UV Properties of Perturbative Supergravity

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Symmetries, unification and the search for quantum gravity

Hermann: “I care what will be with physics 100 years from now”

Outline

- The “on shell supersymmetric” candidate counterterms break genuine local supersymmetry

  (as well as duality)

- To make them genuine supersymmetric, the theory must be deformed to Born-Infeld type N-extended supergravities.

- An explicit N=2 higher-derivative Born-Infeld type supergravity is now available

- There are obstacles on the way to do the same for N = 4,…,8

- Conjecture on a hidden N=4 superconformal symmetry of N=4 supergravity.

Until the actual calculations in N=4,…,8 supergravity were performed, we "knew" that there are infinite number of legitimate counterterm candidates, starting from a certain number of loops. It seemed that they obey all supersymmetry requirements, and therefore it could only be a miracle that the coefficients in front of them vanished.

So when the coefficients in front of several "legitimate counterterms" in N = 4,…,8 were found to disappear, it looked as a miracle, which could happen by chance in 3-loop approximation, maybe in 4-loop approximation, but not forever, unless a really good reason for these miracles is found.

A new perspective:

The candidate countertrms can be made invariant under genuine (off-shell) local supersymmetry for N = 1 and N = 2, where auxiliary fields are known.

Auxiliary fields in N = 4,…,8 are not known, and therefore the genuine invariance of the candidate counterterms was neither investigated nor questioned, in a hope that on-shell invariance is sufficient. There is an accumulating evidence that it is NOT sufficient. At present we DO NOT HAVE ANY candidate counterterms for N = 4,…,8 which are known to be genuine (off shell) locally supersymmetric.

Our previous expectation that these theories MUST have divergences, unless some miracle cancellation happens, was based on an unproven assumption that on-shell supersymmetry is sufficient for consistency of the counterterms. This is not necessarily the case.

The burden of proof that N=4,…,8 supergravities are UV not finite is now on shoulders of those who attempt to present the genuine supersymmetric candidate counterterms!

There is kind of a competition: loop computation of UV divergences by Bern, Dixon et al versus finding legitimate candidate counterterms in supergravity.

The ones we have in N=4,…,8 are not legitimate, their truncation to N=2 has now been shown to miss some important terms which are necessary to make the N=2 on shell counterterms legitimate.

On shell counterterm in N=4,…,8 are known and is interesting only up to terms proportional to classical equations of motion. In background formalism method only such counterterms are independent on the choice of the gauge-fixing conditions in the path integral. The ones, proportional to classical equation of motion are gauge-dependent and therefore are not true UV divergences. In particular, they can be absorbed by a change of variables in the classical action. For example, in pure gravity

$$R^3_{\mu\nu\lambda\delta}$$

Relevant, and is invariant under local general covariance independent as to whether Einstein eqs. are valid or not

$$R^3_{\mu\nu} \qquad R^3$$

Irrelevant, since these counterterms vanish on shell

Therefore over the years there was a confusion, the non-vanishing on shell was viewed as a good property of candidate counterterms. Meanwhile, the local supersymmetry of all known on shell candidate counterterms, starting with

$$R^4_{\mu\nu\lambda\delta} = C^4_{\mu\nu\lambda\delta}$$

can be proven only when classical eqs. of motion are satisfied. Genuine local supersymmetry of the higher derivative action has to be established without the use of equations of motion.

The Null Results

1981, RK; Howe, Stelle, Townsend: N=8 d=4 supergravity is likely to diverge at 3-loops, R^4

Miracle #1 2007 N= 8 d=4 is UV free up to 3-loops

Bern, Carrasco, Dixon, Johansson, Kosower, Roiban

2009, Bossard, Howe, Stelle: N=8 d=5 supergravity is likely to diverge at 4-loops, D^6R^4

Miracle #2 2009 N= 8 d=5 is UV free up to 4-loops

Bern, Carrasco, Dixon, Johansson, Roiban

2011, Bossard, Howe, Stelle, Vanhove, N=4 d=4 supergravity is likely to diverge at 3-loops, R^4

Miracle #3 2012 N= 4 d=4 is UV free up to 3-loops

Bern, Davies, Dennen, Huang : 3-loop d=4 computation in pure supergravity

2012, Tourkine,Vanhove, N=4+matter,
3-loop UV finite, 1-loop UV divergent ?

The most famous null result in history, 1887

The Michelson-Morley experiment produced a **null result**: something that was expected to happen was not observed.

It was aimed at detecting motion of matter relative to the aether

The negative results are generally considered to be the first strong evidence against the then prevalent aether theory, and initiated a line of research that eventually led to special relativity, 1905, in which the classical aether concept has no role

It took 18 years between Michelson-Morley experiment and Special relativity

In d=4 supergravity we need more loop computations before we may say anything with confidence, but it is tempting to start

Old counterterm paradigm

Using the existence of the covariant on-shell superspace (Brink, Howe, 1979) and the background field method in QFT one can use tensor calculus and construct the invariant candidate counterterms for N = 8. RK; Howe, Lindstrom, 1981

Such geometric counterterms are invariant under all known symmetries of the theory, including duality. They start at the L=N loop level.

Tensor calculus = infinite proliferation of candidate counterterms

Linearized ones in N=8 start at the 3-loop level, R^4+... RK; Howe, Stelle, Townsend, 1981

Miracle #1 2007 N= 8 is UV free at 3-loops

RK, 2009: no tensor calculus in N=8 light-cone superspace, no candidate counterterms, all-loop finiteness prediction

Explicit calculations: no UV divergences in N=8 at 4 loops

Bern, Carrasco, Dixon, Johansson, Roiban, 2009

Five-loop progress is continuing but no new results yet (as of August 2012).

Explanation? In d=4 QFT

- Light-cone superspace counterterms are not available at any loop order (prediction of UV finiteness). RK, 2009. This is still the case in 2012

- 3-loop finiteness follows from $E_{7(7)}$ 3-loop counterterms break duality symmetry

Broedel, Dixon
Beisert, Elvang, Friedman, Kiermaier, Morales, Stieberger
Bossard, Howe, Stelle, 2009-2010

If we trust continuous global $E_{7(7)}$ at the 3-loop quantum level, what is the prediction at higher loops?

$E_{7(7)}$ revisited: RK, 2011

Noether-Gaillard-Zumino deformed current conservation is inconsistent with the $E_{7(7)}$ invariance of the candidate counterterms

$E_{7(7)}$ revisited: Bossard-Nicolai, 2011

Yes, NGZ current conservation is inconsistent with the $E_{7(7)}$ invariance of the candidate counterterms. However, one can assume that there is a procedure of deformation of the linear twisted self-duality constraint, which will fix the problem.

$E_{7(7)}$ revisited: Carrasco, RK, Roiban, 2011

BN deformation procedure needs a modification to explain the simplest case of Born-Infeld deformation of the Maxwell theory which conserves the NGZ current. U(1) examples.

N=8 Born-Infeld supergravity?

Miracle #3, 2012 N = 4 supergravity is UV free at 3-loops

- We may apply the duality current conservation argument which was used for $E_{7(7)}$
- In N = 4 pure supergravity the duality group is $SL(2,\mathbb{R}) \times SO(6)$
- Is it possible to explain both N=8 and N=4 computations using a common language?
- Proposal RK 2011: revisit the old counterterm paradigm (reinforced by N=4 computation)

Electro-magnetic duality symmetry, rotating the Bianchi into the vector field equations, is always broken when supersymmetric duality invariant quantum corrections are added to classical extended supergravity with type E7 groups.

A small cloud on the sky: N=4 supergravity has 1-loop global U(1) anomaly, for N>4 duality is anomaly free Marcus (1985). Relevance to 3-loop UV finiteness?

Recent work on New $E_{7(7)}$ invariants and Amplitudes with T. Ortin

We have found an obstruction to BN procedure of deformation of the linear twisted self-duality constraint in the framework of the standard N=8 “on shell superspace”.

On shell superspace does not admit the kind of nonlinear deformation that would be required to incorporate the known candidate counterterms in a way compatible with both N=8 supersymmetry and full non-linear E7 symmetry. (Need a superspace with 56 independent vectors instead of 28)

Conclusion: To fix $E_{7(7)}$ duality it is necessary to deform the superspace simultaneously with deformation of the linear twisted self-duality !

What is the difference between genuine and “on shell” supersymmetry?

Classical action is invariant under local supersymmetry for generic field configurations

$$\int d^4x \frac{\delta S_0}{\delta \varphi(x)} \delta_0 \varphi(x) = 0 \qquad \frac{\delta S_0}{\delta \varphi(x)} \neq 0$$

The counterterms have local supersymmetry under classical supersymmetry transformations under condition that the fields satisfy classical EOM

$$\int d^4x \frac{\delta S_{ct}}{\delta \varphi(x)} \delta_0 \varphi(x) = 0 \quad \text{iff} \quad \frac{\delta S_0}{\delta \varphi(x)} = 0$$

Does it mean that in N=8, …, 4 supergravities the on shell counterterm have genuine local supersymmetry or not?

Is this difference relevant? What does it mean for the higher derivative superinvariant to be genuine versus “on shell” ?

N=2 off shell supersymmetry

recent work with Chemissany, Ferrara and Shahbazi

N=2 Superconformal R^4 de Wit, Katmadas, van Zalk, 2011

What we added:

1. gauge-fix to super-Poincare in presence of higher derivative action

2. eliminate auxiliary fields

3. compare with on shell counterterm

N=2 superconformal action with higher derivatives which upon gauge-fixing extra local symmetries produces N=2 pure supergravity deformed by R^4

$$\boxed{\int d^4\theta \left(S^2 + \lambda \frac{W^2}{S^2} \mathbb{T} \left(\frac{\overline{W^2}}{\overline{S^2}} \right) \right)} \, .$$

$$F = -\frac{i}{4} X^2$$

Simple prepotential

$$\mathcal{H} = \frac{(\mathcal{T}^-)^2}{X^2} \frac{(\mathcal{T}^+)^2}{\bar{X}^2}$$

Simple generalized Kahler potential

Tools: S is a chiral compensator superfield, W is a Weyl superfield. Gauge-fixing requires a choice of the second compensator which we take to be a non-linear vector multiplet.

Classical N=2 superconformal bosonic action, including a second compensator

$$S_{cl} = \frac{1}{8}\int d^4x\, e\left\{4\mathcal{D}^\mu X \mathcal{D}_\mu \bar{X} - 2|X|^2 R - \frac{X}{2}F^+_{ab}\mathcal{T}^{+\ ab} + (F^+)^2 - \frac{Y^2}{2} - \frac{X^2}{16}(\mathcal{T}^+)^2 + h.c\right\}$$

$$+\int d^4x\, e|X|^2\left\{\mathcal{D}^a V_a - \frac{V^a V_a}{2} - \frac{|M|^2}{4} + D^a\Phi^i_\alpha D_a \Phi^\alpha_i\right\}$$

$$F^-_{Sab} = \mathcal{F}^-_{ab} - \frac{\bar{X}}{4}\mathcal{T}^-_{ab}$$

$$\mathcal{F}_{ab} = 2\partial_{[a}W_{b]}$$

Graviphoton

extra superconformal symmetries are gauge-fixed

$$\frac{1}{e}\mathcal{L}_{cl} = -\frac{1}{2}R - \frac{1}{16}\left(F^+_{S\ ab}\mathcal{T}^{+\ ab} + F^-_{S\ ab}\mathcal{T}^{-\ ab}\right) + \frac{1}{8}[(F^-_S)^2 + (F^+_S)^2] - \frac{1}{16\cdot 8}\left((\mathcal{T}^+)^2 + (\mathcal{T}^-)^2\right)$$

$$\mathcal{T}^+_{\mu\nu} = 4\,\mathcal{F}^+_{\mu\nu} \qquad F^-_{Sab} = 0$$

General case

$$
\begin{aligned}
e^{-1}\mathcal{L}_4 = {}& 4\,\mathcal{D}^2 A_\Phi\, \mathcal{D}^2 \bar{A}_\Phi + 8\,\mathcal{D}^\mu A_\Phi\, \big[R_\mu{}^a(\omega,e) - \tfrac{1}{3} R(\omega,e)\, e_\mu{}^a\big]\mathcal{D}_a \bar{A}_\Phi + C_\Phi\, \bar{C}_\Phi \\
& - \mathcal{D}^\mu B_{\Phi\ ij}\, \mathcal{D}_\mu B_\Phi^{ij} + (\tfrac{1}{6} R(\omega,e) + 2\,D)\, B_{\Phi\ ij} B_\Phi^{ij} \\
& - \big[\varepsilon^{ik}\, B_{\Phi\ ij}\, F_\Phi^{+\mu\nu}\, R(\mathcal{V})_{\mu\nu}{}^j{}_k + \varepsilon_{ik}\, B_\Phi^{ij}\, F_\Phi^{-\mu\nu} R(\mathcal{V})_{\mu\nu j}{}^k\big] \\
& - 8\, D\, \mathcal{D}^\mu A_\Phi\, \mathcal{D}_\mu \bar{A}_\Phi + \big(8\,\mathrm{i} R(A)_{\mu\nu} + 2\, \mathcal{T}_\mu{}^{cij}\, T_{\nu cij}\big)\mathcal{D}^\mu A_\Phi\, \mathcal{D}^\nu \bar{A}_\Phi \\
& - \big[\varepsilon^{ij} \mathcal{D}^\mu \mathcal{T}_{bcij} \mathcal{D}_\mu A_\Phi\, F_\Phi^{+bc} + \varepsilon_{ij} \mathcal{D}^\mu T_{bc}{}^{ij} \mathcal{D}_\mu \bar{A}_\Phi\, F_\Phi^{-bc}\big] \\
& - 4\big[\varepsilon^{ij} \mathcal{T}^{\mu b}{}_{ij}\, \mathcal{D}_\mu A_\Phi\, \mathcal{D}^c F^+_{\Phi\ cb} + \varepsilon_{ij} T^{\mu bij}\, \mathcal{D}_\mu \bar{A}_\Phi\, \mathcal{D}^c F^-_{\Phi\ cb}\big] \\
& + 8\,\mathcal{D}_a F_\Phi^{-ab}\, \mathcal{D}^c F^+{}_{\Phi\ cb} + 4\, F_\Phi^{-ac}\, F^+{}_{\Phi\ bc}\, R(\omega,e)_a{}^b + \tfrac{1}{4} \mathcal{T}_{ab}{}^{ij}\, \mathcal{T}_{cdij} F_\Phi^{-ab} F_\Phi^{+cd}
\end{aligned}
$$

Our model $\Phi = \dfrac{W^2}{S^2}$ N=2 Superconformal $\dfrac{(C_{\cdots\cdots})^4}{(X\bar{X})^2}$

$$
\begin{aligned}
A_\Phi &= X^{-2}(\mathcal{T}^-)^2, \\
B_{ij}|_\Phi &= -2\Big[8X^{-2}\varepsilon_{k(i} R(\mathcal{V})^k{}_{j)ab}\, \mathcal{T}^{lmab}\, \varepsilon_{lm} + (\mathcal{T}^-)^2 X^{-3} Y_{ij}\Big], \\
F^-_{ab}|_\Phi &= -16 X^{-2} \mathcal{R}(M)_{cd}{}^{ab} (\mathcal{T}^-)^{cd} - 2(\mathcal{T}^-)^2 X^{-3} F_S^{-\,ab}, \\
C|_\Phi &= (\mathcal{T}_{ab}{}^{ij} \varepsilon_{ij})^2 \left(-2X^{-3} C_S - \frac{3}{2} X^{-4} \left(Y_{ij} Y^{ij} - 2(F^-_S)^2\right)\right) + X^{-2} C_{W^2} \\
&\quad - 16 X^{-3} Y^{ij} \varepsilon_{k(i} R(\mathcal{V})^k{}_{j)ab}\, \mathcal{T}^{lmab}\, \varepsilon_{lm} + 32 X^{-3} \mathcal{R}(M)_{cd}{}^{ab}\, \mathcal{T}^{klcd}\, \varepsilon_{kl} F^-_{Sab},
\end{aligned}
$$

The term quartic in the auxiliary field from the Weyl multiplet is a partner of the term quartic in the Weyl curvature.

$$\lambda(C_{\dots\dots})^4 \qquad\qquad \lambda(\partial\mathcal{T})^4$$

Deformed EOM for the Weyl multiplet auxiliary $\mathcal{T}^{ij}_{\mu\nu}$

$$\boxed{\mathcal{T}^{+def}_{\mu\nu} = 4\mathcal{F}^{+}_{\mu\nu} + \lambda\,[\partial^4\mathcal{T}^3]^{+}_{\mu\nu} + \cdots}$$

Solve recursively: infinite number of higher derivative terms with higher and higher powers of the graviphoton, N=2 supergravity vector

$$\mathcal{T}^{def} = \mathcal{F} + \lambda\,[\partial^4\mathcal{F}^3] + \lambda^2[\partial^4\mathcal{F}^2][\partial^4\mathcal{F}^3] + \cdots$$

The action with auxiliary field eliminated: Born-Infeld with higher derivatives in vectors

$$S^{def} = -\frac{1}{4}\mathcal{F}^2 + \lambda([\partial\mathcal{F}])^4 + \lambda^2[\partial^8\mathcal{F}^6] + \cdots$$

Superconformal Q- and S-supersymmetry

$$\boxed{\delta\psi_\mu^{i\,Superconf} = 2\mathcal{D}_\mu\epsilon^i - \frac{1}{16}\gamma_{ab}\mathcal{T}^{-\,ab}\gamma_\mu\epsilon_i - \gamma_\mu\eta^i}$$

We gauge-fix S-supersymmetry so that the local supersymmetry of supergravity is a combination of Q-supersymmetry and a compensating S-supersymmetry, preserving the gauge

$$\eta_i(\epsilon) = -\left(\gamma^\mu \frac{D_\mu X}{X}\epsilon_i + \frac{1}{4X}F^-_{ab}\gamma^{ab}\epsilon_{ij}\epsilon^j + \frac{Y_{ij}}{2X}\epsilon^j\right),$$

EXACT $$\boxed{\delta\psi_\mu^{i\,Superconf} = 2\mathcal{D}_\mu\epsilon^i - \frac{1}{16}\gamma_{ab}\mathcal{T}^{-\,ab}\gamma_\mu\epsilon_i - \gamma_\mu\eta^i}$$

Deformation of the supergravity local N=2 supersymmetry after S-supersymmetry gauge-fixing and expanding near the classical solution for auxiliary fields

$$\phi_{aux} = \phi^0_{aux} + \Delta\phi_{aux} \qquad \Delta\phi_{aux} = \sum_{n=1}\lambda^n\phi^{(n)}_{aux},$$

$$\delta\psi_\mu^{i\,SG} = 2D_\mu^{SG}\epsilon^i - \frac{1}{16}\gamma_{ab}(\mathcal{T}^{-\,ab})^0\gamma_\mu\epsilon^i$$

$$-\frac{1}{16}\Delta\mathcal{T}^{-\,ab}[\gamma_{ab},\gamma_\mu]\epsilon^i + 3i\Delta A^\nu[\gamma_\mu,\gamma_\nu]\epsilon^i + \Delta(\mathcal{V}_{\mu}{}^{i}_{j} + \frac{1}{2}Y^i{}_j\gamma_\mu)\epsilon^j$$

The deformation of the gravitino supersymmetry due to higher derivative term is

$$\Delta\psi_\mu = -4\lambda[\partial^4\mathcal{F}^3]_\mu{}^\nu\gamma_\nu\,\epsilon^i + \ldots$$

$$\phi_{aux} = \Big\{ \underbrace{(b_\mu, \mathcal{T}_{ab}{}^{ij}\,, \chi^i\,, D\,, A_\mu\,, \mathcal{V}_\mu{}^i{}_j)}_{\text{Weyl multiplet}}\,, \underbrace{(Y_{ij}, \Omega_i)}_{\text{Chiral compensator}}\, \underbrace{(V_a\,, M^{ij}, \lambda^i)}_{\text{Non-linear vector multiplet}} \Big\}$$

The complications due to gauge-fixing procedure of extra superconformal symmetries and due to elimination of all auxiliary fields from a Weyl multiplet and from 2 compensator multiplets do not remove the effect of deformation due to the fact that the auxiliary tensor field of the Weyl multiplet only at the classical level coincides with the graviphoton field strength but leads to a Born-Infeld type structures in supergravity with higher derivatives

$$\mathcal{T}^{+def}_{\mu\nu} = 4\mathcal{F}^{+}_{\mu\nu} + \lambda\,[\partial^4 \mathcal{T}^3]^{+}_{\mu\nu} + \cdots$$

In the recent past it was possible to say: the on shell N=8 superspace deformation to accommodate the required duality deformation is very difficult to make in practice, so we will not even start doing it.

Let us wait till these double copy guys will compute L=5. If they get $D_c = 24/5$ we will say that L=7 is UV divergent, if they find it 26/5 we will say that L=7 may be UV finite, but probably L=8 will be UV divergent.

New facts:

1. N=4 3-loop UV finiteness established
2. Explicit deformation of N=2 on shell superspace in its components form, to match the genuine superspace is established, the difference is known and is not vanishing.

How to establish the existence/non-existence of the consistent order by order deformation of N=4 on shell superspace (or its component form) so that its N=2 truncation fits the genuine N=2 local supersymmetry?

This program became highly motivated and it matches the difficulty of the 4-loop computation in N=4 d=4 supergravity by Bern et al.


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N=4,…,8 on shell superspace and the corresponding candidate counterterms, truncated to N=2 do not have terms  which we found to be required in genuine N=2 superspace and in genuine higher derivative superinvariants.

On shell superspace candidate counterterms in N=4,…,8 break the N=2 part of local supersymmetry. The candidate counterterms have to be constructed in a deformed N=8,…,4 superspace. The available ones are not legitimate.

The set of auxiliary fields can't be finite, the infinite set was never described, and the consistent deformation procedure for N=4,…,8 has not been developed yet. This is a formidable task. Until it is done, we have no reason to predict the UV divergences anymore, due to the established discrepancy with genuine local N=2 supersymmetry.

Performing a consistent order by order deformation of N=8 on shell superspace may be not much easier than computing the UV divergences in higher loops amplitudes.

The on shell Lorentz covariant candidate counterterms with account of the established need for a local N=2 part of supersymmetry deformation acquire the same status as the ones in the light-cone off shell superspace:

Two points of view

1. **Legitimate counterterms are not available yet**
2. **Legitimate counterterms are not available, period**

???

If some L-loop divergences will show up, the story will be likely forgotten soon and our original conclusion from 1981
7- or 8-loop and all higher loops UV divergences will be confirmed.

However, if the UV finiteness will persist in higher loops, one would like to view this as an opportunity to test some new ideas about gravity.

1. Light-cone superspace prediction

2. Duality Noether current conservation prediction

3. Genuine versus "on shell" supersymmetry **New!**

4. Hidden Symmetries of Supergravity **Brand New!**

Hidden Superconformal Symmetry of N=4 Supergravity

recent work with Ferrara and Van Proeyen

Conjecture: the Einstein (super)gravity may be a consistent gauge-fixed version of the (super)conformal theory where there are no dimensionful parameters. The Planck mass M_{PL} is not present in the superconformal (un-gauge-fixed) action, it appears when local conformal symmetry is spontaneously broken: in the unitary gauge it is the gauge-fixed value of the conformal compensator.

Analogy: Mass parameters M_W and M_Z of the massive vector mesons are not present in the gauge invariant (un-gauge-fixed) action of the standard model but show up when the gauge symmetry is spontaneously broken. In the unitary gauge they are present in the action and give an impression of being fundamental. But they are absent in renormalizable gauges, where UV properties are analyzed. **Crucial**: Equivalence theorem (RK, Tyutin 1972), requires absence of local anomalies, and is valid only for the on-shell amplitudes.

Can we treat N=4 Poincare supergravity as a unitary gauge of an N=4 superconformal model? Would it help us explain the absence of divergences?

L=4, N=4, d=4 supergravity UV divergence/finiteness will falsify/support this conjecture.

So much is at stake!

Who would ever trust local Weyl symmetry? Isn't it always anomalous?

We will now argue that N=4 local superconformal model with 6 (wrong sign) vector compensator multiplet, which upon gauge-fixing extra local symmetries:
Weyl symmetry, S-supersymmetry, K-conformal boosts, local SU(4), local U(1)
becomes pure N=4 supergravity, is special.

It has an extremely good chance to be free of N=4 superconformal local anomalies and provide an underlying reason for the UV finiteness of pure N=4 supergravity.

Feature of the superconformal model underlying pure N=4 supergravity: Weyl multiplet with the off shell algebra of local Q- and S-supersymmetry and a Maxwell matter multiplet with the on shell algebra of local supersymmetry.

Bergshoeff, de Roo, van Holten, de Wit and Van Proeyen, 1981; de Roo, 1984

Toy model of Weyl compensator in gravity

$$S^{conf} = \frac{1}{2}\int d^4x\sqrt{-g}\left(\partial_\mu\phi\,\partial_\nu\phi\, g^{\mu\nu} + \frac{1}{6}\phi^2 R\right)$$

Local Weyl symmetry $\qquad g'_{\mu\nu} = e^{-2\sigma(x)} g_{\mu\nu}\,, \qquad \phi' = e^{\sigma(x)}\phi$

Gauge-fix the extra local symmetry $\qquad \phi^2 = \frac{6}{\kappa^2}$

Einstein gravity $\qquad S^{conf}_{gauge-fixed} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\, R$

N=0 Locally conformal R^4 $\qquad \int d^4x\,\sqrt{-g}\,\phi^{-4} C_{\alpha\beta\gamma\delta} C_{\dot\alpha\dot\beta\dot\gamma\dot\delta} C^{\alpha\beta\gamma\delta} C^{\dot\alpha\dot\beta\dot\gamma\dot\delta}$

N=2 superconformal R^4

chiral kinetic action with inverse powers of the compensator superfield S

$$\int d^4\theta \frac{W^2}{S^2}\mathbb{T}\left(\frac{\overline{W^2}}{\overline{S^2}}\right)$$

N=4 superconformal R^4 $\qquad$???

Superconformal coupling of 6 (wrong sign) N=4 vector multiplets to the N=4 Weyl multiplet

De Roo

$$e^{-1}L^{bos}_{s.c.} = -\frac{1}{4}F^{+I}_{\mu\nu}\eta_{IJ}F^{+J}_{\mu\nu}\frac{\phi^1-\phi^2}{\Phi} - \frac{1}{4}\mathcal{D}_\mu\phi^I_{ij}\eta_{IJ}\mathcal{D}_\mu\phi^{ijJ}$$
$$- F^{+I}_{\mu\nu}\eta_{IJ}T^{\mu\nu}_{ij}\phi^{ijJ}\frac{1}{\Phi} - \frac{1}{2}T_{\mu\nu ij}\phi^{ijJ}\eta_{IJ}T^{\mu\nu}_{kl}\phi^{klJ}\frac{\Phi^*}{\Phi}$$
$$- \frac{1}{48}\phi^I_{ij}\eta_{IJ}\phi^{ijJ}\left(E^{kl}E_{kl} + 4D_a\phi^\alpha D^a\phi_\alpha - 12f_\mu{}^\mu\right) + \frac{1}{8}\phi^I_{ij}\eta_{IJ}\phi^{klJ}D^{ij}{}_{kl} + h.c.$$

The conformal boost gauge field $f_\mu{}^a$ is a function of a curvature $\quad f_\mu{}^\mu = -\frac{1}{6}R(\omega)$

$$\Phi = \phi^1 + \phi^2\ , \qquad \Phi^* = \phi_1 - \phi_2\ , \qquad \phi^\alpha\phi_\alpha = 1$$

Classical superconformal action after gauge-fixing

Weyl symmetry, local SU(4), local U(1) (S-supersymmetry, K-conformal boosts)

$$\phi^I_{ij}\eta_{IJ}\phi^{ijJ} = -\frac{6}{\kappa^2} \qquad \varphi_M{}^I(x) = \frac{1}{2\kappa}\delta_M{}^I \qquad \boxed{\mathrm{Im}(\phi_1-\phi_2)=0}$$

is pure N=4 Cremmer-Scherk-Ferrara supergravity

$$\boxed{\frac{1}{4}R - \frac{1}{8}\frac{\partial\tau\partial\bar\tau}{(\mathrm{Im}\tau)^2} + \frac{i}{4}\tau F^{+I}_{\mu\nu}\delta_{IJ}F^{+J\mu\nu} + h.c.}$$

We argue that the higher derivative superconformal actions for N=4 model are not available

- There is only one type of possible matter multiplets in N=4 supersymmetry, i.e. N=4 Maxwell where the scalar has Weyl weight w=1

- It is not possible to construct the N=4 superconformal version of C^4 which we made in N=2

$$\frac{C^4}{(\bar\phi\phi)^2} + \text{superconformal completion} \Rightarrow \int d^4\theta \left(\frac{W^2}{S^2} \mathbb{T} \left(\frac{\overline{W^2}}{\overline{S^2}} \right) \right)$$

which requires an N=4 superfield with the conformal weight w = - 4 to make

$$\frac{C^4}{(\phi^I_{ij}\eta_{IJ}\phi^{ij\,J})^2} + \text{superconformal completion} \Rightarrow \quad ???$$

Algebra closed on W, open on a compensator, the variation of the classical action vanishes off shell! EOM consistent with the open algebra.

The N=2 superconformal tensor calculus allows to use any function of the chiral compensator for building new superinvariants. For example, there are rules for an arbitrary function of the supermultiplet, which makes a new N=2 supermultiplet

$$\mathcal{G}(S)$$

To explain why in N=4 superconformal theory it is not possible to produce superinvariant actions with arbitrary function of superfields consider an example : the off-shell chiral multiplet

$$(z, \chi_L, F)$$

We want to construct an action

$$S = \int d^2\theta\, \mathcal{G}(z)$$

The last component, to be intregrated, is $\mathcal{G}'(z)F - (1/2)\mathcal{G}''(z)\bar{\chi}_L\chi_L$

This transforms to a total derivative $\gamma^\mu\partial_\mu[\mathcal{G}'(z)\chi_L]$

and thus gives a superinvariant action.

However, consider now that we would have only the on-shell multiplet

$$(z, \chi_L, 0)$$

The algebra on spinor leads then to field equation $\gamma^\mu\partial_\mu\chi_L = 0$

We can build the superfield $\mathcal{G}(z) + \bar{\theta}_L\mathcal{G}'(z)\chi_L - (1/2)\mathcal{G}''(z)\bar{\theta}_L\theta_L\bar{\chi}_L\chi_L$

However, the integral over theta gives the last component, which transforms under susy to

$$\boxed{\gamma^\mu\chi_L\partial_\mu\mathcal{G}'(z)}$$

This is not a total derivative (missing a term proportional to the field equation, but that we cannot use to have an invariant action) and THE ACTION IS NOT SUPESYMMETRIC

Anomalies

Anomalies in global symmetries, for example COMPOSITE ANOMALIES IN SUPERGRAVITY, like U(1) 1-loop chiral anomaly in N=4 supergravity

Marcus 1985:

"Aside from the issue of dynamical gauging, it is unclear what, if any, is the significance of these anomalies"

Anomalies in local symmetries are known to be fatal, for example, triangular 1-loop chiral U(1) local anomaly, unless compensated, leads to quantum inconsistency: physical observables in the unitary gauge are not the same as the ones in renormalizable gauge.

RK,Tyutin 1972, Equivalence Theorem

$$\langle|S|\rangle|_{a,b} = \langle|S|\rangle|_{a+\delta a, b+\delta b} + X\langle \int \Lambda^\alpha(x, \phi^i, \delta a, \delta b)\, \mathcal{A}_\alpha(\phi^i)\rangle$$

Consistent anomalies in local symmetries of 2 types

$$\delta_\Lambda \Gamma(\phi^i) = \int d^4x\, \Lambda^\alpha(x)\, \mathcal{A}_\alpha(\phi^i)$$

- A consistent anomaly is available
  But X=0 or X not, depending on model

- A consistent anomaly is not available, so it is not a matter of computing X, as for example, for some orthogonal groups, like $SO(4n+2)$

N=4 consistent local superconformal anomaly

N=1 local superconformal anomalies satisfying Wess-Zumino consistency condition

de Wit, Grisaru 1987

$$\delta\Gamma[\Phi, W^2] = \int \mathrm{d}^4x(\Lambda_{\mathrm{D}}(x) - \tfrac{1}{3}\mathrm{i}\Lambda_A(x))$$
$$\times e\{[W^2]_F + \bar{\psi}_{\mathrm{L}}\cdot\gamma[W^2]_\Psi + 2\bar{\psi}_{\mu\mathrm{L}}\sigma^{\mu\nu}\psi_{\nu\mathrm{R}}[W^2]_A\}$$
$$-2\int \mathrm{d}^4x\, e\bar{\eta}_{\mathrm{R}}(x)\{[W^2]_\Psi + \gamma\cdot\psi_{\mathrm{R}}[W^2]_A\}$$
$$-2\int \mathrm{d}^4x\, e\bar{\varepsilon}_{\mathrm{L}}(x)$$
$$\times\{(\gamma^\mu[W^2]_\Psi + 4\sigma^{\mu\nu}\psi_{\nu\mathrm{R}}[W^2]_A)(b_\mu - \tfrac{1}{3}\mathrm{i}A_\mu) + \gamma\cdot\varphi_{\mathrm{L}}[W^2]A\}$$

Schwimmer, Theisen 2011
Buchbinder, Kuzenko, 1988

$$\delta\Gamma^{ST}(\phi, W^2) = 2(c-a)\int d^8z\,\frac{E^{-1}}{R}\,\delta\Sigma\, W_{\alpha\beta\gamma}W^{\alpha\beta\gamma} + c.c.,$$

The compensator superfield transforms as

$$\phi(x,\theta) \to e^{\,\Sigma(x,\theta)}\,\phi(x,\theta)$$

when its vev is non-vanishing, one may try to define the `dimensionless' Goldstone superfield, which transforms by a superfield shift

$$\delta \ln \phi(x,\theta) \to \delta\Sigma(x,\theta)$$

But $\ln \phi$

does not have a uniform scaling weight w=0, which leads to complication and modification of the scale, chiral and S-supersymmetry transformations.

$$\Gamma^{dWG}(\phi, W^2) = (\ln\phi\, W_{\alpha\beta\gamma}W^{\alpha\beta\gamma})_F + \ldots$$

$$\begin{aligned}\Gamma[\Phi, W^2] = \int \mathrm{d}^4x\, e[&\ln A[W^2]_F + (A^{-1}F + \tfrac{1}{2}A^{-2}\bar{\Psi}_R\Psi_L)[W^2]_A \\ &- \bar{\Psi}_R[W^2]_\Psi \\ &+ \bar{\psi}_L\cdot\gamma \ln A[W^2]_\Psi + \bar{\psi}_L\cdot\gamma\Psi_L[W^2]_A A^{-1} \\ &+ 2\bar{\psi}_{\mu L}\sigma^{\mu\nu}\psi_{\nu R}\ln A[W^2]_A + \text{h.c.}]\end{aligned}$$

$$\Gamma^{ST}(\phi, W^2) = 2(c-a)\int d^8z\frac{E^{-1}}{R}\,\ln\phi\, W_{\alpha\beta\gamma}W^{\alpha\beta\gamma} + c.c.$$

$$\Gamma^{dWG}(\phi, W^2) = (\ln\phi\, W_{\alpha\beta\gamma} W^{\alpha\beta\gamma})_F + \ldots$$

To generalize this action requires to use the log of the compensator, which is possible in N=1, N=2 but not possible in N=4

$$\mathcal{G}(\phi) = \ln\phi$$

We conclude that the status of consistent N=4 superconformal anomaly is the same as higher derivative superinvariants: they are not known to exist

$$\mathcal{G}(\phi) = \phi^{-2(L-1)}$$

Two possibilities:

1. **The consistent N=4 local superconformal anomaly is not available, yet**
2. **The consistent N=4 local superconformal anomaly is not available, period** **???**

‘t Hooft 2011

Can we understand black hole complimentarity using the locally conformal theory?

When gravity is coupled to the system, local conformal invariance should be a spontaneously broken exact symmetry. The argument has to do with the requirement that black holes obey a complementarity principle relating ingoing observers to outside observers

Different observers, outside and inside the event horizon =>

Different gauge-fixing of local Weyl symmetry outside and inside the horizon
Since Ricci curvature is not invariant under conformal transformations, perhaps we may resolve the Hawking radiation issue.

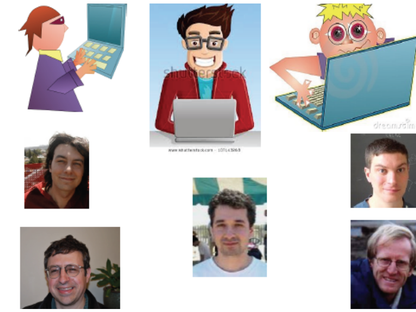
This is the central theme of the black hole complementarity issue.

However, there now is one important complication: conformal anomalies. They should not be allowed to ruin exact local conformal invariance. A local conformal transformation is necessary to transform from one space-time to another where observers disagree.

Superconformal N=4 does not have a local conformal anomaly

Issue way beyond perturbative gravity

If the recent computations tell us something more than perturbative UV finiteness of low-loop supergravity, what is it?



Can people who discovered and developed supergravity foundations start looking into it?

Physics — *Physics* **2**, 70 (2009)

Viewpoint

Vanquishing infinity

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Published August 17, 2009

Quantum field theoretic extensions of Einstein's theory of gravity tend to suffer from incurable infinities, but a theory called N = 8 supergravity may actually avoid them—against expectations held for almost 30 years.

Subject Areas: **Particles and Fields**

A Viewpoint on:
Ultraviolet Behavior of $\mathcal{N} = 8$ Supergravity at Four Loops
Z. Bern, J. J. M. Carrasco, L. J. Dixon, H. Johansson and R. Roiban
Phys. Rev. Lett. 103, 081301 (2009) – Published August 17, 2009

Gravity is different