SO(9) supergravity *

conference on the occasion of
Hermann Nicolai's 60th anniversary

Golm 2012

Henning Samtleben

* in two dimensions

SO(9) supergravity *

[Thomas Ortiz, HS] 2012
[HS, Martin Weidner] 2007

[Hermann Nicolai, HS] 2000

[Hermann Nicolai, Bernard de Wit] 1982

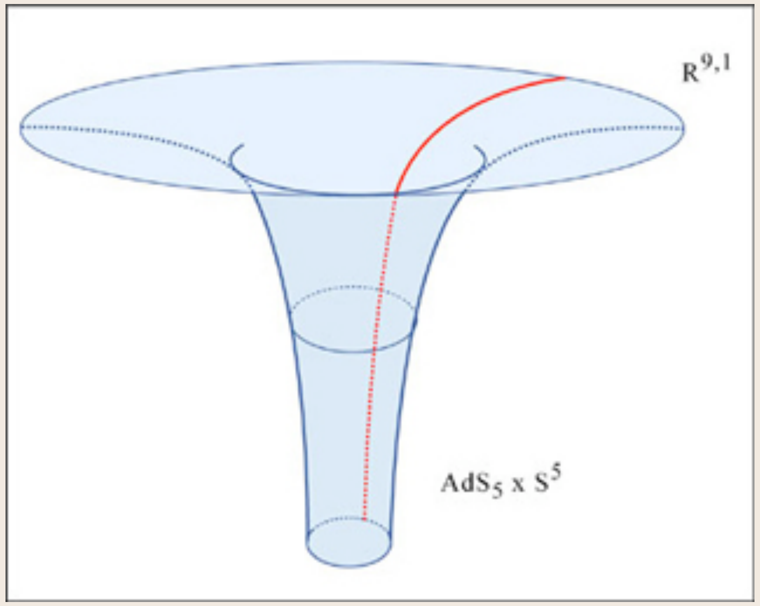
Domain wall / QFT correspondence

holography for Dp-branes

[H.J. Boonstra, K. Skenderis, P. Townsend, 1999]

AdS / CFT case

D3	IIB	$AdS_5 \times S^5$	d=5, SO(6)



gaugings of maximal supergravity

[Günaydin, Romans, Warner, 1985]

Domain wall / QFT correspondence

holography for Dp-branes

[H.J. Boonstra, K. Skenderis, P. Townsend, 1999]

warped

D6	IIA	$AdS_8 \times S^2$	d=8, SO(3)
D5	IIB	$AdS_7 \times S^3$	d=7, SO(4)
D4	IIA	$AdS_6 \times S^4$	d=6, SO(5)
D3	IIB	$AdS_5 \times S^5$	d=5, SO(6)
D2	IIA	$AdS_4 \times S^6$	d=4, SO(7)
F1/D1	IIA/B	$AdS_3 \times S^7$	d=3, SO(8)
D0	IIA	$AdS_2 \times S^8$	d=2, SO(9)

gaugings of maximal supergravity

[Salam, Sezgin, 1984]

[Pernici, Pilch, van Nieuwenhuizen, 1984]

[Günaydin, Romans, Warner, 1985]

[Hull, 1984]

[de Wit, Nicolai, 1982]

??? try to construct the SO(9) theory ...

1999 : constructing SO(9) supergravity

[Hermann Nicolai, HS]

1999 : trying to construct d=2, SO(9) supergravity

two-dimensional supergravity is particularly simple :

$$\mathcal{L} \;=\; -\tfrac{1}{4}\sqrt{-g}\,\rho\Big(-R+\mathrm{tr}[P^\mu P_\mu]\Big) \;+\; \mathcal{L}_{\mathrm{ferm}}(\psi^I,\psi_2^I,\chi^{\dot A})$$

coset space sigma model $E_8/SO(16)$ coupled to dilaton gravity

but has a remarkable structure :

(infinite tower of) dual scalar potentials

$$\partial_\mu Y_M \;\equiv\; \varepsilon_{\mu\nu}\, J^\nu_M$$

conserved E_8 Noether current

etc.

⟶ classical integrability, affine Lie-Poisson symmetry E_9

off-shell symmetry : E_8 can we gauge a subgroup SO(9) ??

2000 : trying to construct d=2, SO(9) supergravity

- introducing vector fields ? (non-propagating in d=2)

$$\delta A_\mu{}^M \;=\; 2\,\mathcal{V}^M{}_{IJ}\,(\bar\epsilon^I\psi^J_\mu) - i\Gamma^I_{A\dot A}\,\mathcal{V}^M{}_A\,(\bar\epsilon^I\gamma_\mu\chi^{\dot A})$$

we can introduce 248 (= dim E8) vector fields on which supersymmetry closes provided that $F_{\mu\nu}{}^M = 0$! (origin : d=3)

- gauging requires Yukawa couplings and fermion shifts

$$\delta\psi^I_\mu = D_\mu\epsilon^I - gA^{IJ}\gamma_\mu\epsilon^J \qquad \text{etc.}$$

with scalar tensors $A^{IJ}(\mathcal{V}, Y_M)$ depending on scalars and dual potentials !

$$D_\mu Y_M \;\equiv\; \varepsilon_{\mu\nu}\,J^\nu_M$$

non-abelian duality relation
implies non-trivial consistency relations $[F_{\mu\nu}, Y] = \varepsilon_{\mu\nu}\,(D_\rho\,J^\rho) \;\propto\; \varepsilon_{\mu\nu}\,\partial_{\mathcal{V}}V_{\rm pot}$

surprise nº 1 : consistent supersymmetric system of field equations !

2000 : trying to construct d=2, SO(9) supergravity

- minimal couplings and the embedding tensor

$$D_\mu \equiv \partial_\mu - g A_\mu{}^M \Theta_{MN} t^N$$

surprise nº 2 : group theory determines all possible gaugings !

$$\Theta_{MN} \subset \mathbf{1} + \mathbf{3875}$$

$\mathrm{SO}(8) \times \mathrm{SO}(8)$ distinguished compact gauge group

$\mathrm{SO}(7,1) \times \mathrm{SO}(7,1)$
$\mathrm{SO}(6,2) \times \mathrm{SO}(6,2)$
$\mathrm{SO}(5,3) \times \mathrm{SO}(5,3)$
$\mathrm{SO}(4,4) \times \mathrm{SO}(4,4)$
$G_{2(2)} \times F_{4(4)}$
$G_2 \times F_{4(-20)}$
$E_{6(6)} \times \mathrm{SL}(3)$
$E_{6(2)} \times \mathrm{SU}(2,1)$
$E_{6(-14)} \times \mathrm{SU}(3)$
$E_{7(7)} \times \mathrm{SL}(2)$
$E_{7(-5)} \times \mathrm{SU}(2)$
$E_{8(8)}$

several maximal subgroups of E_8

no trace of SO(9) ... (no SO(9) singlet in **3875**)

2000 : trying to construct d=2, SO(9) supergravity

- minimal couplings and the embedding tensor

$$D_\mu \;\equiv\; \partial_\mu - g A_\mu{}^M\, \Theta_{MN}\, t^N$$

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$\mathrm{SO}(8) \times \mathrm{SO}(8)$ distinguished compact gauge group

surprise nº 3 : in fact, this is a d=3 theory !

$$D_\mu Y_M \;\equiv\; \varepsilon_{\mu\nu}\, J^\nu_M \quad\longrightarrow\quad \Theta_{MN} F^N_{\mu\nu} \;\equiv\; \varepsilon_{\mu\nu\rho}\, J^\rho_M$$

$[F_{\mu\nu}, Y] = \varepsilon_{\mu\nu}\,(D_\rho\, J^\rho) \;\propto\; \varepsilon_{\mu\nu}\,\partial_Y V_{\mathrm{pot}}$

non-abelian scalar-vector duality in d=3

surprise nº 4 : there is a 'simple' Lagrangian description !

$$\mathcal{L} \;=\; \mathrm{tr}\,[P_\mu P^\mu] + \Theta_{MN}\, A^M \wedge dA^N + \ldots$$

gauged sigma-model with Chern-Simons coupled vector fields !

2000 : trying to construct d=2, SO(9) supergravity

$$D_\mu \;\equiv\; \partial_\mu - g A_\mu{}^M\,\Theta_{MN}\,t^N \qquad\qquad \Theta_{MN}\;\subset\;\mathbf{1}+\mathbf{3875}$$

$$\mathcal{L} \;=\; \mathrm{tr}\,[P_\mu P^\mu] + \Theta_{MN}\,A^M\wedge dA^N + \ldots$$

gauged sigma-model with Chern-Simons coupled vector fields !

$\mathrm{SO}(8)\times\mathrm{SO}(8)$ distinguished compact gauge group

d=3 maximal gauged supergravity

- construction of three-dimensional AdS supergravities
- new AdS vacua
- holography & fluxes
- construction/systematics of higher-dimensional gaugings
- tensor hierarchies of non-abelian p-forms
- rigid supersymmetry, flat target spaces

2000 : trying to construct d=2, SO(9) supergravity

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$$\mathcal{L} \;=\; \mathrm{tr}\,[P_\mu P^\mu] + \Theta_{MN}\, A^M \wedge dA^N + \ldots$$

gauged sigma-model with Chern-Simons coupled vector fields !

$\mathrm{SO}(8) \times \mathrm{SO}(8)$ distinguished compact gauge group

d=3 maximal gauged supergravity

where is the **SO(9)** theory ?

12 years later ...
2012 : Golm, Hermann's 60th birthday ...

2012 : constructing SO(9) supergravity

[with Thomas Ortiz]

2007 : gauging d=2 supergravity

$$D_\mu \equiv \partial_\mu - g A_\mu{}^{\mathcal{M}} \Theta_{\mathcal{M}}{}^{\mathcal{A}} T_{\mathcal{A}} \qquad\qquad \Theta_{\mathcal{M}}{}^{\mathcal{A}} = (T_{\mathcal{B}})_{\mathcal{M}}{}^{\mathcal{N}} \eta^{\mathcal{AB}} \theta_{\mathcal{N}}$$

- d=2 supergravity has an affine symmetry group : $E_9 = \widehat{E_8}$
- vector fields transform in the basic representation of E_9
- the embedding tensor transforms in the basic representation of E_9

$\theta_{\mathcal{M}}$

1

248

$1 + 248 + 3875$

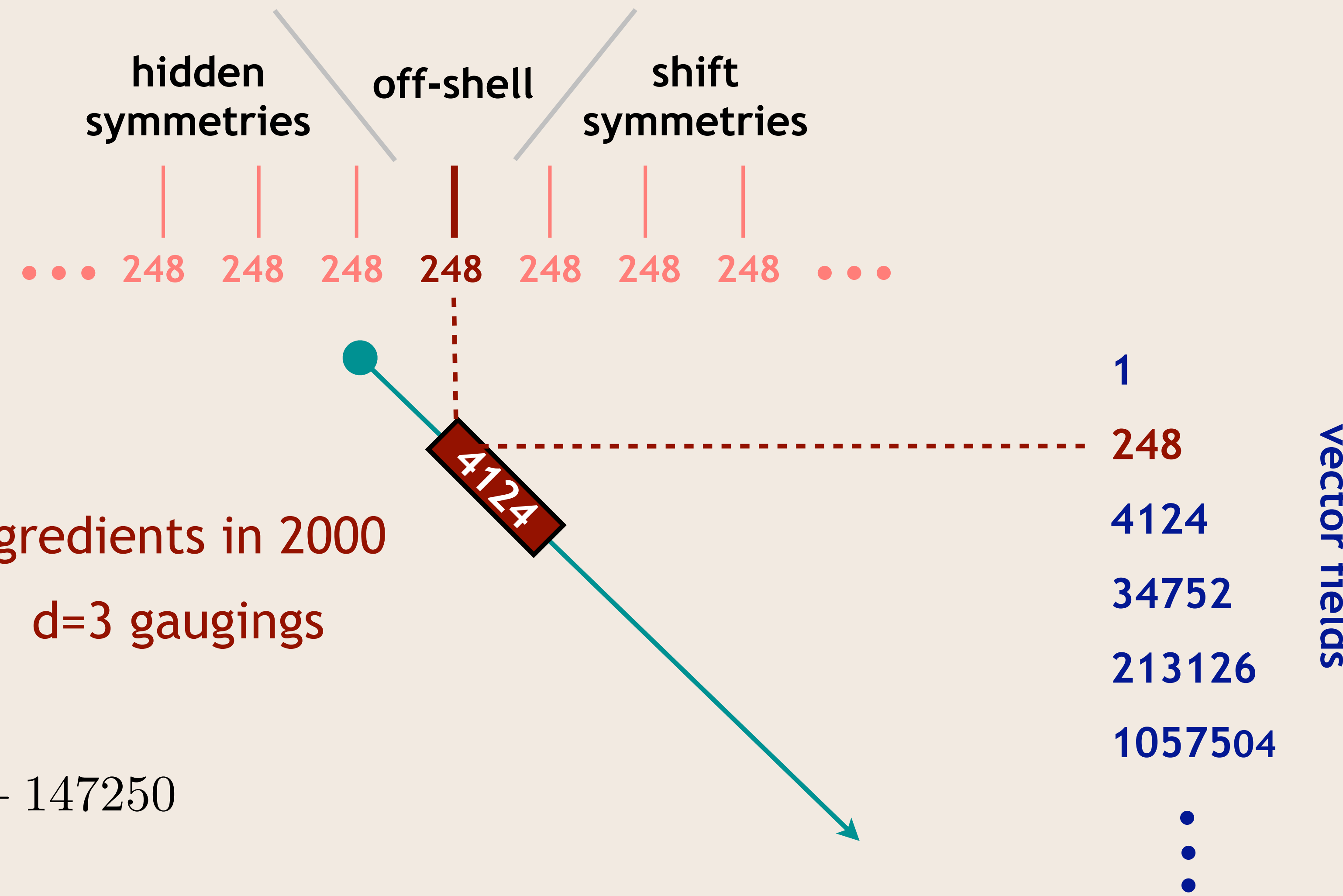
$1 + 2 \cdot 248 + 3875 + 30380$

$2 \cdot 1 + 3 \cdot 248 + 2 \cdot 3875 + 30380 + 27000 + 147250$

⋮

the ingredients in 2000

⟶ d=3 gaugings



2007 : gauging d=2 supergravity

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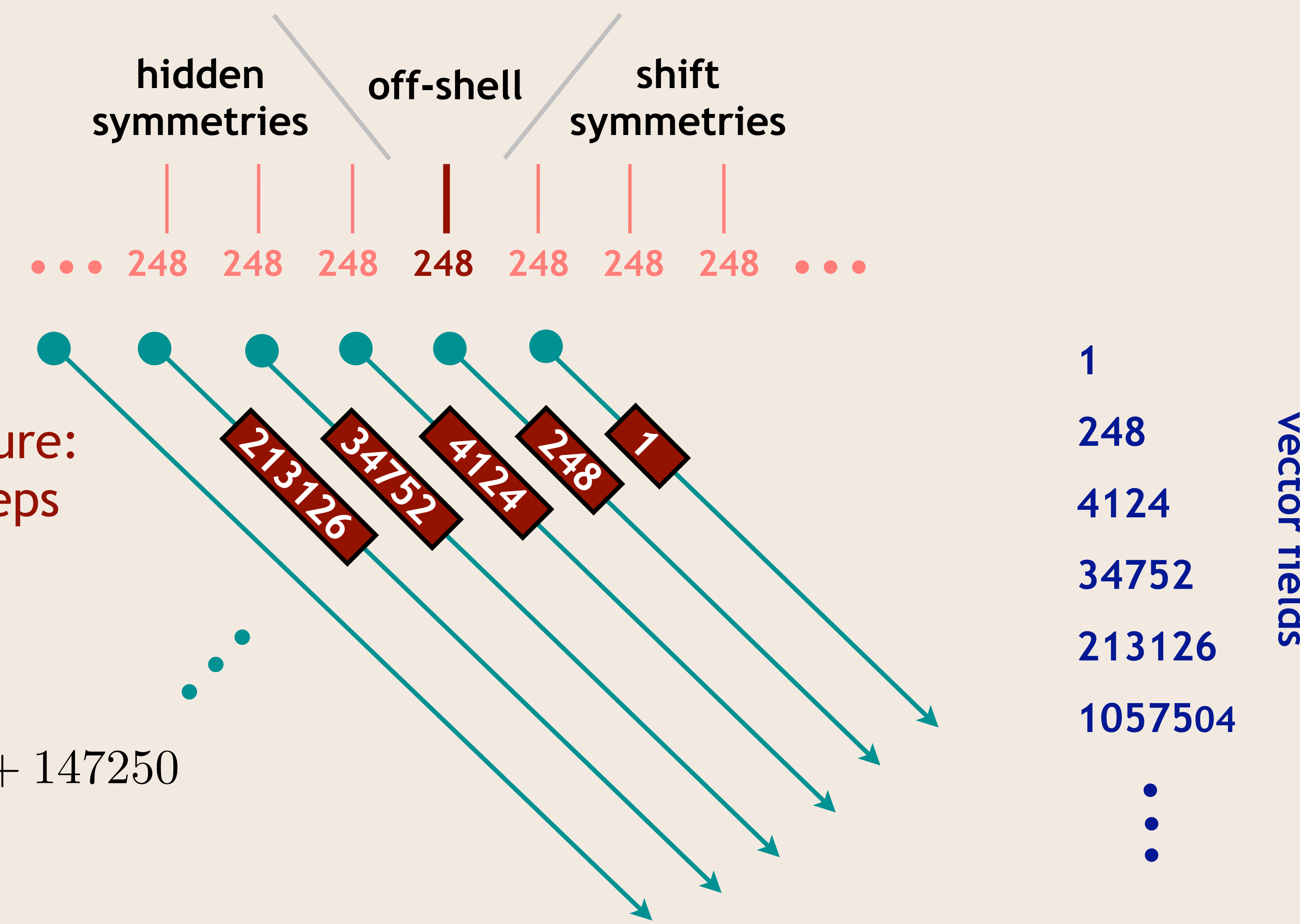
the full structure:
inf-dim HW reps



2012 : constructing SO(9) supergravity

$$D_\mu \equiv \partial_\mu - g A_\mu{}^{\mathcal{M}} \Theta_{\mathcal{M}}{}^{\mathcal{A}} T_{\mathcal{A}} \qquad\qquad \Theta_{\mathcal{M}}{}^{\mathcal{A}} = (T_{\mathcal{B}})_{\mathcal{M}}{}^{\mathcal{N}} \eta^{\mathcal{A}\mathcal{B}} \theta_{\mathcal{N}}$$

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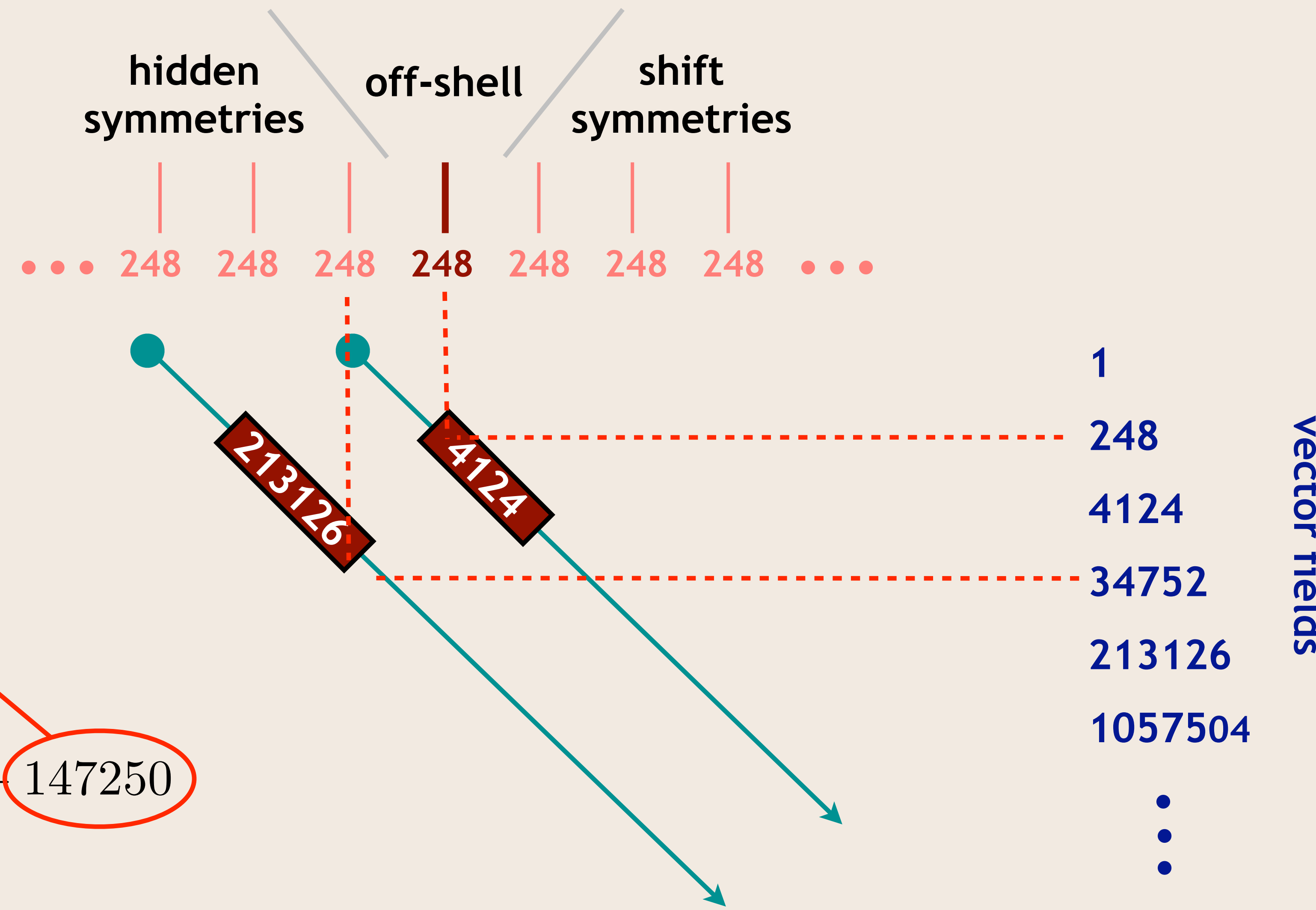
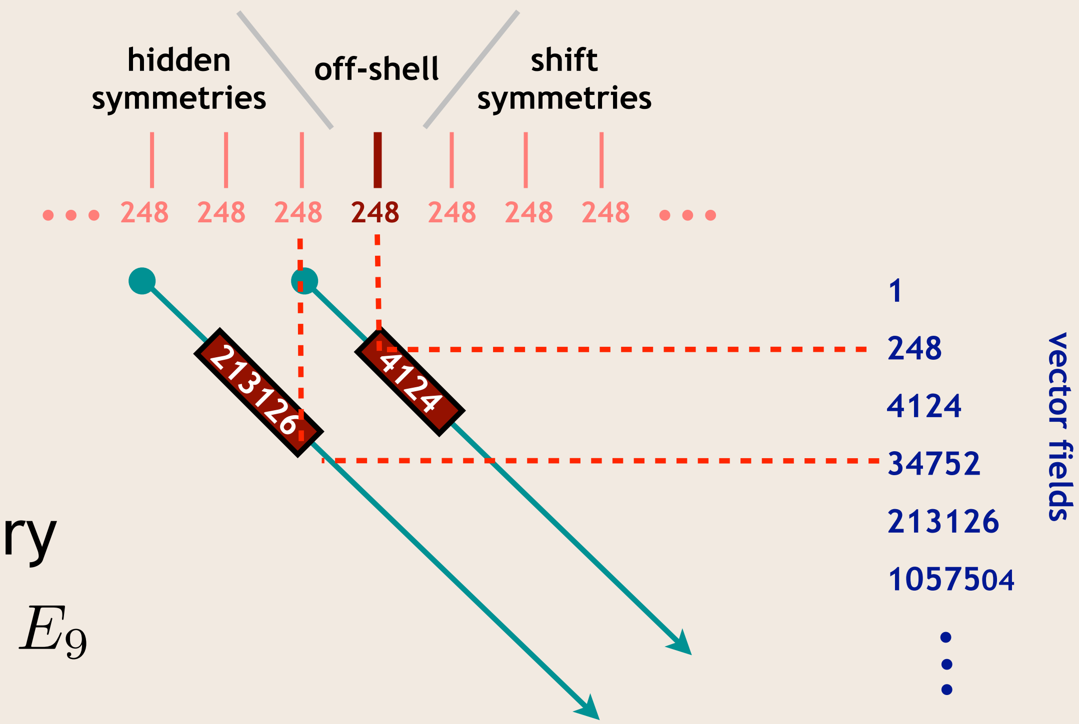
the **SO(9)** theory can be identified !

1



2012 : constructing SO(9) supergravity



- the SO(9) theory is a genuine d=2 theory
  in particular $SO(9) \not\subset E_8$ but $SO(9) \subset E_9$

- the full gauge group is infinite-dimensional (shift symmetries)

- the theory in the "E₈ frame" looks rather miserable

  in particular the gauge group is $G = \underbrace{\mathrm{SO}(8) \ltimes \Big((\mathbb{R}_+^{28} \times \mathbb{R}_+^{8})_0}_{\text{off-shell}} \times \underbrace{(\mathbb{R}_+^{8})_{-1}}_{\text{hidden (on-shell)}} \Big)$

- go to a "T-dual frame" in which SO(9) is among the off-shell symmetries

2012 : constructing SO(9) supergravity

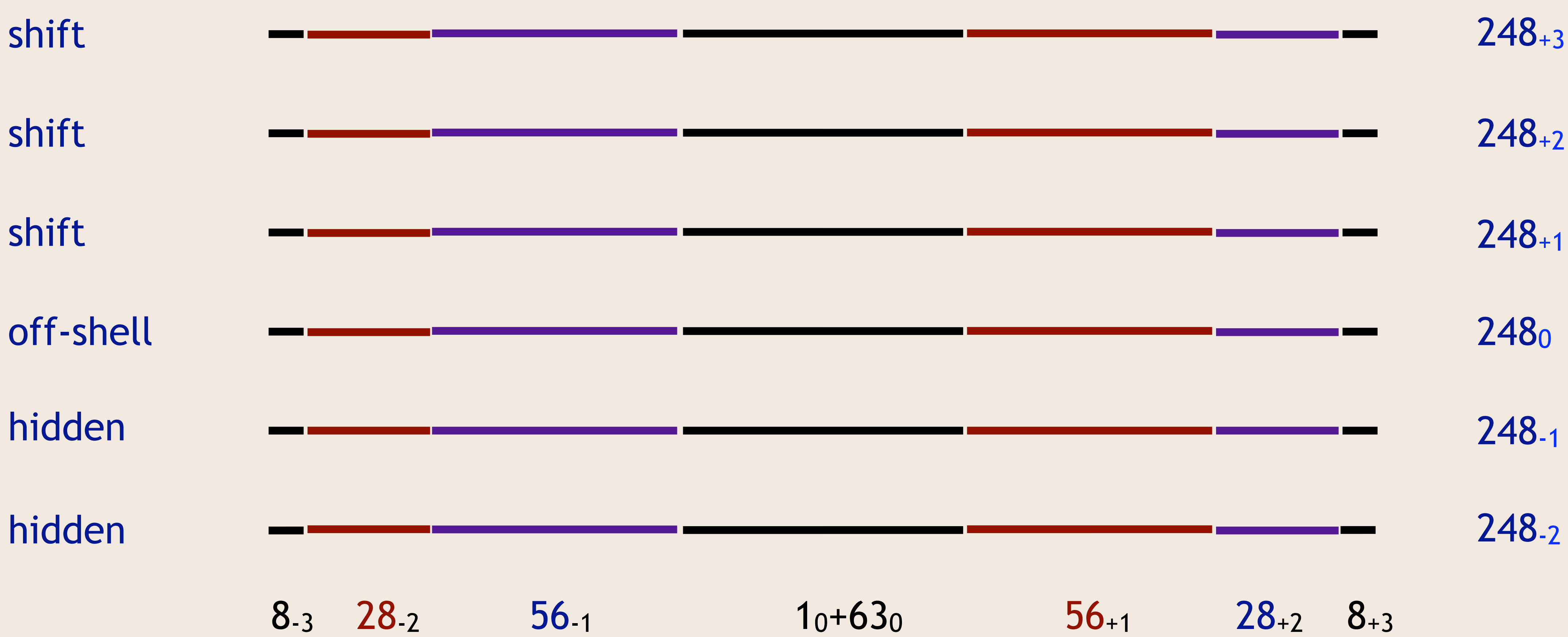
affine E_8 with L_0 grading

shift	248_{+3}
shift	248_{+2}
shift	248_{+1}
off-shell	248_{0}
hidden	248_{-1}
hidden	248_{-2}

2012 : constructing SO(9) supergravity

affine E_8 with L_0 grading

breaking under $SL(8) \times \mathbb{R}^+$

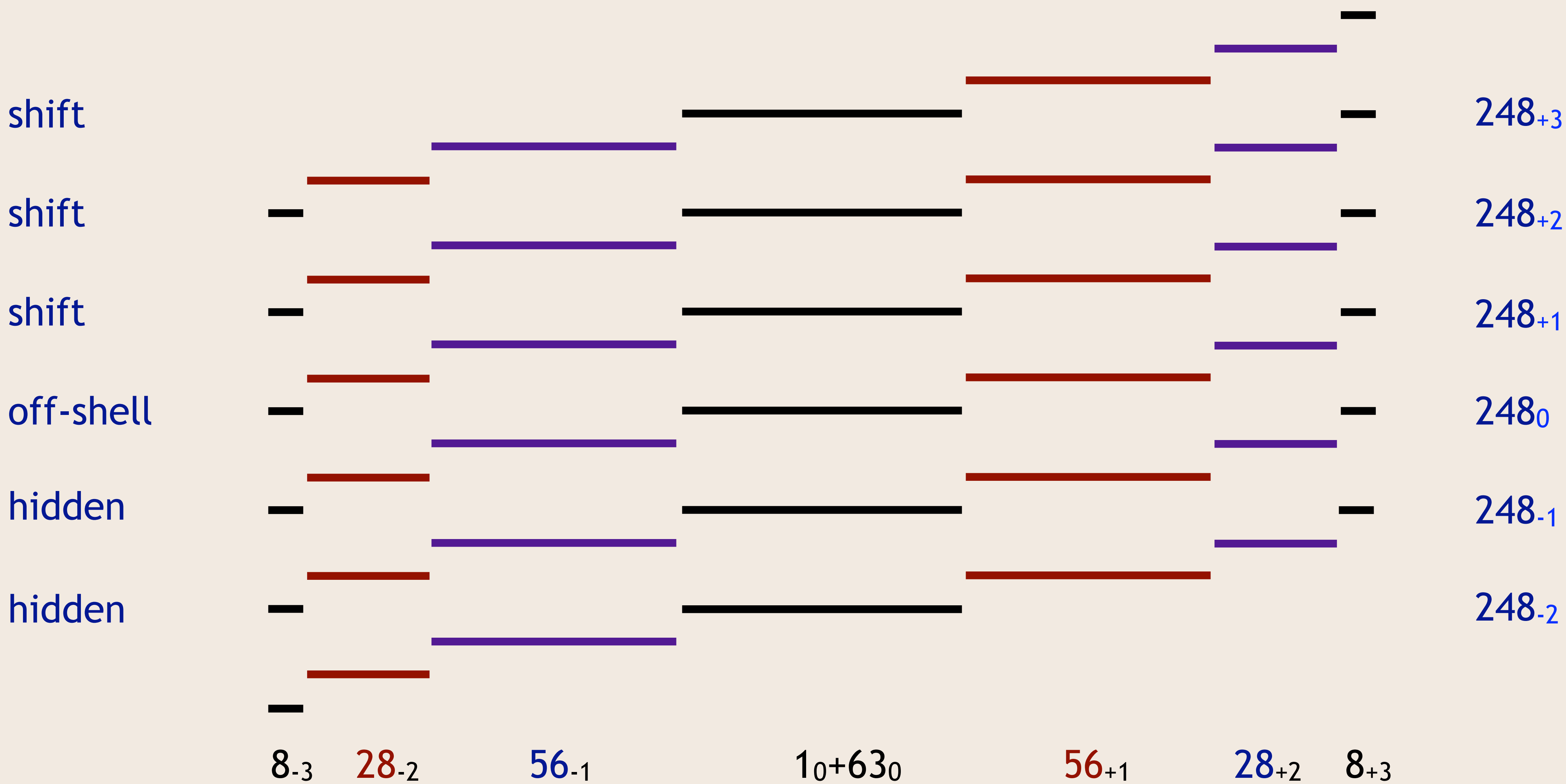


diagonal grading : $\widehat{L_0} \equiv L_0 + \frac{1}{3} q_{\mathbb{R}}$

2012 : constructing SO(9) supergravity

affine E_8 with L_0 grading

breaking under $SL(8) \times \mathbb{R}^+$

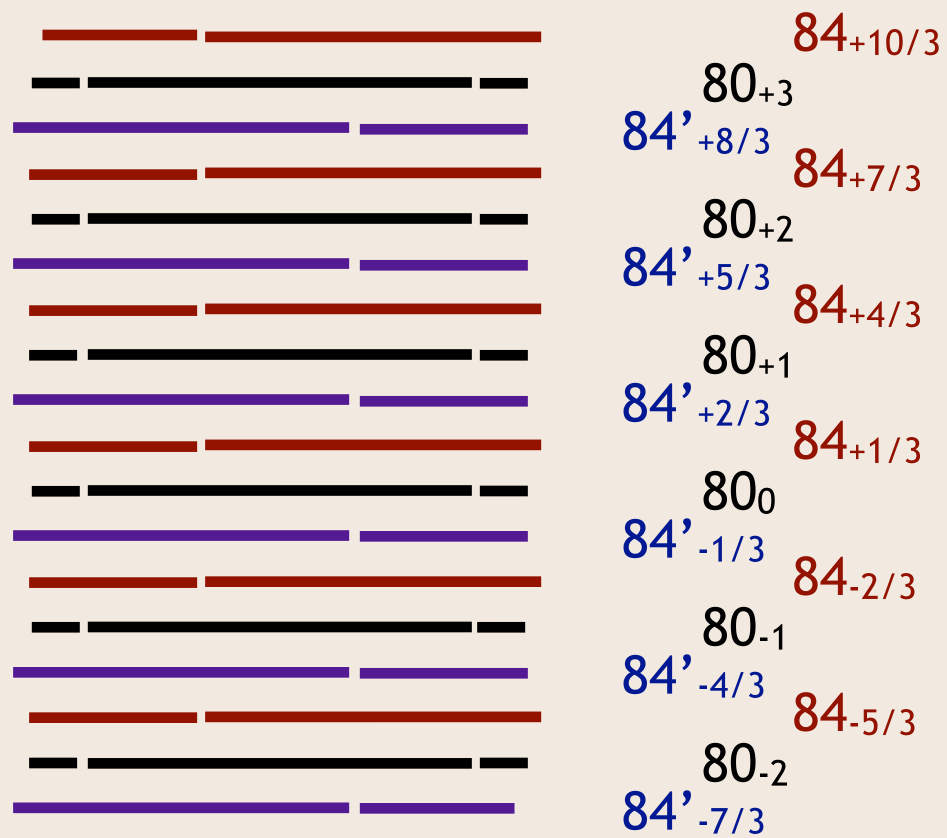


diagonal grading : $\widehat{L_0} \equiv L_0 + \frac{1}{3} q_{\mathbb{R}}$

2012 : constructing SO(9) supergravity

affine E_8 with $\widehat{L_0}$ grading : decomposition under $\widehat{SL}(9)$

off-shell symmetry $SL(9) \ltimes \mathbb{T}^{84}$



"T-dual frame" :

coset sigma model $E_8/SO(16)$ $\longrightarrow$ coset sigma model $\left(SL(9) \ltimes \mathbb{T}^{84}\right)/SO(9)$ with WZW term

2012 : constructing SO(9) supergravity

"T-dual frame" :

coset sigma model $\left(SL(9) \ltimes \mathbb{T}^{84}\right)/SO(9)$ with WZW term 84 ∧ 84 ∧ 84 ⟶ 1

$$\mathcal{L}_0 = -\frac{1}{4}\rho R + \frac{1}{4}\rho\, P^{\mu\, ab} P_\mu^{ab} + \frac{1}{12}\,\rho^{1/3}\, M_{il} M_{jm} M_{kn}\, \partial^\mu \phi^{ijk} \partial_\mu \phi^{lmn}$$
$$+\frac{1}{648}\,\varepsilon^{\mu\nu} \varepsilon_{klmnpqrst}\, \phi^{klm}\, \partial_\mu \phi^{npq}\, \partial_\nu \phi^{rst}$$

in fact this is the d=11 theory reduced on a torus T^9 ...

fermionic part :

$$-\rho e^{-1} \varepsilon^{\mu\nu} \bar{\psi}_2^I D_\mu \psi_\nu^I - \frac{i}{2}\,\bar{\psi}_\nu^I \gamma^\nu \psi_\mu^I\, \partial^\mu \rho - \frac{i}{2}\,\rho\, \bar{\chi}^{aI} \gamma^\mu D_\mu \chi^{aI} + \frac{i}{2}\,\rho^{2/3}\, \bar{\chi}^{aI} \gamma^3 \gamma^\mu \chi^{bJ} \Gamma^c_{IJ}\, \varphi_\mu^{abc} - \frac{i}{24}\,\rho^{2/3}\, \bar{\chi}^{aI} \gamma^3 \gamma^\mu \chi^{aJ} \Gamma^{bcd}_{IJ}\, \varphi_\mu^{bcd}$$
$$-\frac{1}{4}\,\rho^{2/3}\, \bar{\chi}^{aI} \gamma^3 \gamma^\nu \gamma^\mu \psi_\nu^J \Gamma^{bc}_{IJ}\, \varphi_\mu^{abc} - \frac{i}{12}\,\rho^{2/3}\, \bar{\chi}^{aI} \gamma^\mu \psi_2^J \Gamma^{bc}_{IJ}\, \varphi_\mu^{abc} - \frac{1}{2}\,\rho\, \bar{\chi}^{aI} \gamma^\nu \gamma^\mu \psi_\nu^J \Gamma^b_{IJ} P_\mu^{ab} - \frac{i}{2}\,\rho\, \bar{\chi}^{aI} \gamma^3 \gamma^\mu \psi_2^J \Gamma^b_{IJ} P_\mu^{ab}$$
$$+\frac{i}{54}\,\rho^{2/3}\, \bar{\psi}_2^I \gamma^3 \gamma^\mu \psi_2^J \Gamma^{abc}_{IJ}\, \varphi_\mu^{abc} + \frac{1}{24}\,\rho^{2/3}\, \bar{\psi}_2^I \left(\gamma^\mu \gamma^\nu - \frac{1}{3}\gamma^\nu \gamma^\mu\right) \psi_\nu^J\, \Gamma^{abc}_{IJ}\, \varphi_\mu^{abc}$$

off-shell symmetry $SL(9) \ltimes \mathbb{T}^{84}$ | $\supset\ SO(9)$ gauging |

2012 : constructing SO(9) supergravity

"T-dual frame" :

gauged coset sigma model $\left(SL(9)\ltimes\mathbb{T}^{84}\right)/SO(9)$ with WZW term

$$\mathcal{L} \;=\; -\frac{1}{4}\rho R+\frac{1}{4}\rho\,P^{\mu\,ab}P^{ab}_{\mu}+\frac{1}{12}\,\rho^{1/3}\,M_{il}M_{jm}M_{kn}\,D^{\mu}\phi^{ijk}D_{\mu}\phi^{lmn}$$
$$+\frac{1}{648}\,\varepsilon^{\mu\nu}\varepsilon_{klmnpqrst}\,\phi^{klm}\,D_{\mu}\phi^{npq}\,D_{\nu}\phi^{rst}$$

fermion couplings and Yukawa terms

$$e^{-1}\mathcal{L}_{\rm Yuk} \;=\; -\frac{1}{2}\,e^{-1}\rho\,\varepsilon^{\mu\nu}\left(\bar\psi^I_\nu\psi^J_\mu B_{IJ}+\bar\psi^I_\nu\gamma^3\psi^J_\mu\tilde B_{IJ}-2i\bar\psi^I_2\gamma_\nu\psi^J_\mu A_{IJ}\right)+i\rho\,\bar\psi^I_2\gamma^\mu\psi^J_\mu\tilde A_{IJ}$$
$$+\,i\rho\,\bar\chi^{aI}\gamma^\mu\psi^J_\mu C^a_{IJ}\;-i\rho\,\bar\chi^{aI}\gamma^3\gamma^\mu\psi^J_\mu\tilde C^a_{IJ}\;+\rho\,\bar\psi^I_2\psi^J_2 D_{IJ}\;+\rho\,\bar\psi^I_2\gamma^3\psi^J_2\tilde D_{IJ}$$
$$+\,\rho\,\bar\chi^{aI}\psi^J_2 E^a_{IJ}\;+\rho\,\bar\chi^{aI}\gamma^3\psi^J_2\tilde E^a_{IJ}\;+\rho\,\bar\chi^{aI}\chi^{bJ}F^{ab}_{IJ}\;+\rho\,\bar\chi^{aI}\gamma^3\chi^{bJ}\tilde F^{ab}_{IJ}$$

$$A_{IJ} = \frac{7}{9}\,\delta_{IJ}\,b-\frac{5}{9}\,\Gamma^a_{IJ}\,b^a+\frac{1}{9}\,\Gamma^{abcd}_{IJ}\,b^{abcd}\,,$$
$$\tilde A_{IJ} = \frac{2}{9}\,\Gamma^{ab}_{IJ}\,b^{ab}-\frac{4}{9}\,\Gamma^{abc}_{IJ}\,b^{abc}\,,$$
$$B_{IJ} = \Gamma^{ab}_{IJ}\,b^{ab}+\Gamma^{abc}_{IJ}\,b^{abc}\,,$$
$$\tilde B_{IJ} = \delta_{IJ}\,b+\Gamma^a_{IJ}\,b^a+\Gamma^{abcd}_{IJ}\,b^{abcd}\,,$$
$$C^a_{IJ} = \frac{8}{9}\,\delta_{IJ}\,b^a-\frac{1}{9}\Gamma^{ab}_{IJ}\,b^b+\frac{20}{9}\Gamma^{bcd}_{IJ}\,b^{abcd}-\frac{4}{9}\Gamma^{abcde}_{IJ}\,b^{bcde}+c^{ab}\Gamma^b_{IJ}\,,$$
$$\tilde C^a_{IJ} = -\frac{14}{9}\,\Gamma^b_{IJ}\,b^{ab}+\frac{2}{9}\Gamma^{abc}_{IJ}\,b^{bc}+\frac{2}{9}\Gamma^{bc}_{IJ}\,b^{abc}-\frac{1}{9}\Gamma^{abcd}_{IJ}\,b^{bcd}+c^{a,bc}\,\Gamma^{bc}_{IJ}\,,$$
$$D_{IJ} = \frac{14}{81}\,\delta_{IJ}\,b-\frac{70}{81}\,\Gamma^a_{IJ}\,b^a+\frac{8}{81}\,\Gamma^{abcd}_{IJ}\,b^{abcd}\,,$$

$$\tilde D_{IJ} = \frac{22}{81}\,\Gamma^{ab}_{IJ}\,b^{ab}-\frac{20}{81}\,\Gamma^{abc}_{IJ}\,b^{abc}\,,$$
$$E^a_{IJ} = \frac{26}{9}\,\Gamma^b_{IJ}\,b^{ab}-\frac{1}{9}\,\Gamma^{bc}_{IJ}\,b^{abc}-\frac{1}{9}\,c^{a,bc}\,\Gamma^{bc}_{IJ}\,,$$
$$\tilde E^a_{IJ} = \frac{19}{9}\,\delta_{IJ}\,b^a+\frac{28}{9}\,\Gamma^{bcd}_{IJ}\,b^{abcd}-\frac{1}{9}\,c^{ab}\,\Gamma^b_{IJ}\,,$$
$$F^{ab}_{IJ} = -\frac{1}{18}\,\delta^{ab}\,\delta_{IJ}\,b+\frac{1}{2}\,\delta^{ab}_{cd}\,\Gamma^c_{IJ}\,b^d\;[illegible]\;-12\,\Gamma^{cd}_{IJ}\,b^{abcd}-2\,c^{ab}\,\delta_{IJ}$$
$$\tilde F^{ab}_{IJ} = \frac{1}{2}\,\delta^{ab}\,3\Gamma^{cd}_{IJ}\,b^{cd}-\frac{1}{2}\,\delta^{ab}\,\Gamma^{cde}_{IJ}\,b^{cde}\;[illegible]\;-2\,c^{c,ab}\,\Gamma^c_{IJ}\,,$$

$$b = \frac{1}{4}\,\rho^{-2/9}\,T\,,$$
$$b^a = -\rho^{-14/9}\,\mathcal{V}^{-1km}{}_{bc}\,\theta_{ml}\,\varphi^{abc}Y_k{}^l+\frac{1}{144}\,\rho^{-14/9}\,\varepsilon^{bcdefghij}T^{kl}\varphi^{kef}\varphi^{lgh}\varphi^{aij}\varphi^{bcd}$$
$$b^{ab} = \frac{1}{2}\,\rho^{-11/9}\,\mathcal{V}^{-1[km]}{}_{ab}\,\theta_{ml}Y_k{}^l+\frac{1}{144}\,\rho^{-11/9}\,\varepsilon^{abcdefghi}T^{jk}\varphi^{jcd}\varphi^{kef}\varphi^{ghi}\,,$$
$$b^{abc} = \frac{5}{4}\,\rho^{-5/9}\,T^{d[a}\varphi^{bc]d}\,,$$
$$b^{abcd} = -\frac{1}{8}\,\rho^{-8/9}\,T^{ef}\varphi^{e[ab}\varphi^{cd]f}\,,$$
$$c^{ab} = -\frac{1}{2}\,\rho^{-2/9}\left(T^{ab}-\frac{1}{9}\,\delta^{ab}\,T\right),$$
$$c^{a,bc} = \frac{1}{3}\,\rho^{-5/9}\left(T^{da}\varphi^{bcd}+T^{d[b}\varphi^{c]da}\right),$$

2012 : constructing SO(9) supergravity

"T-dual frame" :

gauged coset sigma model $\left(SL(9) \ltimes \mathbb{T}^{84}\right)/SO(9)$ with WZW term

$$\mathcal{L} = -\frac{1}{4}\rho R + \frac{1}{4}\rho\, P^{\mu\, ab} P_{\mu}^{ab} + \frac{1}{12}\,\rho^{1/3}\, M_{il} M_{jm} M_{kn}\, D^{\mu}\phi^{ijk} D_{\mu}\phi^{lmn}$$
$$+\frac{1}{648}\,\varepsilon^{\mu\nu}\varepsilon_{klmnpqrst}\,\phi^{klm}\, D_{\mu}\phi^{npq}\, D_{\nu}\phi^{rst}$$

vector fields couple via

$$\mathcal{L}_F = \varepsilon^{\mu\nu}\, F_{\mu\nu}{}^{mn}\, \mathcal{Y}_{mn}$$

with auxiliary (dual scalar) fields $\mathcal{Y}_{mn}$

scalar potential

$$V_{\text{pot}} = \frac{1}{8}\,\rho^{5/9}\left((\text{tr}\, T)^2 - 2\,\text{tr}(T^2) + 18\,\rho^{-2/3}\, T^{d[a}\varphi^{bc]d} T^{ea}\varphi^{bce} - 16\,\rho^{-2/3}\, T^{d[b}\varphi^{c]ad} T^{eb}\varphi^{cae}\right)$$
$$-\rho^{-13/9}\, T^{ac}T^{bc}\, Y_{ad}Y_{bd} \;+\; \mathcal{O}(\phi^3)$$

$T \equiv \left(\mathcal{V}^{\mathrm{T}}\mathcal{V}\right)^{-1}$

$\varphi \equiv \phi \cdot \mathcal{V}$

$Y \equiv \mathcal{V}^{\mathrm{T}}\mathcal{Y}\,\mathcal{V}$

eighth order polynomial in ϕ

the dilaton powers precisely support the correct DW solution (near horizon of AdS_2 x S^8)

2012 : constructing SO(9) supergravity

different presentations

gauged coset sigma model $\left(SL(9)\ltimes\mathbb{T}^{84}\right)/SO(9)$ with WZW term

$$\mathcal{L} \;=\; -\frac{1}{4}\rho R+\frac{1}{4}\rho\,P^{\mu\,ab}P^{ab}_{\mu}+\frac{1}{12}\,\rho^{1/3}\,M_{il}M_{jm}M_{kn}\,D^{\mu}\phi^{ijk}D_{\mu}\phi^{lmn}$$
$$+\frac{1}{648}\,\varepsilon^{\mu\nu}\varepsilon_{klmnpqrst}\,\phi^{klm}\,D_{\mu}\phi^{npq}\,D_{\nu}\phi^{rst}+\varepsilon^{\mu\nu}\,F_{\mu\nu}{}^{mn}\,\mathcal{Y}_{mn}\;+\;V_{\rm pot}(\rho,\mathcal{V},\phi,\mathcal{Y})$$

integrate out the vector fields $A_{\mu}{}^{mn}$

$$\mathcal{L}_{\rm T} \;=\; -\frac{1}{4}\rho R+G_{ij}(\rho,\mathcal{V},\phi,\mathcal{Y})\,\partial_{\mu}\Phi^{i}\partial^{\mu}\Phi^{j}+\varepsilon^{\mu\nu}\,B_{ij}(\rho,\mathcal{V},\phi,\mathcal{Y})\,\partial_{\mu}\Phi^{i}\partial_{\nu}\Phi^{j}+V_{\rm pot}(\rho,\mathcal{V},\phi,\mathcal{Y})$$

ungauged sigma model on a different target space

integrate out the auxiliary scalars $\mathcal{Y}_{mn}$

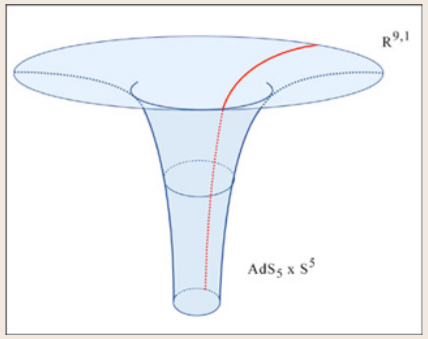
$$\mathcal{L}_{2} \;=\; -\frac{1}{4}\rho R+F_{\mu\nu}{}^{mn}F^{\mu\nu\,kl}\,\mathcal{R}_{mn,kl}(\rho,\mathcal{V},\phi)\;+\;\ldots\;+\;\tilde{V}_{\rm pot}(\rho,\mathcal{V},\phi)$$

gauged sigma model coupled to d=2 SYM

concluding

SO(9) supergravity

- maximally supersymmetric d=2 supergravity with gauge group SO(9)
- last missing gauged supergravity around near-horizon geometries



warped

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D0	IIA	$AdS_2 \times S^8$	d=2, SO(9)	

- mission completed ...
- holography : d=1 supersymmetric matrix quantum mechanics ...!

concluding

SO(9) supergravity

- maximally supersymmetric d=2 supergravity with gauge group SO(9)
- last missing gauged supergravity around near-horizon geometries
- holography : d=1 supersymmetric matrix quantum mechanics ...!

we came here by a wonderful detour via three (and many other) dimensions

with Hermann via the **d=3, SO(8) x SO(8) theory**

which is still awaiting its embedding/interpretation in higher dimensions,
string theory, holography, ... (matrix string theory, double field theory, ...???)

sufficient material for the next anniversaries ...

Happy Birthday, Hermann ! (in all dimensions)