Scalar Fields with Higher Derivatives in Supergravity and Cosmology

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Based on work with Justin Khoury, Michael Köhn and Burt Ovrut

arXiv:1012.3748
arXiv:1103.0003
arXiv:1207.3798
arXiv:1208.0752

See also

Baumann, Green
arXiv:1109.0293

Sasaki, Yamaguchi, Yokoyama
arXiv:1205.1353

Farakos, Kehagias
arXiv:1207.4767

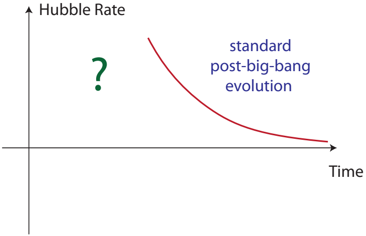
done for vector superfields by Deser & Puzalowski in 1979...

In cosmology, there are situations of interest where higher-derivative kinetic terms are important
Example:

- DBI inflation, k-inflation
  (Silverstein, Tong; Armendariz-Picon, Damour, Mukhanov)

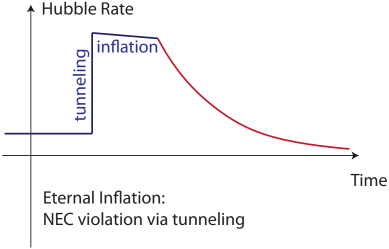
In alternative approaches to early universe cosmology higher-derivative theories also show up

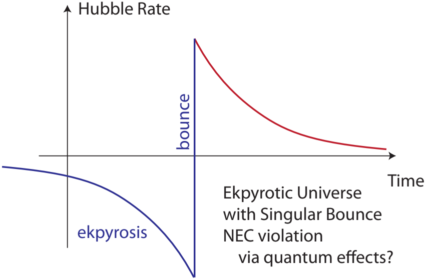
General question:



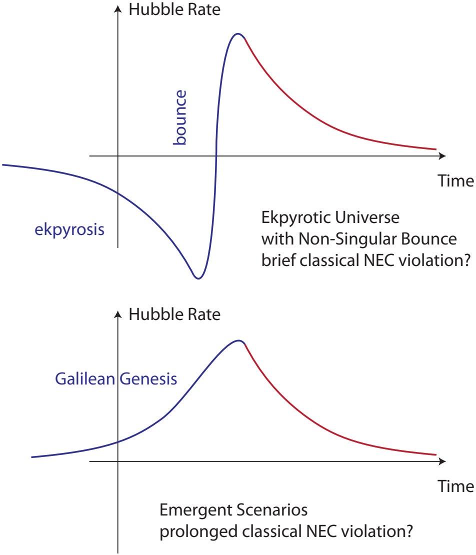
What came before?

Possible answers involving quantum NEC violation:





Possible answers involving classical NEC violation:



Can these models be realized in $\mathcal{N} = 1$ supergravity?

Method:
construct the supersymmetric extension of $(\partial^\mu \phi \partial_\mu \phi)^2 \equiv (\partial\phi)^4$ first

Work in superspace, use a chiral superfield Φ *i.e.* $\bar{\mathcal{D}}_{\dot\alpha}\Phi = 0.$

Components:

$$\begin{array}{ccl} A & \equiv \Phi \mid & \text{Complex scalar} \\ \chi_\alpha & \equiv \frac{1}{\sqrt{2}} D_\alpha \Phi \mid & \text{Spin } \dfrac{1}{2} \text{ fermion} \\ F & \equiv -\frac{1}{4} D^2 \Phi \mid & \text{Auxiliary field} \end{array}$$

Usual action, with $A = \frac{1}{\sqrt{2}}(\phi + \mathrm{i}\xi)$

$$\begin{aligned} \int d^4x d^4\theta \Phi^\dagger \Phi &= \int d^4x (-\partial A \cdot \partial A^* + F^* F) \\ &= \int d^4x \left(-\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\xi)^2 + F^* F \right) \end{aligned}$$

Now we would like to add 2 more fields and 2 more spacetime derivatives

$$\partial_m \sim \{\mathcal{D}, \bar{\mathcal{D}}\}$$

In global susy, there exist only 2 "clean" extensions of $(\partial\phi)^4$:

$$\int d^4\theta\, \mathcal{D}^\alpha \Phi \mathcal{D}_\alpha \Phi \bar{\mathcal{D}}_{\dot{\alpha}} \Phi^\dagger \bar{\mathcal{D}}^{\dot{\alpha}} \Phi^\dagger$$

$$\int d^4\theta\, (\Phi^\dagger - \Phi)^2 \mathcal{D}^m \Phi^\dagger \mathcal{D}_m \Phi$$

In local susy, first version leads to minimal coupling to gravity, while second version contains derivative couplings to gravity, of the form $\xi^2(\partial\phi)^2 R$

$\rightarrow$ focus on first version, which is the unique clean, minimally-coupled extension of $(\partial\phi)^4$

Properties of $\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger$

$$-\frac{1}{8}\int \mathrm{d}^2\Theta 2\mathcal{E}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger\, T(\Phi) + h.c.$$
$$= 16e\left((\partial A)^2(\partial A^*)^2 - 2\partial A\cdot\partial A^* FF^* + (FF^*)^2\right)T(\Phi)|$$

- Scalars appear in combination $(\partial A)^2(\partial A^*)^2$, and not $(\partial A\cdot\partial A^*)^2$ as one might have expected
- F still auxiliary (note: we could have obtained $AA^*\partial F\cdot\partial F^*$, but didn't)
- Equation for F is now cubic – hence there exist three branches of the theory
- For the bosonic part, only the top component is non-zero $\rightarrow$ can multiply by arbitrary scalar function T of Φ and its spacetime derivatives
  $\rightarrow$ can obtain a supergravity extension of any term containing $(\partial\phi)^4$ as a factor
  e.g. can obtain sugra version of $P(X,\phi)$ where $X \equiv -\frac{1}{2}(\partial\phi)^2$

Full Theory

$$\begin{aligned}\mathcal{L} &= \int \mathrm{d}^2\Theta 2\mathcal{E}\Big[\frac{3}{8}(\bar{\mathcal{D}}^2 - 8R)e^{-K(\Phi^i,\Phi^{\dagger k*})/3} + W(\Phi^i)\Big] + h.c. \\ &\quad -\frac{1}{8}\int \mathrm{d}^2\Theta 2\mathcal{E}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}\Phi^i\mathcal{D}\Phi^j\bar{\mathcal{D}}\Phi^{\dagger k*}\bar{\mathcal{D}}\Phi^{\dagger l*}T_{ijk*l*} + h.c.\end{aligned}$$

K Kähler Potential

W Superpotential

$T_{ijk^*l^*}$ Target Space Tensor, Spacetime Scalar

In Components

After Weyl re-scaling & eliminating b_m, M

$$\begin{aligned}\frac{1}{e}\mathcal{L}_{\rm Weyl} \quad &= \quad -\frac{1}{2}\mathcal{R} - g_{ik*}\partial A^i \cdot \partial A^{k*} + g_{ik*}e^{K/3}F^i F^{k*} \\ &\quad + e^{2K/3}[F^i(D_A W)_i + F^{k*}(D_A W)^*_{k*}] + 3e^K W W^* \\ &\quad + 16(\partial A^i \cdot \partial A^j)(\partial A^{k*} \cdot \partial A^{l*})\, T_{ijk*l*\rm Weyl}| \\ &\quad -32\, e^{K/3}F^i F^{k*}(\partial A^j \cdot \partial A^{l*})\, T_{ijk*l*\rm Weyl}| \\ &\quad +16e^{2K/3}F^i F^j F^{k*} F^{l*}\, T_{ijk*l*\rm Weyl}|\end{aligned}$$

Equation of motion for F^i

$$g_{ik*}F^i + e^{K/3}(D_A W)^*_{k*} + 32F^i(e^{K/3}F^j F^{l*} - \partial A^j \cdot \partial A^{l*})T_{ijk*l*\rm Weyl}| = 0$$

algebraic and cubic

New Branches: Potential Without Superpotential

Consider a single chiral superfield, define $T_{1111} \,|\equiv \mathcal{T}$
Set $W = 0$
Then, the equation for F

$$F\big(K_{,AA^*} + 32\mathcal{T}(e^{K/3}|F|^2 - |\partial A|^2)\big) = 0$$

has the non-trivial solution

$$|F_{\rm new}|^2 = -\frac{1}{32\,\mathcal{T}} e^{-K/3} K_{,AA*} + e^{-K/3}\,|\partial A|^2$$

leading to

$$\begin{aligned} \frac{1}{e}\mathcal{L}_{W=0,F_{\rm new}} \quad &= \quad -\frac{1}{2}\mathcal{R} + 16\mathcal{T}\big((\partial A)^2(\partial A^*)^2 - (\partial A \cdot \partial A^*)^2\big) \\ &\quad -\frac{1}{64\mathcal{T}}(K_{,AA^*})^2 \end{aligned}$$

- ordinary kinetic term has disappeared
- new term blows up as $\mathcal{T} \to 0$, new branch is not connected to ordinary branch

Take *e.g.*

$$\mathcal{T} = (K_{,AA^*})^2 v(\phi,\xi)$$

Then, in a flat Robertson-Walker background get

$$\int \mathrm{d}^4 x a^3 \Big(-3\frac{\dot{a}^2}{a^2} + \frac{16}{a^2} v(\phi,\xi)(\xi_{,i}^2\dot{\phi}^2 + \phi_{,i}^2\dot{\xi}^2 - 2\phi_{,i}\xi_{,i}\dot{\phi}\dot{\xi}) \\ + \frac{16}{a^4} v(\phi,\xi)(\phi_{,i}\xi_{,i}\phi_{,j}\xi_{,j} - \phi_{,i}^2\xi_{,j}^2) - \frac{1}{64 v(\phi,\xi)}\Big)$$

Via their interactions, the scalars can generate "ordinary" kinetic terms for each other

For example, if $\xi = \xi(x^i)$ then one must impose $v(\phi,\xi) > 0$ to avoid ghosts

This then leads to a positive potential

$$V_{\mathrm{new}} = \frac{1}{64 v(\phi,\xi)}$$

But because this branch cannot be reached from ordinary branch, focus on ordinary branch now

With Superpotential – Small Higher-Derivative Terms

Solve for F perturbatively, leads to the (ordinary branch) Lagrangian

$$\begin{aligned}\frac{1}{e}\mathcal{L}_{\mathrm{ordinary},\mathcal{T}\to 0} = -\frac{1}{2}\mathcal{R} - K_{,AA^*}|\partial A|^2 - \ e^K(K^{,AA^*}|D_A W|^2 - 3|W|^2) \\ - 32\, e^K K^{,AA^*}|D_A W|^2 K^{,AA^*}|\partial A|^2\, \mathcal{T} \\ + 16\, (\partial A)^2(\partial A^*)^2\, \mathcal{T} \\ + 16 e^{2K}(K^{,AA^*}|D_A W|^2)^2\, (K^{,AA^*})^2 \mathcal{T}\end{aligned}$$

The potential is now given by

$$\begin{aligned} V \quad = \quad & e^K(K^{,AA^*}|D_A W|^2 - 3|W|^2) \\ & -16(e^K K^{,AA^*}|D_A W|^2)^2\, (K^{,AA^*})^2 \mathcal{T}_{\mathrm{no\,der.}}\end{aligned}$$

- Corrections to both kinetic & potential terms
- The potential depends on the value of the higher-derivative kinetic terms

Large Higher-Derivative Terms: Example of the DBI Action

DBI action:

$$
\begin{aligned}
&\frac{1}{e}\mathcal{L}_{\mathrm{DBI}} \\
= \quad & -\frac{1}{f(A,A^*)}\left(\sqrt{\det(g_{mn}+f(A,A^*)\,\partial_m A\partial_n A^*)}-1\right) \\
= \quad & -\frac{1}{f}\left(\sqrt{1+2f\,|\partial A|^2+f^2\,|\partial A|^4-f^2\,(\partial A)^2(\partial A^*)^2}-1\right) \\
= \quad & -|\partial A|^2+(\partial A)^2(\partial A^*)^2\frac{f}{1+f\,|\partial A|^2+\sqrt{(1+f\,|\partial A|^2)^2-f^2\,(\partial A)^2(\partial A^*)^2}}
\end{aligned}
$$

Note: for time-dependent backgrounds

$$
\frac{1}{e}\mathcal{L}_{\mathrm{DBI}}=-\frac{1}{f}\left(\sqrt{1-2f|\dot{A}|^2}-1\right) \qquad f,\ \text{warp factor}
$$

$\rightarrow$ speed limit $|\dot{A}|^2 \leq \frac{1}{2f}$ $\rightarrow$ DBI inflation (Silverstein, Tong)
leads to interesting observational signatures such as equilateral non-gaussianity

DBI in Supergravity

Choose

$$16T = \frac{f(\Phi,\Phi^\dagger)}{1 + f\partial\Phi\cdot\partial\Phi^\dagger e^{K/3} + \sqrt{(1+f\partial\Phi\cdot\partial\Phi^\dagger e^{K/3})^2 - f^2(\partial\Phi)^2(\partial\Phi^\dagger)^2 e^{2K/3}}}$$

includes Weyl compensating factors and gives

$$\begin{aligned}
\frac{1}{e}\mathcal{L} \quad = \quad & -\frac{1}{2}\mathcal{R} + 3e^K|W|^2 \\
& -\frac{1}{f}\left(\sqrt{1+2f\,\partial A\cdot\partial A^* + f^2\,(\partial A\cdot\partial A^*)^2 - f^2\,(\partial A)^2(\partial A^*)^2} - 1\right) \\
& + e^{K/3}|F|^2 + \; e^{2K/3}\big(F(D_A W) + F^*(D_A W)^*\big) \\
& -32\, e^{K/3}|F|^2\partial A\cdot\partial A^*\,\mathcal{T} + 16e^{2K/3}|F|^4\,\mathcal{T}
\end{aligned}$$

still have to eliminate F

Solve for F (consider only ordinary branch):

Small f
$F \approx -e^{K/3}(D_A W)^*$ leads to the potential

$$V_{\mathrm{non-rel.}} = e^K \left(|D_A W|^2 - 3|W|^2 \right)$$

Large f
$F \approx -\left(\frac{(D_A W)^{*2}}{4f\, D_A W} \right)^{1/3}$ leads to the potential

$$\begin{aligned} V_{\mathrm{rel.}} &\approx \frac{3}{2} \frac{e^K |D_A W|^2}{\left(4f\, e^K |D_A W|^2\right)^{1/3}} - 3e^K |W|^2 \\ &\approx -3e^K |W|^2 \end{aligned}$$

$\rightarrow$ potential becomes negative
This is a general feature for these higher-derivative supergravity theories
$\rightarrow$ in particular: DBI inflation is impossible!

DBI Inflation in Supergravity

Couple to a second chiral superfield

$$S = B + \Theta^\alpha \Theta_\alpha F_B$$

with ordinary, two-derivative kinetic term (assume also that $K_{,AB^*} = 0$)
Then to leading order the potential is given by

$$\begin{aligned} V_{\mathrm{non-rel.}} &= e^K\big(|D_A W|^2 + K^{,BB^*}|D_B W|^2 - 3|W|^2\big) \\ V_{\mathrm{rel.}} &= e^K\big(K^{,BB^*}|D_B W|^2 - 3e^K|W|^2\big) \end{aligned}$$

Now choose the following form for the superpotential

$$W = Sw(\Phi) \qquad W \mid= Bw(A)$$

and limit dynamics to $B = 0$ plane where $W|_{B=0} = 0,$ $D_B W \mid_{B=0} \neq 0$

(Kawasaki, Yamaguchi, Yanagida; Kallosh, Linde, Rube)

Idea is to limit dynamics to $B = 0$ plane by choosing an appropriate form for the Kähler potential

For $B = \frac{1}{\sqrt{2}}(b + \mathrm{i}d)$ demand $m_b^2 = m_d^2 \gtrsim H^2$
$\rightarrow K_{,BBB^*B^*} \lesssim -\frac{1}{3}$

Can restrict dynamics further to $Im(A) = \xi = 0$ line by demanding $m_\xi^2 \gtrsim H^2$
$\rightarrow K_{,AA^*BB^*} \lesssim \frac{5}{6}$

$$V = V\left(\frac{\phi}{\sqrt{2}}\right)$$

Arbitrary positive potential

Note a further property: $D_A W|_{B=0} \propto B = 0$, so that $V_{\mathrm{non-rel.}} = V_{\mathrm{rel.}}$
Potential does not change as the higher-derivative terms become important

But:
Need a special Kähler potential, for example

$$K = -\frac{1}{2}(\Phi - \Phi^\dagger)^2 + SS^\dagger + \zeta(SS^\dagger)^2 + \frac{\gamma}{2}SS^\dagger(\Phi - \Phi^\dagger)^2$$

with $\zeta \lesssim -1/12$ and $\gamma \gtrsim 5/6$ (Kallosh, Linde, Rube)

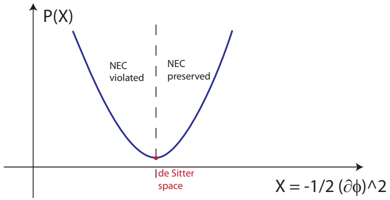
Need a special form for the superpotential

$$W = Sw(\Phi)$$

Can these arise from string theory?

Ghost Condensate

When the kinetic function $P(X)$ has a minimum, develop a time-dependent vev for ϕ : $\phi = t$
(Arkani-Hamed, Cheng, Luty, Mukohyama)



Typical action: $-X + X^2$
Minimum corresponds to dS space
Perturbations around minimum allow stable violations of the Null Energy Condition for short periods of time
Can be used to model dark energy or non-singular bounces

Ghost Condensate in Supergravity

Setting $W = 0$, omitting the second real scalar, and up to quadratic order in fermions the action becomes

$$\begin{aligned}
\frac{1}{e}\mathcal{L}_{g.c.} = &-\frac{1}{2}\mathcal{R} - X + X^2 \\
&+\frac{1}{2}\varepsilon^{klmn}\left[\bar{\psi}_k\bar{\sigma}_l\tilde{\mathcal{D}}_m\psi_n - \psi_k\sigma_l\tilde{\mathcal{D}}_m\bar{\psi}_n\right] \\
&+\frac{\mathrm{i}}{2}\left[\chi\sigma^m\mathcal{D}_m\bar{\chi} + \bar{\chi}\bar{\sigma}^m\mathcal{D}_m\chi\right](1-X) \\
&+\frac{\mathrm{i}}{2}\phi^{,m}\phi_{,n}\left(\bar{\chi}\bar{\sigma}^n(\mathcal{D}_m\chi) + \chi\sigma^n(\mathcal{D}_m\bar{\chi})\right) \\
&+\frac{1}{2}\left(\chi\sigma^m\bar{\sigma}^n\psi^p + \bar{\chi}\bar{\sigma}^m\sigma^n\bar{\psi}^p\right)\left(g_{mp}\phi_{,n} - Xg_{mn}\phi_{,p} + Xg_{np}\phi_{,m}\right)
\end{aligned}$$

The vacuum ($\phi = t$) breaks Lorentz invariance, manifested by a wrong sign spatial gradient term for the goldstino
Mixed mass term for gravitino-goldstino $\rightarrow$ super-BEHK mechanism?

Super-BEHK

Susy transformation

$$\delta\chi = \mathrm{i}\sqrt{2}\sigma^m\bar{\zeta}\mathcal{D}_m A + \sqrt{2}e^{K/6}F\zeta$$

Usual breaking: $\langle A \rangle = 0;\ \langle F \rangle \sim \langle DW \rangle \neq 0$
Gravitino eats goldstino and becomes *massive*

Here there is no superpotential, *but* $\langle \sqrt{2}A \rangle = t,$ hence goldstino again shifts by a constant:

$$\delta\chi = \mathrm{i}\sigma^0\bar{\zeta}$$

But there is no mass term for the gravitino (which would have been proportional to W) – so what happens?

Redefine gravitino to get rid of mixed mass term

$$\tilde{\psi}_{m\alpha} \equiv \psi_{m\alpha} - 2\mathrm{i}\mathcal{D}_m(\phi_{,n}\sigma^n_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}})$$

The action becomes

$$\begin{aligned}\frac{1}{e}\mathcal{L}_{g.c.} = &- \frac{1}{2}\mathcal{R} - X + X^2 \\ &+ \frac{1}{2}\varepsilon^{klmn}\left(\bar{\tilde{\psi}}_k\bar{\sigma}_l\mathcal{D}_m\tilde{\psi}_n - \tilde{\psi}_k\sigma_l\mathcal{D}_m\bar{\tilde{\psi}}_n\right) \\ &+ \frac{\mathrm{i}}{2}\left(\chi\sigma^m\mathcal{D}_m\bar{\chi} + \bar{\chi}\bar{\sigma}^m\mathcal{D}_m\chi\right) \\ &+ \mathrm{i}\phi^{,m}\phi_{,n}\left(\bar{\chi}\bar{\sigma}^n\mathcal{D}_m\chi + \chi\sigma^n\mathcal{D}_m\bar{\chi}\right)\end{aligned}$$

- Gravitino remains massless
- Goldstino is still present, otherwise degrees of freedom would be lost
- The goldstino kinetic term has an unusual normalization relative to its scalar superpartner, which is the indication that supersymmetry really is broken!

Remarks

- The same kind of supersymmetry breaking can apply in different contexts too, *e.g.* for Galileon theories, which are closely related to the ghost condensate, but generally better-behaved
- Perhaps surprisingly, there is nothing wrong with the ghost condense in supergravity *per se* – however, trouble seems to arise once you couple to a superpotential W : then the potential, which includes a term $e^K K^{,AA^*}|D_A W|^2$ blows up in going from an ordinary phase ($K_{,AA^*} > 0$) to a ghost condensate phase ($K_{,AA^*} < 0$)! Again, with Galileons this may be circumvented as one can have $K_{,AA^*} > 0$ throughout.

Summary and Outlook

- Square of ordinary kinetic term, *i.e.* $(\partial\phi)^4$ has a unique clean, minimally-coupled extension to sugra
- Can obtain sugra version of any term containing $(\partial\phi)^4$ as a factor, hence can write out sugra versions of $P(X,\phi)$ theories such as DBI actions
- Generic features:
  – three branches (new branches are exotic, currently no obvious physical significance)
  – potential without superpotential
  – corrections to both ordinary kinetic term and to potential
  – potential negative when higher-derivative terms are important
- Shown how one can get DBI inflation to work by coupling to an additional chiral superfield
- Of all theories giving rise to second-order equations of motion, only term not covered is the DGP-like Galileon term $(\partial\phi)^2\Box\phi$ – work in progress!