The Gravity/Coset Conjecture and a Possible Algebraic Description of Emergent Space

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work with
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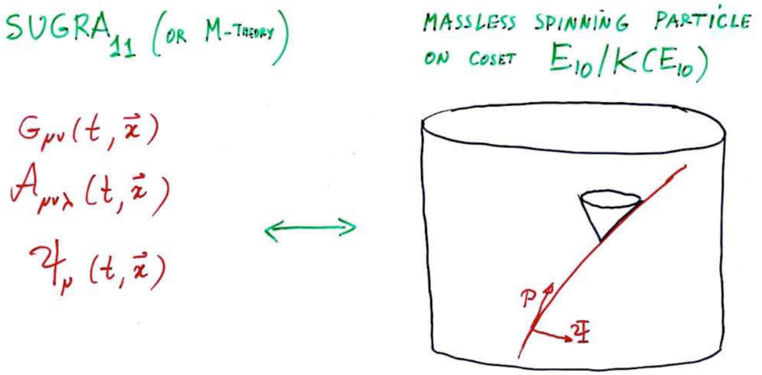
Nicolai Fest, 6–8 September, 2012
AEI, Golm

Gravity/Coset Correspondence

E_{10} : Damour, Henneaux, Nicolai $'02$;
related: Ganor $'99$ $'04$; E_{11} : West $'01$



Gravity/Coset Correspondence

$$G_{\mu\nu}(t,\mathbf{x}), \mathcal{A}_{\mu\nu\lambda}(t,\mathbf{x}), \psi_\mu(t,\mathbf{x})$$

$$g(t) = \exp(h^a_b(t)\, K^b_a)$$
$$\exp\left[\tfrac{1}{3!}\, A_{abc}(t)\, E^{abc} + \tfrac{1}{6!}\, A_{a_1\ldots a_6}(t)\, E^{a_1\ldots a_6} + \tfrac{1}{9!}\, A_{a_0|a_1\ldots a_8}(t)\, E^{a_0|a_1\ldots a_8} + \ldots\right]$$

$$S_{11} = \int d^{11}_x \left\{ \tfrac{E}{4}\, R(G) - \tfrac{E}{48}\, (d\mathcal{A}_3)^2 + \ldots \right\}$$

$$S_1^{\mathrm{COSET}} = \int dt \left\{ \tfrac{1}{4n(t)} \langle P(t), P(t) \rangle - \tfrac{i}{2} (\Psi(t) \mid \mathcal{D}^{\mathrm{vs}}\, \Psi(t))_{\mathrm{vs}} + \ldots \right\}$$

Gradient Expansion (BKL)
($\sim$ Small Tension Expansion: $\alpha' \to \infty$)

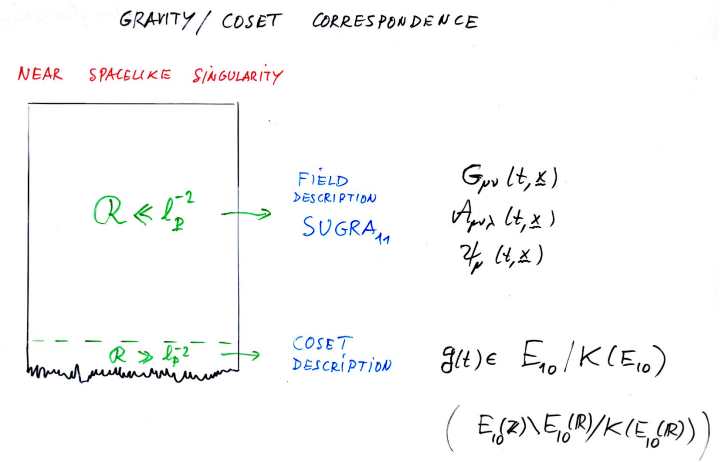
Height Expansion
in Kac-Moody Algebra

$$\partial^{k_1}_{x^1}\, \partial^{k_2}_{x^2} \ldots \partial^{k_{10}}_{x^{10}} \ll \partial_T^{k_1+k_2+\ldots+k_{10}}$$

Root:

$$\alpha = n_1\, \alpha_1 + n_2\, \alpha_2 + \ldots + n_{10}\, \alpha_{10}$$

Idea: Two 'dual' or 'complementary' descriptions



The 'singularity' is 'resolved' by the effective 'disappearance' of space, and the replacement of dynamical fields, $g_{ij}(t, \mathbf{x}), \mathcal{A}_{ijk}(t, \mathbf{x}), \ldots$ by a Lie-algebraic variable $g(t) \in E_{10}/K_{10}$

KAC-MOODY ALGEBRAS

SU(2) ≅ A_1 is SL(2)

$$[J_z, J_+] = +J_+ \qquad [J_z, J_-] = -J_-$$

J_z: CARTAN GENERATOR (DIAGONAL); J_+: RAISING GENERATOR

J_z: CARTAN GENERATOR; J_-: LOWERING GENERATOR

CARTAN SUBALGEBRA : LINEAR SPACE $\mathbb{R}^r$ (r = RANK)

$$\mathfrak{h} = \{\beta^a h_a \,;\, a = 1, \dots, r\}$$

β^a: coordinates in Cartan space: $h = \sum_a \beta^a h_a$

h_a: r independent Cartan gators

$$[h', h''] = 0$$

<u>TRIANGULAR DECOMPOSITION:</u>

$$\mathfrak{g} = \mathfrak{n}^- \oplus \mathfrak{h} \oplus \mathfrak{n}^+$$

$\mathfrak{n}^-$: LOWERING GATORS F_α; $\mathfrak{h}$: CARTAN h; $\mathfrak{n}^+$: RAISING GATORS E_α

DEGENERACY INDEX

$$[h, E_\alpha^{(s)}] = \alpha(h)\, E_\alpha^{(s)}$$

Cartan: $h = \sum_a \beta^a h_a$

RAISING GENERATOR(S)

ROOT $\equiv$ EIGENVALUE OF ad_h AS A LINEAR FORM OF $h \in CSA$

$$h = \beta^a h_a \to \alpha(h) = \alpha_a \beta^a \doteq \alpha(\beta)$$

$$[E_\alpha^{(s)}, E_\beta^{(t)}] = C_{\alpha\beta}^{(stu)} E_{\alpha+\beta}^{(u)}$$

$$[h, F_\alpha^{(s)}] = -\alpha(h)\, F_\alpha^{(s)}$$

LOWERING GENERATORS : $F_\alpha^{(s)} \equiv E_{-\alpha}^{(s)}$

+ JACOBI + SERRE RELATIONS

E_{10} Dynkin Diagram (= Cartan Matrix)

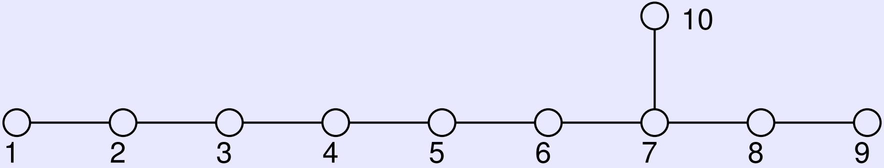
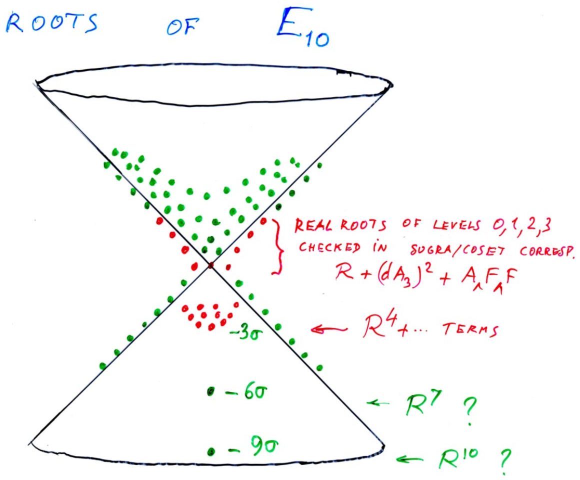


Figure: *Dynkin diagram of E_{10} with numbering of nodes.*

Here, leftmost E_{10} "hyperbolic" node labelled 1

E_{10} Root Diagram



Kinematics on Coset G/K

$$\mathcal{V}(t) \in G/K$$

$$v \equiv \partial_t \mathcal{V}\,\mathcal{V}^{-1} \in \mathrm{Lie}(G)$$

decomposed into $v = \mathcal{P} + \mathcal{Q}$ where

$$\mathcal{P} = v^{\mathrm{sym}} = \frac{1}{2}(v + v^T) : \text{‘coset velocity’}$$

$$\mathcal{Q} = v^{\mathrm{antisym}} = \frac{1}{2}(v - v^T) : \text{‘K angular velocity’}$$

Coset Action :

$$S_{1_{\mathrm{BOS}}}^{\mathrm{coset}} = \int \frac{dt}{n(t)}\,\frac{1}{4}\,\langle \mathcal{P}(t), \mathcal{P}(t) \rangle$$

$n(t)$: coset lapse $\rightarrow$ constraint $\langle \mathcal{P}(t), \mathcal{P}(t) \rangle = 0$

DECOMPOSING E_{10} WRT $GL(10)$ SUBALGEBRA

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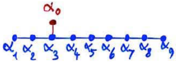


← $SL(10) = A_9$ SUBALGEBRA

"LEVEL" ℓ : $\alpha = \ell\,\alpha_0 + \sum_{j=1}^{9} m^j \alpha_j$

$\ell = 0$ GL(10) GENERATORS $K^a{}_b$ $[K^a{}_b, K^c{}_d] = \delta^c_b K^a{}_d - \delta^a_d K^c{}_b$

$\ell = \pm 1$ $E^{[a_1 a_2 a_3]}$, $F_{[a_1 a_2 a_3]}$

3 INDICES

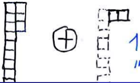
$\ell = \pm 2$ $E^{[a_1 \cdots a_6]}$, $F_{[a_1 \cdots a_6]}$

6 INDICES

$\ell = \pm 3$ $E^{[a_0 | a_1 \cdots a_8]}$, $F_{[a_0 | a_1 \cdots a_8]}$

9 INDICES

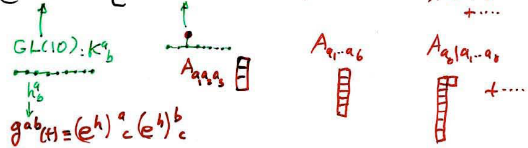
$\ell = \pm 4$



12 INDICES

EXPLICIT PARAMETRIZATION OF $E_{10}/K(E_{10})$

$$g(t) = e^{h^a{}_b(t) K^b{}_a} \; e^{\frac{1}{3!} A_{a_1a_2a_3}(t) E^{a_1a_2a_3} + \frac{1}{6!} A_{a_1\cdots a_6} E^{a_1\cdots a_6} + \frac{1}{9!} A_{a_0|a_1\cdots a_8} E^{a_0|a_1\cdots a_8} + \cdots}$$



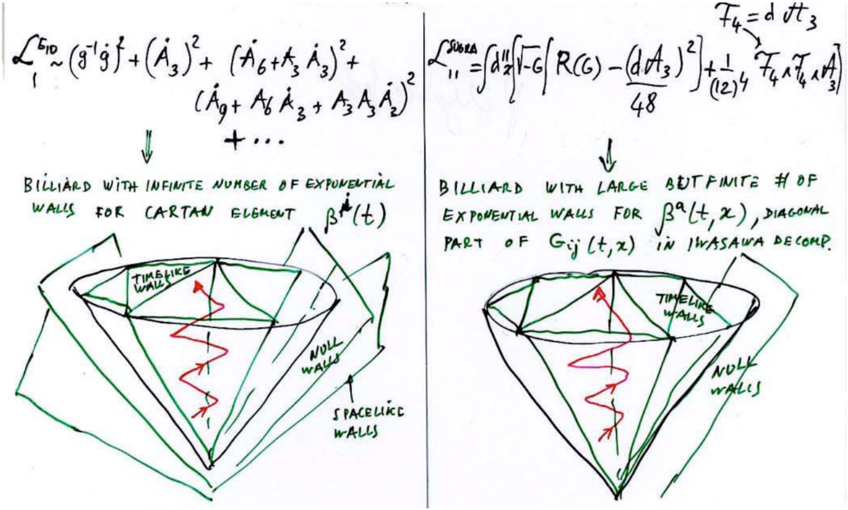
indices raised by g^{ab}

$$S_1^{E_{10}/K(E_{10})} = \int \frac{dt}{n(t)} \left[\frac{1}{4} \left(g^{ac} g^{bd} - g^{ab} g^{cd} \right) \dot{g}_{ab} \dot{g}_{cd} + \frac{1}{2} \frac{1}{3!} \dot{A}_{a_1a_2a_3} \dot{A}^{a_1a_2a_3} + \frac{1}{2} \frac{1}{6!} DA_{a_1\cdots a_6} DA^{a_1\cdots a_6} + \frac{1}{2} \frac{1}{9!} DA_{a_0|a_1\cdots a_8} DA^{a_0|a_1\cdots a_8} + \cdots \right.$$

$$DA_{a_1\cdots a_6} = \dot{A}_{a_1\cdots a_6} + 10\, A_{[a_1\cdots a_3} \dot{A}_{a_4\cdots a_6]}$$

$$DA_{a_0|a_1\cdots a_8} = \dot{A}_{a_0|a_1\cdots a_8} + 42\, A_{\langle a_0 \cdots a_3} \dot{A}_{a_4\cdots a_8\rangle} - 42\, \dot{A}_{\langle a_0\cdots a_3} A_{a_4\cdots a_8\rangle} + 280\, A_{\langle a_0\cdots a_3} A_{a_4\cdots a_6} \dot{A}_{a_7\cdots a_8\rangle}$$

Correspondence Between Coset/Gravity Dynamics



Gravity/Coset Dictionary

$$\mathcal{H}_1 = \frac{1}{2} G^{\mu\nu}\pi_\mu\pi_\nu + \sum c_\alpha(q,p)\, e^{-2\alpha(\beta)}$$

$$\alpha(\beta) = \sum_i m^i \alpha_i(\beta), \quad m^i \in \mathbb{N}$$

SIMPLE ROOTS

$$\mathcal{H}_{1q} = \frac{1}{2} G^{\mu\nu}\pi_\mu\pi_\nu + \sum_A c_A(Q, P, \partial_x\beta, \partial_x Q \ldots) \times e^{-2 w_A(\beta)}$$

$$w_A(\beta) = \sum_i m^i w_i(\beta)$$

DOMINANT WALLS

DICTIONARY

$$g^{ab}(t) = (e^h)^a{}_c (e^h)^b{}_c = G^{ab}(t, \vec{x}_0)$$

← WRT A SPECIAL FRAME

$$\dot{A}_{a_1a_2a_3}(t) = \mathcal{F}_{0a_1a_2a_3}(t, \vec{x}_0) \quad \checkmark \quad \theta^a(x) = e^a{}_i(x)\, dx^i$$

$$DA^{a_1\cdots a_6}(t) = g^{a_1a_1'}\cdots g^{a_6a_6'}\left[\dot{A}_{a_1'\cdots a_6'} + 10\, A_{[3}\dot{A}_{3]}\right] = -\frac{1}{4!}\,\varepsilon^{a_1\cdots a_6 b_1\cdots b_4}\,\mathcal{F}_{b_1\cdots b_4}(t, \vec{x}_0)$$

$$DA^{b|a_1\cdots a_8}(t) = g^{\cdots}\left[\dot{A}_9 + 42\, A_3\dot{A}_6 + 280\, A_3 A_3 \dot{A}_3\right] = +\frac{3}{2}\,\varepsilon^{a_1\cdots a_8 b_1 b_2}\, C^b{}_{b_1b_2}(\vec{x}_0)$$

$$d\theta^a = \frac{1}{2} C^a{}_{bc}\,\theta^b \wedge \theta^c$$

THE CORRESPONDENCE WORKS FOR ALL TERMS OF HEIGHT ≤ 29

$$\sum_i m^i \leq 29 \qquad \sum_i m^i \leq 29$$

Classical Coset Hamiltonian

Bosonic

$$S_1 = \int \frac{dt}{2n(t)} \|(\partial_t \mathcal{V}\,\mathcal{V}^{-1})^{\mathrm{Sym}}\|^2$$

Iwasawa:

$$\mathcal{V}(t) = \underbrace{\exp\left(\beta^a(t)\, H_a\right)}_{\text{Cartan}} \; \underbrace{\exp\left(\sum_{\alpha>0} \nu^\alpha(t)\, E_\alpha\right)}_{\text{Raising (+ multiplicity)}}$$

$$S^{\mathrm{coset}} = \int \frac{dt}{n(t)} \left[\frac{1}{2} G_{ab}\, \dot\beta^a\, \dot\beta^b + \sum_{\alpha>0} \frac{1}{4}\, e^{2\alpha(\beta)} (\dot\nu^\alpha + c\, \nu^{\alpha'}\, \dot\nu^{\alpha''} + \ldots)^2\right]$$

$$H^{\mathrm{coset}}(\beta^a, \pi_a; \nu^\alpha, p_\alpha) = n(t) \left[\frac{1}{2} G^{ab}\, \pi_a\, \pi_b + \sum_{\alpha>0} e^{-2\alpha(\beta)}\, \Pi_\alpha^2(p, \nu)\right]$$

Quantum Gravity $\leftrightarrow$ Quantum Coset Model

Quantum Coset Model: in configuration space β^a, ν^α

$$\Box_{E_{10}/K_{10}} \Psi(\beta^a, \nu^\alpha) = 0$$

$$\left[-G^{ab}\, \partial_{\beta^a}\, \partial_{\beta^b} - \sum_{\alpha > 0} e^{-2\alpha(\beta)}\, \partial^2_{\nu^\alpha} + \dots \right] \Psi(\beta^a, \nu^\alpha) = 0$$

Infinite-dimensional Klein-Gordon type equation: $- + + + + + \dots$

From Hull-Townsend '95 expect $E_{10}(\mathbb{Z})$ symmetry, i.e.

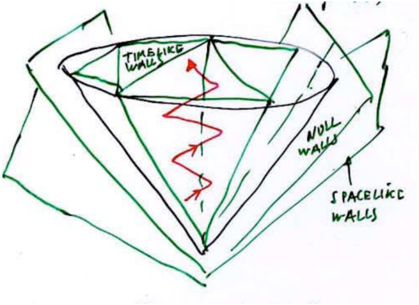
$\Psi(\beta, \nu) =$ automorphic function over $E_{10}(\mathbb{Z}) \backslash E_{10}(\mathbb{R}) / K_{10}(\mathbb{R})$

(Ganor '99; Brown, Ganor, Helfgott '04)

Baby Version of the Coset Model: Billiard

Particle in β (Cartan) space; billiard walls $=$ simple roots of E_{10} (Damour-Henneaux $'00$)

coset billiard $=$ BKL gravity billiard $\simeq$ Kasner mini-superspace
$g_{ab}(t) = e^{-2\beta^a(t)}\,\delta_{ab}$ + effect of leading spatial gradients



Baby Version of the Quantum Coset Model: Quantum Billiard

Kleinschmidt, Nicolai et al. ′09 ′10

Setting $n = 1$ one has to quantize

$$\mathcal{L} = \frac{1}{2}\sum_{a,b=1}^{d} \dot\beta^a G_{ab} \dot\beta^b = \frac{1}{2}\left[\sum_{a=1}^{d}(\dot\beta^a)^2 - \left(\sum_{a=1}^{d}\dot\beta^a\right)^2\right]$$

with null constraint $\dot\beta^a G_{ab} \dot\beta^b = 0$ on billiard domain.

Canonical momenta: $\pi_a = G_{ab}\dot\beta^b \quad \Rightarrow \quad \mathcal{H} = \frac{1}{2}\pi_a G^{ab}\pi_b$.

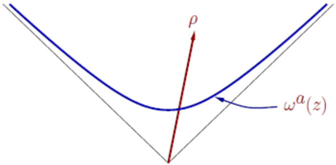
Wheeler–DeWitt (WDW) equation in canonical quantization

$$\mathcal{H}\Psi(\beta) = -\frac{1}{2}G^{ab}\partial_a\partial_b\Psi(\beta) = 0$$

Klein–Gordon ‘inner product’.

Quantum Billiard: Polar Coordinates $\beta^a = \rho\, \omega^a(z)$

Introduce new coordinates ρ and $\omega^a(z)$ from 'radius' and co-ordinates z on unit hyperboloid



$$\beta^a = \rho\omega^a, \quad \omega^a G_{ab}\omega^b = -1$$
$$\rho^2 = -\beta^a G_{ab}\beta^b$$

Timeless WDW equation in these variables

$$\left[-\rho^{1-d}\frac{\partial}{\partial\rho}\left(\rho^{d-1}\frac{\partial}{\partial\rho}\right) + \rho^{-2}\Delta_{\mathrm{LB}}\right]\Psi(\rho, z) = 0$$

Laplace–Beltrami operator on unit hyperboloid

Solving the WDW Equation

$$\left[-\rho^{1-d}\frac{\partial}{\partial\rho}\left(\rho^{d-1}\frac{\partial}{\partial\rho}\right)+\rho^{-2}\Delta_{\mathsf{LB}}\right]\Psi(\rho,z)=0$$

Separation of variables: $\Psi(\rho,z)=R(\rho)F(z)$

For

$$-\Delta_{\mathsf{LB}}F(z)=EF(z)$$

get

$$R_{\pm}(\rho)=\rho^{-\frac{d-2}{2}\pm i\sqrt{E-\left(\frac{d-2}{2}\right)^2}}$$

[Positive frequency coming out of singularity is $R_{-}(\rho)$.]

Left with spectral problem on hyperbolic space.

Boundary Conditions

Classical Billiard suggests Dirichlet boundary conditions at time-like walls

Then yields a Weyl (E_{10})-modular form by using Weyl images

Interesting *arithmetic* aspects of this Weyl billiard

$$\underbrace{W^+(E_{10})}_{\text{even Weyl group}} = \mathrm{PSL}_2(\underbrace{O}_{\text{octavians = integer octonions}}) \text{ (Feingold, Kleinschmidt, Nicolai '08)}$$

generalizes $W^+(AE_3) = \mathrm{PSL}_2(\mathbb{Z})$ for $3+1$ gravity

$\longrightarrow \quad \Psi_{E_{10}}^{\text{billiard}} =$ Maass wave form for $\mathrm{PSL}_2(O)$

Deeper issue of space like boundary condition at $\rho \to \infty$ (singularity)

? need some final-state boundary condition ?
(Horowitz, Maldacena '04)

(Bosonic) Evolution Equations and Constraints

Supergravity	Coset	Name
$\mathcal{G}_{ab} = 0$	$\mathcal{D} \overset{(0)}{P}_{ab} = 0$	Einstein dynamical eq.
$\mathcal{M}^{a_1 a_2 a_3} = 0$	$\mathcal{D} \overset{(1)}{P}_{a_1 a_2 a_3} = 0$	Matter dynamical eq.
$D_{[0} F_{a_1 \dots a_4]} = 0$	$\epsilon_{a_1 \dots a_4 b_1 \dots b_6} \mathcal{D} \overset{(2)}{P}_{b_1 \dots b_6} = 0$	F-Bianchi I
$R_{[0 a b] c} = 0$	$\epsilon_{bcd_1 \dots d_8} \mathcal{D} \overset{(3)}{P}_{a \vert d_1 \dots d_8} = 0$	R-Bianchi I
$\mathcal{G}_{00} = 0$	$\langle \mathcal{P} \mid \mathcal{P} \rangle = 0$	Hamiltonian constraint
$\mathcal{G}_{0a} = 0$	$\epsilon_{ac_1 \dots c_9} \overset{(3)}{\mathcal{C}}_{c_1 \dots c_9} = 0$	Momentum constraint
$\mathcal{M}^{0 a_1 a_2} = 0$	$\epsilon_{b_1 \dots b_{10}} \overset{(4)}{\mathcal{C}}_{b_1 \dots b_{10} \Vert a_1 a_2} = 0$	Gauss constraint
$D_{[c_1} F_{c_2 \dots c_5]} = 0$	$\epsilon_{b_1 \dots b_{10}} \epsilon_{a_1 \dots a_5 c_1 \dots c_5} \overset{(5)}{\mathcal{C}}_{b_1 \dots b_{10} \Vert a_1 \dots a_5} = 0$	F-Bianchi II
$R_{[c_1 c_2 c_3] a_0} = 0$	$\epsilon_{b_1 \dots b_{10}} \epsilon_{a_1 \dots a_7 c_1 c_2 c_3} \overset{(6)}{\mathcal{C}}_{b_1 \dots b_{10} \Vert a_0 \vert a_1 \dots a_7} = 0$	R-Bianchi II

Structure of Known (Bosonic) Constraints

Hamiltonian constraint: $\mathcal{G}_{00} = 0 \to \langle \mathcal{P} \mid \mathcal{P} \rangle = 0$

Other SUGRA constraints in $GL(10)$ decomposition

$$\text{Momentum:} \quad \overset{(3)}{\mathcal{C}}_{a_1 \dots a_9} = \overset{(0)}{P}_{ca_1} \overset{(3)}{P}_{c|a_2 \dots a_9} + 28 \overset{(1)}{P}_{a_1 a_2 a_3} \overset{(2)}{P}_{a_4 \dots a_9},$$

$$\text{Gauss:} \quad \overset{(4)}{\mathcal{C}}_{b_1 \dots b_{10}||a_1 a_2} = \overset{(1)}{P}_{a_1 b_1 b_2} \overset{(3)}{P}_{a_2|b_3 \dots b_{10}} + \frac{21}{5} \overset{(2)}{P}_{a_1 b_1 \dots b_5} \overset{(2)}{P}_{a_2 b_6 \dots b_{10}},$$

$$\text{Bianchi-F:} \quad \overset{(5)}{\mathcal{C}}_{b_1 \dots b_{10}||a_1 \dots a_5} = \overset{(2)}{P}_{a_1 \dots a_4 b_1 b_2} \overset{(3)}{P}_{a_5|b_3 \dots b_{10}},$$

$$\text{Bianchi-R:} \quad \overset{(6)}{\mathcal{C}}_{b_1 \dots b_{10}||a_0|a_1 \dots a_7} = \overset{(3)}{P}_{a_0|b_1 \dots b_8} \overset{(3)}{P}_{b_9|b_{10} a_1 \dots a_7}.$$

The E_{10} Nœther Current

Global E_{10} symmetry of coset action $\Rightarrow$ existence of infinite number of Nœther charges: $\mathcal{J} = n^{-1}\,\mathcal{V}^{-1}\,\mathcal{P}\,\mathcal{V}$.
Expansion of $\mathcal{J} \in \mathrm{Lie}(E_{10})$ along generators (with truncation $\mathcal{J}^{(\ell)} = 0$ for $\ell = -4, -5, -6, \dots$)

$$
\begin{aligned}
\mathcal{J} \;=\;& \tfrac{1}{9!} \overset{(-3)}{J}{}^{m_0|m_1\dots m_8} F_{m_0|m_1\dots m_8} + \tfrac{1}{6!} \overset{(-2)}{J}{}^{m_1\dots m_6} F_{m_1\dots m_6} + \tfrac{1}{3!} \overset{(-1)}{J}{}^{mnp} F_{mnp} + \\
&+ \overset{(0)}{J}{}^{n}{}_{m} K^m{}_n + \tfrac{1}{3!} \overset{(1)}{J}{}_{mnp} E^{mnp} + \tfrac{1}{6!} \overset{(2)}{J}{}_{m_1\dots m_6} E^{m_1\dots m_6} + \dots
\end{aligned}
$$

$$
\begin{aligned}
\overset{(-3)}{J}{}^{m_0|m_1\dots m_8} &= P^{(3)m_0|m_1\dots m_8}\,, \\
\overset{(-2)}{J}{}^{m_1\dots m_6} &= P^{(2)m_1\dots m_6} + \tfrac{1}{3!} A_{pqr} P^{(3)p|qrm_1\dots m_6}\,, \\
\overset{(-1)}{J}{}^{mnp} &= P^{(1)mnp} + \tfrac{1}{3!} A_{rst}\, P^{(2)rstmnp} + \\
&\quad + \left(\tfrac{2}{3}\, A_{r_1\dots r_6} + \tfrac{1}{72}\, A_{r_1r_2r_3} A_{r_4r_5r_6}\right) P^{(3)r_1|r_2\dots r_6 mnp}\,.
\end{aligned}
$$

Hidden Sugawara Structure of Constraints: J J

Remarkably the constraints admit a formulation in terms of conserved Nœther charges $\mathcal{J}$: Hamiltonian constraint: $\overset{(0)}{\mathcal{L}} \equiv \langle \mathcal{J} \mid \mathcal{J} \rangle \approx 0$ (E_{10} singlet), and :

$$\text{Momentum:} \quad \overset{(-3)}{\mathfrak{L}}{}^{n_1 \dots n_9} = 28 \overset{(-1)}{J}{}^{n_1 n_2 n_3} \overset{(-2)}{J}{}^{n_4 \dots n_9} + \overset{(0)}{J}{}^{n_1}{}_p \overset{(-3)}{J}{}^{p|n_2 \dots n_9}$$

$$\text{Gauss:} \quad \overset{(-4)}{\mathfrak{L}}{}^{m_1 \dots m_{10}||n_1 n_2} = \frac{21}{5} \overset{(-2)}{J}{}^{n_1 m_1 \dots m_5} \overset{(-2)}{J}{}^{n_2 m_6 \dots m_{10}} + \overset{(-1)}{J}{}^{n_1 m_1 m_2} \overset{(-3)}{J}{}^{n_2 | m_3 \dots m_{10}} ,$$

$$\text{Bianchi-F:} \quad \overset{(-5)}{\mathfrak{L}}{}^{m_1 \dots m_{10}||n_1 \dots n_5} = \overset{(-2)}{J}{}^{n_1 \dots n_4 m_1 m_2} \overset{(-3)}{J}{}^{n_5 | m_3 \dots m_{10}}$$

$$\text{Bianchi-R:} \quad \overset{(-6)}{\mathfrak{L}}{}^{m_1 \dots m_{10}||n_0|n_1 \dots n_7} = \overset{(-3)}{J}{}^{n_0 | m_1 \dots m_8} \overset{(-3)}{J}{}^{m_9 | m_{10} n_1 \dots n_7}$$

Analogy: String / Kac-Moody Coset

Gauge-fixed action	$S = \int\int d\tau\, d\sigma(\dot{X}^2 - X'^2)$	$S = \int \frac{dt}{n(t)}[(\dot{g}g^{-1})^{\mathrm{sym}}]^2$
Infinite symmetry	$\delta X^\mu = \epsilon_n^\mu\, e^{in(\tau\mp\sigma)}$	$g(t) \to k(t)\, g(t)\, g_0$
Infinite # conserved Nœther charges	j_n^μ	$\mathcal{J} = g^{-1} v^{\mathrm{sym}} g/n$
Integrable	$X_L^\mu = \frac{i\ell}{2}\sum \frac{j_n^\mu}{n}\, e^{-in(\tau-\sigma)}$	$g(t) = e^{t\mathcal{J}}\, g(0)\, k(t)$
∞ # constraints	$L_m \sim \int d\sigma\, e^{-im\sigma}(\partial_- X)^2$	SUGRA constraints inc. spatial gradients
Sugawara structure	$L_m = \frac{1}{2}\sum j_{m-n}^\mu j_n^\mu$	YES, DKN'07'09
Affine KM alg.	$[j_m^a, j_n^b] = f_c^{ab}\, j_{m+n}^c + O(K)$	Hyperbolic KM alg. $\{J_\alpha, J_\beta\} = f_{\alpha\beta}^{\alpha+\beta} J_{\alpha+\beta}$
Constraint alg.	$[L_m, L_n] = (m-n)L_{m+n} + O(c)$	Hyperbolic analog of Virasoro ?

Towards a Generalized Virasoro Algebra

Damour Kleinschmidt Nicolai '11

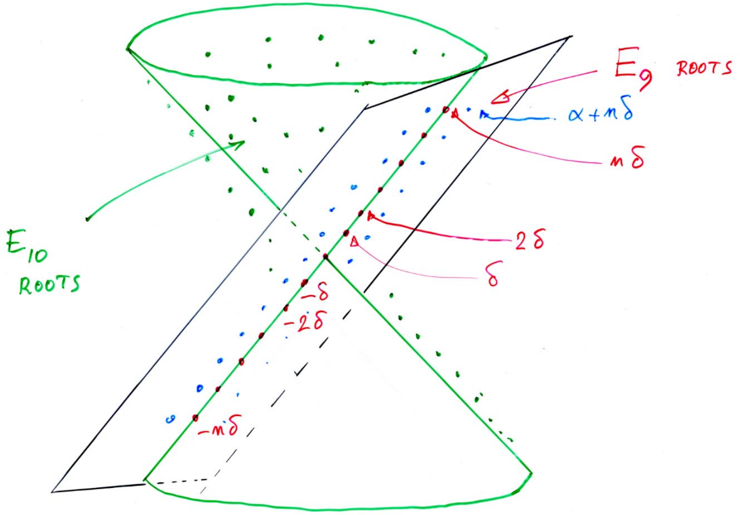
• Usual Virasoro constraints L_n can be considered as labelled by a string of null roots $L_{n\delta} := \sum_{\beta\in\Delta^{\rm aff}} T_{n\delta-\beta} T_\beta$ in terms of affine K^M generators T_α : $[T_{\alpha_1}, T_{\alpha_2}] = f_{\alpha_1\,\alpha_2}{}^{\alpha_1+\alpha_2} T_{\alpha_1+\alpha_2} + \kappa_{\alpha_1,\alpha_2} K$

• Similarly one can label the Gravity/Coset constraints $\mathcal{L}$ in terms of E_{10} roots.

e.g. $GL(10)$ multiplet $\overset{(-3)}{\mathcal{L}}{}^{[n_1 \dots n_9]}$: highest weight $\overset{(-3)}{\mathcal{L}}{}^{2345678910}$, associated with negative, null root (which is the fundamental weight Λ_1 associated with the leftmost E_{10} "hyperbolic" node)

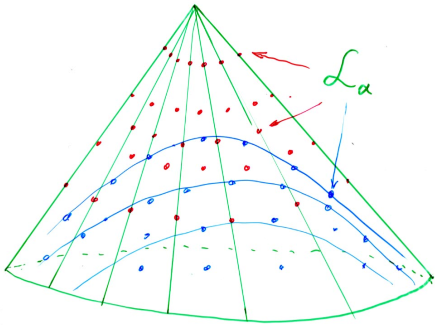
$$\alpha = \Lambda_1 = -\delta^{(3)} = -(\alpha_2 + 2\alpha_3 + 3\alpha_4 + 4\alpha_5 + 5\alpha_6 + 6\alpha_7 + 4\alpha_8 + 2\alpha_9 + 3\alpha_{10})$$

Affine Kac-Moody (E_9) versus Hyperbolic (E_{10})



Full Root Support of Coset Constraints

Expect: root support of fundamental rep. $L(\Lambda_1)$ + spatial gradients which add multiples $-n\,\delta$



root support = cone of all past-directed null and time-like roots

'Sugawara Content' of Constraints

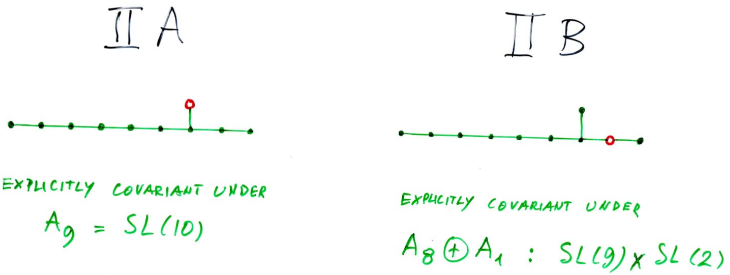
$$\mathcal{L}_\alpha = \sum_{\beta, s, s'} M_{s,s'}(\alpha, \beta)\, J^{(s)}_{\alpha-\beta}\, J^{(s')}_{\beta}$$

? numerical coefficients: $M_{s,s'}(\alpha, \beta)$

? 'constituent roots': $\alpha = \beta_1 + \beta_2$ with $\beta_1 = \alpha - \beta$, $\beta_2 = \beta$

Universality vs Non-Universality of Constraints

While $[\mathfrak{L}_{IIA}]_{\text{real roots}} = [\mathfrak{L}_{IIB}]_{\text{real roots}}$, $[\mathfrak{L}_{IIA}]_{\text{imaginary roots}} \neq [\mathfrak{L}_{IIB}]_{\text{imaginary roots}}$
because of different manifest covariantization groups



Universality vs Non-Universality of Constraints (2)

First possibility: try to combine the different covariantizations to define 'more complete' coset constraints: 'see saw' technique.

Second possibility: admit that the two different choices of (maximal) parabolic subgroups

$$IIA = \mathrm{Lie}(E_{10}) - \{f_{10}\} \quad \neq \quad IIB = \mathrm{Lie}(E_{10}) - \{f_8\}$$

correspond to two different 'gauge fixings' of the underlying E_{10}-invariant coset action $\Rightarrow$ two different additional constraints

$$S_{IIA} = \int dt \left[\frac{\|P\|^2}{n(t)} + \lambda^{\alpha}_{IIA}(t)\, \mathcal{L}^{IIA}_{\alpha} \right] \quad \neq \quad S_{IIB} = \int dt \left[\frac{\|P\|^2}{n(t)} + \lambda^{\alpha}_{IIB}(t)\, \mathcal{L}^{IIB}_{\alpha} \right]$$

Algebra of constraints: Generalized Virasoro alg.

For each choice of (maximal) parabolic subgroup (IIA or IIB) $\leftrightarrow$ associated level decomposition (ℓ_{IIA} counts e_{10}, ℓ_{IIB} counts e_8), and

$$\overset{-(\ell+1)}{\mathfrak{L}} = \{ \overset{(-1)}{J} , \overset{-(\ell)}{\mathfrak{L}} \}$$

$$\overset{-(\ell)}{\mathfrak{L}} = \sum_{p+q=\ell} \overset{(-p)}{J} \cdot \overset{(-q)}{J}$$

$\Rightarrow$ Hyperbolic generalization of usual Virasoro, and (hopefully) of gravity's diffeomorphism algebra:

$$\{\mathcal{L}_\alpha, \mathcal{L}_\beta\} \sim \sum_\gamma J_{\alpha+\beta-\gamma}\, \mathcal{L}_\gamma$$

$$[L_{m\delta}, L_{n\delta}] = 2(m-n)\, K\, L_{(m+n)\delta}$$

$$[\mathcal{H}_\mu(x), \mathcal{H}_\nu(y)] \sim g(x)\, \partial\delta(x-y)\, \mathcal{H}_\lambda(y)$$

Conclusions

• The "near cosmological singularity limit" suggests that there exists a "holographic" correspondence between $D = 11$ SUGRA (or M-theory) and a $D = 1$ spinning particle dynamics on the infinite-dimensional coset space E_{10}/K_{10}.

• The quantum dynamics of the coset model could provide both a background-independent formulation of M-theory, and a description of the (de-)emergence of space at a big-bang (big-crunch) singularity.

• There is an "arithmetic chaos" structure contained in the coset model (*via* Weyl group $W(E_{10})$) that could play an important role.

• From the coset geodesic dynamics part, one expects the quantum wave function Ψ to be an automorphic function under $E_{10}(\mathbb{Z})$, solution of $\Box\Psi = 0$.

• $\exists$ probably too many d.o.f. in the coset. One expects the existence of an infinite tower of constraints $\mathcal{L}_\alpha \Psi = 0$, possibly linked (for IIA) with the Λ_1 representation and leading to a semi-open algebra of the type, $[\mathcal{L}_\alpha, \mathcal{L}_\beta] = \sum_\gamma J_{\alpha+\beta-\gamma}\, \mathcal{L}_\gamma$, which (hopefully) is a generalization of the diffeomorphism algebra: $[\mathcal{H}_\mu(x), \mathcal{H}_\nu(y)] \sim g(x)\, \partial\delta(x-y)\, \mathcal{H}_\lambda(y)$.