CONSTRAINTS ON SUPERSTRING AMPLITUDES

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NICOLAI FEST

Symmetries, Unification and the Search for Quantum Gravity

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HAPPY BIRTHDAY HERMANN !!

Thank you for your intellectual depth, your imaginative insights, your leadership and your many impressive achievements.

And, above all, thank you for your welcoming and friendly manner.

STRING THEORY AND ITS LOW ENERGY LIMIT

Closed String Scattering in Flat Space – Maximal SUSY:

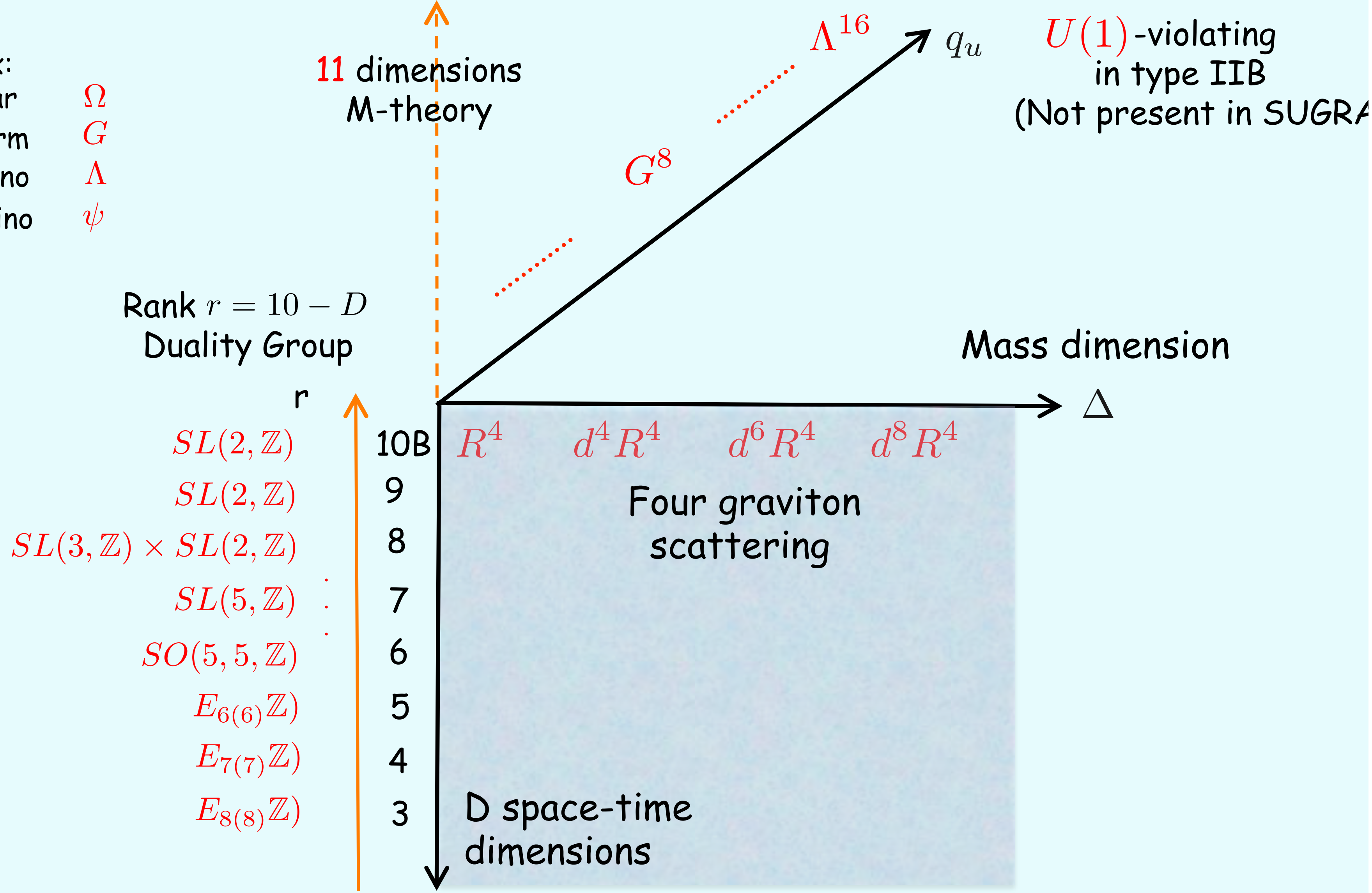
- Low Energy, or Large Distance, expansion.
  - infinite series of higher-derivative interactions
- Rich dependence on moduli (scalar fields)
  - interplay of perturbative and nonperturbative (instanton) effects.
- Comparison of perturbative maximal supergravity with String/M-Theory.

Recent work with: **Stephen Miller, Jorge Russo, Pierre Vanhove**

Other work: Gutperle, Obers, Pioline, Kiritsis, Basu, Sethi, Waldron, ...

THE LOW ENERGY EXPANSION OF TYPE IIB STRING THEORY

Riemann curvature	R
Complex:	
scalar	Ω
3-form	G
dilatino	Λ
gravitino	ψ



11 dimensions
M-theory

Λ^{16} q_u

$U(1)$-violating
in type IIB
(Not present in SUGRA

G^{8}

Rank $r = 10 - D$
Duality Group

Mass dimension Δ

Duality Group	r
$SL(2,\mathbb{Z})$	10B
$SL(2,\mathbb{Z})$	9
$SL(3,\mathbb{Z}) \times SL(2,\mathbb{Z})$	8
$SL(5,\mathbb{Z})$	7
$SO(5,5,\mathbb{Z})$	6
$E_{6(6)}\mathbb{Z})$	5
$E_{7(7)}\mathbb{Z})$	4
$E_{8(8)}\mathbb{Z})$	3

R^4 d^4R^4 d^6R^4 d^8R^4

Four graviton
scattering

D space-time
dimensions

Moduli spaces for toroidal compactification of 11-dim. supergravity on a $d+1$-torus to $D=10-d$ dimensions. Scalars parameterise coset space

$G(\mathbb{R})/K(\mathbb{R})$ ← maximal compact subgroup

	Dimension $D=10-d$	Group G	Maximal Compact Subgroup
	D	$E_{d+1}(\mathbb{R})$	$K_{d+1}(\mathbb{R})$
11-Dim. SUGRA	11	1	1
	10A	$\mathbb{R}^+$	1
	10B	$SL(2,\mathbb{R})$	$SO(2)$
	9	$SL(2,\mathbb{R})\times\mathbb{R}^+$	$SO(2)$
	8	$SL(3,\mathbb{R})\times SL(2,\mathbb{R})$	$SO(3)\times SO(2)$
	7	$SL(5,\mathbb{R})$	$SO(5)$
	6	$SO(5,5,\mathbb{R})$	$SO(5)\times SO(5)$
	5	$E_{6(6)}(\mathbb{R})$	$USp(8)/\mathbb{Z}_2$
N=8 SUGRA	4	$E_{7(7)}(\mathbb{R})$	$SU(8)/\mathbb{Z}_2$
	3	$E_{8(8)}(\mathbb{R})$	$Spin(16)/\mathbb{Z}_2$

CONTINUOUS OR DISCRETE DUALITY SYMMETRY?

In supergravity perturbation theory the continuous duality group $G(\mathbb{R})$ is unbroken - non-anomalous. No dependence on moduli.

BUT

- Presence of non-perturbative charged black holes and instantons is consistent with Dirac quantization lattice breaking continuous symmetry when D < 10

$$G(\mathbb{R}) \to G(\mathbb{Z})$$

Discrete arithmetic subgroup

- Only a discrete arithmetic subgroup is a symmetry in string theory - even at tree level.
- Note: $SL(2,\mathbb{R})$ symmetry of type IIB SUGRA in D=10 is ANOMALOUS - broken to $SL(2,\mathbb{Z})$ in perturbation theory.

Should also be true for breaking of $SL(2,\mathbb{R})$ to $SL(2,\mathbb{Z})$ in $\mathcal{N}=4$ SUGRA ??

Higher derivative interactions have rich dependence on moduli, which live in coset spaces

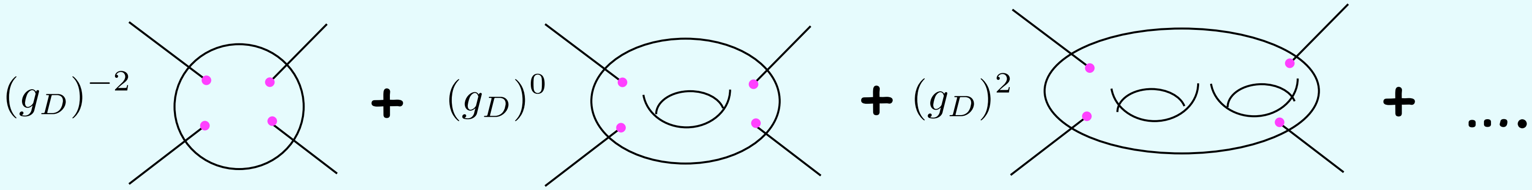
$$G(\mathbb{Z})\backslash G(\mathbb{R})/K(\mathbb{R})$$

Discrete identifications of scalar fields

SCATTERING AMPLITUDES IN STRING THEORY

Some generalities :

- Closed string perturbation theory for N-particle scattering is a sum over Riemann surfaces of genus h with N punctures embedded in the target space.



$(g_D)^{-2}$ + $(g_D)^{0}$ + $(g_D)^{2}$ + ...

- Non-perturbative effects are crucial for completion of the series. Implementing the duality symmetries of the amplitude (see later).
- There are **No Ultraviolet Divergences** in String Perturbation Theory.
- Compactify on a d -torus to $10-d$ dimensions. Rich dependence on moduli
- Can UV divergences be seen in SUGRA limit? At what loop order?

How Powerful Are Constraints Imposed by SUSY, Duality And Unitarity ?

FOUR-GRAVITON SCATTERING IN TYPE II STRING THEORY

$3 \leq D \leq 10$

$$A_D(s,t,u;\mu) = A_D^{analytic}(s,t,u;\mu) + A_D^{nonan}(s,t,u;\mu)$$

moduli

$$A_D^{analytic}(s,t,u;\mu) = \mathcal{R}^4\, T_D(s,t,u;\mu)$$

$\mathcal{R}$ Linearized Weyl curvature (supersymmetry)

Symmetric polynomial in $s\,,t\,,u$ (with $s+t+u=0$)

has expansion in power series of $\sigma_2 \sim s^2$ and $\sigma_3 \sim s^3$

$\sigma_2 = s^2 + t^2 + u^2$

$\sigma_3 = s^3 + t^3 + u^3 = 3stu$

$$T_D(s,t,u) = \sum_{p,q} \mathcal{E}^{(D)}_{(p,q)}\, \sigma_2^p\, \sigma_3^q$$

$\sim s^{2p+3q} + \ldots$

Coefficients are duality invariant functions of scalar fields (moduli, or coupling (constants)).

e.g.

$D = 10$ PERTURBATIVE terms

TREE-LEVEL (VIRASORO) AMPLITUDE:

$$A_{10}^{tree} = e^{-2\phi}\,\mathcal{R}^4\, T_{10}^{tree}(s,t,u)$$

dilaton ↗ $g = e^{\phi}$ coupling

$$T_{10}^{tree}(s,t,u) = \frac{1}{stu}\,\frac{\Gamma(1-\alpha' s\,\Gamma(1-\alpha' t)\,\Gamma(1-\alpha' u)}{\Gamma(1+\alpha' s)\,\Gamma(1+\alpha' t)\,\Gamma(1+\alpha' u)}$$

Derivative expansion: Easy to expand in an infinite series of powers of s, t, u

SUPERGRAVITY tree ↘ $\quad\mathcal{R}^4$ ↙ $\quad\partial^4\,\mathcal{R}^4$ ↓ $\quad\partial^6\,\mathcal{R}^4$ ↓

$$T_{10}^{tree} = \frac{3}{\sigma_3} + 2\zeta(3)\alpha'^3 + \zeta(5)\alpha'^5\,\sigma_2 + \frac{2\zeta(3)^2}{3}\alpha'^6\,\sigma_3$$

$\partial^8\,\mathcal{R}^4$ ⟶

$$+\frac{\zeta(7)}{2}\alpha'^7\,\sigma_2^2 + \frac{2\zeta(3)\zeta(5)}{3}\alpha'^8\,\sigma_2\sigma_3$$

⟵ $\partial^{10}\,\mathcal{R}^4$

$$+\frac{\zeta(9)}{4}\alpha'^9\,\sigma_2^3 + \frac{2}{27}\left(2\zeta(3)^3 + \zeta(9)\right)\alpha'^9\,\sigma_3^2 + \ldots$$

$\partial^{12}\,\mathcal{R}^4$

$\sigma_2 = s^2 + t^2 + u^2$

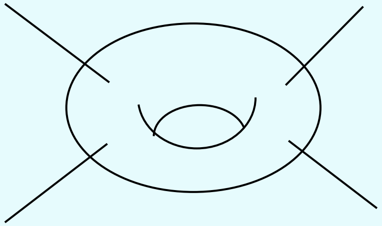
$\sigma_3 = s^3 + t^3 + u^3 = 3stu$

INFINITE SERIES of $d^{2k}R^4$ terms. Coefficients are powers of ζ values (no multi-zeta values) with rational coefficients.

e.g.

$D=10$ PERTURBATIVE terms

ONE-LOOP AMPLITUDE



$$A_{10}^{1-loop} = \mathcal{R}^4\, I(s,t,u)$$

$$I = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2}\, F(\tau,\bar{\tau};s,t,u)$$

Integral over world-sheet modulus
(complex structure of torus)

Low energy expansion is difficult !

(MBG, Russo, Vanhove)

$$I_{an}^{(D=10)} = \frac{\pi}{3}\left(1 \underset{d^4R^4}{\boxed{+\,0\,\sigma_2}} + \frac{\zeta(3)}{3}\,\sigma_3 \underset{d^8R^4}{\boxed{+\,0\,\sigma_2^2}} + \frac{97\zeta(5)}{1080}\,\sigma_2\sigma_3 + \frac{\zeta(3)^2}{30}\,\sigma_2^3 + \frac{61\zeta(3)^2}{1080}\,\sigma_3^2 + \dots\right)$$

BEYOND ONE LOOP:

- 2-loop amplitude is explicit but little studied.
- Technical difficulties constructing 3-loops and beyond !

N-POINT FUNCTIONS:

Some intriguing results for N-point functions, generalizing those of SUSY Yang-Mills and SUGRA.

- N-point trees: Motivic multi-zeta values Mafra, Stieberger, Schlotterer; Stieberger, Schlotterer
- N-point 1-loop Richards; Mafra, Schlotterer

WHAT IS NONPERTURBATIVE COMPLETION ??

Supersymmetry and Dualities and consistency with Unitarity provide strong constraints on coefficients of $s^k\,\mathcal{R}^4$ terms.

- Invariance under duality symmetries implies relations between perturbative and nonperturbative terms in the S-matrix.

DUALITY-INVARIANT EFFECTIVE IIB ACTION

(Einstein frame)

Einstein-Hilbert

Higher-derivative interactions

Non-local

$$S = \frac{1}{\ell_D^{D-2}} \int d^D x \sqrt{-G^{(D)}}\, R + S^{local} + \ldots$$

$$S_D^{local} = \ell_D^{8-D} \int d^D x \sqrt{-G^{(D)}} \left(\mathcal{E}^{(D)}_{(0,0)}\, R^4 + \ell_D^4\, \mathcal{E}^{(D)}_{(1,0)}\, d^4 R^4 + \ell_D^6\, \mathcal{E}^{(D)}_{(0,1)}\, d^6 R^4 + \ldots \right)$$

Planck length

1/2 BPS

1/4 BPS

1/8 BPS

Is this the complete list of "protected" terms??

$\mathcal{E}^{(D)}_{(p,q)}$ - duality-invariant coefficients functions of moduli

Strongly constrained by supersymmetry and dualities.

(I) D=10 DIMENSIONS – SL(2,Z) DUALITY SYMMETRY

SUPERSYMMETRY: Find $\delta\Phi$ such that $\delta\, S = 0$ (MBG, S.Sethi)

and $\epsilon_1\epsilon_2[\delta\,,\delta] = (\epsilon_1\gamma\epsilon_2)\cdot\partial + \text{eqs. of motion}$

Classical action

higher powers of α'

$$S = S^{(0)} + S^{(3)} + S^{(5)} + S^{(6)} + \dots$$

$$\delta = \delta^{(0)} + \delta^{(3)} + \delta^{(5)} + \delta^{(6)} + \dots$$

Classical supersymmetry

At order $(\alpha')^3$

$$\delta^{(0)}\, S^{(3)} + \delta^{(3)}\, S^{(0)} = 0$$

$$[\delta^{(0)}\,,\delta^{(3)}] = 0$$

$S^{(3)}$ contains component interactions differing by $q = 2n$ units of U(1) charge, .

dilatino

$$\mathcal{E}^{0}_{(0.0)}\, R^4\,, \quad \underbrace{\mathcal{E}^{2}_{(0.0)}\, G^2\, R^3\,, \ldots, \mathcal{E}^{24}_{(0.0)}\, \Lambda^{16}}_{U(1)-\text{ violating processes}}$$

$$D\mathcal{E}^{q}_{(0,0)} = a\,\mathcal{E}^{q+2}_{(0,0)}\,, \qquad \bar{D}\mathcal{E}^{q+2}_{(0,0)} = b\,\mathcal{E}^{q}_{(0,0)}$$

Simultaneous first order diff. equations.

With U(1)-covariant derivatives

$$\bar{D} = \Omega_2 \left(-i \frac{\partial}{\partial\, \bar{\Omega}} + \frac{q}{4} \right) \qquad D = \Omega_2 \left(i \frac{\partial}{\partial \Omega} + \frac{q}{4} \right)$$

Holomorphic SL(2) weight $\frac{q}{2}$

Anti-holomorphic SL(2) weight $-\frac{q}{2}$

Iterating gives LAPLACE EQUATION:

$$\Delta\, \mathcal{E}^{q}_{(0,0)} = 4\, D\bar{D}\mathcal{E}^{q}_{(0,0)} = 4ab\, \mathcal{E}^{q}_{(0,0)}$$

Examples:

Functions of one (complex) modulus

$\Omega = \Omega_1 + i\,\Omega_2$

Parameterizes $SU(2)/SO(2)$

$\Omega_2 = e^{-\phi} = \frac{1}{g_B}$

$\Delta_\Omega = \Omega_2^2\,(\partial^2_{\Omega_2} + \partial^2_{\Omega_1})$

$\mathcal{R}^4$ $\qquad \Delta_\Omega\,\mathcal{E}^{(10)}_{(0,0)} = \frac{3}{4}\,\mathcal{E}^{(10)}_{(0,0)}$

$\partial^4\,\mathcal{R}^4$ $\qquad \Delta_\Omega\,\mathcal{E}^{(10)}_{(1,0)} = \frac{15}{4}\,\mathcal{E}^{(10)}_{(1,0)}$

Consequences of maximal supersymmetry

Solutions: **(non-holomorphic)** SL(2,Z) EISENSTEIN SERIES

$s = (2p+3)/2$ $\qquad \mathcal{E}^{(10)}_{(p,0)} = E_s = \sum_{(m,n)\neq(0,0)} \frac{\Omega_2^s}{|m+n\Omega|^{2s}}$

$$E_s = \sum_{\gamma\in\Gamma_\infty\backslash SL(2,\mathbb{Z})} (\mathrm{Im}\,\gamma\Omega)^s$$

(sum over $SL(2,\mathbb{Z})$)

Satisfies Laplace equation $\Delta_\Omega\,E_s = s(s-1)\,E_s$

$s = \frac{3}{2}$ For $\mathcal{R}^4$, $\qquad s = \frac{5}{2}$ for $\partial^4\,\mathcal{R}^4$

FOURIER EXPANSION: $E_s = \sum_{(m,n)\neq(0,0)} \frac{\Omega_2^s}{|m+n\Omega|^{2s}} = \sum_k \mathcal{F}_k \, e^{2\pi i k \Omega_1}$

$$\sim \overbrace{2\zeta(2s)\Omega_2^s + (\dots)\zeta(2s-1)\Omega_2^{1-s}}^{\text{constant term } \mathcal{F}_0} + \sum_{k\neq 0} \overbrace{\sigma_{2s-1}(k) e^{-2\pi|k|\Omega_2} e^{2\pi i k \Omega_1}}^{\text{non-zero modes } \mathcal{F}_k} \left(1 + O(\Omega^{-1})\right)$$

TWO power-behaved (perturbative) terms

TREE-level term

$\Omega_2 = g_B^{-1}$

GENUS-$\left(s - \frac{1}{2}\right)$ term

D-INSTANTON terms

Infinite no. of perturbative corrections.

Interesting measure

$\sigma_n(k) = \sum_{p|k} p^n$

- NON-RENORMALIZATION - NO HIGHER LOOP perturbative terms.
- Fascinating structure of INSTANTON terms – match in detail with Yang-Mills instantons via AdS/CFT (Montonen-Olive duality).

examples: $E_{\frac{3}{2}} \, \mathcal{R}^4$ $\quad E_{\frac{5}{2}} \, \partial^4 \mathcal{R}^4 \sim (s^2 + t^2 + u^2) \, \mathcal{R}^4$

TREE-level + ONE-loop $\quad$ TREE-level + TWO-loop

SUSY at order $(\alpha')^6\,\partial^6\,\mathcal{R}^4$

$$\delta^{(0)}\,S^{(6)}+\delta^{(3)}\,S^{(3)}+\delta^{(6)}\,S^{(6)}=0$$

Mixing with intermediate terms responsible
for source term in Laplace equation

$\mathcal{E}^{(D=10)}_{(0,1)}\,\partial^6\,\mathcal{R}^4$

$$(\Delta_\Omega-12)\,\mathcal{E}^{(D=10)}_{(0,1)}=E_{\frac{3}{2}}\,E_{\frac{3}{2}}$$

Source term is quadratic in
the coefficient of $\mathcal{R}^4$

Solution is a novel automorphic function (not Eisenstein series)
– contains 0,1,2,3 loop perturbation terms

i.e. Non-renormalization of $\partial^6\,\mathcal{R}^6$ beyond THREE LOOPS

(II) $3 \leq D \leq 10$ DIMENSIONS – HIGHER-RANK DUALITY GROUPS

MBG, Miller, Russo, Vanhove

Duality symmetry

$$\mathcal{E}^{(D)}_{(p,q)}(\gamma \cdot \varphi) = \mathcal{E}^{(D)}_{(p,q)}(\varphi); \qquad \gamma \in E_{d+1}(\mathbb{Z})$$

$\mathcal{R}^4$
$$\left(\Delta^{(D)} - \frac{3(11-D)(D-8)}{D-2}\right) \mathcal{E}^{(D)}_{(0,0)} = \boxed{6\pi\, \delta_{D-8,0}}$$

$\partial^4\,\mathcal{R}^4$
$$\left(\Delta^{(D)} - \frac{5(12-D)(D-7)}{D-2}\right) \mathcal{E}^{(D)}_{(1,0)} = \boxed{40\zeta(2)\, \delta_{D-7,0}}$$

$\partial^6\,\mathcal{R}^4$
$$\left(\Delta^{(D)} - \frac{6(14-D)(D-6)}{D-2}\right) \mathcal{E}^{(D)}_{(0,1)} = -\left(\mathcal{E}^{(D)}_{(0,0)}\right)^2 + \boxed{120\zeta(3)\, \delta_{D-6,0}}$$

- Note Kroneker delta terms contribute in "critical" dimensions

(where supergravity has log UV divergences)

- Also require a number of boundary conditions in certain limits.

SOLUTIONS: Maximal Parabolic LANGLANDS EISENSTEIN SERIES

(for $\mathcal{R}^4$, $\partial^4\mathcal{R}^4$)

generalisations of SL(2) Eisenstein series to higher rank duality groups

For a group G associated with **a maximal parabolic subgroup** labeled by a simple root, β, with elements

$$P_\beta = L_\beta \times U_\beta \subset E_{d+1}$$

"LEVI subgroup"
(block diagonal)

"Unipotent radical"
(upper triangular)

Cartan elements

$$E^G_{\beta;s} \;:=\; \sum_{\gamma\,\in\, P_\beta(\mathbb{Z})\backslash G(\mathbb{Z})} e^{2\,s\,\langle \omega_\beta\,, H(\gamma g)\rangle}\;,$$

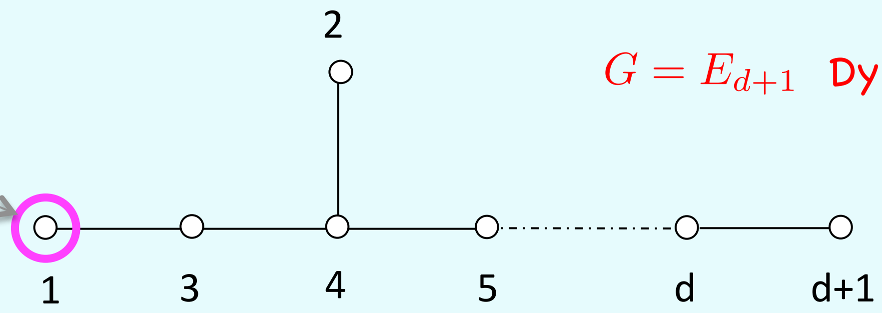
c.f. $SL(2)$ case

$$E_s = \sum_{\gamma\in\Gamma_\infty\backslash SL(2,\mathbb{Z})} (\mathrm{Im}\,\gamma\Omega)^s$$

fundamental weight vector ω_β
for a simple root β.

For $G = E_6\,, E_7\,, E_8$
Consistency suggests
choice $\beta = \alpha_1$

$G = E_{d+1}$ Dynkin diagram



Striking simplifications when $s = \frac{3}{2}\,,\quad s = \frac{5}{2}$ (for $\mathcal{R}^4\,,\quad \partial^4\mathcal{R}^4$)

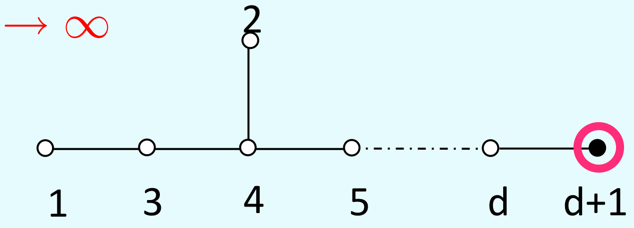
Power Behaved

A) Zero Fourier Terms agree with perturbation expansions:

- expansions in various limits associated with various subgroups

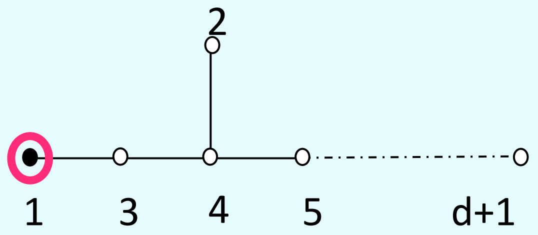
i) Decompactify from D to D+1 dimensions, $r_d \to \infty$

$$P_{\alpha_{d+1}} = GL(1)\times E_d \times U_{\alpha_{d+1}}$$



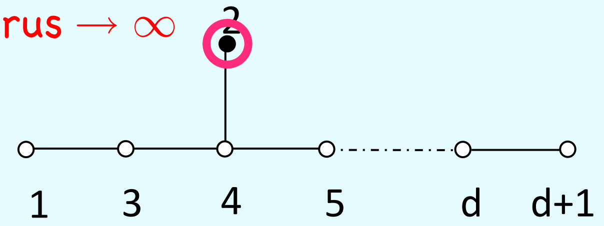
ii) String perturbation theory, $g_D \to 0$

$$P_{\alpha_1} = GL(1)\times SO(d,d)\times U_{\alpha_1}$$



iii) 11-dim. SUGRA limit, Vol. of M-theory (d+1) torus $\to \infty$

$$P_{\alpha_2} = GL(1)\times SL(d)\times U_{\alpha_2}$$



- Precise agreement in all limits
- Extended to $E_9\,,\ E_{10}\,,\ E_{11}$ (Fleig, Kleinschmidt)

THE INSTANTON TERMS – NON-ZERO FOURIER MODES

(MBG, Miller, Vanhove)

In string/M theory fractional BPS instanton arise from wrapping euclidean world-volumes of p-branes around closed (p+1)-cycles -the circles of a T^d torus in string theory (or T^{d+1} in M-theory).

- The prototype is the D-instanton which arises in $D = 10$ type IIB.
- The spectrum of instantons depends on which perturbative limit is being considered –

(i) Decompactification; $\sim \exp(-C\, r_d)$

(ii) String perturbation theory; $\sim \exp(-C\, /g_D)$

(iii) M-theory in 11 dimensions. $\sim \exp(-C\, \mathcal{V})$ ← Vol. of M-theory torus

THE INSTANTON TERMS – NON-ZERO FOURIER MODES

The instantons fill out orbits under the action of L_{α_i} that are
$\frac{1}{2}-$, $\frac{1}{4}-$ and $\frac{1}{8}-$ BPS.

- $\frac{1}{2}-$ BPS orbits are "MINIMAL ORBITS" contained in $E^{E_{d+1}}_{\alpha_1;\frac{3}{2}}$

  (Kazhdan, Savin; Ginzburg, Rallis Sudry)

- $\frac{1}{4}-$ BPS orbits are **"NEXT-TO-MINIMAL ORBITS"** contained in $E^{E_{d+1}}_{\alpha_1;\frac{5}{2}}$

  (MBG, Miller, Vanhove 2011)

- In decompactification limit a charge $k = d \times q$ instanton in D dimensions (almost always) identified with euclidean world-line of a charge q BPS black hole in $D+1$ dimensions wrapped d times around the circle of radius r_d.

Connection with Supergravity UV divergences

MBG, Russo, Vanhove

Maximal SUGRA has $\log\,\Lambda$ UV divergences in "Critical" dimensions

$$D = D_c$$

Protected terms

These can be determined by duality + supersymmetry.

One loop in D=8	$\mathcal{R}^4$
Two loops in D=7	$\partial^4\,\mathcal{R}^4$
Three loops in D=6	$\partial^6\,\mathcal{R}^4$
Four loops in D=11/2	$\partial^8\mathcal{R}^4$
Five loops in D=???	$\boxed{\partial^{??}\mathcal{R}^4}$

$$D_c = 4 + \frac{6}{L}$$

Is $d^8\,R^4$ Protected ???

COMPARE PERTURBATIVE STRING THEORY AND SUPERGRAVITY:

Perturbative supergravity in D dimensions has fixed D-dimensional Planck length, l_D , whereas in string theory l_s is fixed. These two Scales are related by the powers of the string coupling, g_D .

string length scale

$$\ell_D = g_D^{\frac{2}{D-2}}\,\ell_s$$

string coupling in D dimensions

Log thresholds

Extra contribution to analytic term

$$\log(s\,\ell_D^2) = \log(s\,\ell_s^2) + \frac{4}{D-2}\log g_D$$

Supergravity threshold

String theory threshold

String theory coefficients have corresponding $\log g_D$ contributions

- even though there are no UV divergences in string theory,

(i) D=8 $\mathcal{R}^4$

$$\mathcal{E}^{(8)}_{(0,0)} = \frac{2\zeta(3)}{g_8^2} + 2(E_1(T) + E_1(U)) + \frac{4\pi}{3}\log g_8 + \mathrm{non-pert.}$$

tree 1-loop 1-loop logarithm

(ii) D=7 $\partial^4\,\mathcal{R}^4$

$$\mathcal{E}^{(7)}_{(1,0)} = \frac{\zeta(5)}{g_7^4} + \frac{1}{g_7^2}(\dots) + (\dots) + \frac{16\pi^2}{15}\log g_7 + \mathrm{non-pert.}$$

tree 1-loop 2-loop 2-loop logarithm

(iii) D=6 $\partial^6\,\mathcal{R}^4$

$$\mathcal{E}^{(6)}_{(0,1)} = \frac{2\zeta(3)^2}{3g_6^6} + \frac{1}{g_6^4}(\dots) + \frac{1}{g_6^2}(\dots) + (\dots) + 30\zeta(3)\log g_6 + \mathrm{non-pert.}$$

tree 1-loop 2-loop 3-loop 3-loop logarithm

Factor of 6 discrepency??

CAN MAXIMAL SUPERGRAVITY BE OBTAINED AS A LIMIT OF STRING THEORY ?

- String perturbation theory has fixed ℓ_s and $g_D \to 0 \quad (\ell_D \ll \ell_s)$.
- Supergravity perturbation theory has fixed ℓ_D.
- Decoupling string modes $\ell_s \to 0$ implies $\boxed{g_D \to \infty}$ since $\ell_D = g_D^{\frac{2}{D-2}}\,\ell_s$

In the limit in which massive string states develop large mass and decouple ($\ell_s \to 0$), towers of massive "non-perturbative" states become massless since the string coupling $g_D \to \infty$.

non-perturbative masses $\sim \frac{1}{g_D}$

(MBG, Ooguri, Schwarz)

Strongly suggests that standard perturbative maximal supergravity cannot be obtained as a limit of string theory.

COMMENTS ON $\partial^8\,\mathcal{R}^4$:

- Four-loop supergravity UV divergence in $D = \frac{11}{2}$ dimensions

(Bern, Carrasco, Dixon, Johansson, Roiban)

Is there a FIVE-LOOP contribution ??

- Indications of five-loop contribution to $\partial^8\,\mathcal{R}^4$

Duality argument; pure spinor argument; possible supersymmetric counterterm

- Suggests $\partial^8\,\mathcal{R}^4$ is a "D-term" - contributions from all loops - not protected from renormalisation

Would lead to **7-loop** UV divergence in $D = 4 \quad \mathcal{N} = 8$

But the $E_{7(7)}$-invariant 7-loop counterterm is an integral over 7/8 of superspace. Does this provide higher-loop protection ??

(Bossard, Howe, Stelle, Vanhove)

A PECULIAR POSSIBILITY

There is a small (and unpersuasive!!) hint from duality arguments that:

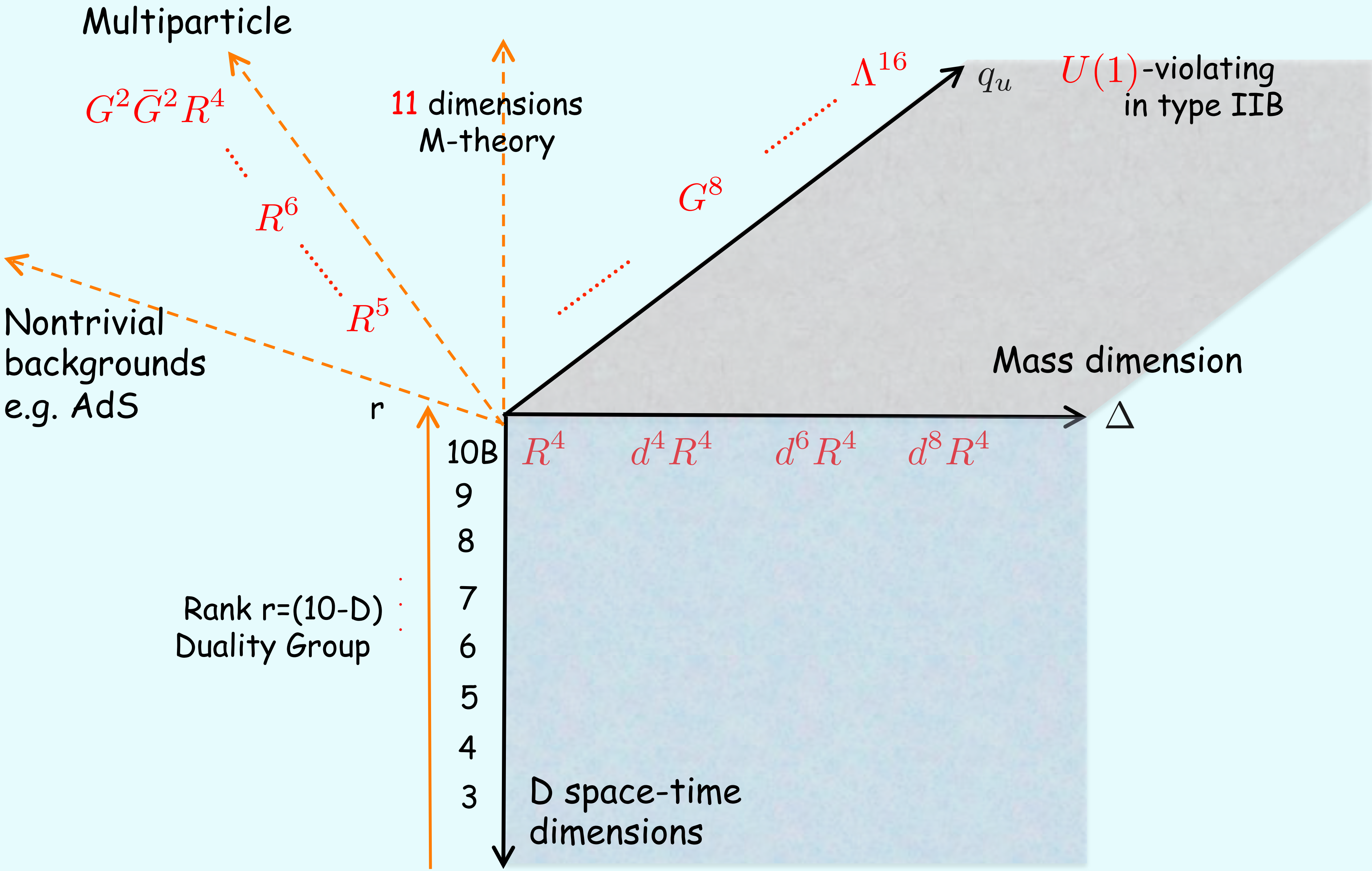
$\partial^8 \mathcal{R}^4$ may get a 5-loop contribution but no higher contributions;

$\partial^{10} \mathcal{R}^4$ may get perturbative contributions up to 7 loops;

$\partial^{12} \mathcal{R}^4$ may get perturbative contributions up to 9 loops.

That would result in the first divergence in $D = 4$ maximal supergravity ($\mathcal{N} = 8$ supergravity) at 9 loops.

MORE DIRECTIONS OF TYPE II STRING THEORY



Multiparticle

$G^2\bar{G}^2R^4$

R^6

R^5

11 dimensions
M-theory

G^8

Λ^{16}

q_u

$U(1)$-violating
in type IIB

Nontrivial
backgrounds
e.g. AdS

Mass dimension

r

Δ

10B

R^4 d^4R^4 d^6R^4 d^8R^4

9
8
7
6
5
4
3

Rank r=(10-D)
Duality Group

D space-time
dimensions