From Loop Quantum Cosmology to the multiverse

(thanks Francesca !)

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Work made, in particular, with Thomas Cailleteau and Julien Grain

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Loop Quantum Gravity 1

Why do we need QG ? More than unification, the issue is consistency ! GR breaks down.

Canonical quantization is known to be the royal road toward quantization
→ 3+1 decomposition :

$$ds^2 = (-N^2 + N_a N^a)dt^2 + 2N_a dt dx^a + q_{ab} dx^a dx^b$$

q_ab as the conf. variable and K_ab, the extrinsic curvature, as the conjugate momentum.

One can use instead triads and the spin connection.

$$q_{ab} = E_i^a E_j^b \delta^{ij}$$

The constraints associated with the shift form a vector. The constraint associated with the lapse (scalar) represents deformations. The theory has 6 configuration degrees of freedom (g^ij) and 4 constraints.

Ashtekar gravity : SU(2) Y-M connection Aia (the conf. var.) and the densitized triads Eia (the canonically conjugate momenta).
E→ space metric
A→ extrinsic curvature

$$L = \frac{1}{8\pi G\gamma}\int d^3x\left(E_i^a \dot{A}_a^i + N\varepsilon_{ijk}E_i^a E_j^b F_{ab}^k + N^a E_i^b F_{ab}^i + \lambda^i (D_a E^a)^i\right)$$

3 set of constraints (7 constraints and 9 conf. var.)

Quantization. Naively (reverse to WdW) :

$$\hat{A}_a^i \Psi(A) = A_a^i \Psi(A)$$

$$\hat{E}_i^a \Psi(A) = -i \frac{\delta \Psi(A)}{\delta A_a^i}$$

But one now needs to promote the constaints to operators. This raises many problems (espacially with Hamiltonian constraints (in addition to f.o.) :

-No geometric meaning

-No clear inner product

-Wrong density weight)

→Loop representation (Giles theorem : traces give the gauge invariant connection)

→The diffo constraints is easily solved

→Intuitively : Faraday, curvature … + diff. Invariance !

$$\Psi[A] = \sum_\gamma \Psi[\gamma] W_\gamma[A]$$

$$W_\gamma[A] = Tr\left(P\left[\exp\left(-\oint_\gamma \dot{\gamma}(s) A_a(s) ds\right)\right]\right)$$

The structure constants of the algebra are the Levi-Civita symbols. But one can choose any representation of SU(2) to construct the connection. The parallel transport is a Matrix and one can tie the indices at the intersection (with intertiwiners). → Spin network.

→ CONSISTENT THEORY. CONVERGENCE OF APPROACHES. AREAS AND VOLUMES QUANTIZED. BH ENTROPY RECOVERED.

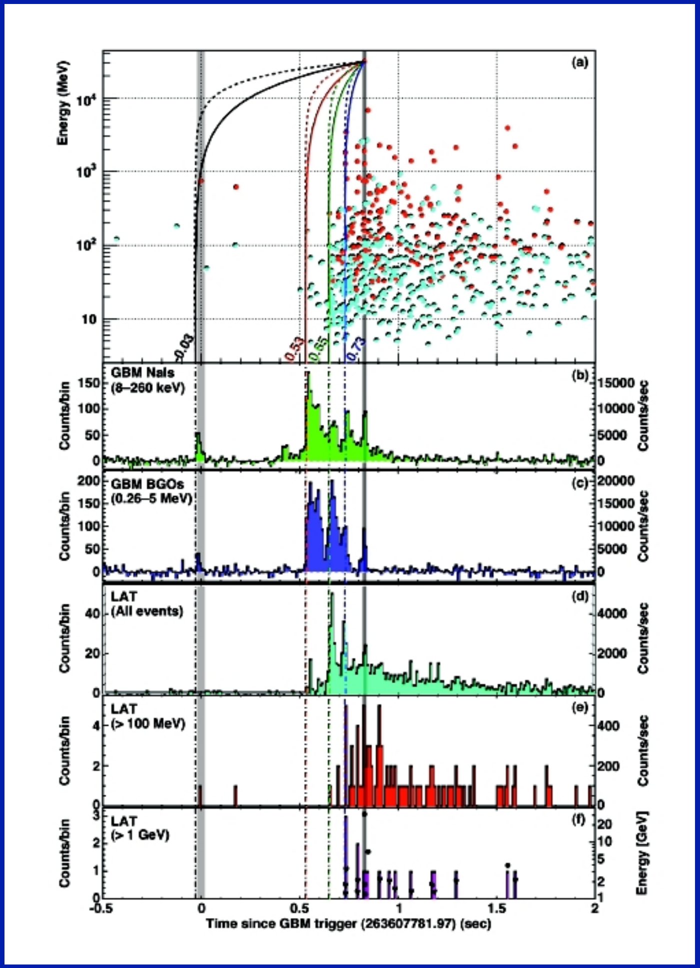
Loop Quantum Gravity 2

- Think of lattice QCD
- Define a graph and the Hilbert space : L2(G^L/G^N). The Fock space is obtained by taking the appropriate limit.
- In gravity one can do the same : HΓ = L2[SU(2)^L/SU(2)^N]. Then H̃Γ = HΓ/ ~ (automorphism group)
- Define « natural » operators on L2[SU2]
- Gauge invariance + Penrose theorem lead to a simple geometrical interpretation in the classical limit.
- Define the spin-network basis (diagonolizes the area and volume operators)

See *e.g.* Zakopane lectures

Let’s take LQG seriously. Experimental tests ?

- **High energy gamma-ray** (Amélino-Camelia et al.)



Not very conclusive however

Experimental tests

- Discrete values for areas and volumes

- Observationnal cosmology

LQC :

- IR limit
- UV limit (bounce)
-inflation

Toward LQC

Following Ashtekar

Within the Wheeler, Misner and DeWitt QGD, the BB singularity is not generically resolved → could it be different in the specific quantum theory of Riemannian geometry called LQG?

KEY questions:

- How close to the BB does smooth space-time make sense ? Is inflation safe ?
- Is the BB singularity solved as the hydrogen atom in electrodynamics (Heinsenberg)?
- Is a new principle/boundary condition at the BB essential ?
- Do quantum dynamical evolution remain deterministic through classical singularities ?
- Is there an « other side » ?

The Hamiltonian formulation generally serves as the royal road to quantum theory. But absence of background metric → constraints, no external time.

- Can we extract, from the arguments of the wave function, one variable which can serve as emergent time ?
- Can we cure small scales and remain compatible with large scale ? 14 Myr is a lot of time ! How to produce a huge repulsive force @ 10^94 g/cm^3 and turn it off quickly.

LQC: a few results

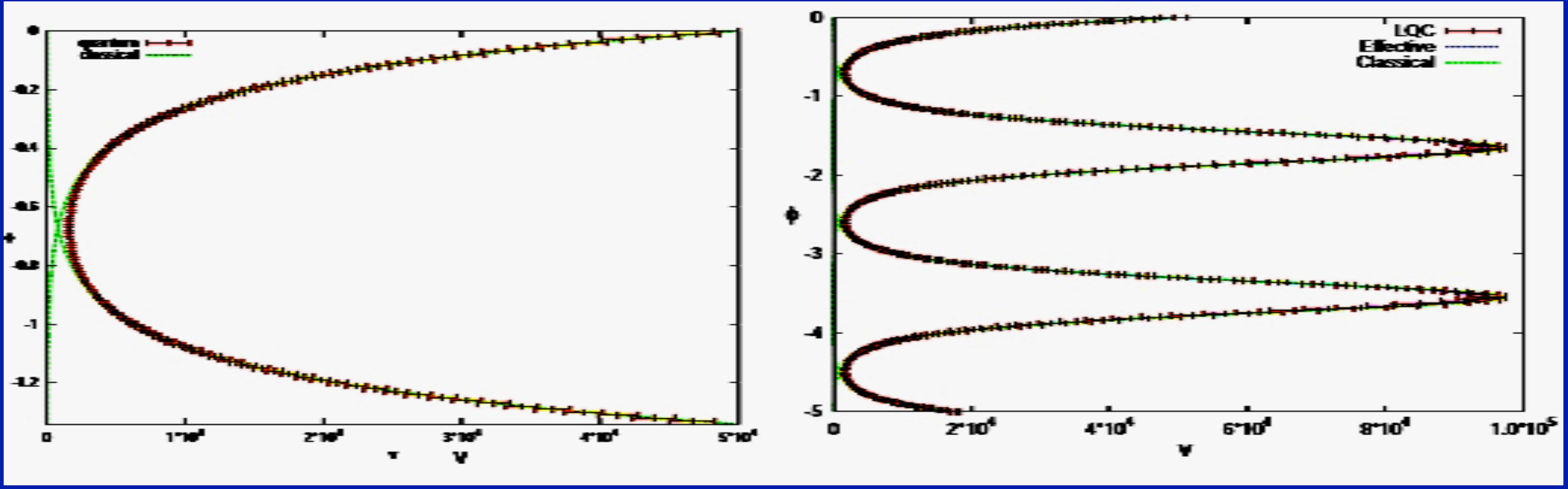
von Neumann theorem ? OK in non-relativistic QM. Here, the holonomy operators fail to be weakly continuous → no operators corresponding to the connections! → new QM

$$\Theta_o \Psi(v,\phi) = -F(v)\ \left(C^+(v)\,\Psi(v+4,\phi) + C^o(v)\,\Psi(v,\phi) + C^-(v)\,\Psi(v-4,\phi)\right)$$

Dynamics studied:

- **Numerically**
- **With effective equations**
- **With exact analytical results**

- **Trajectory defined by expectation values of the observable V is in good agreement with the classical Friedmann dynamics for $\rho<\rho_{Pl}/100$**
- **When $\rho \rightarrow \rho_{Pl}$ quantum geometry effects become dominant. Bounce at $0.41\rho_{Pl}$**



Plots from Ashtekar

LQC: a few results

- The volume of the Universe take its minimum value at the bounce and scales as p(Φ)
- The recollapse happens at Vmax which scales as p(Φ)^(3/2). GR is OK.
- The states remain sharply peaked for a very large number of cycles. Determinisme is kept even for an infinite number of cycles.
- The dynamics can be derived from effective Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = (8\pi G\,\rho/3)\,\left(1 - \frac{\rho}{\rho_{\rm crit}}\right)$$

- The LQC correction naturally comes with the correct sign. This is non-trivial.
- Furthermore, one can show that the upper bound of the spectrum of the density operator coincides with ρ_{crit}

The matter momentum and instantaneous volumes form a complete set of Dirac observables. The density and 4D Ricci scalar are bounded. → precise BB et BC singularity resolution. No fine tuning of initial conditions, nor a boundary condition at the singularity, postulated from outside. No violation of energy conditions (What about Penrose-Hawking th ? → LHS modified !). Quantum corrections to the matter hamiltonian plays no role. Once the singularity is resolved, a new « world » opens.

→ Role of the high symmetry assumed ? (string entropy ?)

LQC & inflation

-Inflation

- success (paradoxes solved, perturbations, etc.)
- difficulties (no fundamental theory, initial conditions, etc.)

-LQC

- success (background-independant quantization of GR, BB Singularity resolution, good IR limit)
- difficulties (very hard to test !)

Could it be that considering both LQC and inflation within the same framework allows to cure simultaneously all the problems ?

Bojowald, Hossain, Copeland, Mulryne, Numes, Shaeri, Tsujikawa, Singh, Maartens, Vandersloot, Lidsey, Tavakol, Mielczarek …….

First point :
The “bounce” is not the only LQC effect

« standard » inflation background

Bojowald & Hossain, Phys. Rev. D 77, 023508 (2008)

A.B. & Grain, Phys. Rev. Lett. , 102, 081321 (2009)

J. Grain, A.B., A. Gorecki, Phys. Rev. D , 79, 084015 (2009)

J. Grain, T. Cailleteau, A.B., Phys. Rev. D, 81, 024040 (2010)

-decouples the effects

-happens after superinflation

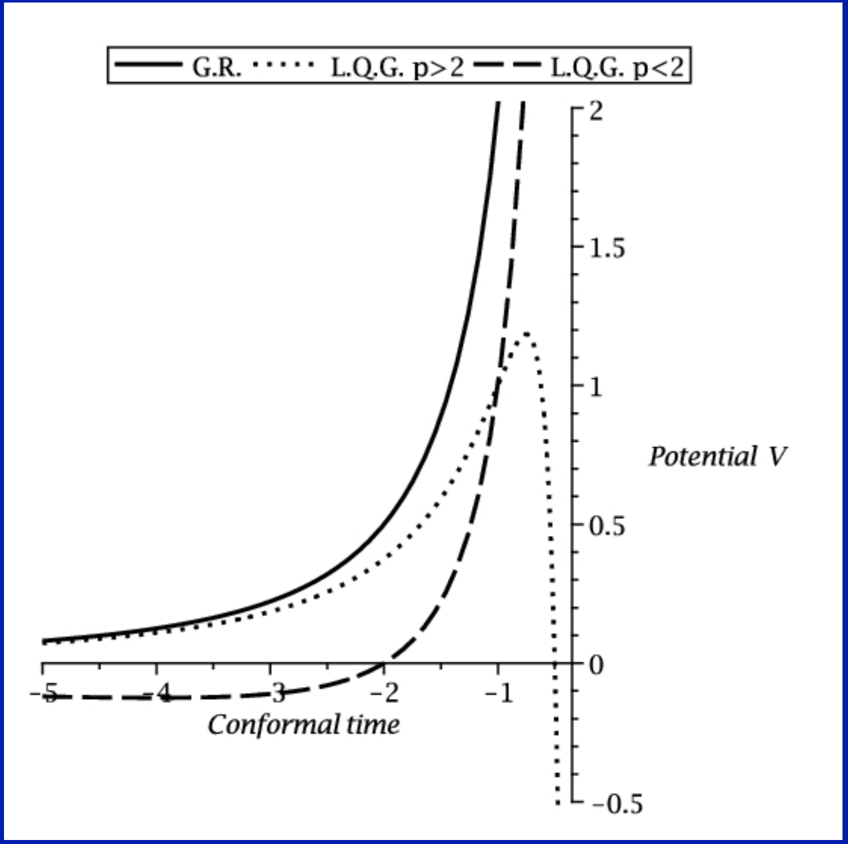
Holonomy corrections

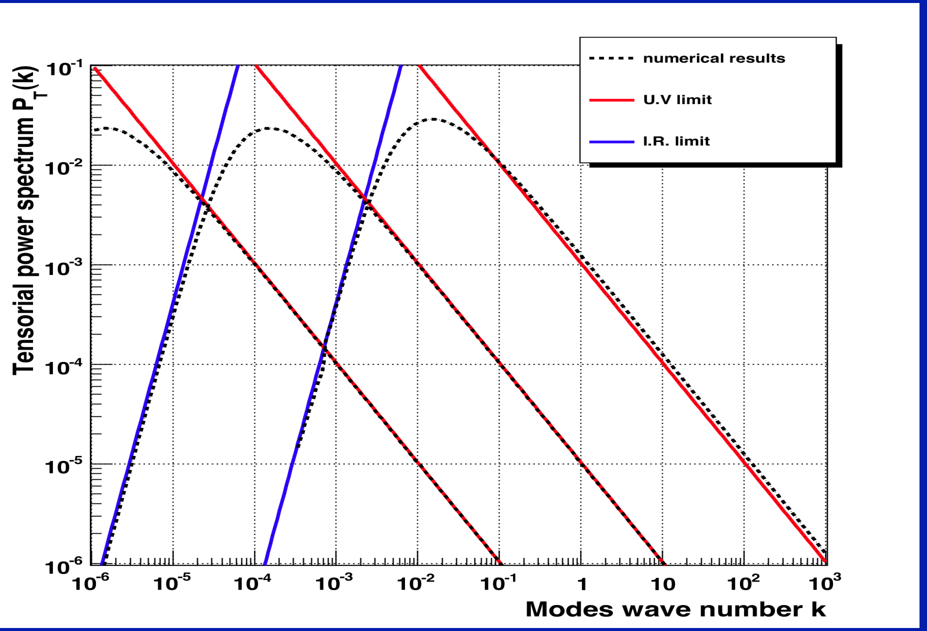
Bojowald & Hossain, Phys. Rev. D 77, 023508 (2008)

$$\left[\frac{\partial^2}{\partial\eta^2}+\left(\frac{\sin(2\gamma\bar{\mu}\bar{k})}{\gamma\bar{\mu}}\right)\frac{\partial}{\partial\eta}-\nabla^2-2\gamma^2\bar{\mu}^2\left(\frac{\bar{p}}{\bar{\mu}}\frac{\partial\bar{\mu}}{\partial\bar{p}}\right)\left(\frac{\sin(\gamma\bar{\mu}\bar{k})}{\gamma\bar{\mu}}\right)^4\right]h_a^i=16\pi G S_a^i$$

$$\bar{p}=a^2 \qquad \gamma\bar{k}=\bar{E}_a^i;\bar{k}=\dot{a}/a \qquad \gamma\bar{k}\longrightarrow\frac{\sin(\bar{\mu}\gamma\bar{k})}{\bar{\mu}}$$

Potential in the effective Schrödinger equation





Grain & Barrau, Phys. Rev. Lett. 102,081301 (2009)

Was also done with Inverse Volume corrections

Inverse-volume + holonomy corrections

$$H_G^{\mathrm{Phen}}[N] = \frac{1}{2\kappa}\int_\Sigma \mathrm{d}^3x \bar{N}\alpha\Bigg[-6\sqrt{\bar{p}}\left(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma}\right)^2 - \frac{1}{2\bar{p}^{3/2}}\left(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma}\right)^2(\delta E^c_j\delta E^d_k\delta^k_c\delta^j_d)$$
$$+\sqrt{\bar{p}}(\delta K^j_c\delta K^k_d\delta^c_k\delta^d_j) - \frac{2}{\sqrt{\bar{p}}}\left(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma}\right)(\delta E^c_j\delta K^j_c) - \frac{1}{\bar{p}^{3/2}}(\delta_{cd}\delta^{jk}E^c_j\delta^{ef}\partial_e\partial_f E^d_k)\Bigg]$$

$$H_{matter}[\bar{N}] = \int_\Sigma d^3x\left(\frac{1}{2}D(q)\frac{p_\Phi^2}{\bar{p}^{\frac{3}{2}}} + \bar{p}^{\frac{3}{2}}V(\Phi)\right).$$

J. Grain, T. Cailleteau, A.B., A. Gorecki, Phys. Rev. D. , 2009

$$\frac{1}{2}\left[\ddot{h}^i_a + 2S\left(\frac{sin(2\bar{\mu}\gamma\bar{k})}{2\bar{\mu}\gamma}\right)\dot{h}^i_a\left(1-\frac{\bar{p}}{S}\frac{\partial S}{\partial\bar{p}}\right) - S^2\nabla^2 h^i_a + S^2 T_Q h^i_a\right] + S\mathcal{A}^i_a = \kappa S\Pi^i_{Q_a},$$

$$T_Q = -2\left(\frac{\bar{p}}{\bar{\mu}}\frac{\partial\bar{\mu}}{\partial\bar{p}}\right)(\bar{\mu}\gamma)^2\left(\frac{sin(\bar{\mu}\gamma\bar{k})}{\bar{\mu}\gamma}\right)^4,$$

$$\Pi^i_{Q_a} = \frac{1}{3V_0}\frac{\partial H_{matter}}{\partial\bar{p}}\left(\frac{\delta E^c_j\delta^j_a\delta^i_c}{\bar{p}}\right)cos(2\bar{\mu}\gamma\bar{k}) + \frac{\delta H_{matter}}{\delta(\delta E^a_i)},$$

$$\mathcal{A}^i_a = \frac{1}{2}\sqrt{\bar{p}}\frac{\delta S}{\delta(\delta E^a_i)}[...] - \bar{p}\frac{\partial S}{\partial\bar{p}}cos(2\bar{\mu}\gamma\bar{k})\left(\frac{sin(\bar{\mu}\gamma\bar{k})}{\bar{\mu}\gamma}\right)^2 h^i_a.$$

$$
\begin{aligned}
E_k(\eta) &= S^2k^2 = \left[1 + 2\lambda_s \left(\frac{l_{PL}}{l_0}\right)^s |\eta|^{s(1+\epsilon)}\right] k^2, \\
V(\eta) &= \frac{2+3\epsilon}{\eta^2} + \frac{6}{\kappa}\frac{1}{\rho_c}\frac{(1+4\epsilon)}{l_0^2}|\eta|^{-2(1-\epsilon)} \\
&+ \lambda_s \left(\frac{l_{PL}}{l_0}\right)^s \left[-\frac{12}{\kappa}\frac{1}{\rho_c}\frac{(1+4\epsilon)}{l_0^2}|\eta|^{s-2+\epsilon(s+2)} + s(1+2\epsilon)|\eta|^{s(1+\epsilon)-2} - \frac{1}{2}s(s-1+\epsilon(2s-1))|\eta|^{s(1+\epsilon)-2}\right]
\end{aligned}
$$

$$
P_T^{IR}(k) = 16\pi^3 \left(\frac{l_{PL}}{l_0}\right)^2 (Z(1-4\omega))^{-\frac{3}{2}} k^3 e^{\pi\sqrt{\frac{Z}{8}}\frac{(1-4\omega)}{k}}
$$

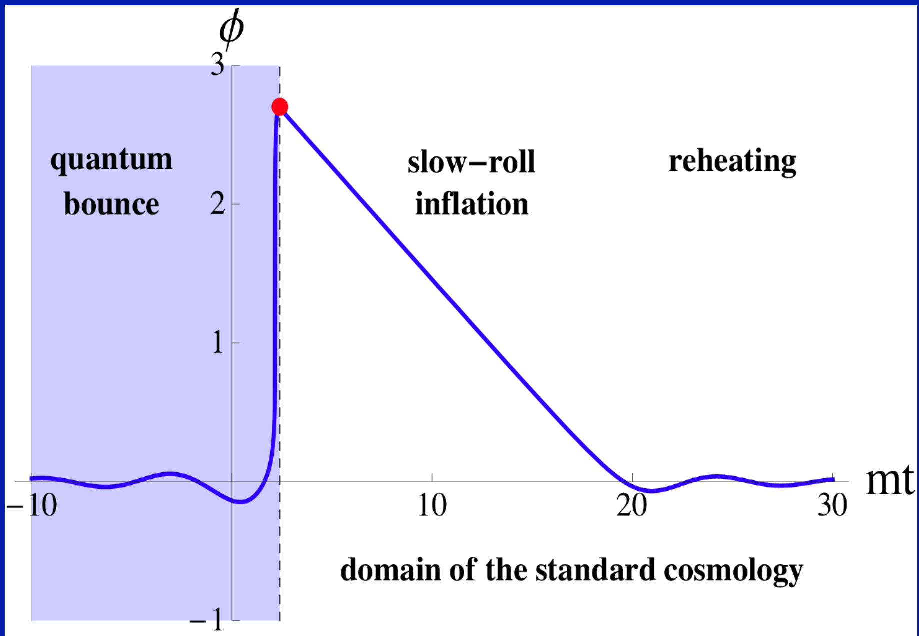
$$
P_T^{UV}(k) = 16\pi^3 \left(\frac{l_{PL}}{l_0^2}\right)^2 \left(1 + \frac{3}{2}\frac{Z}{k^2}(1-4\epsilon)\right) k^{-\frac{4}{3}\omega}
$$

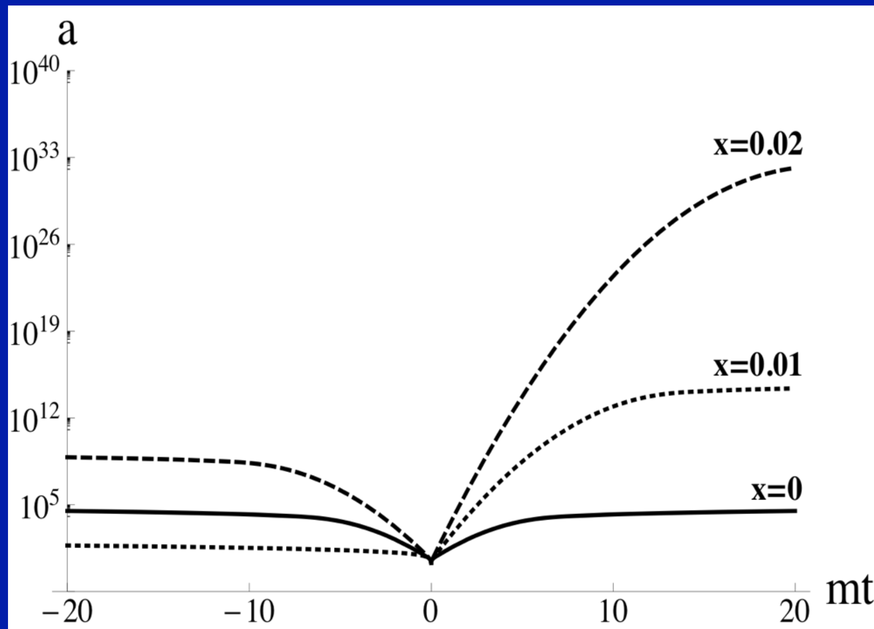
Holonomies dominate the background and inverse-volume dominate the modes

Main point : Taking into account the background modifications

H changes sign in the KG equation $\phi'' + 3H\phi' + m2\phi = 0$

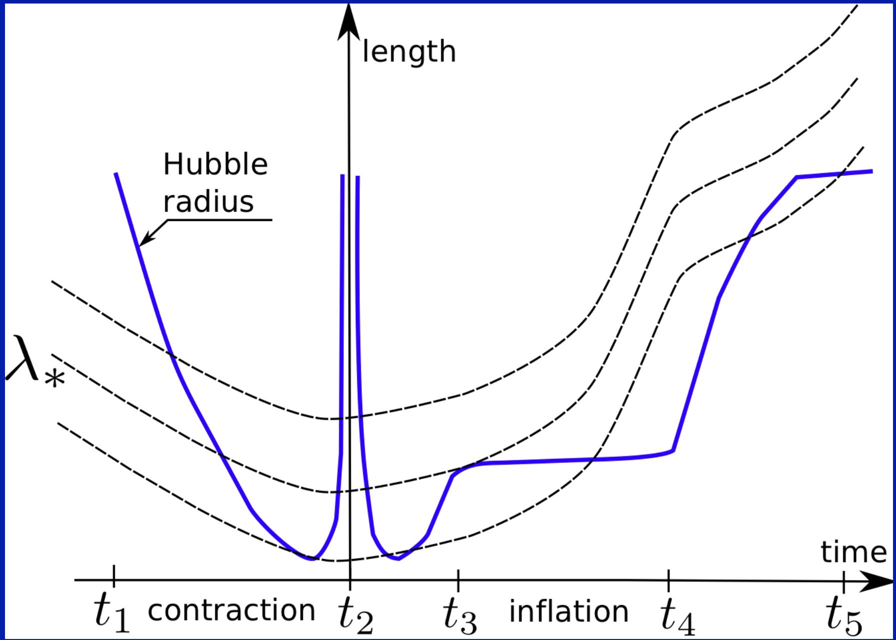
→ Inflation naturally occurs !



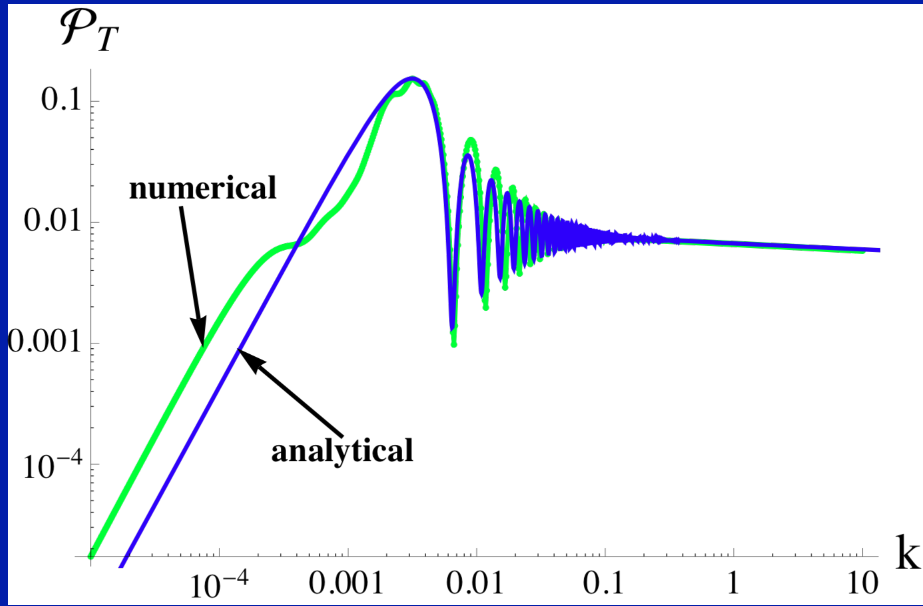


Mielczarek, Cailleteau, Grain, A.B., Phys. Rev. D, 81, 104049, 2010

A tricky horizon history…



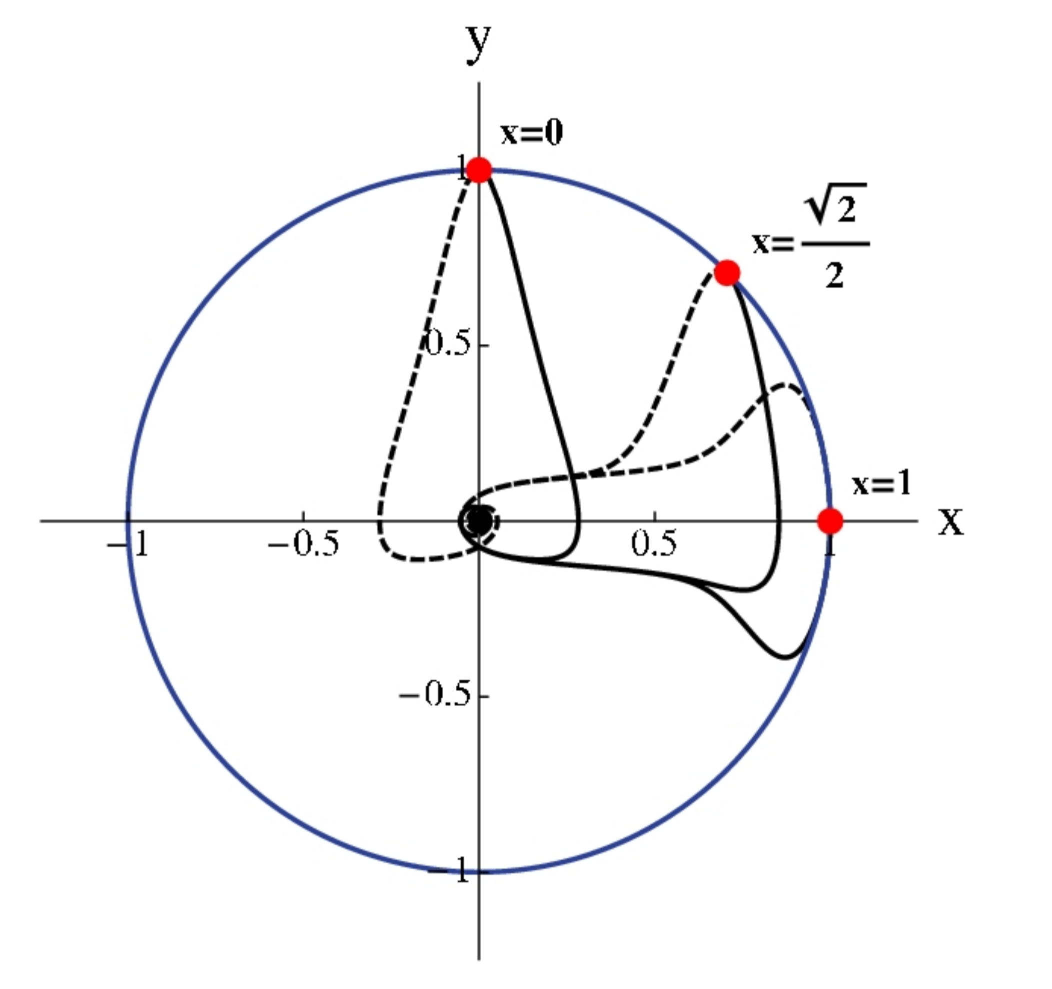
Physical modes may cross he horizon several times…



Computation of the primordial power spectrum:
-Bogolibov transformations
-Full numerical resolution

-The power is suppressed in the infra-red (IR) regime. This is a characteristic feature associated with the bounce
-The UV behavior agrees with the standard general relativistic picture.
-Damped oscillations are superimposed with the spectrum around the ”transition” momentum k* between the suppressed regime and the standard regime.
-The first oscillation behaves like a ”bump” that can substantially exceed the UV asymptotic value.

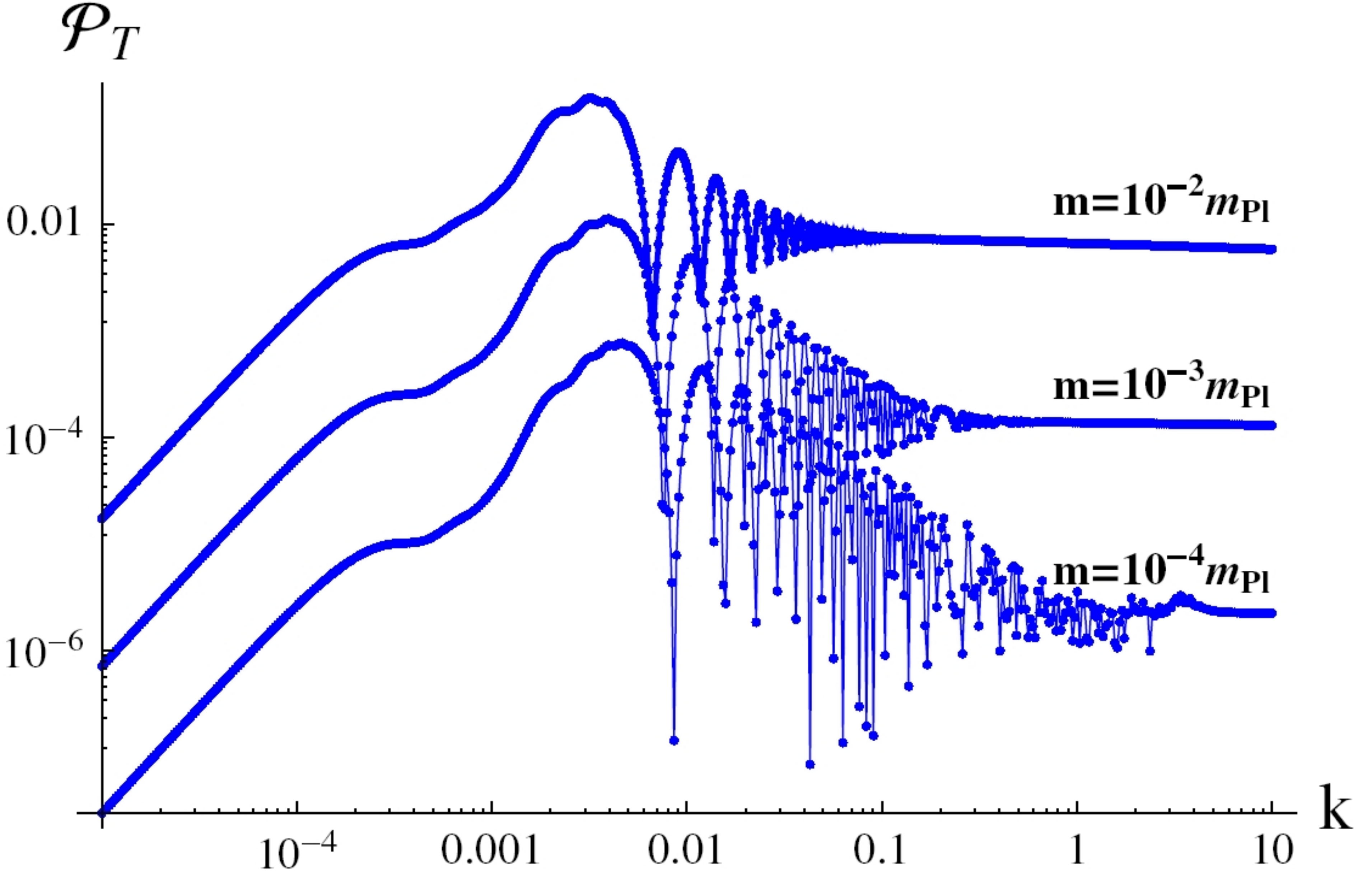
 Mielczarek, Cailleteau, Grain, A.B., Phys. Rev. D, 81, 104049, 2010



Effective description with a Bogoliubov transformation :
- Frequency of the oscillations controlled by Delta(eta), the width of the bounce
- Amplitude of the oscillations controlled by k0, the effective mass at the bounce

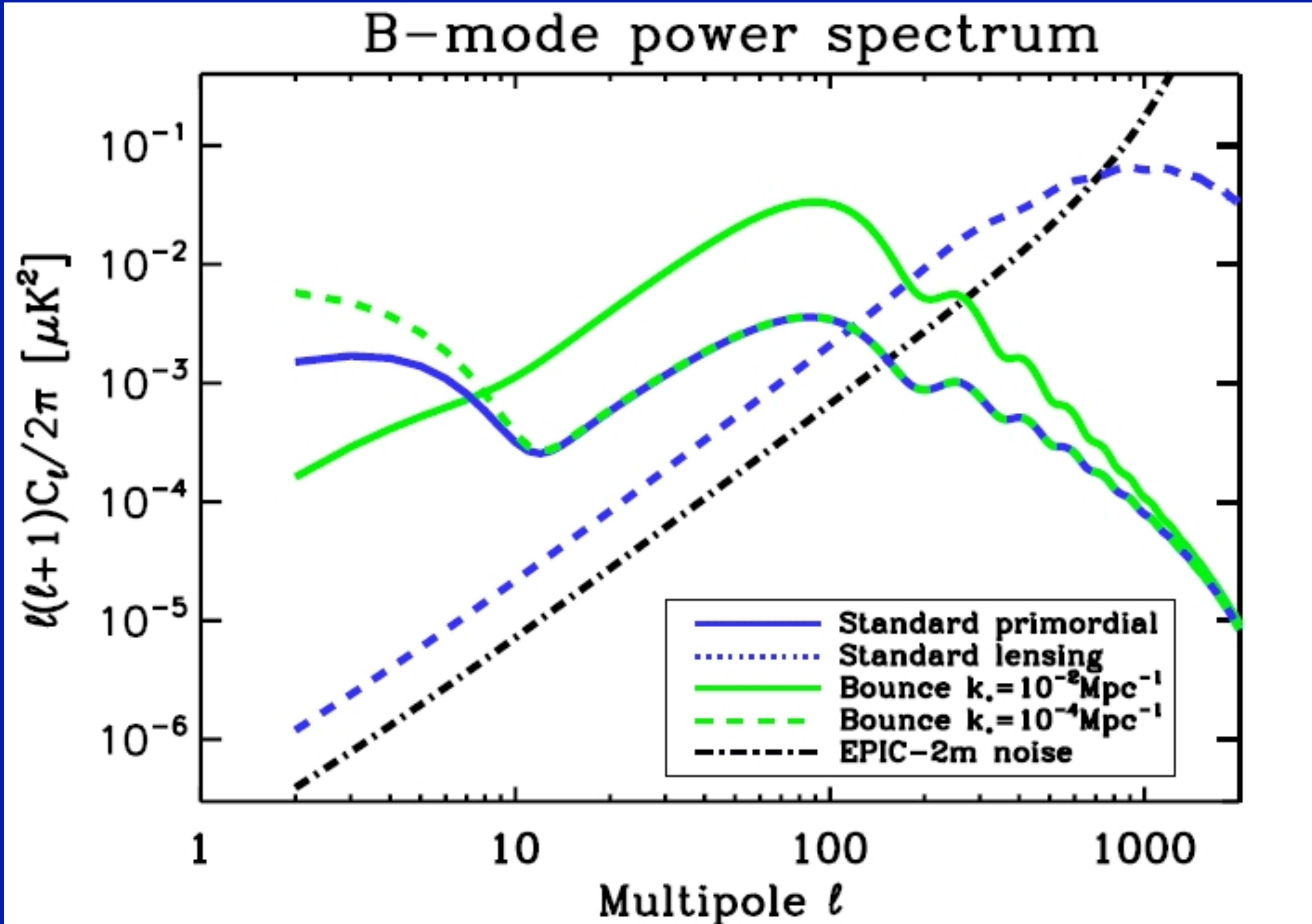
Fundamental description :
-R driven my the field mass
-k* driven by initial conditions

Initial conditions are critical

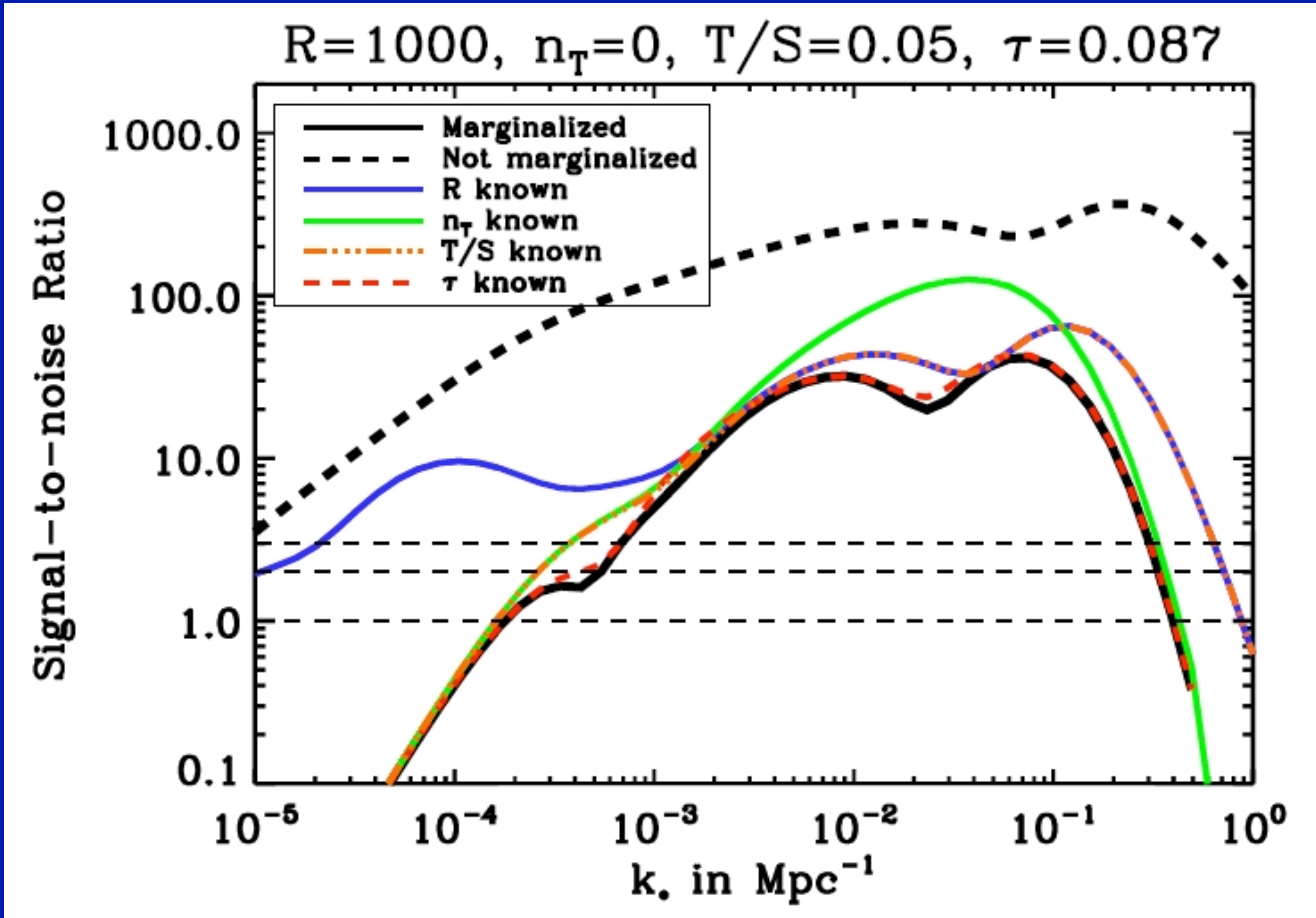


$\mathcal{P}_T$
0.01
10^{-4}
10^{-6}
$\mathbf{m=10^{-2}}m_{\mathrm{Pl}}$
$\mathbf{m=10^{-3}}m_{\mathrm{Pl}}$
$\mathbf{m=10^{-4}}m_{\mathrm{Pl}}$
10^{-4}
0.001
0.01
0.1
1
10
k

CMB consequences…

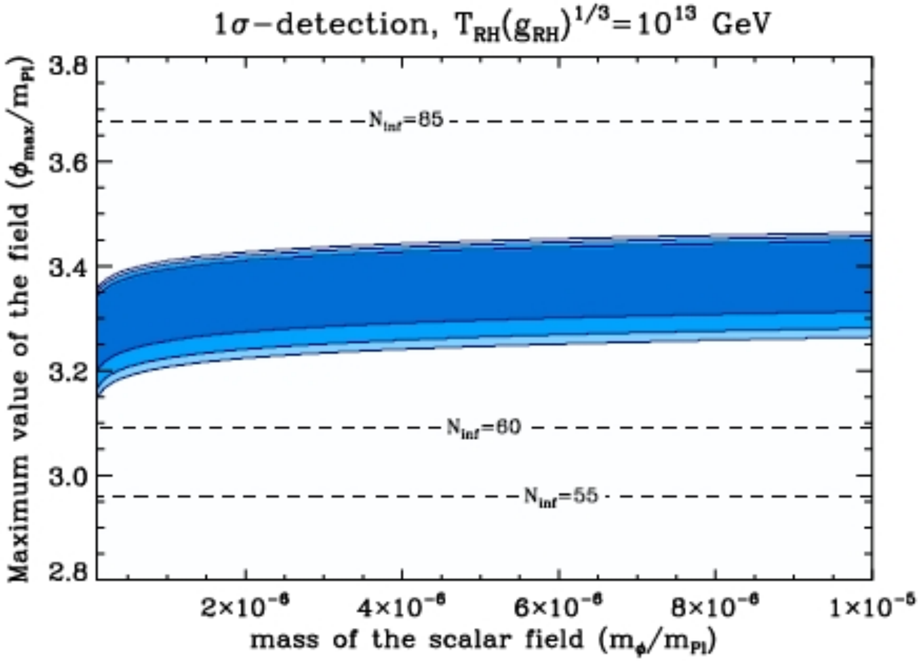


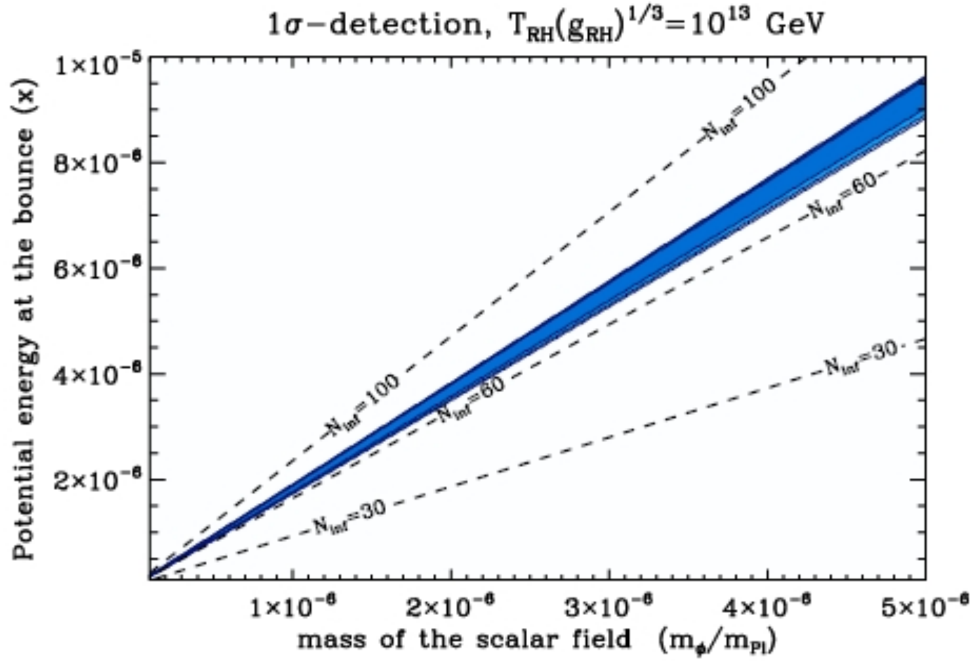
Grain, A.B., Cailleteau, Mielczarek, Phys. Rev. D, 82, 123520 (2010)



Grain, A.B. et al.

CMB consequences





If the scalar spectrum is assumed not to be affected : one needs x<2E-6 to probe the model

Grain, A.B., Cailleteau, Mielczarek, Phys. Rev. D, 82, 123520 (2010)

Is a N>78 inflation probable ? What is the probability to be compatible with WMAP data ?

-As H = 0 at the bounce, (purely geometric) superinflation unavoidably occurs. However, not sufficiant → matter+potential required.

-Standard inflation violates the strong energy condition and could then escape Penrose-Hawking theorem. However the Borde-Guth-Vilenkin version holds if H>0 at all times. Eternal inflation is past incomplete.

-LQC solves the singularity as long as the inflaton potential is bounded from below.

-Probabilities are poorly defined in the standard picture (dependant upon the measure and upon the chosen time.)

-In LQC, gauge-independant measure defined at the bounce. Comparison with WMAP data gives 1-P=10^-6 (Ashtekar and Sloan).

Anomaly-free vector algebra for holonomy corrections

Very important general question of consistency for effective approaches : are the constraints consistent with the evolution they generate ?

Smeared constraints:

$$
\begin{aligned}
\mathcal{C}_1 &= G[N^i] = \frac{1}{2\kappa}\int_\Sigma d^3x\, N^i C_i, \\
\mathcal{C}_2 &= D[N^a] = \frac{1}{2\kappa}\int_\Sigma d^3x\, N^a C_a, \\
\mathcal{C}_3 &= S[N] = \frac{1}{2\kappa}\int_\Sigma d^3x\, N C,
\end{aligned}
$$

$$\{G[N^i] + D[N^a] + S[N], G[M^i] + D[M^a] + S[M]\} \approx 0.$$

$$\{\mathcal{C}_I, \mathcal{C}_J\} = f^K{}_{IJ}(A^j_b, E^a_i)\mathcal{C}_K.$$

← First class algebra. However, when going to the quantum version anomalies usually appear.

$$\{\mathcal{C}^Q_I, \mathcal{C}^Q_J\} = f^K{}_{IJ}(A^j_b, E^a_i)\mathcal{C}^Q_K + \mathcal{A}_{IJ}.$$

Mielszarek, Cailleteau, A.B., Grain, CQG 2012

This issue is especially important when dealing with perturbations around the cosmological background.

$$A_a^i = \gamma \bar{k} \delta_a^i + \delta A_a^i \quad \text{and} \quad E_i^a = \bar{p} \delta_i^a + \delta E_i^a,$$

In bouncing cosmologies vector modes can be important.

Already derived for inverse-triad corrections and for holonomies up to fourth order in k. This is not enough to go through the bounce.

We follow the usual prescription with an arbitrary n integer. We don't restrict a prioris the mu dependance upon p.

$$\bar{k} \to \frac{\sin(n\bar{\mu}\gamma\bar{k})}{n\bar{\mu}\gamma}$$

→ Defined as K[n]

One has to write down vector perturbations in the canonical formulation. Then the quantum holonomy corrected hamiltonian constraint

$$S^Q[N] = \frac{1}{2\kappa}\int_\Sigma d^3x \left[\bar{N}(C^{(0)} + C^{(2)})\right],$$

where

$$\begin{aligned}
C^{(0)} &= -6\sqrt{\bar{p}}\,(\mathbb{K}[1])^2\,, \\
C^{(2)} &= -\frac{1}{2\bar{p}^{3/2}}\,(\mathbb{K}[1])^2\,(1+\alpha_1)(\delta E^c_j \delta E^d_k \delta^k_c \delta^j_d) \\
&+ \sqrt{\bar{p}}(\delta K^j_c \delta K^k_d \delta^c_k \delta^d_j) \\
&- \frac{2}{\sqrt{\bar{p}}}\,(\mathbb{K}[v_1])\,(1+\alpha_2)(\delta E^c_j \delta K^j_c).
\end{aligned}$$

Quantum holonomy corrected diffeomorphism constraint.

$$\begin{aligned}
D^Q[N^a] &= \frac{1}{\kappa}\int_\Sigma d^3x \delta N^c \left[-\bar{p}(\partial_k \delta K^k_c)\right. \\
&- \left.(\mathbb{K}[v_2])\,\delta^k_c(\partial_d \delta E^d_k)\right],
\end{aligned}$$

In LQG, it keeps its standard expression.

In order to investigate the algebra of constraints, the Poisson brackets have to be calculated.

$$\begin{aligned}
\{S^Q[N_1], S^Q[N_1]\} &= 0, & (18)\\
\{D^Q[N_1^a], D^Q[N_2^a]\} &= 0, & (19)\\
\{S^Q[N], D^Q[N^a]\} &= \frac{\bar{N}}{\sqrt{\bar{p}}}\mathcal{B}D^Q[N^a] \\
&+ \frac{\bar{N}}{\kappa\sqrt{\bar{p}}}\int_\Sigma d^3x\delta N^c\delta_c^k(\partial_d\delta E_k^d)\delta E_k^d\mathcal{A},
\end{aligned}$$

$$\mathcal{B} := (1+\alpha_2)\mathbb{K}[v_1] + \mathbb{K}[v_2] - 2\mathbb{K}[2]$$

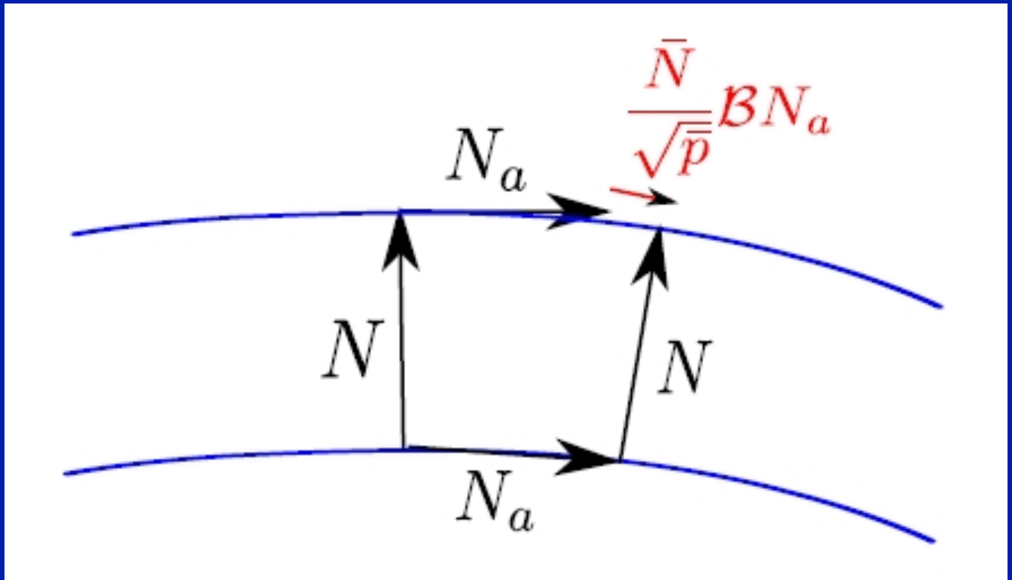
$$\begin{aligned}
\mathcal{A}_1 &= \mathcal{B}\mathbb{K}[v_2],\\
\mathcal{A}_2 &= 2\mathbb{K}[2]\bar{p}\frac{\partial\mathbb{K}[v_2]}{\partial\bar{p}} - \frac{1}{2}(\mathbb{K}[1])^2\cos(v_2\bar{\mu}\gamma\bar{k})\\
&- 2\mathbb{K}[1]\bar{p}\frac{\partial\mathbb{K}[1]}{\partial\bar{p}}\cos(v_2\bar{\mu}\gamma\bar{k})\\
&+ (1+\alpha_2)\mathbb{K}[v_1]\mathbb{K}[v_2] - \frac{1}{2}\mathbb{K}[1]^2(1+\alpha_1).
\end{aligned}$$

In order to investigate the algebra of constrains, the Poisson brackets have to be calculated.

One needs A=0. If this can be achieved, the algebra will be closed but deformed.

$$\{S^Q[N], D^Q[N^a]\} = D^Q\left[\frac{\bar{N}}{\sqrt{\bar{p}}}\mathcal{B}N^a\right]$$

The Hamiltonian and diffeomorphism constraints generate gauge transformations in directions respectively normal and parallel to te hypersurface



1) **Without counterterms. One is led to a (Pell-type) Diophantine equations in v1 and v2. Infinite number of solutions, but up to 4th order only.**
2) **General case. A=0 →**

$$\alpha_1 = -1 + 4(1+\alpha_2)\frac{\mathbb{K}[v_1]\mathbb{K}[v_2]}{\mathbb{K}[1]^2} - 4(1+\beta)\frac{\mathbb{K}[2]\mathbb{K}[v_2]}{\mathbb{K}[1]^2} + 2\frac{\mathbb{K}[v_2]^2}{\mathbb{K}[1]^2} + (4\beta-1)\cos(v_2\bar{\mu}\gamma\bar{k}).$$

$$H_{\mathrm{m}}[N] = \bar{H}_{\mathrm{m}} + \delta H_{\mathrm{m}} = \int_\Sigma d^3x \bar{N}(C^{(0)}_{\mathrm{m}} + C^{(2)}_{\mathrm{m}}), \quad (37)$$

where

$$C^{(0)}_{\mathrm{m}} = \bar{p}^{3/2}\left[\frac{1}{2}\frac{\bar{\pi}^2}{\bar{p}^3} + V(\bar{\varphi})\right]. \quad (38)$$

The value of $C^{(2)}_{\mathrm{m}}$ is given by

$$C^{(2)}_{\mathrm{m}} = \frac{1}{2}\frac{\delta\pi^2}{\bar{p}^{3/2}} + \frac{1}{2}\sqrt{\bar{p}}\delta^{ab}\partial_a\delta\varphi\partial_b\delta\varphi + \frac{1}{2}\bar{p}^{3/2}V_{,\varphi\varphi}(\bar{\varphi})\delta\varphi^2 + \left(\frac{1}{2}\frac{\bar{\pi}^2}{\bar{p}^{3/2}} - \bar{p}^{3/2}V(\bar{\varphi})\right)\frac{\delta^k_c\delta^j_d\delta E^c_j\delta E^d_k}{4\bar{p}^2}, \quad (39)$$

where we have used the condition $\delta^i_a\delta E^a_i = 0$. The matter diffeomorphism constraint is given by:

$$D_{\mathrm{m}}[N^a] = \int_\Sigma d^3x\delta N^a\bar{\pi}(\partial_a\delta\varphi). \quad (40)$$

At this stage, ambiguities remain. Matter has to be introduced

←

The matter Ham. does not depend on the Ashtekar connection : no holo cor.

$$\{S_{\rm tot}[N_1], S_{\rm tot}[N_1]\} = 0,$$
$$\{D_{\rm tot}[N_1^a], D_{\rm tot}[N_2^a]\} = 0,$$
$$\{S_{\rm tot}[N], D_{\rm tot}[N^a]\} = \frac{\bar{N}}{\sqrt{\bar{p}}}\mathcal{B}D^Q[N^a]$$
$$+\frac{\bar{N}}{\kappa\sqrt{\bar{p}}}\int_\Sigma d^3x \delta N^c \delta_c^k (\partial_d \delta E_k^d)\delta E_k^d \mathcal{A}$$
$$+[\cos(v_2\bar{\mu}\gamma\bar{k}) - 1]\frac{\sqrt{\bar{p}}}{2}\left(\frac{\bar{\pi}^2}{2\bar{p}^3} - V(\bar{\varphi})\right)\times$$
$$\times \int_\Sigma d^3x \bar{N}\partial_c(\delta N^a)\delta_a^j \delta E_j^c$$
$$+\frac{\bar{\pi}}{\bar{p}^{3/2}}\int_\Sigma d^3x \bar{N}(\partial_a \delta N^a)\delta\pi$$
$$-\bar{p}^{3/2}V_\varphi(\bar{\varphi})\int_\Sigma d^3x \bar{N}(\partial_a \delta N^a)\delta\varphi.$$

←The total Poisson brackets can now be calculated.

The algebra can be closed without ambiguity.

The free Hamiltonian reads as :

$$S^Q_{\rm free}[N] = \frac{1}{2\kappa}\int_\Sigma d^3x \left[\bar{N}(C^{(0)}_{\rm free} + C^{(2)}_{\rm free})\right], \quad (46)$$

where

$$C^{(0)}_{\rm free} = -6\sqrt{\bar{p}}\,(\mathbb{K}[1])^2, \quad (47)$$
$$C^{(2)}_{\rm free} = -\frac{1}{2\bar{p}^{3/2}}\left[4(1-\beta)\mathbb{K}[2]\bar{k} - 2\bar{k}^2 + (4\beta - 1)\mathbb{K}[1]^2\right]\times$$
$$\times\ (\delta E_j^c \delta E_k^d \delta_c^k \delta_d^j) + \sqrt{\bar{p}}(\delta K_c^j \delta K_d^k \delta_k^c \delta_j^d)$$
$$-\ \frac{2}{\sqrt{\bar{p}}}\left(2\mathbb{K}[2] - \bar{k}\right)(\delta E_j^c \delta K_c^j). \quad (48)$$

Anomaly-free scalar algebra for holonomy corrections

Scalar perturbations are the more important ones from the observational viewpoint. Developping and anomaly-free and gauge-invariant framework for holonomy corrections has been an open issue.

→FLRW background with scalar perturbations. 4 background variables (k,p, phi, pi) and 4 perturbed variables (dK, dE, dPhi, dPi).

→The full Poisson Bracket can be decomposed in 4 main terms

→The usual replacement k→sin is performed

Cailleteau, Mielzcarek, A.B., Grain, CQG 2012

The holonomy-modified Hamiltonian constraint reads as

$$H_G^Q[N] = \frac{1}{2\kappa}\int_\Sigma d^3x \left[\bar{N}(\mathcal{H}_G^{(0)} + \mathcal{H}_G^{(2)}) + \delta N \mathcal{H}_G^{(1)}\right],$$

where

$$\mathcal{H}_G^{(0)} = -6\sqrt{\bar{p}}(\mathbb{K}[1])^2,$$

$$\mathcal{H}_G^{(1)} = -4\sqrt{\bar{p}}\left(\mathbb{K}[s_1] + \alpha_1\right)\delta_j^c \delta K_c^j - \frac{1}{\sqrt{\bar{p}}}\left(\mathbb{K}[1]^2 + \alpha_2\right)\delta_c^j \delta E_j^c$$
$$+\frac{2}{\sqrt{\bar{p}}}(1+\alpha_3)\partial_c\partial^j \delta E_j^c,$$

$$\mathcal{H}_G^{(2)} = \sqrt{\bar{p}}(1+\alpha_4)\delta K_c^j \delta K_d^k \delta_k^c \delta_j^d - \sqrt{\bar{p}}(1+\alpha_5)(\delta K_c^j \delta_j^c)^2$$
$$-\frac{2}{\sqrt{\bar{p}}}\left(\mathbb{K}[s_2] + \alpha_6\right)\delta E_j^c \delta K_c^j - \frac{1}{2\bar{p}^{3/2}}\left(\mathbb{K}[1]^2 + \alpha_7\right)\delta E_j^c \delta E_k^d \delta_c^k \delta_d^j$$
$$+\frac{1}{4\bar{p}^{3/2}}\left(\mathbb{K}[1]^2 + \alpha_8\right)(\delta E_j^c \delta_c^j)^2 - \frac{1}{2\bar{p}^{3/2}}(1+\alpha_9)\delta^{jk}(\partial_c \delta E_j^c)(\partial_d \delta E_k^d).$$

The standard holo corrections are parametrized by 2 integers s1 and s2. The alpha_i are counter-terms, which are introduced to remove anomalies (vanishing in the mu→0 limit). The diffeo constraint holds its classical form :

$$D_G[N^a] = \frac{1}{\kappa}\int_\Sigma d^3x \delta N^c \left[\bar{p}\partial_c(\delta_k^d \delta K_d^k) - \bar{p}(\partial_k \delta K_c^k) - \bar{k}\delta_c^k(\partial_d \delta E_k^d)\right].$$

Poisson Brackets.

$$\left\{H_G^Q[N], D_G[N^a]\right\} = -H_G^Q\left[\delta N^a \partial_a \delta N\right] + \mathcal{B}\ D_G[N^a]$$

$$+\frac{\sqrt{\bar{p}}}{\kappa}\int_\Sigma d^3x \delta N^a(\partial_a \delta N)\mathcal{A}_1 + \frac{\bar{N}\sqrt{\bar{p}}\bar{k}}{\kappa}\int_\Sigma d^3x \delta N^a(\partial_i \delta K_a^i)\mathcal{A}_2$$

$$+\frac{\bar{N}}{\kappa\sqrt{\bar{p}}}\int_\Sigma d^3x \delta N^i(\partial_a \delta E_i^a)\mathcal{A}_3 + \frac{\bar{N}}{2\kappa\sqrt{\bar{p}}}\int_\Sigma d^3x(\partial_a \delta N^a)(\delta E_i^b \delta_b^i)\mathcal{A}_4,$$

where

$$\mathcal{B} = \frac{\bar{N}}{\sqrt{\bar{p}}}\left[-2\mathbb{K}[2] + \bar{k}(1+\alpha_5) + \mathbb{K}[s_2] + \alpha_6\right],$$

and

$$\mathcal{A}_1 = 2\bar{k}(\mathbb{K}[s_1] + \alpha_1) + \alpha_2 - 2\mathbb{K}[1]^2,$$

$$\mathcal{A}_2 = \alpha_5 - \alpha_4,$$

$$\mathcal{A}_3 = -\mathbb{K}[1]^2 - \bar{p}\frac{\partial}{\partial\bar{p}}\mathbb{K}[1]^2 - \frac{1}{2}\alpha_7$$

$$+\bar{k}(-2\mathbb{K}[2] + \bar{k}(1+\alpha_5) + 2\mathbb{K}[s_2] + 2\alpha_6),$$

$$\mathcal{A}_4 = \alpha_8 - \alpha_7.$$

{H,H} introduces 4 more anomalies
{D,D} is vanishing

Including matter

is

$$D_M[N^a] = \int_\Sigma \delta N^a \bar{\pi}(\partial_a \delta\varphi).$$

The scalar matter Hamiltonian can be expressed as:

$$H_M^Q[N] = H_M[\bar{N}] + H_M[\delta N],$$

where

$$\begin{aligned}
H_M[\bar{N}] &= \int_\Sigma d^3x \bar{N} \left[\left(\mathcal{H}_\pi^{(0)} + \mathcal{H}_\varphi^{(0)}\right) + \left(\mathcal{H}_\pi^{(2)} + \mathcal{H}_\nabla^{(2)} + \mathcal{H}_\varphi^{(2)}\right) \right], \\
H_M[\delta N] &= \int_\Sigma d^3 \delta N \left[\mathcal{H}_\pi^{(1)} + \mathcal{H}_\varphi^{(1)}\right].
\end{aligned}$$

$$\begin{aligned}
\mathcal{H}_\pi^{(0)} &= \frac{\bar{\pi}^2}{2\bar{p}^{3/2}}, \\
\mathcal{H}_\varphi^{(0)} &= \bar{p}^{3/2} V(\bar{\varphi}), \\
\mathcal{H}_\pi^{(1)} &= \frac{\bar{\pi}\delta\pi}{\bar{p}^{3/2}} - \frac{\bar{\pi}^2}{2\bar{p}^{3/2}} \frac{\delta_c^j \delta E_j^c}{2\bar{p}}, \\
\mathcal{H}_\varphi^{(1)} &= \bar{p}^{3/2} \left[V_{,\varphi}(\bar{\varphi})\delta\varphi + V(\bar{\varphi}) \frac{\delta_c^j \delta E_j^c}{2\bar{p}} \right], \\
\mathcal{H}_\pi^{(2)} &= \frac{1}{2} \frac{\delta\pi^2}{\bar{p}^{3/2}} - \frac{\bar{\pi}\delta\pi}{\bar{p}^{3/2}} \frac{\delta_c^j \delta E_j^c}{2\bar{p}} + \frac{1}{2} \frac{\bar{\pi}^2}{\bar{p}^{3/2}} \left[\frac{(\delta_c^j \delta E_j^c)^2}{8\bar{p}^2} + \frac{\delta_c^k \delta_d^j \delta E_j^c \delta E_k^d}{4\bar{p}^2} \right], \\
\mathcal{H}_\nabla^{(2)} &= \frac{1}{2} \sqrt{\bar{p}} (1 + \alpha_{10}) \delta^{ab} \partial_a \delta\varphi \partial_b \delta\varphi, \\
\mathcal{H}_\varphi^{(2)} &= \frac{1}{2} \bar{p}^{3/2} V_{,\varphi\varphi}(\bar{\varphi}) \delta\varphi^2 + \bar{p}^{3/2} V_{,\varphi}(\bar{\varphi}) \delta\varphi \frac{\delta_c^j \delta E_j^c}{2\bar{p}} \\
&\quad + \bar{p}^{3/2} V(\bar{\varphi}) \left[\frac{(\delta_c^j \delta E_j^c)^2}{8\bar{p}^2} - \frac{\delta_c^k \delta_d^j \delta E_j^c \delta E_k^d}{4\bar{p}^2} \right].
\end{aligned}$$

One can now compute the total constraints

$$\{D_{tot}[N_1^a], D_{tot}[N_2^a]\} = 0.$$

$$\begin{aligned}\{H_{tot}[N], D_{tot}[N^a]\} &= \left\{H_M^Q[N], D_{tot}[N^a]\right\} + \left\{H_G^Q[N], D_G[N^a]\right\} \\ &+ \left\{H_G^Q[N], D_M[N^a]\right\}.\end{aligned}$$

$$\begin{aligned}\{H_{tot}[N_1], H_{tot}[N_2]\} &= \left\{H_G^Q[N_1], H_G^Q[N_2]\right\} + \{H_M[N_1], H_M[N_2]\} \\ &+ \left[\left\{H_G^Q[N_1], H_M[N_2]\right\} - (N_1 \leftrightarrow N_2)\right].\end{aligned}$$

$$\begin{aligned}&\left\{H_G^Q[N_1], H_M[N_2]\right\} - (N_1 \leftrightarrow N_2) = \\ &= \frac{1}{2}\int_\Sigma d^3x \bar{N}(\delta N_2 - \delta N_1)\left(\frac{\bar{\pi}^2}{2\bar{p}^3} - V(\bar{\varphi})\right)(\partial_c\partial^j\delta E_j^c)\mathcal{A}_9 \\ &+3\int_\Sigma d^3x \bar{N}(\delta N_2 - \delta N_1)\left(\frac{\bar{\pi}\delta\pi}{\bar{p}^2} - \bar{p}V_\varphi(\bar{\varphi})\delta\varphi\right)\mathcal{A}_{10} \\ &+\int_\Sigma d^3x \bar{N}(\delta N_2 - \delta N_1)(\delta_j^c\delta K_j^c)\left(\frac{\bar{\pi}^2}{2\bar{p}^3} - V(\bar{\varphi})\right)\bar{p}\mathcal{A}_{11} \\ &+\frac{1}{2}\int_\Sigma d^3x \bar{N}(\delta N_2 - \delta N_1)(\delta_c^j\delta E_j^c)\left(\frac{\bar{\pi}^2}{2\bar{p}^3}\right)\mathcal{A}_{12} \\ &+\frac{1}{2}\int_\Sigma d^3x \bar{N}(\delta N_2 - \delta N_1)(\delta_c^j\delta E_j^c)V(\bar{\varphi})\mathcal{A}_{13},\end{aligned}$$

where

$$\begin{aligned}\mathcal{A}_9 &= \frac{\partial\alpha_3}{\partial\bar{k}}, \\ \mathcal{A}_{10} &= \mathbb{K}[2] - \mathbb{K}[s_1] - \alpha_1, \\ \mathcal{A}_{11} &= -\frac{\partial}{\partial\bar{k}}(\mathbb{K}[s_1] + \alpha_1) + \frac{3}{2}(1+\alpha_5) - \frac{1}{2}(1+\alpha_4), \\ \mathcal{A}_{12} &= -\frac{1}{2}\frac{\partial}{\partial\bar{k}}(\mathbb{K}[1]^2 + \alpha_2) + 5(\mathbb{K}[s_1] + \alpha_1) - 5\mathbb{K}[2] + \mathbb{K}[s_2] + \alpha_6, \\ \mathcal{A}_{13} &= \frac{1}{2}\frac{\partial}{\partial\bar{k}}(\mathbb{K}[1]^2 + \alpha_2) + \mathbb{K}[s_1] + \alpha_1 - \mathbb{K}[2] - \mathbb{K}[s_2] - \alpha_6.\end{aligned}$$

This introduces 5 more anomalies.

Anomaly freedom : A_i=0

It is indeed possible ! And uniquely determined under reasonable assumptions. In addition, it requires beta=-1/2.

$$
\begin{aligned}
\alpha_1 &= \mathbb{K}[2] - \mathbb{K}[s_1], \\
\alpha_2 &= 2\mathbb{K}[1]^2 - 2\bar{k}\mathbb{K}[2], \\
\alpha_3 &= 0, \\
\alpha_4 &= \Omega - 1, \\
\alpha_5 &= \Omega - 1,
\end{aligned}
$$

$$
\begin{aligned}
\alpha_6 &= 2\mathbb{K}[2] - \mathbb{K}[s_2] - \bar{k}\Omega, \\
\alpha_7 &= -4\mathbb{K}[1]^2 + 6\bar{k}\mathbb{K}[2] - 2\bar{k}^2\Omega, \\
\alpha_8 &= -4\mathbb{K}[1]^2 + 6\bar{k}\mathbb{K}[2] - 2\bar{k}^2\Omega, \\
\alpha_9 &= 0, \\
\alpha_{10} &= \Omega - 1.
\end{aligned}
$$

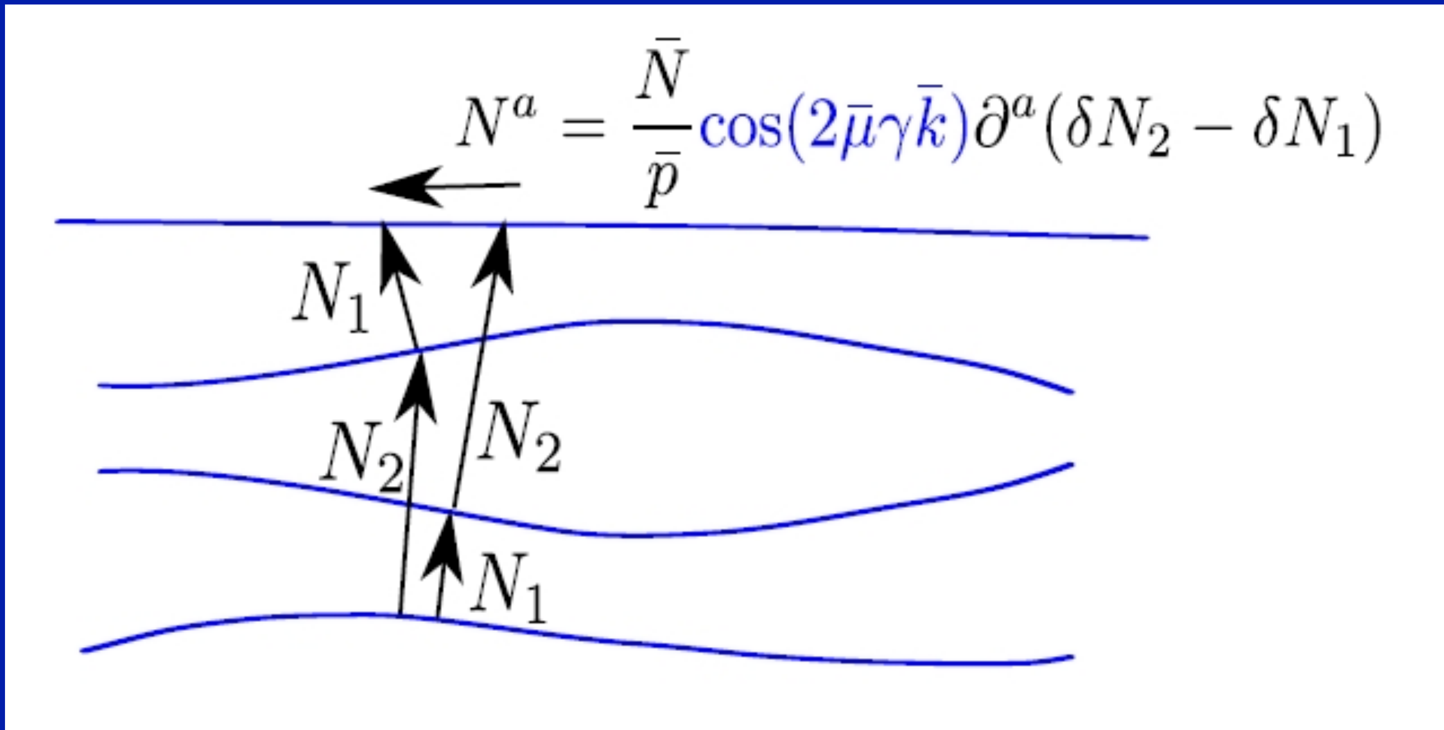
Omega = dK[2]/dk = cos(2 mu gamma k) = 1-2*rho/rho_c

The final Hamiltonian do *not* depend on s1 and s2.

Full algebra of constraints:

$$
\begin{aligned}
\{D_{tot}[N_1^a], D_{tot}[N_2^a]\} &= 0, \\
\{H_{tot}[N], D_{tot}[N^a]\} &= -H_{tot}[\delta N^a \partial_a \delta N], \\
\{H_{tot}[N_1], H_{tot}[N_2]\} &= D_{tot}\left[\Omega \frac{\bar{N}}{\bar{p}} \partial^a (\delta N_2 - \delta N_1)\right]
\end{aligned}
$$

Although the algebra is closed, there are modifications with respect to the classical case, due to the presence of the Omega factor. Therefore, not only the dynamics (as a result of the modification of the Hamiltonian constraint) is modified but hte very structure of spacetime itself is deformed.



The equations of motion can now be derived

1) For the background (as usual)

2) For the perturbed variables

Interestingly the perturbations undergo a change of signature for rho>rho_c/2. This corresponds to an effective euclidean space-time.

This also leads to a modification of the previously derived algebra for tensor mode.

Potential important observational effects.

Recently, 2 new approches were suggested

-On ne the one hand, Agullo, Ashtekar and Nelson suggest a new way to deal with the transplanckian issue. The main physical effects are captured by a « dressed » metric.

-On the other hand, Bojowald et al. suggest to use technniques analogue from those useful to study Bose-Einstein condensates (solitons). This might have experimental consequences.

A multiverse

Although, as G. Ellis pointed it out several times, the term multiverse is not completely well defined, LQC predicts a kind of multiverse. And allows predictions in the multiverse.

QG si strongly related to the multiverse.
-Say « gravity » : in 2 geometries, space is infinite
-Say « quantum » : Everett interpretation
-Say « quantum gravity » and it gets worst ☺ :
- LQC example
- String theory leads to an even richer image (in relation with inflation)

Is it still science ?
- Is this Popper compatible ? I think yes. Because the multiverse is *not* a theory but a consequence of a theory.
- Anyway, should science refuse to have its own rule evolving ? I think no.
- Finally, is the Popper criterion the more appropriate ?

Aurélien Barrau LPSC-Grenoble (CNRS / UJF)

What if the more efficient espistemologists were not epistemologists ?

On the « French Theory » side, I think Deleuze deserves a special attention.

On the analytic philosophy side, I would like to advocate for Goodman's approach to science.

- deny of reductionism
- extreme relativism
- constructivism
- beyond truth (correctness)

WoW : a) composition and decomposition
b) weighting
c) ordering
d) deletion and supplementation
e) deformation

→ Judicious vacillation

Aurélien Barrau LPSC-Grenoble (CNRS / UJF)

Is there a link with quantum gravity ?

Not specifically, but archetypically.
« pluralistic universe » (James)

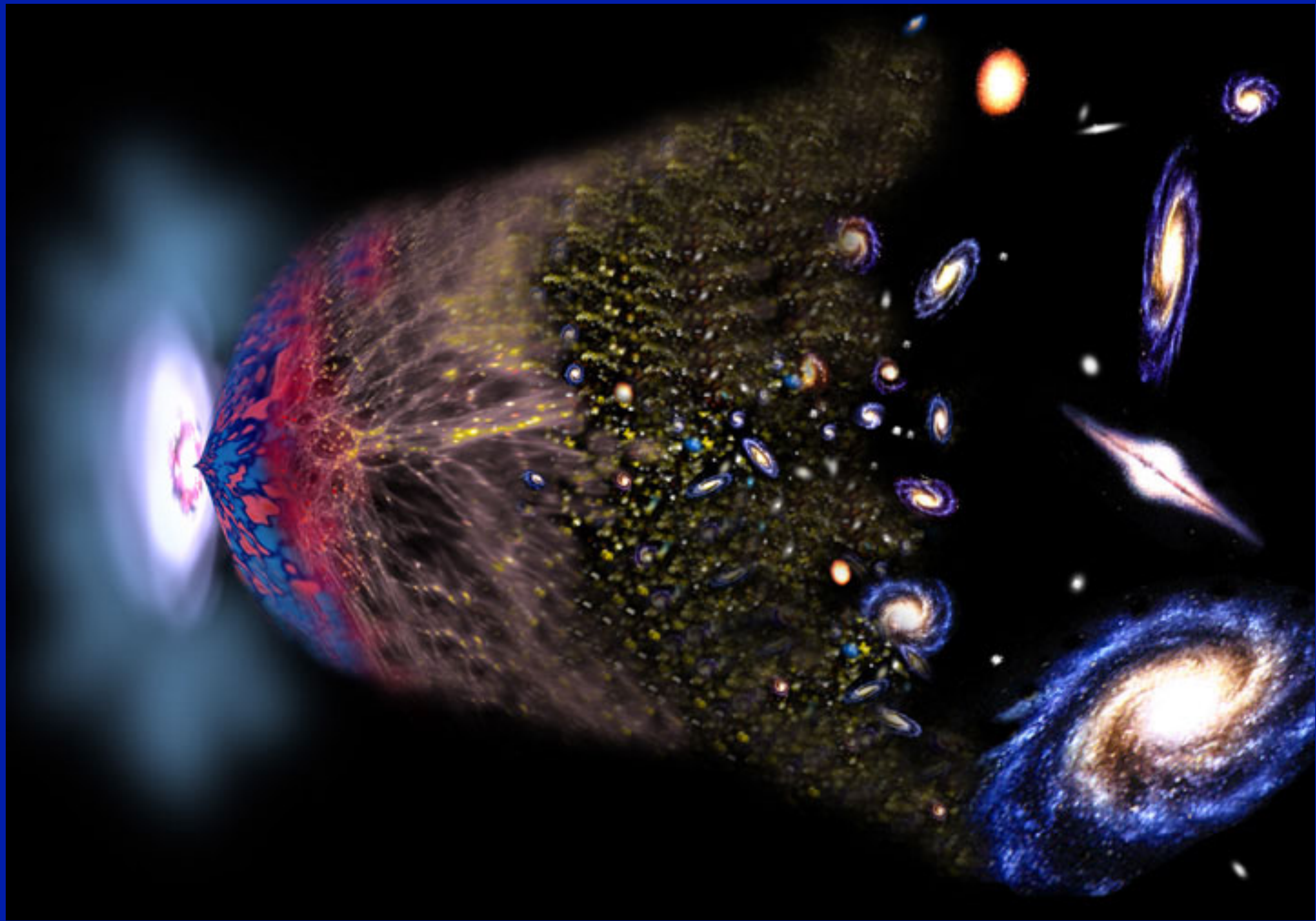
Philosophically, it might be interesting to address an intrinsic and irreductible plurality.

-Either one relies an « unreachable » final theory : all that we say is ontologically totally wrong (each new paradigm is radically different from the previous one)

-Or one takes reriously the unfalsified proposals as « correct »

Goodman is efficient on this road. Then, one will be able to « deconstruct » physics ☺.

And, keep in mind that, as Bennigton says, « deconstruction is not what you think » ! (Cf, Royle's book « deconstructions, a users guide »: everything but deconstruction and physics !)



Toward a constructive deconstruction of quantum cosmology ? ☺