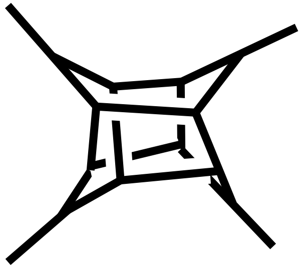
UV Surprises in Half-Maximal Supergravity

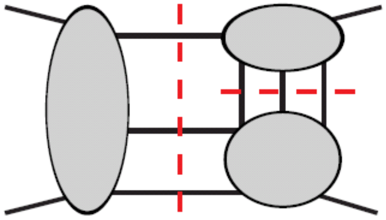
September 7, 2012

Hermann-Fest

Zvi Bern, UCLA

Based on papers with John Joseph Carrasco, Scott Davies, Tristan Dennen, Lance Dixon, Yu-tin Huang, Henrik Johansson and Radu Roiban.





Outline

1) Lightning review of sugra UV properties

2) Duality between color and kinematics.

3) Gravity as a double copy of gauge theory.

4) A two-loop surprise in $D = 5$ half maximal supergravity.

5) A three-loop surprise in $N = 4$ supergravity.

6) Consequences and prospects for future.

A Birthday Present

“If $N = 8$ supergravity is UV finite to all orders the reason must be sought beyond maximal supersymmetry and $E_{7(7)}$.”

Bossard and Nicolai (2011)

I’m here to deliver birthday presents to Hermann:

1) Supergravity puzzle: Vanishing divergences in half-maximal supergravity with no obvious susy or duality reasons.

ZB, Davies, Dennen, Huang (to appear) + Bossard, Howe and Stelle (to appear)

see talk from Bossard

2) A curious reason why $N = 8$ and other supergravity theories might be finite based on symmetries beyond the standard ones.

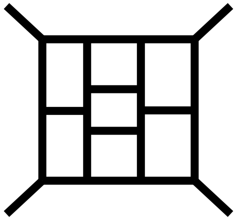
ZB, Davies, Dennen, Huang

Recent Status of Divergences

We already heard a very nice introduction and discussion of supergravity divergences from Lance, Guillaume and Renata.

Consensus that in N = 8 supergravity trouble starts at 5 loops and by 7 loops we have valid UV counterterm in D = 4 under all known symmetries (suggesting divergences) .

Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove ; Green and Bjornsson ; Bossard , Hillmann and Nicolai; Ramond and Kallosh; Broedel and Dixon; Elvang and Kiermaier; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger



For N = 8 sugra in D = 4:

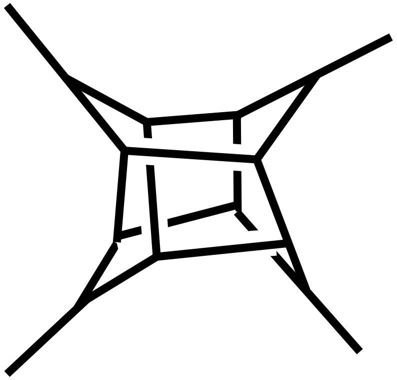
- **All counterterms ruled out until 7 loops!**
- **But D^8R^4 apparently available at 7 loops (1/8 BPS) under all known symmetries. (No known nonrenormalization theorem)**

Bossard, Howe, Stelle and Vanhove

Based on this a reasonable person would conclude that N = 8 supergravity almost certainly diverges at 7 loops in D = 4

N = 8 Sugra 5 Loop Calculation

ZB, Carrasco, Dixon, Johannson, Roiban



~500 such diagrams with ~1000s terms each

Being reasonable and being right are not the same

Place your bets:

- **At 5 loops in $D = 24/5$ does $N = 8$ supergravity diverge?**
- **At 7 loops in $D = 4$ does $N = 8$ supergravity diverge?**

$D^8 R^4$ **counterterms**

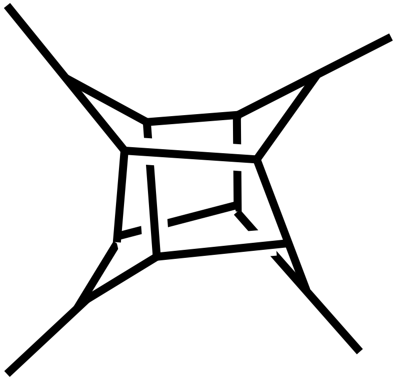
5 loops

Kelly Stelle:
British wine
"It will diverge"

Zvi Bern:
California wine
"It won't diverge"

$\mathcal{N} = 8$ Sugra 5 Loop Calculation

ZB, Carrasco, Dixon, Johannson, Roiban



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$D^8 R^4$ **counterterms**

7 loops

David Gross:
California wine
"It will diverge"

Zvi Bern:
California wine
"It won't diverge"

Recent Status of Divergences

We know of hints that more is going on than suggested by the standard symmetries:

1) **Nontrivial cancellations visible in certain unitarity cuts.**
   ZB, Dixon and Roiban

2) **Strange relations between UV divergences of $N = 4$ sYM and $N = 8$ supergravity in higher dimensions.** **See Lance's talk**
   ZB, Carrasco, Dixon, Johansson, Roiban

3) **BCJ loop-level relations between gravity and gauge-theory amplitudes.**
   ZB, Carrasco and Johansson

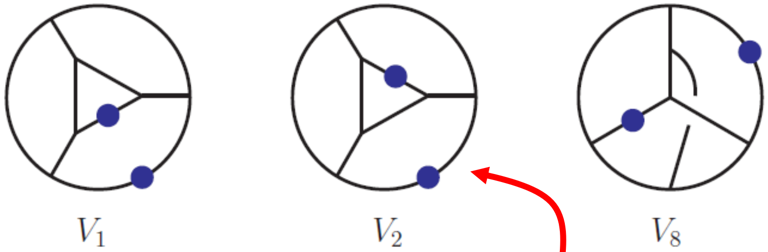
In the present talk I'll focus on the third point

New Four-Loop $N = 8$ Surprise

ZB, Carrasco, Dixon, Johansson, Roiban (2012)

Described in Lance's talk

Critical dimension D =11/2.
Express UV divergences
in terms of vacuum like integrals.



doubled propagator

gauge theory

$$\mathcal{A}_4^{(4)}(1,2,3,4)\Big|_{\text{pole}}^{SU(N_c)} = -6\,g^{10}\,\mathcal{K}\,N_c^2\Big(N_c^2\,V_1 + 12\,\boxed{(V_1+2\,V_2+V_8)}\Big)$$

$$\times\Big(s\,(\mathrm{Tr}_{1324}+\mathrm{Tr}_{1423}) + t\,(\mathrm{Tr}_{1243}+\mathrm{Tr}_{1342}) + u\,(\mathrm{Tr}_{1234}+\mathrm{Tr}_{1432})\Big)$$

same divergence

gravity

$$\mathcal{M}_4^{(4)}\Big|_{\text{pole}} = -\frac{23}{8}\left(\frac{\kappa}{2}\right)^{10} stu(s^2+t^2+u^2)^2\,M_4^{\text{tree}}\,\boxed{(V_1+2V_2+V_8)}$$

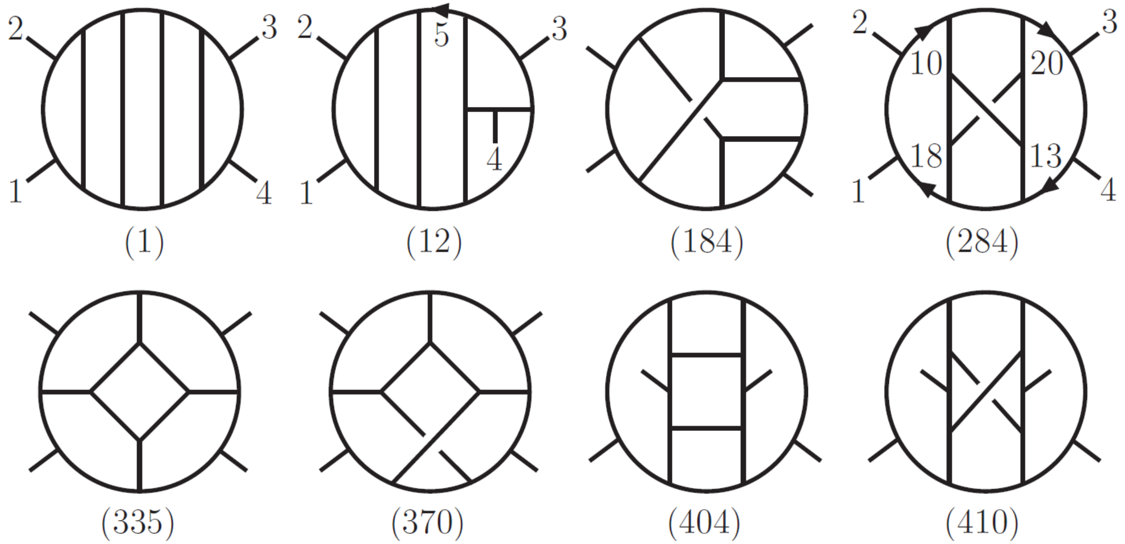
- **Gravity UV divergence is directly proportional to subleading color single-trace divergence of $N = 4$ super-Yang-Mills theory.**
- **Same happens at 1-3 loops.**

Calculation of $N = 4$ sYM 5 Loop Amplitude

ZB, Carrasco, Johansson, Roiban (July 2012)

Key step for $N = 8$ supergravity is construction of complete 5 loop integrand of $N = 4$ sYM theory.

416 such diagrams with ~100s terms each



$$s = (k_1 + k_2)^2$$

diagram numerators

$$N_1 = s^4 , \qquad N_{12} = 2s^3 k_3 \cdot l_5 ,$$
$$N_{284} = 2s^2((l_{10} \cdot l_{20})^2 + (l_{13} \cdot l_{18})^2)$$

We are trying to figure out a BCJ form. If we can get it we should have supergravity finished soon!
(Unclear how long it will take to get this form.)

Fine, but do we have any examples where a divergence vanishes but where known symmetries suggest valid counterterms?

Yes!

Two examples in half-maximal supergravity :

- ***D* = 5 at 2 loops.**
- ***D* = 4 at 3 loops.**

See Bossard's talk on susy and duality constraints

How to proceed?

"At this stage of our understanding of the theory, there is unfortunately no 'royal path' to finiteness cutting short explicit calculations of the type performed in [20]."

Bossard and Nicolai (2011)

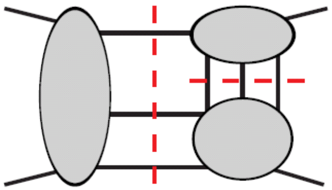
Need:

- **Powerful calculational tools.**
- **New ideas for studying this question.**

See Lance's talk

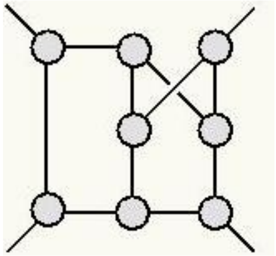
Constructing Multiloop Amplitudes

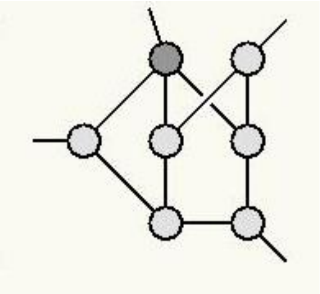
We do have powerful tools for complete calculations including nonplanar contributions:



- **Unitarity Method.**

  ZB, Dixon, Dunbar, Kosower





- **Method of Maximal Cuts**

  ZB, Carrasco, Johansson , Kosower

- **Duality between color and kinematics**

  ZB, Carrasco and Johansson

- **Advanced loop integration technology**

Chetyrkin, Kataev and Tkachov; A. V. Smirnov; V. A. Smirnov, Vladimirov; Marcus, Sagnotti; Cazkon; etc

In this talk we will explain how the duality between color and kinematics allows us to present examples where seemingly "valid" counterterms are, in fact, not present.

Duality Between Color and Kinematics

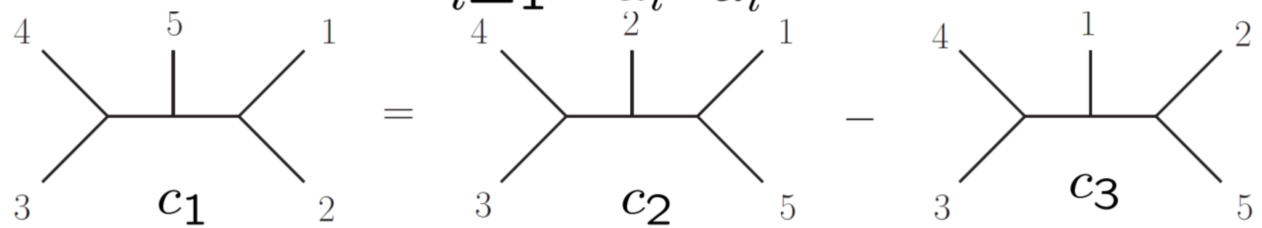
Consider five-point tree amplitude: ZB, Carrasco, Johansson (BCJ)

color factor

kinematic numerator factor

Feynman propagators

$$\mathcal{A}_5^{\text{tree}} = \sum_{i=1}^{15} \frac{c_i\, n_i}{\prod_{\alpha_i} p^2_{\alpha_i}}$$



$$c_1 \equiv f^{a_3 a_4 b} f^{b a_5 c} f^{c a_1 a_2}, \quad c_2 \equiv f^{a_3 a_4 b} f^{b a_2 c} f^{c a_1 a_5}, \quad c_3 \equiv f^{a_3 a_4 b} f^{b a_1 c} f^{c a_2 a_5}$$

$$n_i \sim k_4 \cdot k_5\, k_2 \cdot \varepsilon_1\, \varepsilon_2 \cdot \varepsilon_3\, \varepsilon_4 \cdot \varepsilon_5 + \cdots$$

$$\boxed{c_1 - c_2 + c_3 = 0 \Leftrightarrow n_1 - n_2 + n_3 = 0}$$

Claim: At n-points we can always find a rearrangement so color and kinematics satisfy the same algebraic constraint equations.

Nontrivial constraints on amplitudes in field theory and string theory

BCJ, Bjerrum-Bohr, Feng, Damgaard, Vanhove, ; Mafra, Stieberger, Schlotterer; Cachazo; Tye and Zhang; Feng, Huang, Jia; Chen, Du, Feng; Du, Feng, Fu; Naculich, Nastase, Schnitzer

Gravity and Gauge Theory

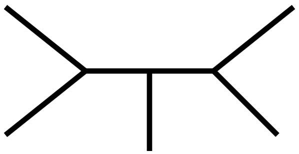
kinematic numerator color factor

gauge theory: $$\frac{1}{g^{n-2}}\mathcal{A}_n^{\text{tree}}(1,2,3,\ldots,n) = \sum_i \frac{n_i\, c_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$ **sum over diagrams with only 3 vertices**

$$c_i \sim f^{a_1 a_2 b_1} f^{b_1 b_2 a_5} f^{b_2 a_4 a_5}$$

Assume we have:

$$c_1 + c_2 + c_3 = 0 \quad \Leftrightarrow \quad n_1 + n_2 + n_3 = 0$$



Then: $c_i \Rightarrow \tilde{n}_i$ **kinematic numerator of second gauge theory**

Proof: ZB, Dennen, Huang, Kiermaier

gravity: $$\boxed{-i\left(\frac{2}{\kappa}\right)^{(n-2)} \mathcal{M}_n^{\text{tree}}(1,2,\ldots,n) = \sum_i \frac{n_i\, \tilde{n}_i}{\prod_{\alpha_i} p_{\alpha_i}^2}}$$ **Encodes KLT tree relations**

Gravity numerators are a double copy of gauge-theory ones.

This works for ordinary Einstein gravity and susy versions.

Cries out for a unified description of the sort given by string theory!

Gravity From Gauge Theory

BCJ

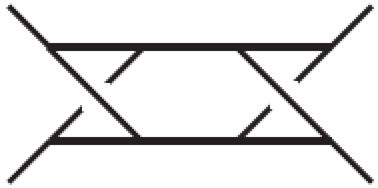
$$-i\left(\frac{2}{\kappa}\right)^{(n-2)}\mathcal{M}_n^{\text{tree}}(1,2,\ldots,n)=\sum_i\frac{n_i\,\tilde{n}_i}{\prod_{\alpha_i}p^2_{\alpha_i}}$$

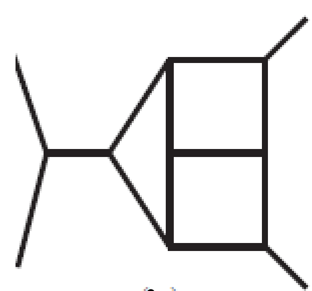
	n		$\tilde{n}$
N **= 8 sugra:**	**(*N* = 4 sYM)**	$\times$	**(*N* = 4 sYM)**
N **= 4 sugra:**	**(*N* = 4 sYM)**	$\times$	**(*N* = 0 sYM)**
N **= 0 sugra:**	**(*N* = 0 sYM)**	$\times$	**(*N* = 0 sYM)**

N **= 0 sugra: graviton + antisym tensor + dilaton**

Loop-Level Conjecture

ZB, Carrasco, Johansson (2010)





$$c_i + c_j + c_k = 0$$
$$n_i + n_j + n_k = 0$$

sum is over diagrams

kinematic numerator

color factor

$$\frac{(-i)^L}{g^{n-2+2L}}\mathcal{A}_n^{\text{loop}} = \sum_j \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_j} \frac{n_j c_j}{\prod_{\alpha_j} p_{\alpha_j}^2}$$

gauge theory

propagators

$$\frac{(-i)^{L+1}}{(\kappa/2)^{n-2+2L}}\mathcal{M}_n^{\text{loop}} = \sum_j \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_j} \frac{n_j \tilde{n}_j}{\prod_{\alpha_j} p_{\alpha_j}^2}$$

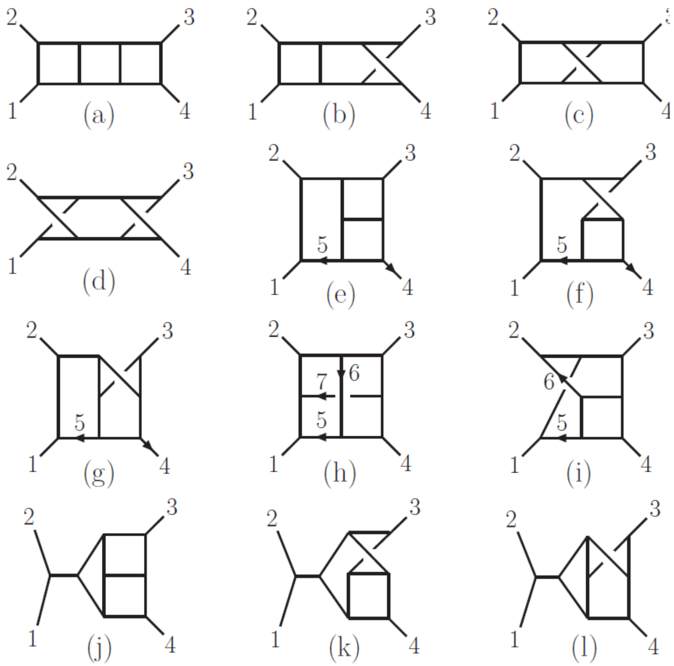
gravity

symmetry factor

Loop-level is identical to tree-level one except for symmetry factors and loop integration.
Double copy works if numerator satisfies duality.

Explicit Three-Loop Construction

ZB, Carrasco, Johansson (2010)



$$c_i = c_j - c_k \;\Rightarrow\; n_i = n_j - n_k$$

For N=4 sYM we have the abililty to go to high loop orders. Go to 3 loops. (1 & 2 loops works.)

Calculation very similar to earlier one with Dixon and Roiban, except now make the duality manifest.

Integral $I^{(x)}$	$\mathcal{N}=4$ Super-Yang-Mills ($\sqrt{\mathcal{N}=8 \text{ supergravity}}$) numerator
(a)–(d)	s^2
(e)–(g)	$\left(s\,(-\tau_{35}+\tau_{45}+t) - t\,(\tau_{25}+\tau_{45}) + u\,(\tau_{25}+\tau_{35}) - s^2\right)/3$
(h)	$\left(s\,(2\tau_{15}-\tau_{16}+2\tau_{26}-\tau_{27}+2\tau_{35}+\tau_{36}+\tau_{37}-u)\right.$ $\left.+t\,(\tau_{16}+\tau_{26}-\tau_{37}+2\tau_{36}-2\tau_{15}-2\tau_{27}-2\tau_{35}-3\tau_{17}) + s^2\right)/3$
(i)	$\left(s\,(-\tau_{25}-\tau_{26}-\tau_{35}+\tau_{36}+\tau_{45}+2t)\right.$ $\left.+t\,(\tau_{26}+\tau_{35}+2\tau_{36}+2\tau_{45}+3\tau_{46}) + u\,\tau_{25} + s^2\right)/3$
(j)-(l)	$s(t-u)/3$

$$\tau_{ij} = 2k_i \cdot l_j$$

- **Duality works!**
- **Double copy works!**

Generalized Gauge Invariance

BCJ
ZB, Dennen, Huang, Kiermaier
Tye and Zhang

gauge theory

$$\frac{(-i)^L}{g^{n-2+2L}}\mathcal{A}_n^{\text{loop}} = \sum_j \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_j} \frac{n_j c_j}{\prod_{\alpha_j} p_{\alpha_j}^2}$$

$$n_i \to n_i + \Delta_i \qquad \sum_j \int \frac{d^{DL} p}{(2\pi)^{DL}} \frac{1}{S_j} \frac{\Delta_j c_j}{\prod_{\alpha_j} p_{\alpha_j}^2} = 0$$

$$(c_\alpha + c_\beta + c_\gamma) f(p_i) = 0$$

Above is just a definition of generalized gauge invariance

gravity

$$\frac{(-i)^{L+1}}{(\kappa/2)^{n-2+2L}}\mathcal{M}_n^{\text{loop}} = \sum_j \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_j} \frac{n_j \tilde{n}_j}{\prod_{\alpha_j} p_{\alpha_j}^2}$$

$$n_i \to n_i + \Delta_i \qquad \sum_j \int \frac{d^{DL} p}{(2\pi)^{DL}} \frac{1}{S_j} \frac{\Delta_j \tilde{n}_j}{\prod_{\alpha_j} p_{\alpha_j}^2} = 0$$

- **Gravity inherits generalized gauge invariance from gauge theory!**
- **Double copy works even if only one of the two copies has duality manifest!**
- **Very useful for $\mathcal{N} \geq 4$ supergravity amplitudes.**

ZB, Boucher-Veronneau and Johansson; Boucher-Veroneau and Dixon; ZB, Davies, Dennen, Haung

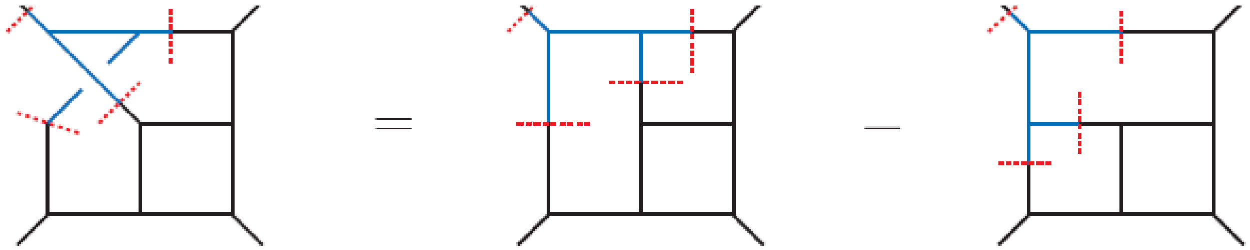
Generalized Gauge Invariance

- Generalized gauge invariance means symmetries of gauge theory inherited by gravity.

- If we see a UV cancellation in a gauge theory we should expect a corresponding cancellation in gravity.

Gravity from Gauge Theory Amplitudes

BCJ



$$c_k = c_i - c_j$$
$$n_k = n_i - n_j$$

If you have a set of duality satisfying numerators. To get:

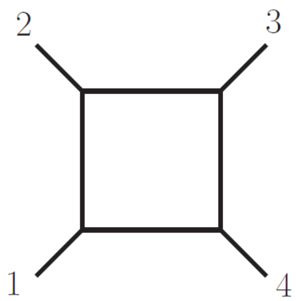
gauge theory → gravity theory

simply take

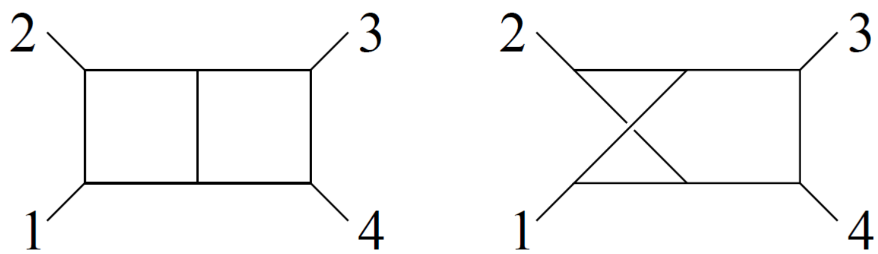
color factor → kinematic numerator

Gravity loop integrands are free!

Half Maximal Supergravity in $D > 4$



ZB, Davies, Dennen, Huang



No surprises at one loop:

- **Finite for $D < 8$**
- **R^4 divergence in $D = 8$**
- **F^4 four-matter multiplet amplitude diverges in $D = 4$ -**

Very instructive to understand from double-copy vantage point

A two-loop surprise:

- **Finite in D =5 with seemingly valid R^4 counterterm.**

See Bossard's talk.

A three loop surprise:

- **Finite for D =4 with seemingly valid R^4 counterterm.**

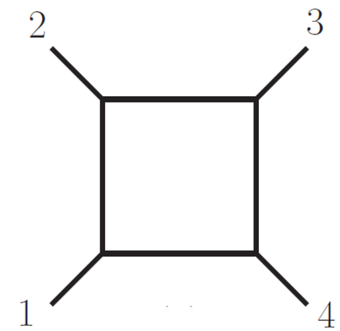
We now go through these examples

One-Loop Warmup in Half-Maximal sugra

ZB, Davies, Dennen, Huang

Generic color decomposition:

$$\mathcal{A}_Q^{(1)} = ig^4\Big[c_{1234}^{(1)} A_Q^{(1)}(1,2,3,4) + c_{1342}^{(1)} A_Q^{(1)}(1,3,4,2) + c_{1423}^{(1)} A_Q^{(1)}(1,4,2,3)\Big]$$



Q = # supercharges **Q = 0 is pure non-susy YM**

To get Q +16 supergravity take 2nd copy *N* = 4 sYM

_N_ = 4 sYM numerators independent of loop momenta

$$n_{1234} = n_{1342} = n_{1423} = st A_{Q=16}^{\rm tree}(1,2,3,4) \qquad c_{1234}^{(1)} \to n_{1234}$$

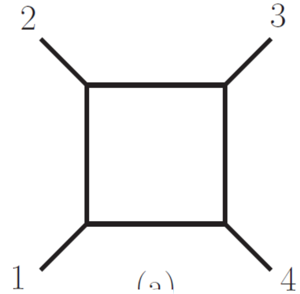
$$\mathcal{M}_{Q+16}^{(1)} = i\left(\frac{\kappa}{2}\right)^4 st A_{Q=16}^{\rm tree}(1,2,3,4)\Big[A_Q^{(1)}(1,2,3,4) + A_Q^{(1)}(1,3,4,2) + A_Q^{(1)}(1,4,2,3)\Big]$$

Note *exactly* the same combination as in *U*(1) decoupling identity.

One-loop divergences in pure YM

Go to a basis of color factors:

ZB, Davies, Dennen, Huang



$C_A = 2\,N_c$
for $SU(N_c)$

$b_1^{(0)}$ and $b_2^{(0)}$: tree color tensors

$b_1^{(1)}$: independent 1-loop color tensor

$$\mathcal{A}_Q^{(1)} = ig^4\Big[b_1^{(1)}\Big(A_Q^{(1)}(1,2,3,4) + A_Q^{(1)}(1,3,4,2) + A_Q^{(1)}(1,4,2,3)\Big) - \frac{1}{2}C_A b_1^{(0)} A_Q^{(1)}(1,3,4,2) - \frac{1}{2}C_A b_2^{(0)} A_Q^{(1)}(1,4,2,3)\Big]$$

***Q* supercharges (mainly interested in *Q* = 0)**

***D* = 4 *F*^{2} counterterm: 1-loop color tensor *not* allowed.**
***D* = 6 *F*^{3} counterterm: 1-loop color tensor *not* allowed.**

$$F^3 = f^{abc} F^{a\mu}_{\nu} F^{b\nu}_{\sigma} F^{c\sigma}_{\mu}$$

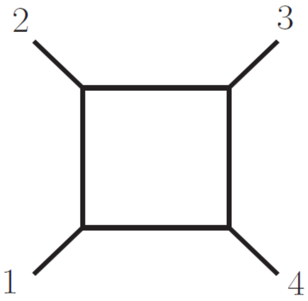
$$A_Q^{(1)}(1,2,3,4) + A_Q^{(2)}(1,3,4,2) + A_Q^{(1)}(1,4,2,3)\Big|_{D=4,6 \text{ div.}} = 0$$

$$M_{Q+16}^{(1)}(1,2,3,4)\Big|_{D=4,6\,\text{div.}} = 0$$

One-Loop Warmup in Half-Maximal Sugra

ZB, Davies, Dennen, Huang

$$\mathcal{M}^{(1)}_{Q+16} = i\left(\frac{\kappa}{2}\right)^4 st A^{\text{tree}}_{Q=16}(1,2,3,4)\Big[A^{(1)}_Q(1,2,3,4) + A^{(1)}_Q(1,3,4,2) + A^{(1)}_Q(1,4,2,3)\Big]$$



Cases where one-loop color tensor appear.
These give supergravity divergences.

$$\frac{1}{\epsilon} c^{abcd} F^{a\mu\nu} F^{b}{}_{\mu\sigma} F^{c\sigma\rho} F^{d}{}_{\rho\mu} \qquad\qquad c^{abcd} \equiv \tilde{f}^{a\, e_1 e_2}\, \tilde{f}^{b\, e_2 e_3}\, \tilde{f}^{c\, e_3 e_4}\, \tilde{f}^{d\, e_4 e_1}$$

one-loop color tensor allowed so no cancellations

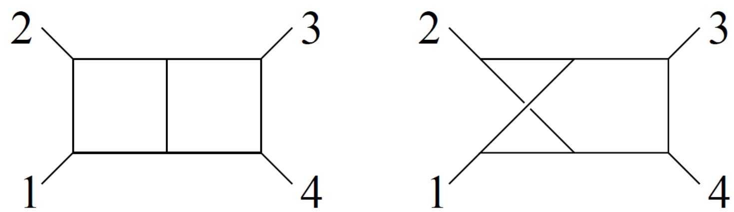
***D* = 8: *F*^{4} YM divergence ⟷ *R*^{4} sugra divergence**

With matter: $$\frac{1}{\epsilon} c^{abcd} \phi^a \phi^b \phi^c \phi^d$$

***D* = 4: ϕ^4 YM divergence ⟷ *F*^{4} matter sugra divergence**

(shown long ago by Fischler)

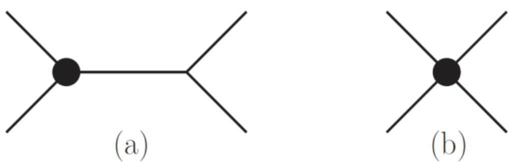
Two Loop Half Maximal Sugra in $D = 5$



ZB, Davies, Dennen, Huang

$$\mathcal{A}_Q^{(2)} = -g^6 \Big[c^{\mathrm{P}}_{1234} A^{\mathrm{P}}_Q(1,2,3,4) + c^{\mathrm{P}}_{3421} A^{\mathrm{P}}_Q(3,4,2,1) + c^{\mathrm{NP}}_{1234} A^{\mathrm{NP}}_Q(1,2,3,4) + c^{\mathrm{NP}}_{3421} A^{\mathrm{NP}}_Q(3,4,2,1) + \text{ cyclic} \Big]$$

$D = 5$ F^3 counterterm: 1,2-loop color tensors forbidden!
Demand this and plug into double copy:



1) **Go to color basis.**
2) **Demand no forbidden color tensors in pure YM divegence.**
3) **Plug into the BCJ double copy formula.**

$$\mathcal{M}^{(2)}_{16+Q}(1,2,3,4)\Big|_{D=5\,\mathrm{div.}} = 0$$

Half maximal supergravity four-point divergence vanishes because forbidden color tensor cancels in pure YM theory

Half Maximal Supergravity in $D = 5$

In a bit more detail:

gauge theory

$$\mathcal{A}_Q^{(2)} = -g^6\Big[c^{\rm P}_{1234}A_Q^{\rm P}(1,2,3,4) + c^{\rm P}_{3421}A_Q^{\rm P}(3,4,2,1) + c^{\rm NP}_{1234}A_Q^{\rm NP}(1,2,3,4) + c^{\rm NP}_{3421}A_Q^{\rm NP}(3,4,2,1) + \text{ cyclic}\Big]$$

gravity

$$\mathcal{M}_{Q+16}^{(2)} = -i\left(\frac{\kappa}{2}\right)^6 st A^{\rm tree}_{Q=16}(1,2,3,4)\Big[s\Big(A_Q^{(\rm P)}(1,2,3,4) + A_Q^{(\rm NP)}(1,2,3,4) + A_Q^{(\rm P)}(3,4,2,1) + A_Q^{(\rm NP)}(3,4,2,1)\Big) + \text{cyclic}\Big]$$

Equations that eliminate forbidden 2 loop color tensor:

$$\begin{aligned}
0 = {} & t(A_Q^{\rm P}(1,3,4,2) + A_Q^{\rm P}(1,4,2,3) + A_Q^{\rm P}(3,1,4,2) + A_Q^{\rm P}(3,2,1,4) \\
& + A_Q^{\rm NP}(1,3,4,2) + A_Q^{\rm NP}(1,4,2,3) + A_Q^{\rm NP}(3,1,4,2) + A_Q^{\rm NP}(3,2,1,4) \\
& + s(A_Q^{\rm P}(1,3,4,2) + A_Q^{\rm P}(3,1,4,2) + A_Q^{\rm NP}(1,3,4,2) + A_Q^{\rm NP}(3,1,4,2))\Big|_{D=5\text{ div.}}\,, \\
0 = {} & s(A_Q^{\rm P}(1,2,3,4) + A_Q^{\rm P}(1,3,4,2) + A_Q^{\rm P}(3,1,4,2) + A_Q^{\rm P}(3,4,2,1) \\
& + A_Q^{\rm NP}(1,2,3,4) + A_Q^{\rm NP}(1,3,4,2) + A_Q^{\rm NP}(3,1,4,2) + A_Q^{\rm NP}(3,4,2,1)) \\
& + t(A_Q^{\rm P}(1,3,4,2) + A_Q^{\rm P}(3,1,4,2) + A_Q^{\rm NP}(1,3,4,2) + A_Q^{\rm NP}(3,1,4,2))\Big|_{D=5\text{ div.}}
\end{aligned}$$

Plug into supergravity double copy formula:

$$\mathcal{M}_{16+Q}^{(2)}(1,2,3,4)\Big|_{D=5\text{ div.}} = 0$$

A Conjecture

Conjecture: (Q + 16) supercharge supergravity amplitudes are finite when divergences in corresponding Q supercharge YM amplitudes have only tree color tensors.

Corollary: $N \geq 4$ supergravity in $D = 4$ is ultraviolet finite.

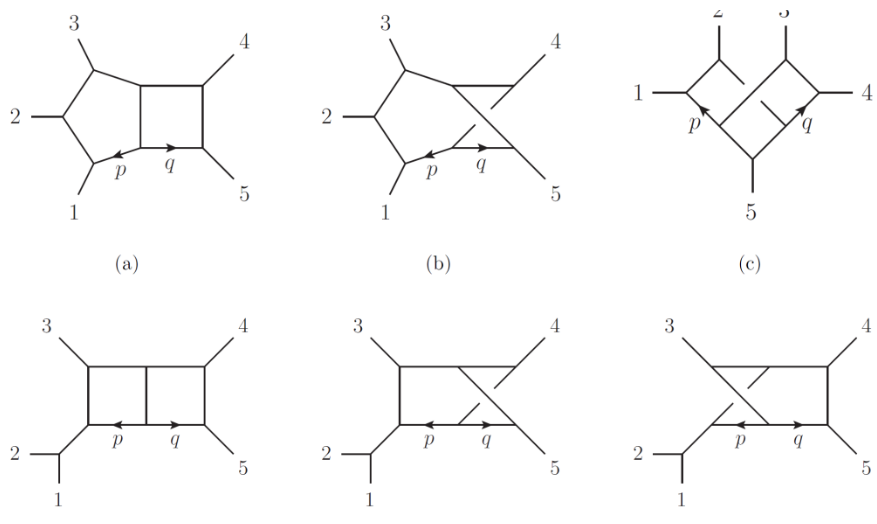
Conjecturing is easy. The nontrivial part is to prove (or disprove!).

Remains a challenge to prove beyond above 1,2 loop examples because loop momenta appear in numerators of both copies.

But we still have the power to calculate and to check!

Two loops and five points

Might the above have to do with the special property of no numerator loop momentum in *N* = 4 sYM (in *D* = 5)? Two loop five point doesn't have this property.


(a) (b) (c) (d) (e) (f)

Carrasco and Johansson Give us maximal sYM in BCJ format.

Half maximal sugra: Take other copy to be pure YM Feynman diagrams.

***D* = 5**

$\mathcal{I}^{(x)}$	$\mathcal{N}=4$ Super-Yang-Mills numerator
(a),(b)	$\frac{1}{4}\Big(\gamma_{12}(2s_{45}-s_{12}+\tau_{2p}-\tau_{1p})+\gamma_{23}(s_{45}+2s_{12}-\tau_{2p}+\tau_{3p}) + 2\gamma_{45}(\tau_{5p}-\tau_{4p})+\gamma_{13}(s_{12}+s_{45}-\tau_{1p}+\tau_{3p})\Big)$
(c)	$\frac{1}{4}\Big(\gamma_{15}(\tau_{5p}-\tau_{1p})+\gamma_{25}(s_{12}-\tau_{2p}+\tau_{5p})+\gamma_{12}(s_{34}+\tau_{2p}-\tau_{1p}+2s_{15}+2\tau_{1q}-2\tau_{2q}) + \gamma_{45}(\tau_{4q}-\tau_{5q})-\gamma_{35}(s_{34}-\tau_{3q}+\tau_{5q})+\gamma_{34}(s_{12}+\tau_{3q}-\tau_{4q}+2s_{45}+2\tau_{4p}-2\tau_{3p})\Big)$
(d)-(f)	$\gamma_{12}s_{45}-\frac{1}{4}\Big(2\gamma_{12}+\gamma_{13}-\gamma_{23}\Big)s_{12}$

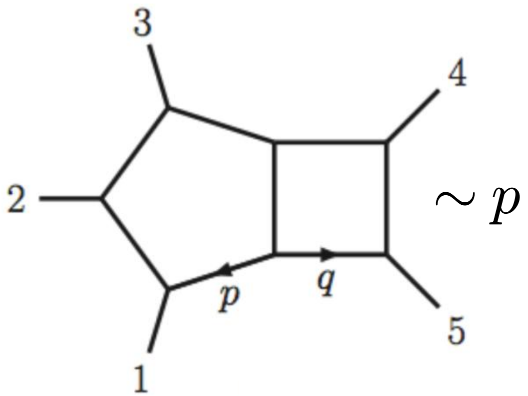
$$\tau_{ip}=2k_i\cdot p$$

Two loops five points double copy in $D = 5$

ZB, Davies, Dennen, Huang

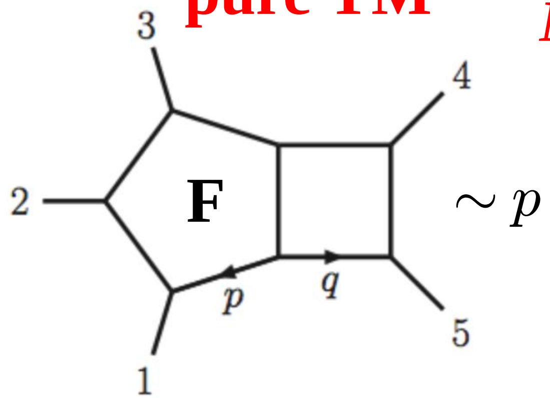
Half maximal supergravity: (N = 4 sYM) x (N = 0 YM)

N = 4 sYM



BCJ representation

pure YM



Feynman representation

N = 4 sugra quadratic divergent

$$\int (d^5 l)^2 \frac{k^6 l^8}{l^{16}}$$

$$D = 5$$

- **To extract UV expand in small external momenta.**
- **Integrals have subdivergences which causes complications. But this was well understood 30 years ago by Vladimirov and by Marcus and Sagnotti.**

Five Points Two loops $D = 5$ half-max sugra

ZB, Davies, Dennen, Huang

Evaluation of the integrals gives the UV divergences:

Graph	(divergence)$/(i\,\gamma_{12}\,\varepsilon_1\cdot\varepsilon_3\,\varepsilon_4\cdot\varepsilon_5\,k_1\cdot\varepsilon_2\,s_{12})$
(a)	$\frac{-64497+925D_s}{362880\sqrt{2}}\frac{1}{\epsilon}$
(b)	$\frac{820641-149788D_s}{1451520\sqrt{2}}\frac{1}{\epsilon}$
(c)	$\frac{-27555+8116D_s}{80640\sqrt{2}}\frac{1}{\epsilon}$
(d)	$\left(\frac{20605+912D_s}{53760\sqrt{2}}+\frac{-38+D_s}{240\sqrt{2}}\frac{s_{14}}{s_{13}}+\frac{655-161D_s}{1680\sqrt{2}}\frac{s_{23}}{s_{13}}+\frac{-5171-148D_s}{6720\sqrt{2}}\frac{s_{24}}{s_{13}}\right)\frac{1}{\epsilon}$
(e)	$\left(\frac{-71986+4511D_s}{241920\sqrt{2}}+\frac{935+6D_s}{6720\sqrt{2}}\frac{s_{14}}{s_{13}}+\frac{-907+342D_s}{6720\sqrt{2}}\frac{s_{23}}{s_{13}}+\frac{27859+844D_s}{60480\sqrt{2}}\frac{s_{24}}{s_{13}}\right)\frac{1}{\epsilon}$
(f)	$\left(\frac{-31847-8615D_s}{241920\sqrt{2}}+\frac{129-34D_s}{6720\sqrt{2}}\frac{s_{14}}{s_{13}}+\frac{-1713+302D_s}{6720\sqrt{2}}\frac{s_{23}}{s_{13}}+\frac{2335+61D_s}{7560\sqrt{2}}\frac{s_{24}}{s_{13}}\right)\frac{1}{\epsilon}$

$D = 5$

Sum over diagrams vanishes

D_s: state counting regularization parameter

- **While more complicated we see the same cancellations as we saw at four points (where no integration required).**
- **Potential R^4 and ϕR^4 counterterms in $D = 5$ half-maximal supergravity have vanishing coefficients.**

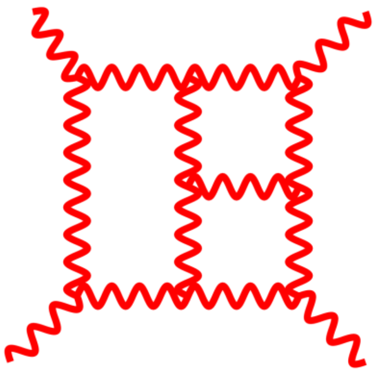
see Bossard's talk for duality + susy puzzles

$N = 4$ Supergravity in $D = 4$

Fine, but do we have a $D = 4$ example?

$N = 4$ sugra at 3 loops ideal $D = 4$ test case to study.

ZB, Davies, Dennen, Huang



Consensus had it that a valid R^4 counterterm exists for this theory in $D = 4$. Analogous to 7 loop counterterm of $N = 8$.

Bossard, Howe, Stelle; Bossard, Howe, Stelle, Vanhove

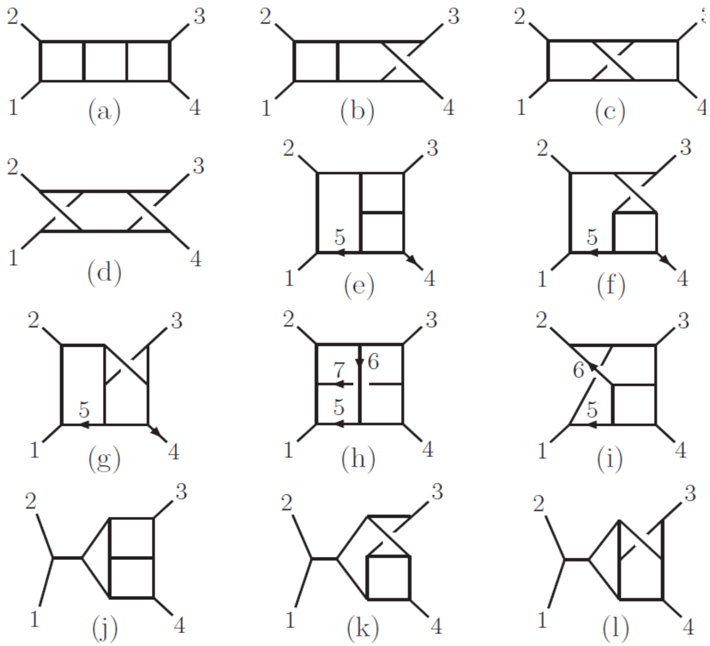
See talk from Bossard on latest "superstatus" of counterterm

Duality between color and kinematics gives us the ability to do the calculation.

Three-loop Construction

ZB, Davies, Dennen, Huang

N = 4 sugra : (N = 4 sYM) x (N = 0 YM)



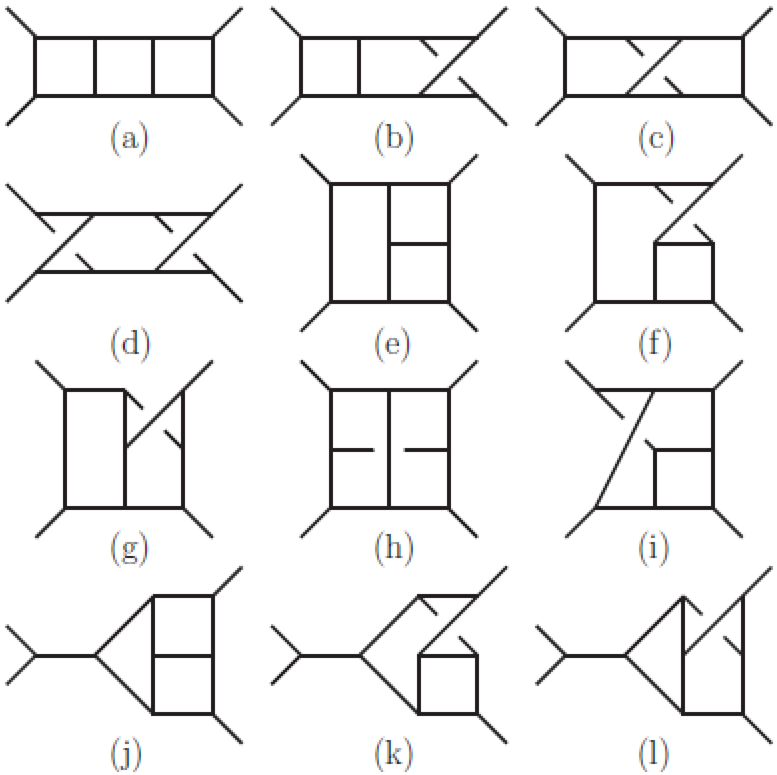
- **For N = 4 sYM copy use known BCJ representation.**
- **Use Feynman diagrams for the pure YM copy.**

Integral $I^{(x)}$	$\mathcal{N}=4$ Super-Yang-Mills ($\sqrt{\mathcal{N}=8 \text{ supergravity}}$) numerator
(a)–(d)	s^2
(e)–(g)	$\big(s(-\tau_{35}+\tau_{45}+t)-t(\tau_{25}+\tau_{45})+u(\tau_{25}+\tau_{35})-s^2\big)/3$
(h)	$\big(s(2\tau_{15}-\tau_{16}+2\tau_{26}-\tau_{27}+2\tau_{35}+\tau_{36}+\tau_{37}-u)$ $+t(\tau_{16}+\tau_{26}-\tau_{37}+2\tau_{36}-2\tau_{15}-2\tau_{27}-2\tau_{35}-3\tau_{17})+s^2\big)/3$
(i)	$\big(s(-\tau_{25}-\tau_{26}-\tau_{35}+\tau_{36}+\tau_{45}+2t)$ $+t(\tau_{26}+\tau_{35}+2\tau_{36}+2\tau_{45}+3\tau_{46})+u\,\tau_{25}+s^2\big)/3$
(j)-(l)	$s(t-u)/3$

BCJ form of the N = 4 sYM integrand

The $N = 4$ Supergravity UV Cancellation

ZB, Davies, Dennen, Huang



Graph	(divergence)$/(\langle 12\rangle^2[34]^2 st A^{\text{tree}}(\frac{\kappa}{2})^8)$
(a)-(d)	0
(e)	$\frac{263}{768}\frac{1}{\epsilon^3} + \frac{205}{27648}\frac{1}{\epsilon^2} + \left(-\frac{5551}{768}\zeta_3 + \frac{326317}{110592}\right)\frac{1}{\epsilon}$
(f)	$-\frac{175}{2304}\frac{1}{\epsilon^3} - \frac{1}{4}\frac{1}{\epsilon^2} + \left(\frac{593}{288}\zeta_3 - \frac{217571}{165888}\right)\frac{1}{\epsilon}$
(g)	$-\frac{11}{36}\frac{1}{\epsilon^3} + \frac{2057}{6912}\frac{1}{\epsilon^2} + \left(\frac{10769}{2304}\zeta_3 - \frac{226201}{165888}\right)\frac{1}{\epsilon}$
(h)	$-\frac{3}{32}\frac{1}{\epsilon^3} - \frac{41}{1536}\frac{1}{\epsilon^2} + \left(\frac{3227}{2304}\zeta_3 - \frac{3329}{18432}\right)\frac{1}{\epsilon}$
(i)	$\frac{17}{128}\frac{1}{\epsilon^3} - \frac{29}{1024}\frac{1}{\epsilon^2} + \left(-\frac{2087}{2304}\zeta_3 - \frac{10495}{110592}\right)\frac{1}{\epsilon}$
(j)	$-\frac{15}{32}\frac{1}{\epsilon^3} + \frac{9}{64}\frac{1}{\epsilon^2} + \left(\frac{101}{12}\zeta_3 - \frac{3227}{1152}\right)\frac{1}{\epsilon}$
(k)	$\frac{5}{64}\frac{1}{\epsilon^3} + \frac{89}{1152}\frac{1}{\epsilon^2} + \left(-\frac{377}{144}\zeta_3 + \frac{287}{432}\right)\frac{1}{\epsilon}$
(l)	$\frac{25}{64}\frac{1}{\epsilon^3} - \frac{251}{1152}\frac{1}{\epsilon^2} + \left(-\frac{835}{144}\zeta_3 + \frac{7385}{3456}\right)\frac{1}{\epsilon}$

Spinor helicity used to clean up table, but calculation for all states

All divergences cancel completely!

4-point 3-loop $N = 4$ sugra UV finite contrary to expectations

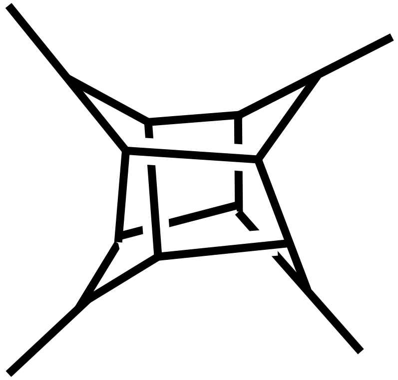
Tourkine and Vanhove have understood this result by extrapolating from the two-loop heteroric string amplitudes. See also talk from Kallosh.

Obvious Next Steps

1) **Five Loops: Compute the coefficient of the D^8R^4 five-loop counterterm of $N = 8$ supergravity in $D = 24/5$.**

2) **Four Loops: Find the coefficient of the D^2R^4 four-loop counterterm of half maximal supergravity $Q = 16$ in $D = 4$.**

3) **Can we prove that in general the supergravity cancellations are tied to cancellations of forbidden color factors in gauge-theory divergences?**



Summary

- A new duality conjectured between color and kinematics.
- When duality between color and kinematics manifest, gravity integrands follow *immediately* from gauge-theory ones.
- Double copy gives us a powerful way to explore the UV properties of gravity theories.
- Surprises, contrary to symmetry expectations:
  — $Q = 16$ supergravity in D=5 has no 2-loop 4-point divergences.
  — $N = 4$ sugra in $D = 4$ has no 3-loop 4-point divergences.
- Power counting using known symmetries and their known consequences can be misleading. Concrete example.
- Concrete examples directly linking cancellations in half-maximal sugra to ones in forbidden color factor of pure YM theory.

The double copy formalism gives us reasons to believe that $N \geq$ 4 pure supergravity theories are perturbatively UV finite. More importantly it give us the tools to decisively test this.

Happy Birthday Hermann!!