

As we have realized one day with Slava chez Jean-Pierre, wine-theorists have also independently discovered **branes**.

If you type brane in weekipedia, that’s what you get:*

Brane may refer to:

- Membrane (M-theory), a spatially extended mathematical concept that appears in M-theory
- Brane, archaic name of Bordeaux wine producer Château Brane-Cantenac
- Brane-Mouton, archaic name of Bordeaux wine producer Château Mouton Rothschild

* Our branes have been discovered later, and have not yet been produced in the laboratory, so they are cited first !

HAPPY BIRTHDAY HERMANN

AdS4 / CFT3 & localization of gravity

Costas Bachas

(Ecole Normale, Paris)

based on:

CB, J. Estes, *arXiv:1103.2800 [hep-th]*

B. Assel, CB, J. Estes, J. Gomis, *1106.4253 [hep-th]*

1209.xxxx [hep-th]

Standard Hypothesis:

4D Graviton

→ is **massless**

→ couples **universally**

to all forms of matter & radiation

Can it be otherwise ?

These properties are automatic in any **compactification** of string theory *(or higher-dim. theory of gravity).*

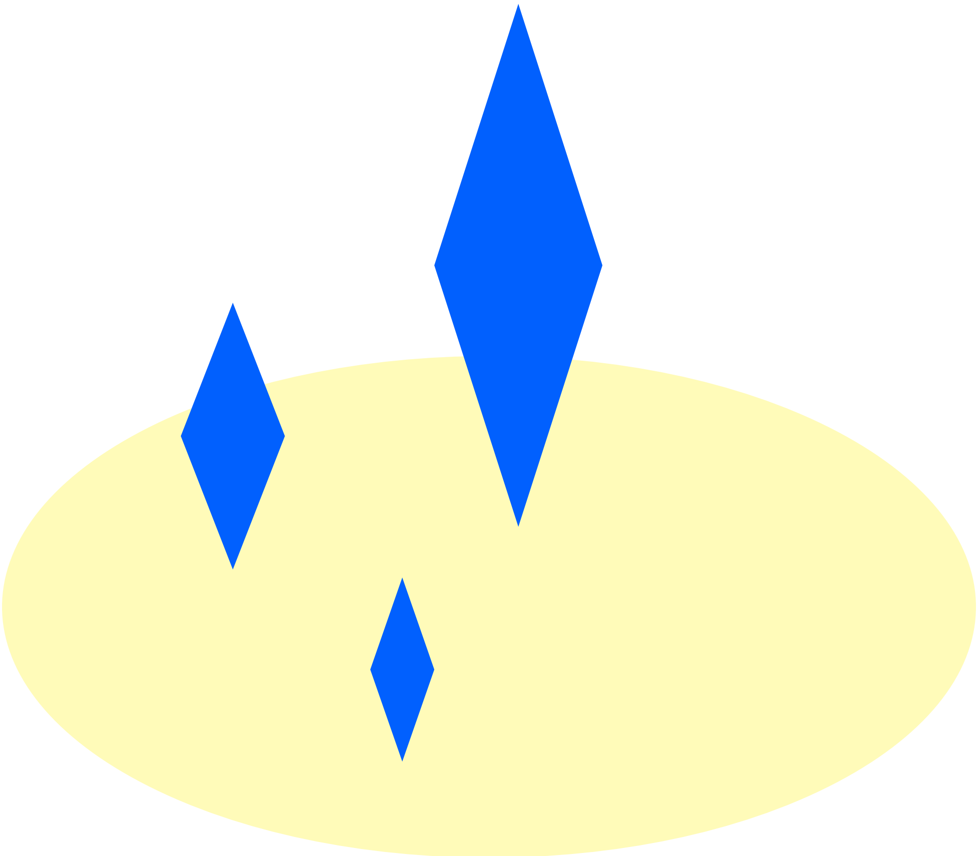
[Note: I will stick with classical 2-derivative (super)gravities]

KK reduction for spin 2:

Consider *warped*-(A)dS vacuum,

$$\widehat{ds^2} = e^{2A(y)} \boxed{\bar{g}_{\mu\nu}(x)\, dx^\mu dx^\nu} + \boxed{\hat{g}_{ab}(y)\, dy^a dy^b}$$

$$\bar{\mathcal{M}}_4 = \mathrm{AdS}_4,\ \mathbb{M}_4,\ \mathrm{dS}_4$$
$$k = -1, 0, 1$$



Consistent reduction of (spin-2) metric perturbations:

$$ds^2 = e^{2A}\,(\bar{g}_{\mu\nu} + h_{\mu\nu})\,dx^\mu dx^\nu + \hat{g}_{ab}\,dy^a dy^b \;\;,$$

with $\qquad h_{\mu\nu}(x,y) \;=\; h^{[\mathrm{tt}]}_{\mu\nu}(x)\,\psi(y)$

where $\qquad \boxed{(\bar{\Box}^{(2)}_x - \lambda)\,h^{[\mathrm{tt}]}_{\mu\nu} = 0 \qquad \text{and} \qquad \bar{\nabla}^\mu h^{[\mathrm{tt}]}_{\mu\nu} = \bar{g}^{\mu\nu} h^{[\mathrm{tt}]}_{\mu\nu} = 0\;.}$

Pauli-Fierz equations $\qquad (\lambda = m^2 + 2k)$

Linearizing the Einstein equations $R_{MN} - \frac{1}{2} g_{MN} R = T_{MN}$

leads to a **universal Schrödinger problem** in the 6D transverse space:

depends only on geometry, not on "matter" fields

Brandhuber, Sfetsos;
Csaki, Erlich, Hollowood, Shirman;
CB, Estes

$$\boxed{-\frac{e^{-2A}}{\sqrt{[\hat{g}]}} \left(\partial_a \sqrt{[\hat{g}]}\, \hat{g}^{ab} e^{4A} \partial_b\right) \psi = m^2 \psi}$$

mass operator $\mathcal{M}$

Using the standard norm $\|\psi\|^2 = \int [dy] e^{2A} \psi^* \psi$ one finds :

$$\langle \psi, \mathcal{M}^2 \psi \rangle = \int [dy] e^{4A} \partial_a \psi^* \partial^a \psi$$

$$\boxed{\mathcal{M}^2 \geq 0 \quad \text{and} \quad \mathcal{M}^2 = 0 \longrightarrow \psi_0 = constant}$$

- A massless 4D graviton requires $\int [dy]\, e^{2A} < \infty$ (automatic for smooth compactification)

- Its couplings are **universal**: $\int [dy]\, e^{2A}\, \psi_0 \phi_i^* \phi_j = \psi_0\, \delta_{ij}$

(To be contrasted to fields with lower spin, whose 4D massless modes can have non-trivial dependence on the "internal" space)

To avoid these conclusions:

- 4D graviton must be massive

- transverse space must be non-compact

A model for this was proposed ten years ago by **Karch & Randall:**

Thin AdS4 brane in AdS5 bulk

cf also Randall, Sundrum;
Dvali, Gabadadze, Porrati; .....

$$I_{\rm KR} = -\frac{1}{2\kappa_5^2}\int d^4x\, dy\, \sqrt{g}\left(R + \frac{12}{L^2}\right) + \lambda \int d^4x\, \sqrt{[g]_4}\ ,$$

AdS5 radius

brane tension

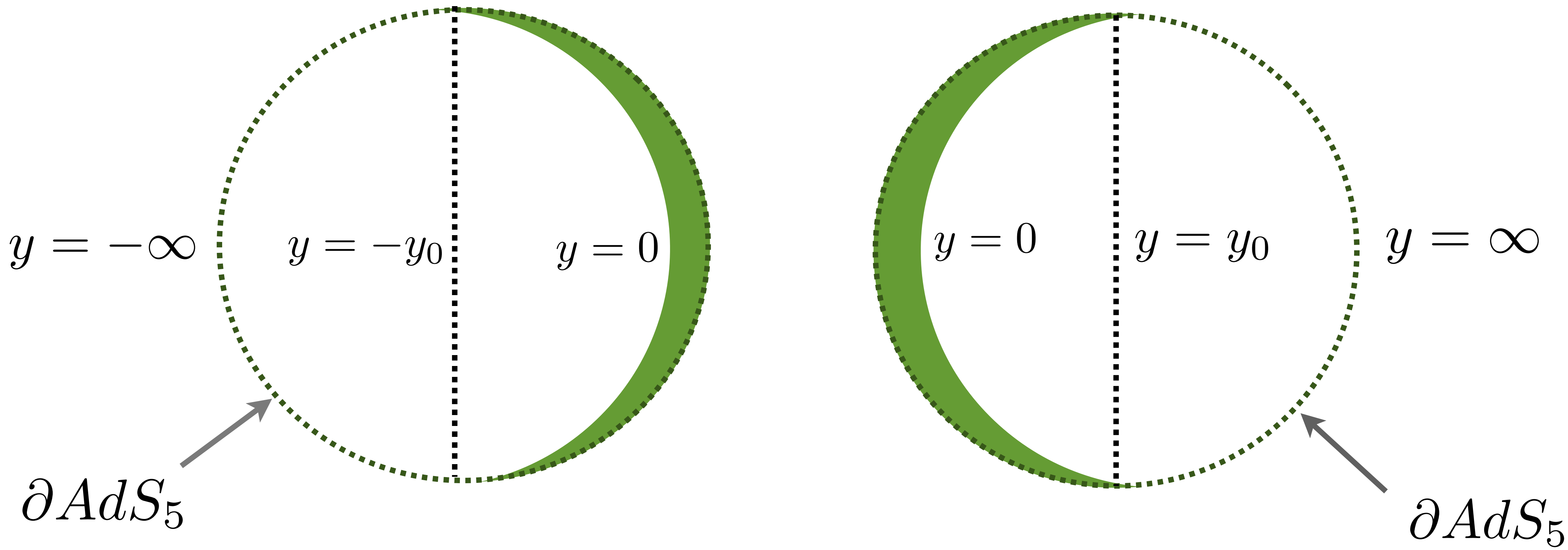
$$ds^2 = L^2 \cosh^2\left(\frac{y_0 - |y|}{L}\right)\bar{g}_{\mu\nu}dx^\mu dx^\nu + dy^2$$

AdS4 foliation

Brane radius of curvature: $\ell = L\cosh(y_0/L) = \dfrac{L}{\sqrt{1-\kappa_5^2\lambda L/6}} \gg L$

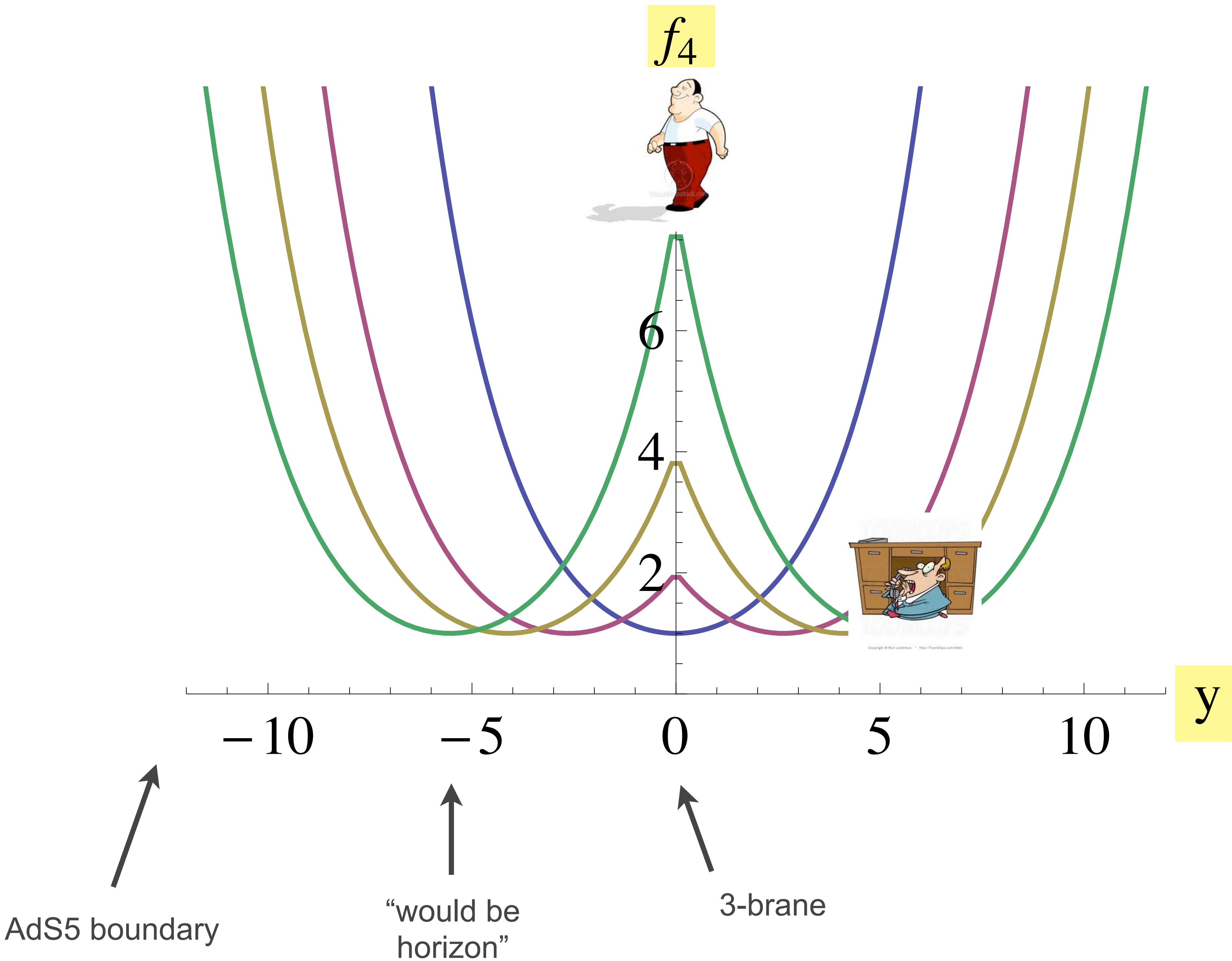
can be tuned so that

Two slices of AdS5 glued along a AdS4 brane:



(cut away green slices and glue)

Warp factor as function of transverse coordinate:



(Randall-Sundrum II)

Spectrum :

one nearly-constant, **nearly massless** mode

vanishes near bottom of warp factor wells

$$\ell^2 m_0^2 \simeq \frac{3L^2}{2\ell^2}$$

two infinite towers of nearly AdS5 modes

supported at bottom of wells

$$\ell^2 m^2 \simeq (2n+1)(2n+4) \qquad n = 0, 1, \cdots$$

Their wavefunctions are **exponentially suppressed** at the brane position, so they are **hidden from 4D gravity:**

$$\int [dy] e^{2A}\, \psi_0 \phi_h^* \phi_h \ll \psi_0$$

No conventional effective 4D theory, but physics does look 4D at

$$L \ll r \ll \ell$$

Newton's law:

$$8\pi G_N \simeq \kappa_5^2/L$$

as in usual KK

$$V_{\rm Newton} + \Delta V \simeq -\frac{G_N m_1 m_2}{r}\left(1 + \gamma\frac{L^2}{r^2} + \cdots\right)$$

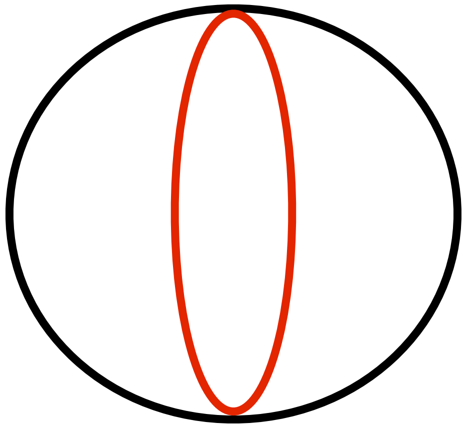
unlike standard KK

Warning: this is not a model for the Universe, since $\Lambda_{\rm eff} < 0$

But **any** long-distance modification of Einstein gravity is (potentially) interesting!

Can the KR model be embedded in string theory ?

(holographic duals to conformal domain walls of CFT_4)



S^3

Boundary in global coordinates

Two beautiful developments got us started:

Lunin; Gomis, Romelsberger

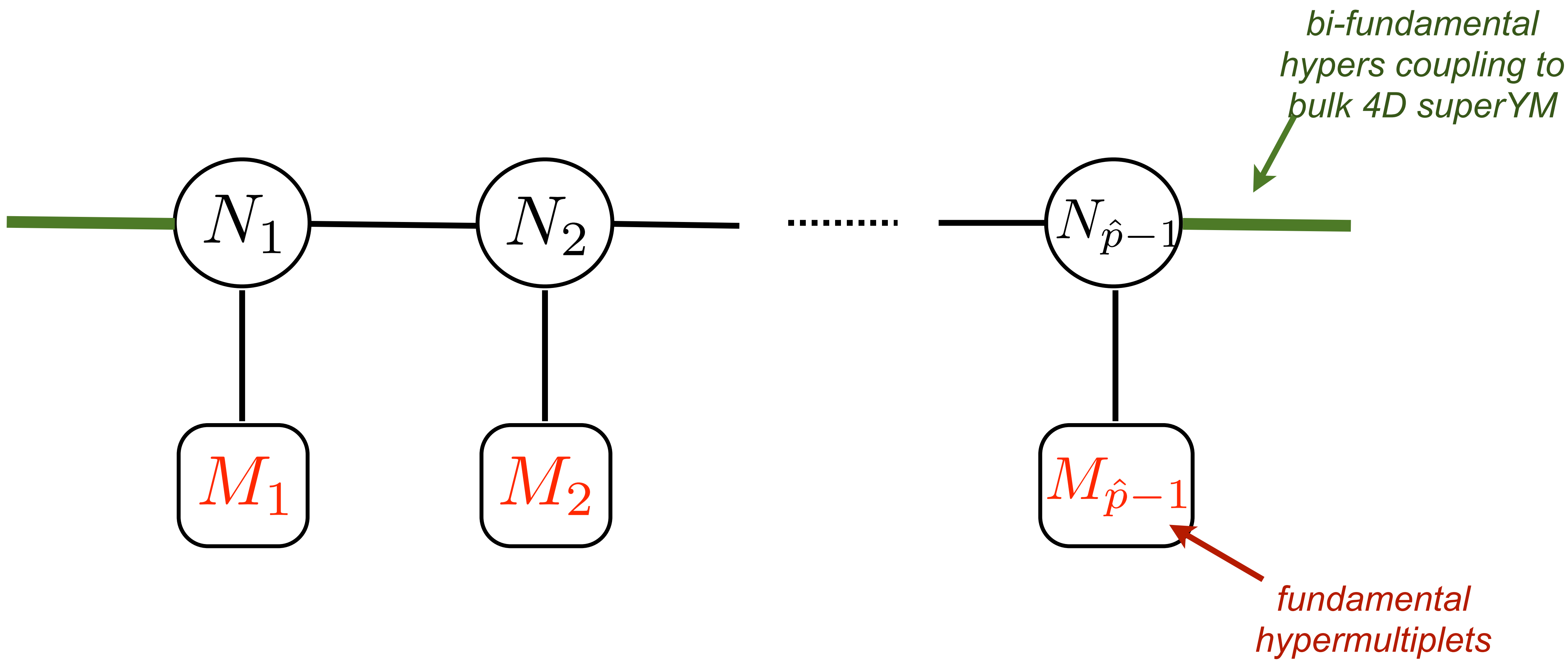
D'Hoker, Estes & Gutperle '07 found the **general local form** of type-IIB supergravity solutions with $OSp(2,2|4) \supset SO(2,3) \times SO(3) \times SO(3)$ supersymmetry.

This is the symmetry of conformal 1/2-BPS domain walls in N=4 D=4 super-Yang-Mills theory, which were classified by **Gaiotto & Witten '08**

DeWolfe, Freedman, Ooguri; .....

To address our question, we established the precise dictionary between these two works. Bonus: some new results on AdS4/CFT3, which is for several reasons the most interesting of holographic dualities.

The Gaiotto-Witten domain walls support **linear-quiver N=4 CFT3s**

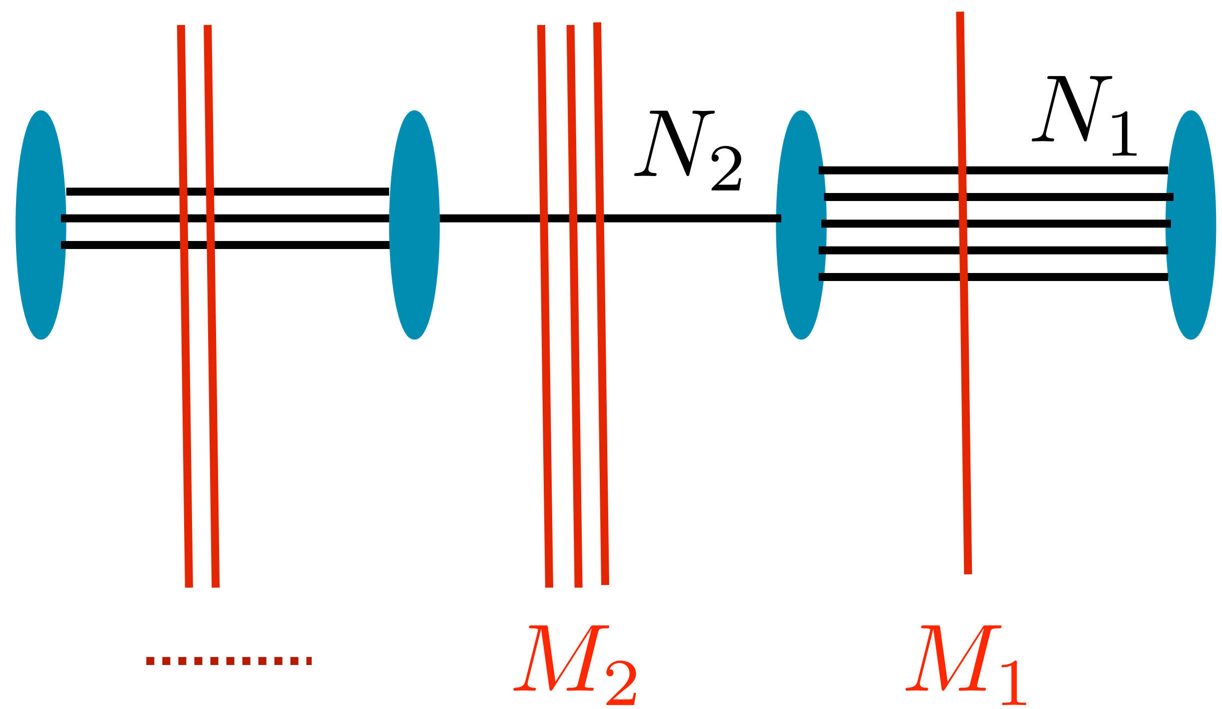


$U(N_1) \times U(N_2) \times \cdots \times U(N_{\hat{p}-1})$ gauge group

$U(M_1) \times U(M_2) \times \cdots \times U(M_{\hat{p}-1})$ **manifest** global symmetry

$U(\hat{M}_1) \times U(\hat{M}_2) \times \cdots \times U(\hat{M}_{p-1})$ **hidden** global symmetry

Engineered on D3-branes suspended/intersecting NS5- and D5- branes



Hanany-Witten

Mirror symmetry exchanges NS5 with D5 (Intriligator, Seiberg; de Boer, Hori, Ooguri, Oz; .....)

(Almost) invariant description in terms of **linking numbers**:

i th D5	ℓ_i	= (net # of D3-branes ending on it from left) + (# of NS5 to its right)
j th NS5	$\hat{\ell}_j$	= (net # of D3-branes ending on it from right) + (# of D5 to its left)

Focus on 3D gauge theories (coupling to 4D YM: slight complication)

Gaiotto-Witten conjectured that these **flow to non-trivial SCFT** whenever

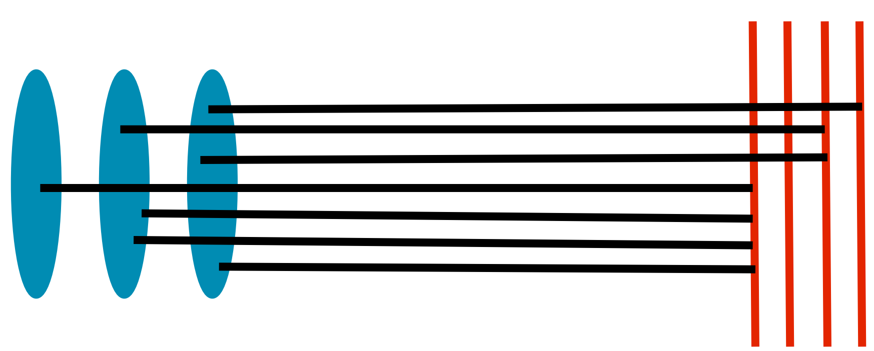
$$\# \ hypers \geq 2N_a$$ for each gauge group

(complete Higgsing)

These conditions translate to $\hat{\ell}_1 \geq \hat{\ell}_2 \cdots \geq \hat{\ell}_{\hat{p}} > 0$ (labelled from right to left)

while also $\ell_1 \geq \ell_2 \cdots \geq \ell_p > 0$ (labelled from left to right)

Move all NS5 to the left of all D5 $\Longrightarrow$ $\boxed{\sum \ell_i = \sum \hat{\ell}_j = N}$



$\downarrow$

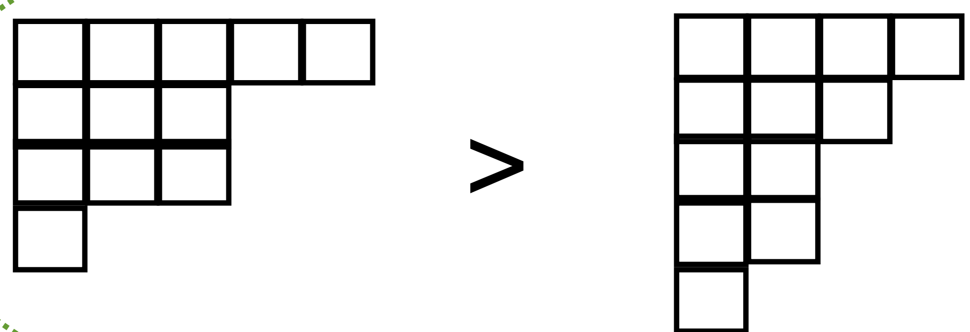
two partitions ρ & $\hat{\rho}$

Supersymmetry (s-rule) implies the (partial ordering) constraints

$$\rho^T > \hat{\rho}$$

(# of boxes in first n columns of ρ **>** # boxes in first n rows of $\hat{\rho}$ $\quad \forall\ n$)

e.g.



arise in many contexts related to Nahm's equations

Nakajima; Kronheimer; CB, Hoppe, Pioline; ....

Summarize: **non-trivial N=4 SCFT3** $T^{\hat{\rho}}_{\rho}(SU(N))$

conjectured for any pair of partitions obeying these constraints

The IIB solutions are **AdS4 x S2 x S2** fibrations over a Riemann surface basis Σ . Local solutions are determined by two **harmonic functions** h_1 and h_2 . Modulo an **SL(2,R) rotation**:

metric :

$$ds^2 = f_4^2 ds^2_{\mathrm{AdS}_4} + f_1^2 ds^2_{\mathrm{S}_1^2} + f_2^2 ds^2_{\mathrm{S}_2^2} + 4\rho^2 dz d\bar{z} \ ,$$

$$f_4^8 = 16 \frac{N_1 N_2}{W^2} \ , \quad f_1^8 = 16\, h_1^8 \frac{N_2 W^2}{N_1^3} \ , \quad f_2^8 = 16\, h_2^8 \frac{N_1 W^2}{N_2^3}$$

$$\rho^8 = \frac{N_1 N_2 W^2}{h_1^4 h_2^4}$$

dilaton : $e^{4\phi} = \frac{N_2}{N_1}$

where

$$W = \partial h_1 \bar{\partial} h_2 + \bar{\partial} h_1 \partial h_2 = \partial\bar{\partial}(h_1 h_2) \ ,$$

$$N_1 = 2h_1 h_2 |\partial h_1|^2 - h_1^2 W \ , \qquad N_2 = 2h_1 h_2 |\partial h_2|^2 - h_2^2 W \ .$$

There are also p-form backgrounds: $F_5\,,\; H_3\,,\; F_3$

expressed in terms of the dual harmonic functions:

$$h_1 = -i(\mathcal{A}_1 - \bar{\mathcal{A}}_1) \quad \longrightarrow \quad h_1^D = \mathcal{A}_1 + \bar{\mathcal{A}}_1$$

$$h_2 = \mathcal{A}_2 + \bar{\mathcal{A}}_2 \quad \longrightarrow \quad h_2^D = i(\mathcal{A}_2 - \bar{\mathcal{A}}_2)$$

and the (normalized) volume forms of the (pseudo)spheres:

ω^{0123}	AdS_4
ω^{45}	S_1^2
ω^{67}	S_2^2

<u>Note</u>: the constant parts of $Re(\mathcal{A}_1)$ and $Im(\mathcal{A}_2)$ are pure gauge.

3-forms :
$$H_{(3)} + iF_{(3)} = \omega^{45} \wedge db_1 + i\,\omega^{67} \wedge db_2$$

where:
$$b_1 = 2ih_1 \frac{h_1 h_2(\partial h_1 \bar{\partial} h_2 - \bar{\partial} h_1 \partial h_2)}{N_1} + 2h_2^D$$
$$b_2 = 2ih_2 \frac{h_1 h_2(\partial h_1 \bar{\partial} h_2 - \bar{\partial} h_1 \partial h_2)}{N_2} - 2h_1^D$$

5-form :
$$F_{(5)} = -4\, f_4^4\, \omega^{0123} \wedge \mathcal{F} + 4\, f_1^2 f_2^2\, \omega^{45} \wedge \omega^{67} \wedge (*_2 \mathcal{F}) \ ,$$

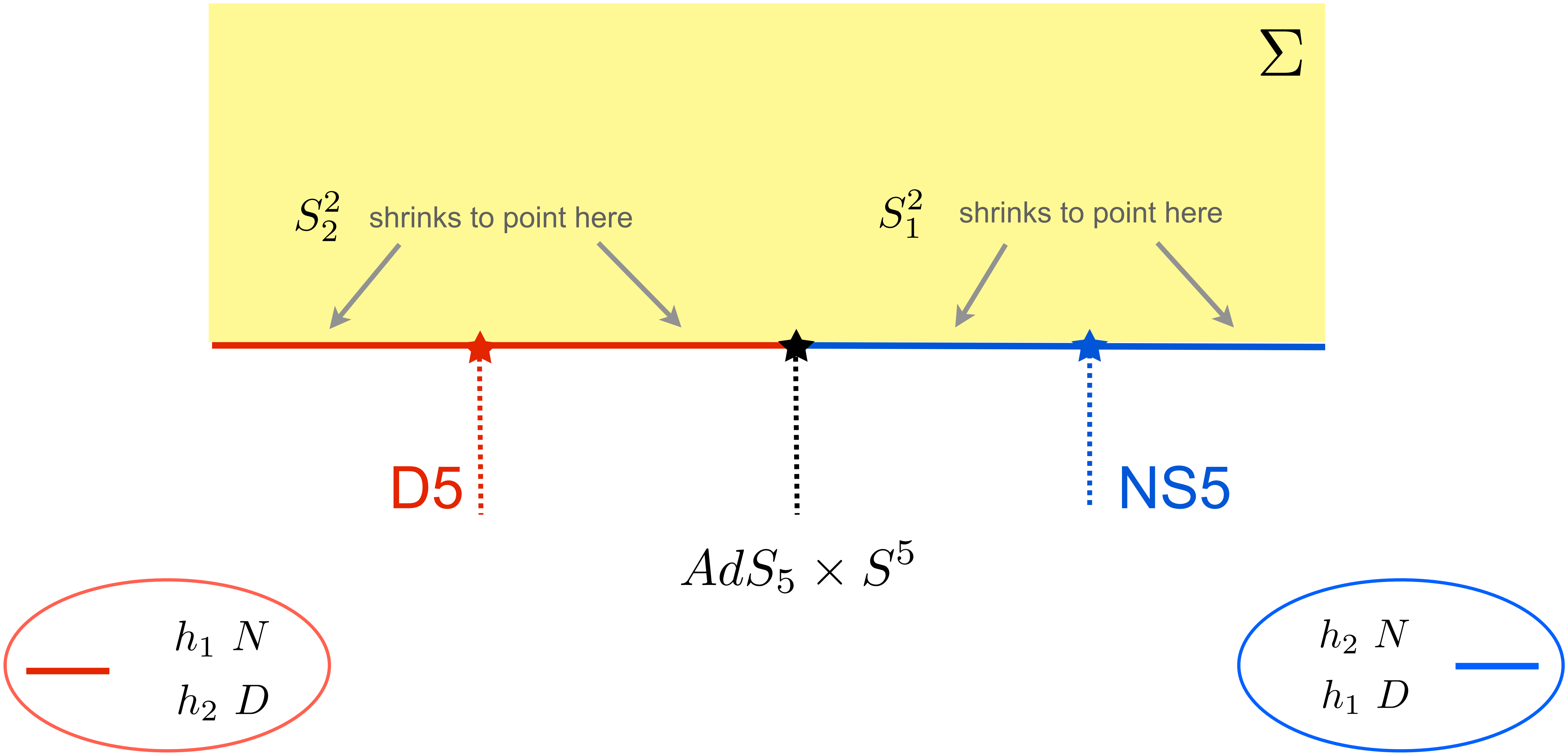
where

$$f_4^4 \mathcal{F} = dj_1 \qquad \text{with} \qquad j_1 = 3\mathcal{C} + 3\bar{\mathcal{C}} - 3\mathcal{D} + i\frac{h_1 h_2}{W}(\partial h_1 \bar{\partial} h_2 - \bar{\partial} h_1 \partial h_2)$$

$$\partial \mathcal{C} = \mathcal{A}_1 \partial \mathcal{A}_2 - \mathcal{A}_2 \partial \mathcal{A}_1 \qquad\qquad \mathcal{D} = \bar{\mathcal{A}}_1 \mathcal{A}_2 + \mathcal{A}_1 \bar{\mathcal{A}}_2$$

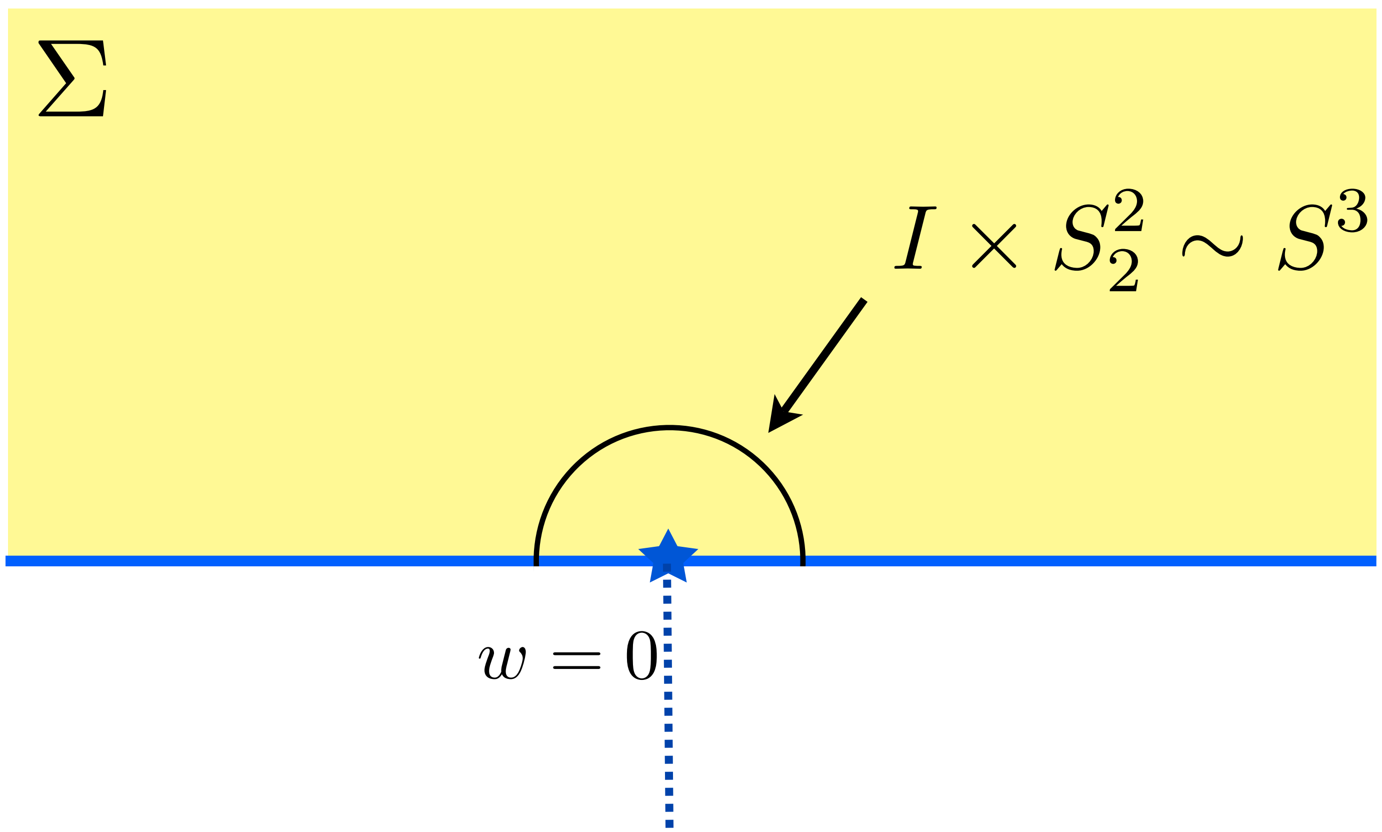
The surface Σ can have **boundaries** which are **interior points** of the 10D geometry. For this, require either h_1 to have a **Dirichlet** condition and h_2 a **Neumann** condition, or vice versa. One 2-sphere then shrinks to zero, and the geometry is (locally) $\sim AdS_4 \times S^2 \times \mathcal{D}^2$

Asymptotic regions arise as singularities on boundary:



Local form of **NS5 singularity**:

(For D5-brane: exchange roles of $\mathcal{A}_1$ and $\mathcal{A}_2$)



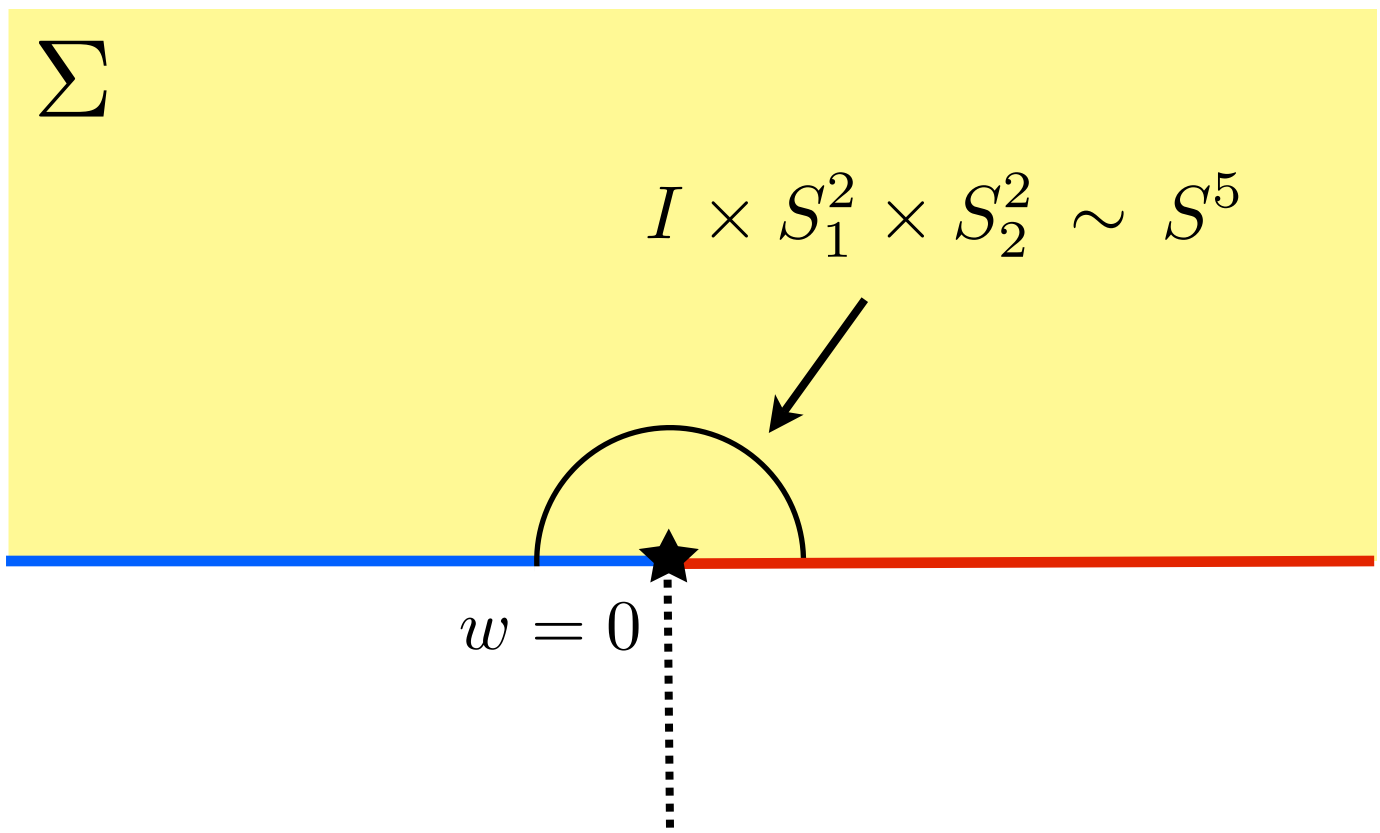
$$\mathcal{A}_1 \simeq \pi\hat{\ell} + \cdots$$

$$\mathcal{A}_2 \simeq -\hat{M}\log w + \cdots$$

$$\hat{M} = \frac{1}{16\pi^2}\int_{S^3} H_3$$ ← **# NS5 branes**

$$\hat{M}\hat{\ell} = \frac{1}{(4\pi)^4}\int_{S^3\times S_1^2}(F_5 + C_2 \wedge H_3)$$ ← **D3-brane Page charge**

Local form of $AdS_5 \times S^5$:



$$\mathcal{A}_1 = \frac{1}{\sqrt{w}}(a_1 + b_1 w + \cdots)$$

$$\mathcal{A}_2 = \frac{1}{\sqrt{w}}(a_2 + b_2 w + \cdots)$$

$$n = \frac{1}{(4\pi)^4}\int_{S^5} F_5 = \frac{(a_2 b_1 - a_1 b_2)}{2\pi}$$ ← D3 charge

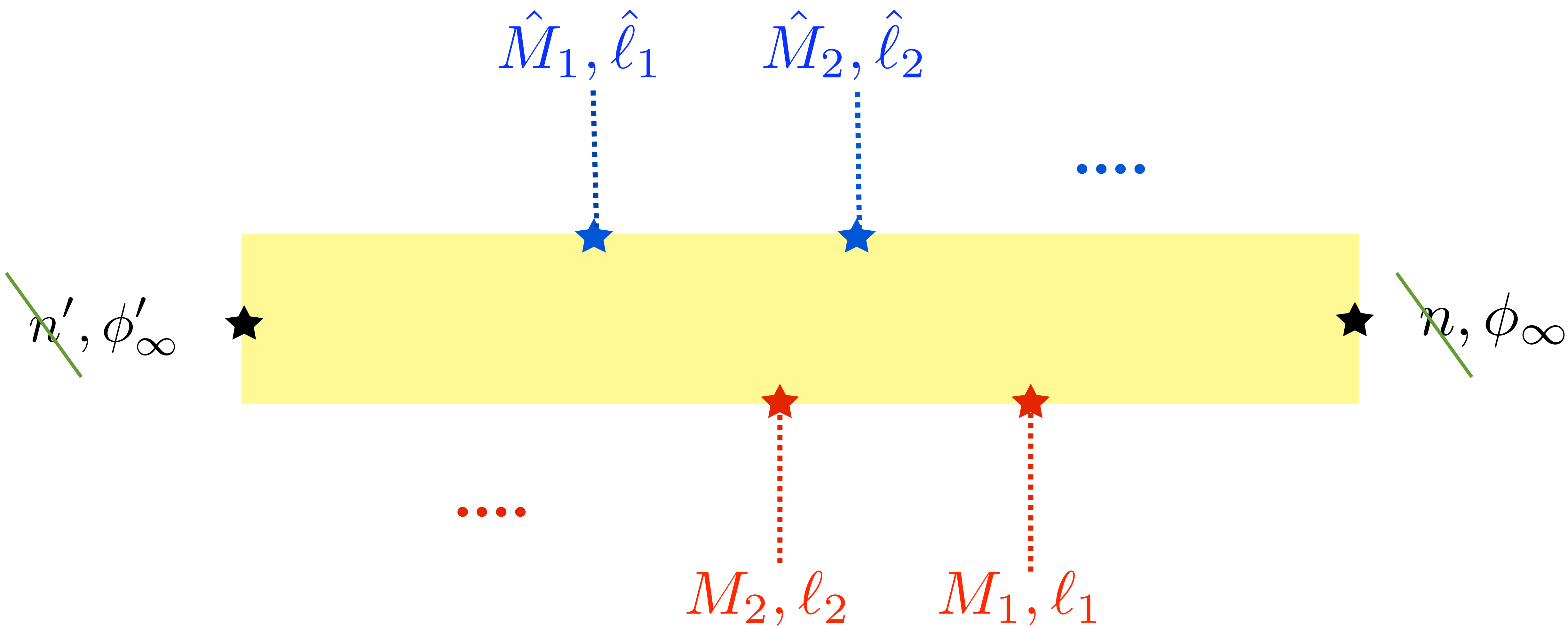
Limit $a_1, a_2 \to 0$ gives a smooth capping-off of geometry

Aharony, Berdichevsky, Berkooz, Shamir; ABEG

Solution for **linear quivers**

$$h_1 = \left[\cancel{-i\alpha \sinh(z-\beta)} - \sum_{a=1}^{q} \gamma_a \ln\left(\tanh\left(\frac{i\pi}{4} - \frac{z-\delta_a}{2}\right)\right)\right] + c.c.$$

$$h_2 = \left[\cancel{\hat{\alpha} \cosh(z-\hat{\beta})} - \sum_{b=1}^{\hat{q}} \hat{\gamma}_b \ln\left(\tanh\left(\frac{z-\hat{\delta}_b}{2}\right)\right)\right] + c.c.$$



Parameter count: $\{M_a, \ell_a, \hat{M}_b, \hat{\ell}_b\} \longleftrightarrow \{\gamma_a, \delta_a, \hat{\gamma}_b, \hat{\delta}_b\}$

May have **zero, one** or **two** asymptotic $AdS_5 \times S^5$ regions,
more leads to conical singularities.

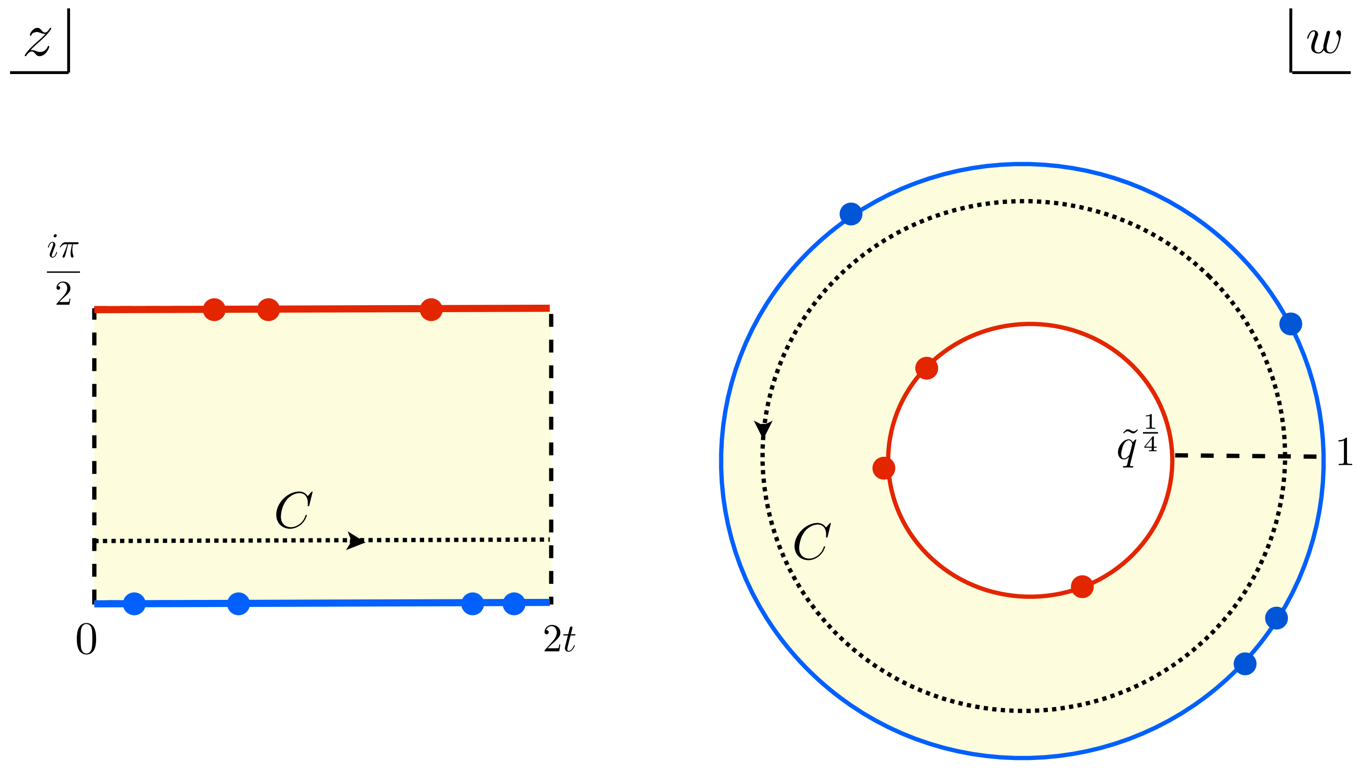
All parameters in first case are discrete: no N=4 moduli .

Global symmetries of CFT3 realized as gauge symmetries on 5-branes

Can show that linking numbers define two partitions obeying the
Young-tableau inequalities

One-to-one correspondence with Gaiotto-Witten SCFT3s

Solution for **circular quivers**



$$\mathcal{A}_1 = -i \sum_{a=1}^{p} \gamma_a \ln \left(\frac{\vartheta_1\left(q|\nu_a\right)}{\vartheta_2\left(q|\nu_a\right)} \right) + \varphi_1 \qquad i\,\nu_a = -\frac{z-\delta_a}{2\pi} + \frac{i}{4}$$

$$\mathcal{A}_2 = -\sum_{b=1}^{\hat{p}} \hat{\gamma}_b \ln \left(\frac{\vartheta_1\left(q|\hat{\nu}_b\right)}{\vartheta_2\left(q|\hat{\nu}_b\right)} \right) + i\varphi_2 \qquad i\,\hat{\nu}_b = \frac{z-\hat{\delta}_b}{2\pi}$$

Annulus modulus $\longleftrightarrow$ K = # of winding D3 branes

Cannot couple to 4D sYM, i.e. add asymptotic $AdS_5 \times S^5$ regions

Modified Young tableaux inequalities, guaranteeing complete Higgsing:

$$\hat{\rho} < \rho^{\,T} + K$$

Several calculations/observations concerning this rich set of holographic dualities, that I have no time to discuss here. Just mention some:

- Free energy on S3 from sugra = from CFT:

checked that for $T(SU(N))$ and some related theories

$$F = \frac{1}{2}N^2 \ln N + \mathcal{O}(N^2)$$

non-trivial

Assel, Estes, Yamazaki

using results of Benvenuti & Pasquetti; Nishioka,Tachikawa, Yamazaki

- Large K corresponds to $t \longrightarrow 0$; T-dual to ABJM-like theories.

- SL(2, Q) transformations generalize mirror symmetry in planar limit

Let's return now to the original question

To create the hierarchy $\boxed{\ell \gg L}$ must look at degeneration limits

AdS_4 (for ℓ) $\qquad AdS_5$ (for L)

- Factorize 5-brane singularities
  Pinch annulus $t \to \infty$ } **"Worm-branes"** (Janus throats)

- $\boxed{\text{Cap-off } AdS_5 \times S^5 \text{ (small rank for bulk D=4 SYM group)}}$

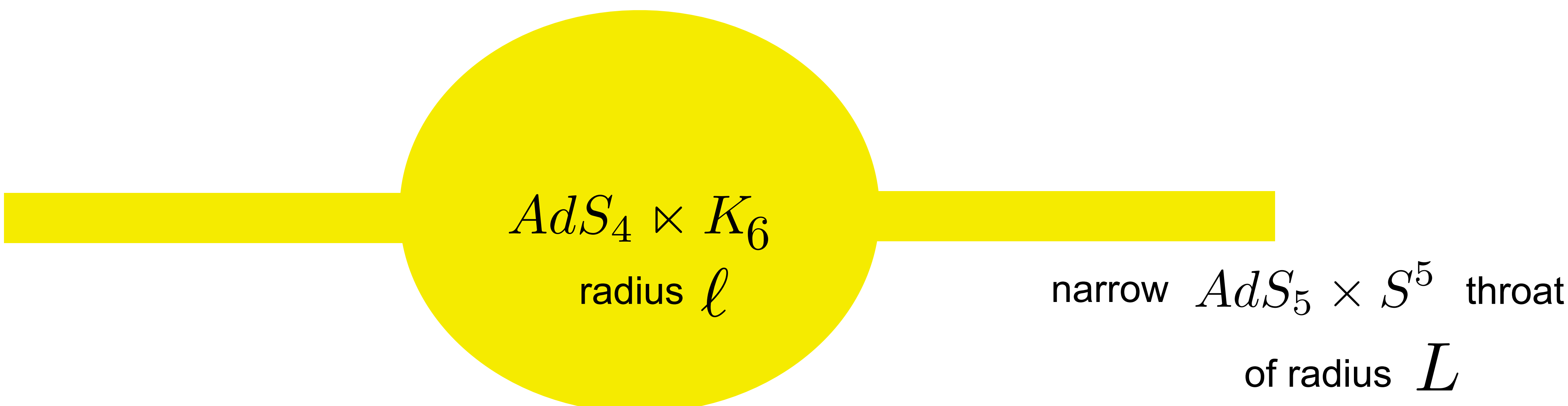
  $\alpha, \hat{\alpha} \to 0$

This is possible, provided there are both NS5s and D5s *(to stabilize dilaton) ;*

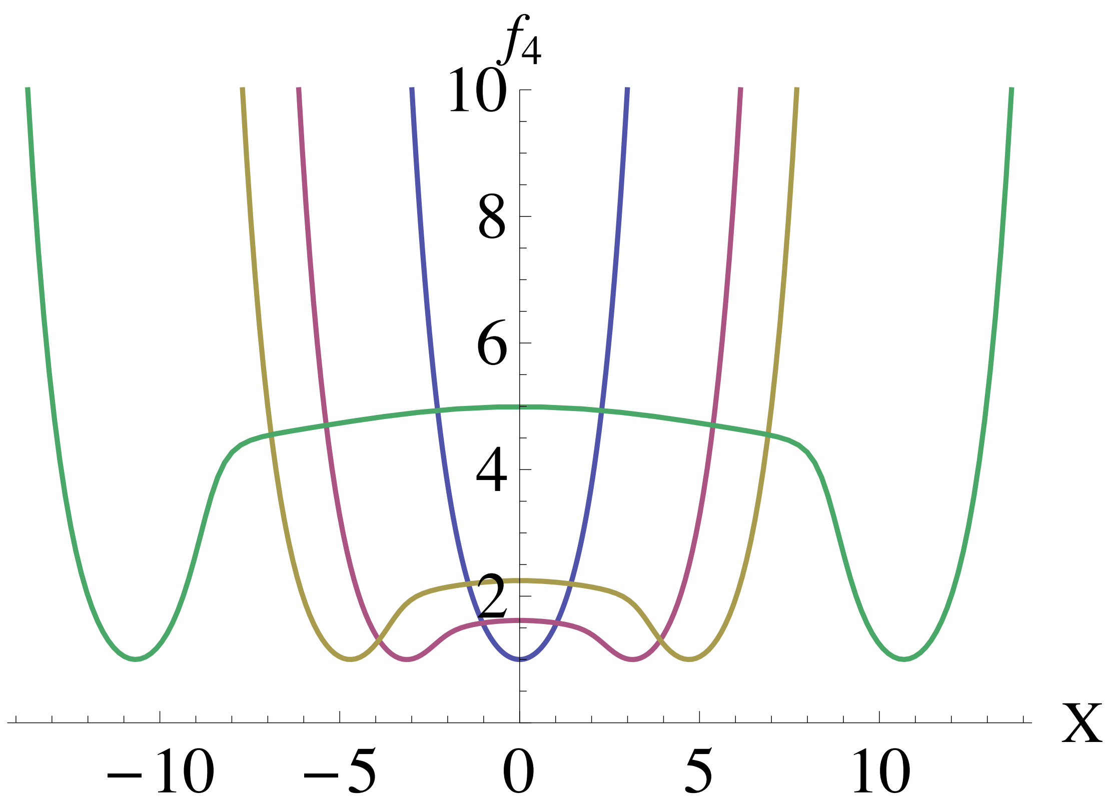
The **lowest** AdS4 **spin-2 mode** has a (tiny) mass and non-universal couplings.

(Assel, CB, unpublished)

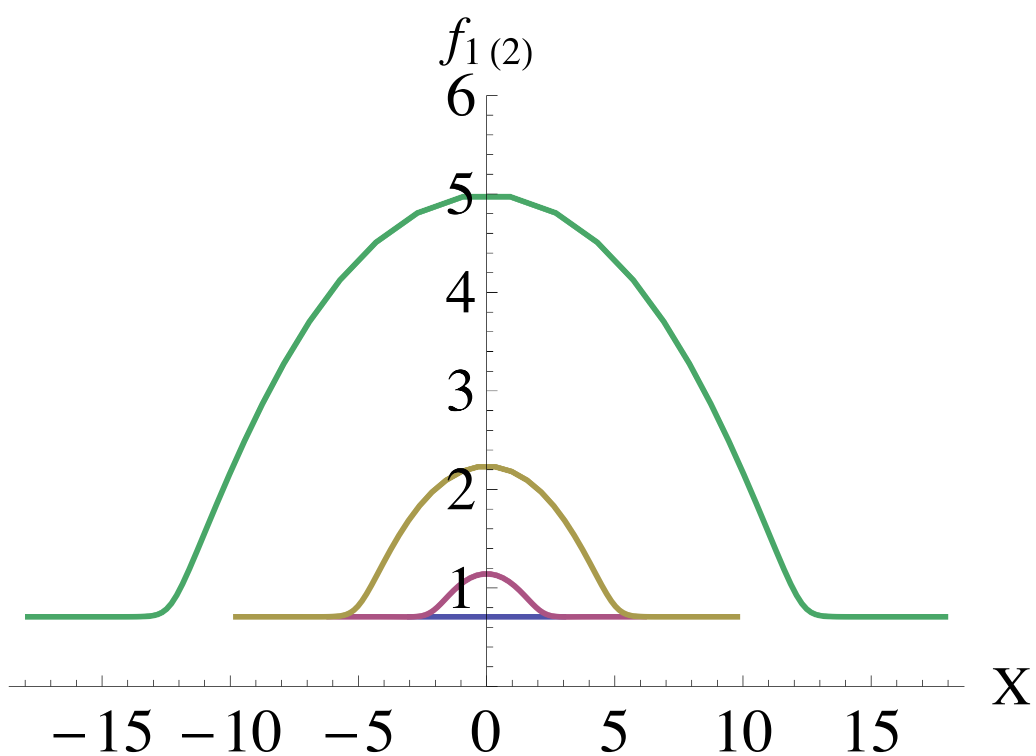
But, as in other efforts to create classical hierarchies, **spacetime decompactifies**



Gravity is never 4-dimensional !



warp factor



sphere radii

What we need:

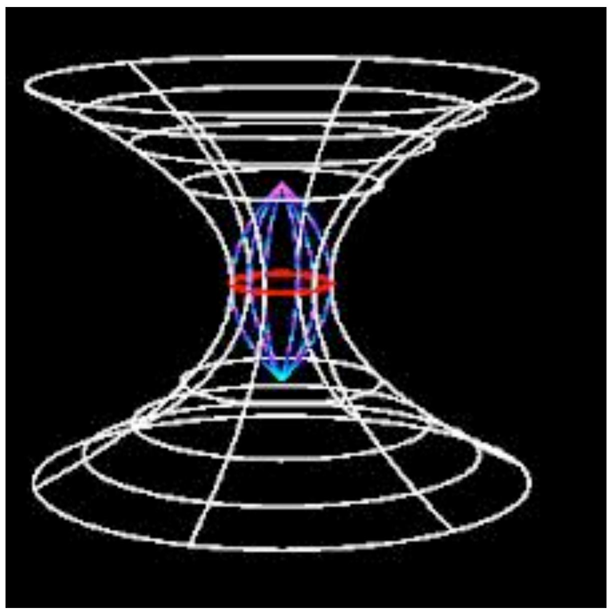
- An AdS4/CFT3 pair for which the **compact space stays small** ($T_{\mu\nu}$ and few other operators, then **large gap**)

interesting, and hard problem by itself

cf Tsimpis

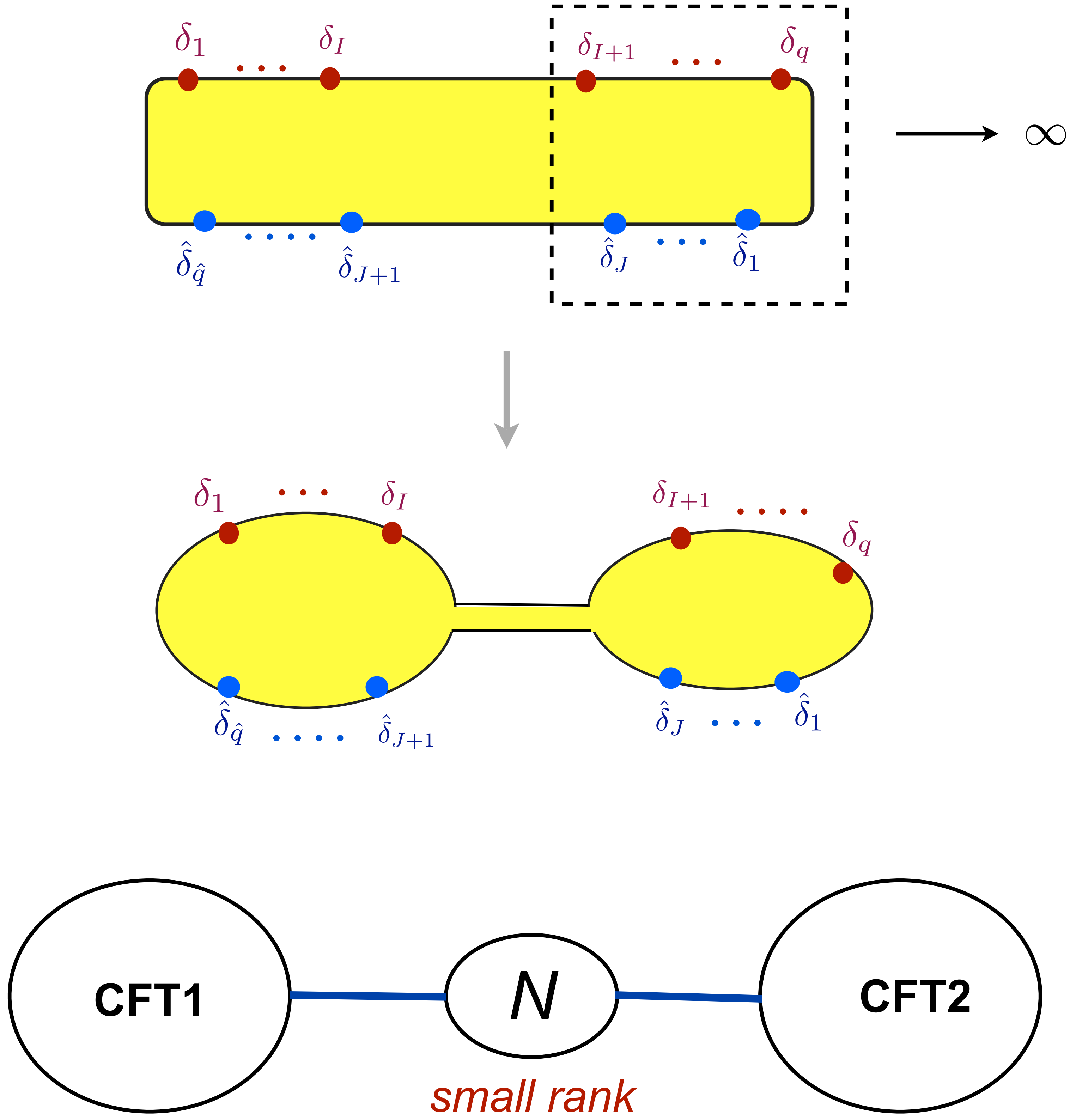
- Couple CFT3 to SYM4 **without destabilizing fixed point**

need understanding of geometric singularities



More generally, the solutions presented here are examples of Einstein-Rosen extended bridges ("worm-branes")

whose properties can be studied using gauge theory



δ_1
δ_I
δ_{I+1}
δ_q
∞
$\hat{\delta}_{\hat{q}}$
$\hat{\delta}_{J+1}$
$\hat{\delta}_J$
$\hat{\delta}_1$
δ_1
δ_I
δ_{I+1}
δ_q
$\hat{\delta}_{\hat{q}}$
$\hat{\delta}_{J+1}$
$\hat{\delta}_J$
$\hat{\delta}_1$
CFT1
N
CFT2
small rank

Summary

Found a **large class of type-IIB backgrounds**, that are duals of the Gaiotto-Witten N=4 SCFT3s , and of their circular-quiver extensions.

[These are AdS4 compactifications with localized 5-branes]

Degeneration limits correspond to **couplings via small-rank** gauge groups.

Failed to localize 4D gravity, because the brane fattens to 10D spacetime.

Interesting worm-brae geometries.

as opposed to local multitrace couplings,e.g.
Kiritsis, Niarchos; Aharony, Clark, Karch