

# BEAUTY, CONSISTENCY AND ETERNAL BLISS



**SYMMETRIES, UNIFICATION  
AND THE SEARCH FOR  
QUANTUM GRAVITY**

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*Hermann and I wrote 26 joint papers, of which the first 14 ones, written during 1980-1987, dealt exclusively with N=8 supergravity and its possible higher-dimensional origin.*

*This effort was based on the believe that N=8 supergravity was so beautiful that it has to play a role in the physical world.*

*In working together during all these years I learned that Hermann has a strong appreciation of the relation between*

**TRUTH & BEAUTY**

*It is certainly true that N=8 supergravity is an exceptional 4D theory.*

## TRUTH & BEAUTY

Of course one must be cautious with arguments based on esthetics and is easy to be carried away as the following trivial example illustrates:

*Hermann Nicolai (AEI) was born in 1952*

*Friedrich der Grosse (Sanssouci) was born in 1712*

Their age difference is therefore

$$\mathcal{F} - \mathcal{H} = 240 \text{ years}$$

Surprisingly, the symplectic group  $S_8(2)$  and the unitary group  $2.U_4(2)$  both have a permutation representation on 240 points.

**How could this be a coincidence .....**

# ***$N=8$ Supergravity***

## **Truly beautiful theory:**

- \* Eight supersymmetries (maximal)
- \* Contains gravity and **28** abelian gauge fields
- \* Non-linearly realized  $E_{7(7)}$  symmetry, optionally broken by an  $SO(8)$  gauge group
- \* Follows from  *$D=11$  supergravity* by dimensional compactification on a 7-sphere

*dW, Freedman, 1977*

*dW, 1979*

*Cremmer, Julia, 1979*

*Brink, Howe, 1979*

*dW, Nicolai, 1982*

## **What we did not know at the time:**

- \* Many more gauge groups are possible  
(not related to non-compact or contracted versions of  $SO(8)$  )

*dW, Samtleben, Trigiante, 2007*

- \*  $N = 8$  Supergravity may be a finite quantum field theory !

*Bern, Carrasco, Dixon, Johansson, Kosower, Roiban, 2007*





# QUALITIES OF THE UNIFIED FIELD

## LOCATED IN THE LAGRANGIAN OF THE UNIFIED FIELD



- ### QUALITIES
- ALL POSSIBILITIES
  - FREEDOM
  - UNBOUNDEDNESS
  - SELF-SUFFICIENCY
  - BLISS
  - INTEGRATING
  - SELF-REFERRAL
  - INVINCIBILITY
  - PERFECT BALANCE
  - FULLY AWAKE WITHIN ITSELF
  - TOTAL POTENTIAL OF NATURAL LAW
  - SIMPLICITY
  - UNMANIFEST
  - HARMONIZING
  - INFINITE CORRELATION
  - INFINITE DYNAMISM
  - INFINITE SILENCE
  - PURE KNOWLEDGE
  - INFINITE ORGANIZING POWER
  - PERFECT ORDERLINESS
  - INFINITE CREATIVITY
  - PURIFYING
  - IMMORTALITY
  - NOURISHING
  - EVOLUTIONARY

### LAGRANGIAN

$$\mathcal{L} =$$

$$-\frac{1}{2}eR(e, \omega)$$

$$-\frac{1}{2}e^{\mu\nu\sigma} (\bar{\psi}_\mu \gamma_\nu D_\sigma \psi_{\sigma i} - \bar{\psi}_\mu \tilde{D}_\sigma \gamma_\nu \psi_{\sigma i})$$

$$-\frac{1}{8}e F_{\mu\nu IJ} F^{+ \mu\nu IJ}$$

$$-\frac{1}{12}e (\bar{\chi}^{\mu k} \gamma^\mu D_\mu \chi_{\mu k} - \bar{\chi}^{\mu k} \tilde{D}_\mu \gamma^\mu \chi_{\mu k})$$

$$-\frac{1}{96}e \mathcal{A}_{\mu\nu}^{\alpha\beta\gamma\delta} \mathcal{A}_{\alpha\beta\gamma\delta}$$

$$-\frac{1}{4}e [F_{\mu\nu IJ} S^{IJ, KL} F^{+ \mu\nu KL} + \text{h.c.}]$$

$$-\frac{1}{2}e [F_{\mu\nu IJ} S^{IJ, KL} O^{+ \mu\nu KL} + \text{h.c.}]$$

$$-\frac{1}{4}e [O_{\mu\nu}^{+ IJ} (S^{IJ, KL} + u^{\nu IJ} v_{\nu KL}) O^{+ \mu\nu KL} + \text{h.c.}]$$

$$-\frac{1}{24}e [\bar{\chi}_{\mu k} \gamma^\nu \gamma^\mu \psi_{\nu i} (\mathcal{A}_{\mu\nu}^{\alpha\beta\gamma\delta} + \mathcal{A}_{\mu\nu}^{\alpha\beta\gamma\delta}) + \text{h.c.}]$$

$$+ ge [\sqrt{2} A_{1\mu} \bar{\psi}_\mu \sigma^{\mu\nu} \psi_\nu + \frac{1}{2} A_{2i}^{\mu\nu} \bar{\psi}_\mu \gamma^\mu \chi_{\nu k}$$

$$+ \frac{\sqrt{2}}{144} \eta e^{\mu\nu\alpha\beta\gamma\delta} A_{2\mu\nu}^{\alpha\beta} \bar{\chi}_{\mu k} \chi_{\nu mn} + \text{h.c.}]$$

$$+ g^2 e [\frac{1}{2} |A_1^i|^2 - \frac{1}{24} |A_{2ij}^k|^2]$$

$$-\frac{1}{2}e \bar{\psi}_\mu \gamma^\mu \psi_\nu \bar{\psi}_\nu \gamma^\nu \psi_\mu$$

$$+ \frac{\sqrt{2}}{4} e [\bar{\psi}_\mu \sigma^{\mu\nu} \gamma^\lambda \chi_{\mu k} \bar{\psi}_\nu \psi_\nu^k + \text{h.c.}]$$

$$+ e [\frac{1}{144} \eta e^{\mu\nu\alpha\beta\gamma\delta} \bar{\chi}^{\mu k} \sigma^{\mu\nu} \chi^{\alpha mn} \bar{\psi}_\mu \psi_\nu$$

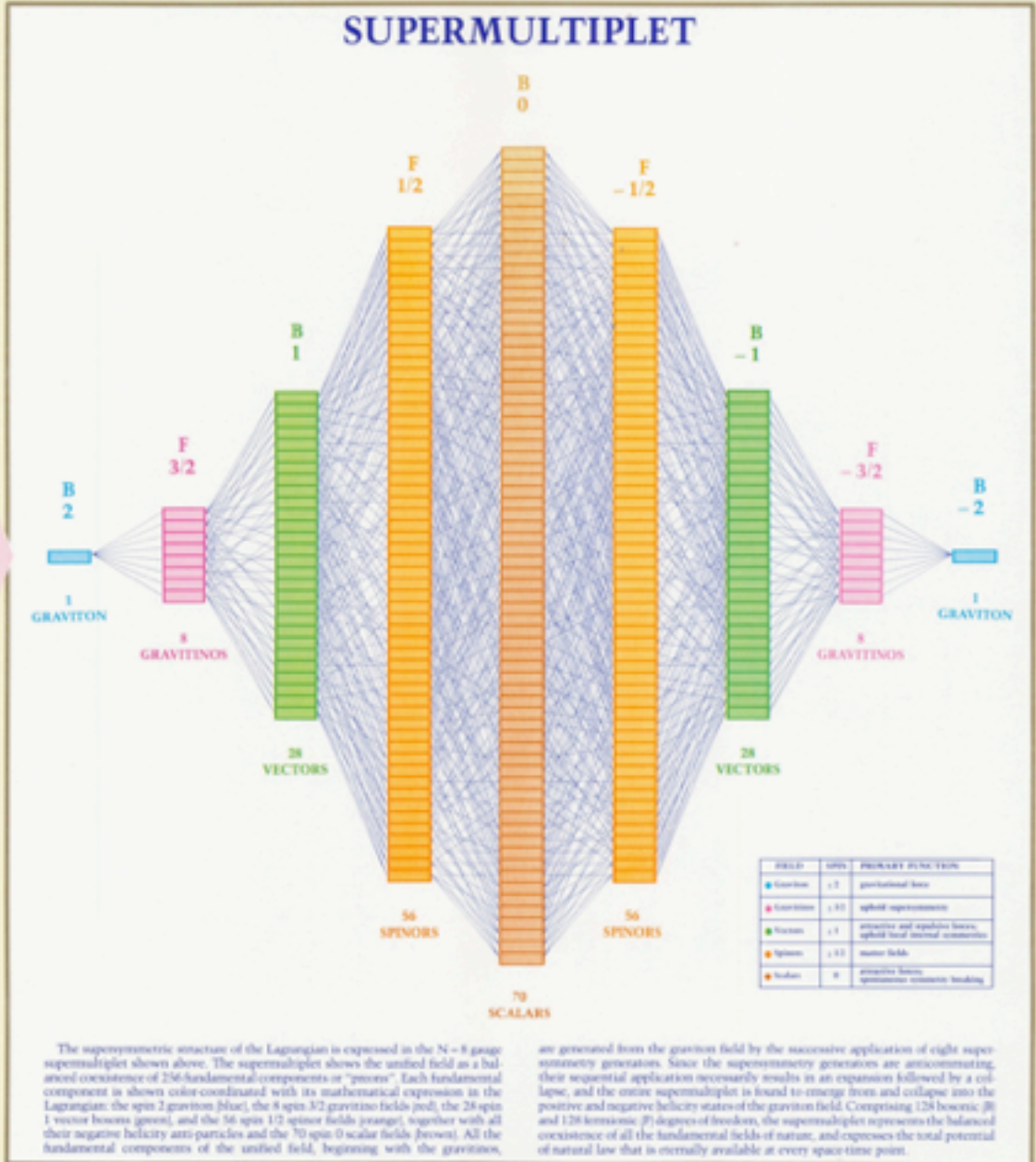
$$+ \frac{1}{8} \bar{\psi}_\mu \sigma^{\mu\nu} \gamma^\lambda \chi_{\mu k} \bar{\psi}_\nu \gamma_\nu \chi^{\lambda k} + \text{h.c.}]$$

$$+ \frac{\sqrt{2}}{6 \cdot 144} \eta e [e^{\mu\nu\alpha\beta\gamma\delta} \bar{\chi}_{\mu k} \sigma^{\mu\nu} \chi_{\alpha mn} \bar{\psi}_\mu \gamma_\nu \chi_{\beta qr} + \text{h.c.}]$$

$$+ \frac{1}{32} e \bar{\chi}^{\mu k} \gamma^\mu \chi_{\mu k} \bar{\chi}^{\alpha mn} \gamma_\mu \chi_{\alpha mn}$$

$$-\frac{1}{96} e (\bar{\chi}^{\mu k} \gamma^\mu \chi_{\mu k})^2$$

The N=8 supergravity theory offers the most complete description of the unified field to emerge from the study of modern high energy physics. The essence of this theory is contained in the N=8 supergravity Lagrangian shown above. The Lagrangian is the most compact mathematical expression of the complete structure of the unified field—its symmetries, components and self-interaction. Each color represents one of the fundamental components of the unified field: the graviton (blue), the gravitinos (pink), the vector bosons (green), the spinors (orange), and the scalars (brown). The first five rows contain the essential dynamics of each of these spin types, while the rest of the Lagrangian describes their mutual interactions.



Because the unified field is the foundation of natural law, all qualities in the universe have their origin in the unified field. This chart presents a few key characteristics of the unified field derived by Dr. John Hagelin, Professor of Physics at Maharishi International University, from the Lagrangian of N=8 supergravity theory as recently formulated by Dr. Bernard de Wit and Dr. Hermann Nicolai. These qualities provide a glimpse of the benefits that arise when the unified field is embodied in individual and collective consciousness through the Maharishi Technology of the Unified Field.

- ALL POSSIBILITIES:** All possible local gauge-invariant operators are generated by non-perturbative quantum gravitational effects at the Planck scale.
- FREEDOM:** The graviton remains a free, unbound particle in the physical spectrum, and the entire supermultiplet becomes asymptotically free at the Planck scale.
- UNBOUNDEDNESS:** The translational invariance of the Lagrangian density, also expressed by the graviton, which is the gauge field of an infinite range force.

- SELF-SUFFICIENCY:** The graviton does not participate in the activity of proton binding and is a singlet with respect to the internal SO(8) and SU(8) symmetries of the Lagrangian.
- BLISS:** Expressed by the continuous effluence of topological fluctuations at the Planck scale and by the universally attractive nature of the graviton field.
- INTEGRATING:** The gravitino fields dynamically uphold local supersymmetry, which integrates the different spin components of the supermultiplet maintaining the unbroken wholeness of the superfield.
- SELF-REFERRAL:** The non-Abelian property of self-interaction of the vector fields that uphold the local SO(8) symmetry. The property of self-interaction is also present in the graviton, gravitino, spinor, and scalar fields, and therefore in the entire supermultiplet.
- INVINCIBILITY:** A non-Abelian gauge field dynamically upholds its own invariance under local symmetry transformations.
- PERFECT BALANCE:** Supersymmetry—perfect balance of bosonic and fermionic

- degrees of freedom.
- FULLY AWAKE WITHIN ITSELF:** The zero-point motion of the quantum fields reaches its ultimate level of dynamism at the Planck scale.
- TOTAL POTENTIAL OF NATURAL LAW:** All the fundamental field types are fully embodied as dynamical degrees of freedom at the Planck scale.
- SIMPLICITY:** All of the fundamental components together comprise a single irreducible representation of the symmetry group.
- UNMANIFEST:** The fundamental components of the supermultiplet, the protons, do not appear as manifest particles.
- HARMONIZING:** The gravitino is the gauge field of local supersymmetry, which unites completely opposite values—boson and fermi fields.
- INFINITE CORRELATION:** Expressed by the terms which uphold the local SO(8) gauge invariance of the Lagrangian.
- INFINITE DYNAMISM:** The bilinear and quartic couplings describe the dynamical interaction of the proton fields.

- INFINITE SILENCE:** The bilinear and quartic couplings preserve the invariance of the Lagrangian under local supersymmetry transformations.
- PURE KNOWLEDGE:** The Lagrangian is the most compact mathematical expression of the complete structure of the laws of nature.
- INFINITE ORGANIZING POWER:** The Hamiltonian operator, derived from the Lagrangian by a Legendre transformation, governs all activity in the universe.
- PERFECT ORDERLINESS:** The SO(8), SU(8) and extended super-Poincaré symmetries of the Lagrangian.
- INFINITE CREATIVITY:** The foundation of natural law—from this unified source all the particles and forces of nature are generated through the process of dynamical symmetry breaking.
- PURIFYING:** The symmetries of the Lagrangian, which are broken at macroscopic distances, are spontaneously restored at the Planck scale.
- IMMORTALITY:** The time-translational invariance of the Lagrangian density.

- NOURISHING:** The supermultiplet is a gauge field which dynamically upholds the unified structure of all its individual components.
- EVOLUTIONARY:** The Hamiltonian operator generates the time-evolution of the universe.

All these beautiful, evolutionary qualities spontaneously blossom in individual and collective life through the Maharishi Technology of the Unified Field, which opens human awareness to the direct experience of consciousness in its self-referential state, transcendental consciousness, where consciousness is found identified with the unified field of all the laws of nature. The embodiment of all these qualities in world consciousness was beautifully demonstrated by the improved quality of world events when 7000+ experts in the Maharishi Technology of the Unified Field (the square root of one percent of the world's population) gathered at Maharishi International University from December 17, 1983 to January 6, 1984. This historic assembly verified the practical formula to create a unified field based ideal civilization.



## COMPARISON TO $D=5$ MAXIMAL SUPERGRAVITY

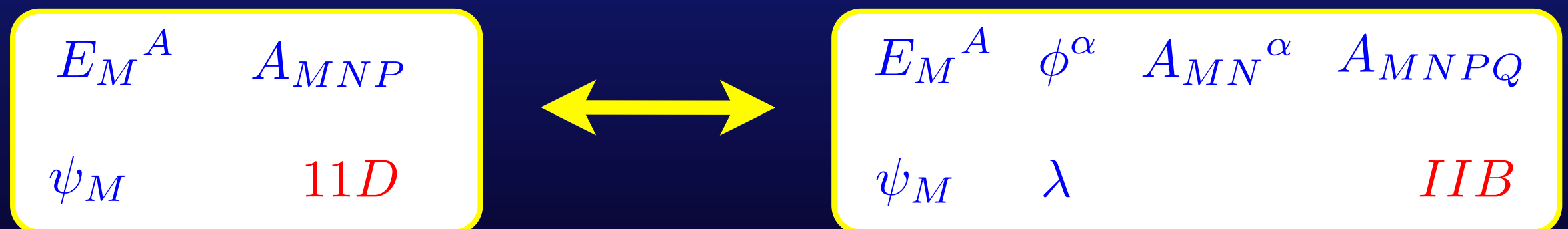
In five space-time dimensions maximal supergravity has similar features. The ungauged theory has a non-linearly realized  $E_{6(6)}$  symmetry.

*Günaydin, Romans, Warner, 2005*

Its field representation depends critically on the particular gauging, a phenomenon related to tensor-vector duality. All possible gaugings are in principle known and encoded in a so-called embedding tensor.

*dW, Samtleben, Trigiante, 2005*

The  $5D$  theory is more 'beautiful' than IIB supergravity in ten space-time dimensions, from which it can be obtained by dimensional reduction. The latter is based on a more reducible field configuration as compared to  $11D$  supergravity.



Of course there is no argument at all for believing that the  $5D$  theory should be less relevant than the  $4D$  one, just because of 'esthetical shortcomings' !

Rather, I want to take this opportunity to review some of what we did at the time for  $N=8$  supergravity, and to consider applying the same strategy for  $5D$  maximal supergravity.

The conclusion will be that we were helped by the symmetric structure of  $D=11$  supergravity. Our strategy was never applied to any other theory. Exploring this strategy for another theory, such as IIB supergravity, is a worthwhile exercise, because the structure of that theory is less rigid. The application to IIB supergravity should also be interesting in its own right.

An important ingredient in our work was the construction of a new formulation of  $D=11$  supergravity.

## CONSISTENCY & EMBEDDING

Compactifying the eleven-dimensional space on a seven-sphere one can truncate the degrees of freedom of  $D=11$  supergravity to a 'massless'  $N=8$  supermultiplet. This multiplet coincides with the supermultiplet on which  $N=8$  supergravity is based.

In attempting to prove that this truncation is consistent, we constructed an alternative formulation of  $D=11$  supergravity, which does not truncate the number of degrees of freedom, and is locally  $SU(8)$  invariant.

*dW, Nicolai, 1986*

It is constructed by writing the higher-dimensional theory in the form of a lower-dimensional one, still retaining the dependence on the extra coordinates, by changing the tangent space of the theory and thus the R-symmetry group.

Hence the spinor fields are converted to spinors in a lower-dimensional space-time. This is a crucial step.



## THE $11 = 4 + 7$ SPLIT :

Kaluza-Klein-type ansatz:

$$E_M^A = \begin{pmatrix} e_\mu^\alpha & B_\mu^n e_n^a \\ 0 & e_m^a \end{pmatrix} \quad \begin{array}{l} \alpha = 0, 1, 2, 3 \\ a = 1, 2, \dots, 7 \end{array}$$

*(Weyl rescaling  $\rightarrow$  Einstein frame)*

Tangent-space group:  $SO(10, 1) \longrightarrow SO(3, 1) \times SO(7)$

Fermion decomposition:  $\psi_M \longrightarrow \psi_\mu + \psi_a$

$$\text{Spin}(10, 1) \longrightarrow \text{Spin}(3, 1) \times \text{Spin}(7)$$

$$\longrightarrow \text{Spin}(3, 1) \times \text{Spin}(8)$$

**8 + 56** 4D spinors

*Majorana extension: use  $\gamma_5$*

**chiral SU(8): 8 + 56**

More precisely:

$$\Gamma_{ab} \oplus i\Gamma_a \oplus \gamma_5 \Gamma_{abc} \quad a = 1, 2, \dots, 7$$

so(7)

— so(8) —

———— su(8) —————

Spinor representation of  $\psi_a$ :  $\chi_{ABC} \propto \Gamma^a_{[AB} \psi_{aC]}$

Cremmer, Julia, 1979

$$\delta B_\mu^m = \frac{1}{8} \sqrt{2} i \Delta^{-1/2} \Gamma^m_{AB} (2\sqrt{2} \bar{\epsilon}^A \psi_\mu^B + \bar{\epsilon}_C \gamma_\mu \chi^{ABC}) + \text{h.c.}$$

Generalized vielbein:  $e^m_{AB} = i \Delta^{-1/2} e_a^m \Gamma^a_{AB}$

Complexified by a compensating local SU(8) transformation on all the spinors. Hence, one regards the results obtained from 11D as a gauge-fixed version of a new underlying theory and includes a uniform SU(8)/SO(7) phase factor.

The generalized vielbein satisfies a large number of algebraic and differential constraints.


Besides the local  $SU(8)$  invariance, the formulation exhibits also  $E_{7(7)}$  covariant (not invariant) features, in a way that is very similar to the structure of the purely  $4D$  theory !

For instance, the supersymmetry variation of the generalized vielbein takes the form

$$\delta e_{AB}^m = -\sqrt{2} \Sigma_{ABCD} e^{mCD}$$

$$\Sigma_{ABCD} = \bar{\epsilon}_{[A} \chi_{BCD]} + \varepsilon_{ABCDEFGH} \bar{\epsilon}^E \chi^{FGH}$$

$SU(8)$  covariant (by construction ?)



$\in \mathfrak{e}_{7(7)}/\mathfrak{su}(8)$

The supersymmetry variations of the spinor fields lead to a number of generalized ‘connections’ (such as an  $SU(8)$  connection) which can be used to define generalized covariant derivatives. The differential constraints on the generalized vielbeine imply that they are covariantly constant with respect to these derivatives. This condition is known as the ‘generalized vielbein postulate’.

This formulation still describes the full  $D=11$  theory. The distribution of the degrees of freedom is very subtle and depends on the background considered.

For instance, in the  $S^7$  compactification, the vector fields  $B_\mu^m$  describe 28 vector gauge fields associated with the  $S^7$  isometries, but only 7 ones in the toroidal compactification. In that case additional vector fields will be provided by the three-rank tensor field  $A_{MNP}$ .



It is fair to say that the full geometrical and the dynamical implications of this new formulation of  $D=11$  supergravity have not been fully explored.

On the other hand, on the basis of this formulation, it was possible to fully understand the **consistent embedding** of  $SO(8)$  gauged supergravity at the full non-linear level (i.e. for the field configuration space, beyond the isolated points that correspond to specific solutions).

*dW, Nicolai, 1987*

*Nicolai, Pilch, 2012*

In this truncation the dependence on the  $S^7$  coordinates of the non-linearly redefined fields is captured in terms of  $S^7$  Killing spinors and vectors.

But it is crucial that this truncation is applied on the **redefined** fields, including the scale factor and the local  $SU(8)/SO(7)$  phase factor. The presence of this phase factor was in line with earlier findings where we consistently extrapolated between two solutions of  $N=8$  supergravity, and correspondingly, between the two uplifted solutions of  $11D$  supergravity.

*To explore this strategy further it is of interest to consider its application to other theories. A prime candidate for this is IIB supergravity. Here I will present some results from a preliminary analysis.*

# IIB SUPERGRAVITY

The existence of this theory was inferred from IIB superstring theory. The theory has a non-linearly realized  $SL(2) \cong SU(1, 1)$  symmetry. Its field configuration contains the vielbein and a complex chiral gravitino, a complex chiral fermion, a complex scalar, and a number of antisymmetric tensor gauge fields:

$$E_M^A \quad \phi^\alpha \quad A_{MN}{}^\alpha \quad A_{MNPQ}$$

$$\psi_M \quad \lambda$$

*Green, Schwarz, 1982*

*Schwarz, West, 1983*

*Schwarz, 1983*

## Upon truncation:

Its compactification on a five-torus is expected to lead to ungauged  $5D$  maximal supergravity.

*Cremmer, 1980*

Its compactification on the five-sphere is expected to lead to  $SO(6)$  gauged supergravity.

*Günaydin, Romans, Warner, 1986*

## THE 10 = 5 + 5 SPLIT :

Kaluza-Klein-type ansatz:

$$E_M^A = \begin{pmatrix} e_\mu^\alpha & B_\mu^n e_n^a \\ 0 & e_m^a \end{pmatrix} \quad \begin{array}{l} \alpha = 0, 1, \dots, 4 \\ a = 1, 2, \dots, 5 \end{array}$$

*(Weyl rescaling  $\rightarrow$  Einstein frame)*

Tangent space group:  $SO(9, 1) \longrightarrow SO(4, 1) \times SO(5)$

Fermion decomposition:  $\psi_M + \lambda \longrightarrow \psi_\mu + \psi_a + \lambda$

$$\text{Spin}(9, 1) \times U(1) \longrightarrow \text{USp}(4, 1) \times \text{USp}(4) \times U(1)$$

$$\longrightarrow \text{Spin}(4, 1) \times \text{SU}(4) \times U(1)$$

**4 + 4 + 20 + 20 + 4 + 4** 5D spinors

*Further extension: use*  $U(1)$

**USp(8) : 8 + 48**



USp(8) generators for gravitini :

$$T \equiv i\mathbf{1}_4 \otimes \sigma_3, \quad T_a \equiv i\Gamma_a \otimes \sigma_3,$$

$$T_{ab}^0 \equiv \Gamma_{ab} \otimes \mathbf{1}_2, \quad T_{ab}^1 \equiv \Gamma_{ab} \otimes \sigma_1, \quad T_{ab}^2 \equiv \Gamma_{ab} \otimes \sigma_2.$$

(symplectic matrix :  $\Omega = \Omega_{(4)} \otimes \sigma_1$  )

$$T : \text{U}(1) \text{ generator : } \begin{aligned} \psi_M &\rightarrow e^{i\Lambda/2} \psi_M \\ \lambda &\rightarrow e^{-3i\Lambda/2} \lambda \end{aligned}$$

$T_a \oplus T_{ab}^0$  generators of SU(4)

$$T_{ab}^1 \oplus T_{ab}^2 \quad \Delta T = \pm 1$$

4 × 4 gamma matrices :

$$\gamma_\alpha \Big|_{\text{spacetime}} = i\Gamma_\alpha \Gamma_6 \Gamma_7 \Gamma_8 \Gamma_9 \Gamma_{10}$$

$$\Gamma_a \Big|_{\text{internal}} = i\Gamma_{a+5} \Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4 \Gamma_5$$

The  $USp(8)$  transformations on the 48 spin-1/2 is much more subtle. One has to assemble the fields  $\psi_a \oplus \lambda$  and their complex conjugates into a single three-index tensor

$$\delta B_\mu{}^m = \frac{1}{2} \Delta^{-1/3} e_a{}^m \left( i\bar{\epsilon} \Gamma^a \psi_\mu + \bar{\epsilon} \gamma_\mu (\delta^a_b + \frac{1}{3} \Gamma^a \Gamma_b) \psi^b + \text{h.c.} \right)$$

incomplete representation  $T = \pm 1/2$



This raises intriguing questions about the role of the generalized vielbein! It might be that the above structure will have to be complemented with terms appearing in the variation of the vector fields that arise under the reduction of the tensor fields.

It is clear that things are more involved in this case, and to elucidate the full structure will require more time. Nevertheless, we have already understood how a large part of the program as it was applied to 11D supergravity, can be carried out.

*A very intriguing question concerns the overall structure that will be obtained, in particular with regard to the generalized vielbein postulate.*

*The potential results can be used to show that  $SO(6)$  gauged supergravity can be embedded consistently into IIB supergravity.*

*With regard to other possible applications, it is important to realize that the new formulation still contains the full tower of Kaluza-Klein supermultiplets. Perhaps this could eventually be exploited in the context of AdS/CFT.*

# ETERNAL BLISS

IN ALL THESE YEARS WE WERE NOT JUST DOING  
SUPERGRAVITY AND OTHER SUPERSTUFF.....

WE SPEND MANY PLEASANT TIMES TOGETHER, ALSO  
WITH OUR FAMILIES, WE MET ALL OVER THE GLOBE.....

WE SHARED IN MORE THAN OUR JOINT SCIENTIFIC  
EXCITEMENT.....

AND WE PLAYED THE PIANO !



Happy Birthday

And many happy returns !

