The Emergence of Spacetime in the Condensed Matter Approach to Quantum Gravity

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1. The Condensed Matter Approach to QG.
2. Effective Field Theories.
3. How Spacetime Might Emerge from a Condensate.
4. A Concept of Emergence for EFTs.
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1. The Condensed Matter Approach to QG.

- *Goal*: To construct an *effective field theory* (EFT) of a condensate that mimicks GR and Standard Model.

- *Types of condensate*: bosonic or fermionic superfluid (Barceló *et al.* 2005), (Volovik 2003); quantum Hall liquid (Zhang & Hu 2001).

- *Informal claims related to emergence*:

  - "emergent gravitational features in condensed matter systems"; "emergent spacetime symmetries". (Barceló *et al.* 2005, p. 59, 62.)
  - "...an effective electrodynamics emerges from an underlying fermionic condensed matter system." (Dziarmaga 2002, p. 274.)
  - "emergent relativistic quantum field theory and gravity"; "emergent nontrivial spacetimes". (Volovik 2003, preface.)
  - "emergence of relativity". (Zhang & Hu 2001, p. 825.)

1. The Condensed Matter Approach to QG.

- *Issues*:
  - What could be meant by the claim that spacetime emerges in the low-energy sector of a condensate?
  - In what sense can a fundamental condensate be said to be non-spatiotemporal?
  - If spacetime emerges from a fundamentally non-spatiotemporal reality, what does this imply about the fundamentality of laws of nature and causation?

2. Effective Field Theories.

- An *effective field theory (EFT)* = a description of a physical system at energies low compared to a given cutoff.
  - Construction of an EFT = process of elimination of degrees of freedom from a "high-energy" theory.
  - A *degree of freedom (DOF) of theory T* = a parameter that needs to be assigned a value in order to provide a dynamical state description of a physical system described by T.
  - A *dynamical state description* = a description of a physical system at an instant in time that, in conjunction with an equation of motion, determines a future or a past state.

Ex. 1. A free classical particle governed by a 2nd-order differential equation of motion (*e.g.*, Newton's Second Law).

- *Dynamical state description is specified by values of position and momentum.*
- *Six DOF (in 3D).*

2. Effective Field Theories.

- An *effective field theory (EFT)* = a description of a physical system at energies low compared to a given cutoff.
  - Construction of an EFT = process of elimination of degrees of freedom from a "high-energy" theory.
  - A *degree of freedom (DOF) of theory T* = a parameter that needs to be assigned a value in order to provide a dynamical state description of a physical system described by T.
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Ex. 2. A free classical field $\phi(x)$ governed by a 2nd-order differential equation of motion (*e.g.*, Klein-Gordon equation).

- *Dynamical state description is specified by values of $\phi(x)$ and $\partial_\mu\phi(x)$ at every point x of spacetime.*
- *Infinite DOF.*

2. Effective Field Theories.

How to construct an EFT (Polchinski 1993)

Given a "high-energy" Lagrangian $\mathcal{L}[\phi(x)]$:

(I) Identify and eliminate high-energy DOF.

- Choose a cutoff Λ and decompose $\phi(x) = \phi_H(x) + \phi_L(x)$.
- Perform integration over $\phi_H(x)$:

$$Z = \int \mathcal{D}\phi_L \mathcal{D}\phi_H e^{i\int d^D x \mathcal{L}[\phi_L, \phi_H]} = \int \mathcal{D}\phi_L e^{i\int d^D x \mathcal{L}_{eff}[\phi_L]}$$

(II) Construct local operator expansion of $\mathcal{L}_{eff}[\phi_L(x)]$.

$$\mathcal{L}_{eff} = \mathcal{L}_0 + \sum_i g_i \mathcal{O}_i$$

2. Effective Field Theories.

Characteristics

(1) $\mathcal{L}_{eff}[\phi_L]$ is formally distinct from $\mathcal{L}[\phi]$.

(2) $\mathcal{L}_{eff}$ is not simply a perturbative approximation of $\mathcal{L}$.

> *Folk Theorem*
> "...if one writes down the most general possible Lagrangian, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition, and the assumed symmetry principles." (Weinberg 1979)

(3) $\mathcal{L}_{eff}$ is obtained by imposing a constraint *directly* on $\mathcal{L}$ (as opposed to a set of equations of motion).

2. Effective Field Theories.

Suggests:

(a) *Failure of law-like deducibility*: The phenomena described by an EFT are not deducible consequences of the laws of a high-energy theory.

Why?

- Since $\mathcal{L}_{eff}$ and $\mathcal{L}$ are formally distinct, they have formally distinct Euler-Langrange equations of motion.
- *Suppose*: The laws of a theory encoded in a Lagrangian density are understood to be its Euler-Lagrange equations of motion.

2. Effective Field Theories.

Suggests:

(a) *Failure of law-like deducibility*: The phenomena described by an EFT are not deducible consequences of the laws of a high-energy theory.

(b) *Ontological distinctness*: The DOF of an EFT characterize physical systems that are ontologically distinct from physical systems characterized by the DOF of a high-energy theory.

Why?

- Since $\mathcal{L}_{eff}$ and $\mathcal{L}$ have distinct equations of motion, their DOF characterize *dynamically distinct* physical systems.
- *Moreover*: The DOF of $\mathcal{L}_{eff}$ are encoded in formally distinct field variables $\phi_L(x)$ than those $\phi(x)$ of $\mathcal{L}$.

2. Effective Field Theories.

Suggests:

(a) *Failure of law-like deducibility*: The phenomena described by an EFT are not deducible consequences of the laws of a high-energy theory.

(b) *Ontological distinctness*: The DOF of an EFT characterize physical systems that are ontologically distinct from physical systems characterized by the DOF of a high-energy theory.

(c) *Ontological dependence*: Physical systems described by an EFT are ontologically dependent on physical systems described by a high-energy theory.

Why?

- The DOF of $\mathcal{L}_{eff}$ are precisely the low-energy DOF of $\mathcal{L}$.

2. Effective Field Theories.

Ex. 1: EFT for superfluid ^3He-A.

- High-energy DOF are fermionic ^3He atoms arranged in Cooper pairs:

$$\mathcal{L} = \Psi^\dagger\{i\partial_t - (\partial_i^2/2m + \mu)\}\tau_3\Psi + \mathcal{L}_{int}[\Psi, \Delta] \qquad (1)$$

  - *Non-relativistic* Lagrangian density. (Schakel 1998)
  - Ψ encodes creation/annihilation operators for ^3He atoms.
  - Order parameter Δ encodes ^3He-A Cooper pair interaction.

2. Effective Field Theories.

Ex. 1: EFT for superfluid ^3He-A.

- Low-energy DOF are massless fermions coupled to a Maxwell field:

$$\mathcal{L}_{eff} = \bar{\Psi}\gamma^{\mu}(\partial_{\mu} - qA_{\mu})\Psi + \mathcal{L}_{Max} \qquad (2)$$

  - *Relativistic* Lagrangian density. (Volovik 2003)
  - Ψ encodes creation/annihilation operators for ^3He atoms.
  - γ-matrices are determined by a Lorentz-signature "metric" $g^{\mu\nu}$ that encodes ^3He-A Cooper pair degrees of freedom.
  - qA_{μ} encodes position of "Fermi points" in 4-momentum space.

2. Effective Field Theories.

Comparison

$$\mathcal{L} = \Psi^\dagger\{i\partial_t - (\partial_i^2/2m + \mu)\}\tau_3\Psi + \mathcal{L}_{int}[\Psi, \Delta] \quad (1)$$

$$\mathcal{L}_{eff} = \bar{\Psi}\gamma^\mu(\partial_\mu - qA_\mu)\Psi + \mathcal{L}_{Max} \quad (2)$$

(a) *Failure of law-like deducibility.*

- High-energy theory (1) is a non-relativistic QFT.
- EFT (2) is a relativistic QFT.

(b) *Ontological distinctness.*

- (1) describes non-relativistic ^3He atoms.
- (2) describes relativistic fermions coupled to a Maxwell field.

(c) *Ontological dependence.*

- DOF of (2) are precisely the low-energy DOF of (1).

2. Effective Field Theories.

Ex. 2. EFT for 2-dim Quantum Hall Liquid.

- High-energy DOF are electrons coupled to an external magnetic field $A_i(x)$ and a Chern-Simons field $a_\mu(x)$.

$$\mathcal{L} = -\psi^\dagger\{\partial_t - ie(A_0 - a_0)\}\psi - \frac{1}{2m}\psi^\dagger\{\partial_i - ie(A_i + a_i)\}\psi + \mu\psi^\dagger\psi + \vartheta\varepsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda \quad (3)$$

  - *Non-relativistic* Lagrangian density. (Schakel 1998)
  - ϑ chosen so that electrons $\psi(x)$ have an even number of magnetic fluxes ("composite" fermions).
  - Quantum Hall Effect: $\sigma = conductivity = \nu(e^2/h)$,

$$\nu = \frac{(\#\, electrons)}{(\#\, states\ per\ energy\ level)} = \text{integer or fraction}$$

2. Effective Field Theories.

Ex. 2. EFT for 2-dim Quantum Hall Liquid.

- Low-energy DOF of bulk are two Chern-Simons fields:

$$\mathcal{L}_{\textit{eff-bulk}} = \vartheta\varepsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda + \vartheta'\varepsilon^{\mu\nu\lambda}(A_\mu + a_\mu)\partial_\nu(A_\lambda + a_\lambda) \qquad (4)$$

  - *Topological* quantum field theory. (Schakel 1998)
  - a_μ, $(A_\mu + a_\mu)$ are two Chern-Simons fields.

2. Effective Field Theories.

Ex. 2. EFT for 2-dim Quantum Hall Liquid.

- Low-energy DOF of edge are bosonic sound waves $\phi(x)$:

$$\mathcal{L}_{\textit{eff-edge}} = (1/8\pi)\{(\partial_t\phi)^2 - (\partial_x\phi)^2\} \tag{5}$$

  - *Relativistic* (1+1)-dim Lagrangian density. (Wenn 1990)

Extention to (3+1)-dim.

- Edge of 4-dim QH liquid contains stable bosonic states that satisfy (3+1)-dim zero rest mass field equations. (Zhang & Hu 2001)
- To obtain GR and Standard Model, reformulate in terms of twistor theory. (Sparling 2002)

2. Effective Field Theories.

Comparison

$$\mathcal{L} = -\psi^\dagger\{\partial_t - ie(A_0 - a_0)\}\psi - \frac{1}{2m}\psi^\dagger\{\partial_i - ie(A_i + a_i)\}\psi + \mu\psi^\dagger\psi + \vartheta\varepsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda \tag{3}$$

$$\mathcal{L}_{\textit{eff-bulk}} = \vartheta\varepsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda + \vartheta'\varepsilon^{\mu\nu\lambda}(A_\mu + a_\mu)\partial_\nu(A_\lambda + a_\lambda) \tag{4}$$

$$\mathcal{L}_{\textit{eff-edge}} = (1/8\pi)\{(\partial_t\phi)^2 - (\partial_x\phi)^2\} \tag{5}$$

(a) *Failure of law-like deducibility.*

- (3) is a non-relativistic QFT; (4) is a topological QFT; (5) is a relativistic QFT.

(b) *Ontological distinctness.*

- (3) describes non-relativistic composite fermions; (4) describes topological Chern-Simons fields; (5) describes relativistic scalar fields.

(c) *Ontological dependence.*

- DOF of (5) and (4) are precisely the low-energy DOF of (1).

3. How Spacetime Might Emerge from a Condensate.

Claim 1. Relativistic spatiotemporal structure emerges in the low-energy sector of a non-relativistic fundamental condensate (fermionic or bosonic).

- *Background dependence*: spatiotemporal structure of fundamental condensate is fixed *a priori*.
- *Neutral* as to whether emergent spatiotemporal structure should be interpreted *relationally* or *substantivally*.
- *Does not suggest* that relativistic spacetime structure emerges from a more fundamental non-spatiotemporal reality.
  - *Hence*: If causation and laws of nature assume spatiotemporal structures, then Claim 1 does not suggest they are not fundamental.

3. How Spacetime Might Emerge from a Condensate.

Claim 2. Relativistic spatiotemporal structure emerges in the low-energy sector of the edge of a 4-dim quantum Hall liquid.

- *Background independence*: spatiotemporal (*viz.*, metrical) structure of fundamental condensate is not fixed *a priori*.
- *Neutral* as to whether emergent spatiotemporal structure should be interpreted *relationally* or *substantivally*.
- *Suggests* that relativistic spatiotemporal structure emerges from a more fundamental *twistor reality*.

3. How Spacetime Might Emerge from a Condensate.

Claim 2. Relativistic spatiotemporal structure emerges in the low-energy sector of the edge of a 4-dim quantum Hall liquid.

Claim 2a: Relativistic spatiotemporal structure emerges from a more fundamental non-spatiotemporal twistor reality.

- Twistors encode structure of conformally flat Lorentzian spacetimes.
- *So*: If twistors are to be viewed as non-spatiotemporal, so must conformal structure.
- *But*: Claim 2a is typical of twistor advocates in general, irrespective of the role twistors may play in the condensed matter approach.

3. How Spacetime Might Emerge from a Condensate.

Claim 2. Relativistic spatiotemporal structure emerges in the low-energy sector of the edge of a 4-dim quantum Hall liquid.

Claim 2b: Relativistic spatiotemporal structure is encoded in twistors, which emerge from the edge of a non-spatio-temporal fundamental QH liquid.

- If a QH liquid is to be viewed as non-spatiotemporal, so must topological and differentiable structure.
- Claim 2b seems more appropriate in the condensed matter approach, in which reference to a fundamental condensate plays an essential role.

3. How Spacetime Might Emerge from a Condensate.

Claim 2b: Relativistic spatiotemporal structure is encoded in twistors, which emerge from the edge of a non-spatiotemporal fundamental QH liquid.

Implications for laws of nature and causation:

- *Relevant question*: What type of structure underwrites laws of nature and causation?
  - *Suppose*: Laws of nature are represented by topologically well-behaved differential equations.
  - *Then*: Claim 2b allows laws to be fundamental.

3. How Spacetime Might Emerge from a Condensate.

Claim 2b: Relativistic spatiotemporal structure is encoded in twistors, which emerge from the edge of a non-spatiotemporal fundamental QH liquid.

Implications for laws of nature and causation:

- *Relevant question*: What type of structure underwrites laws of nature and causation?
  - *Suppose*: Causation requires conformal structure.
  - *Then*: Claim 2b entails causation is not fundamental.

3. How Spacetime Might Emerge from a Condensate.

Claim 2b: Relativistic spatiotemporal structure is encoded in twistors, which emerge from the edge of a non-spatiotemporal fundamental QH liquid.

Implications for laws of nature and causation:

- *Relevant question*: What type of structure underwrites laws of nature and causation?
  - *Suppose*: Laws and causation require metrical structure.
  - *Then*: Claim 2b entails laws and causation are not fundamental.

4. A Concept of Emergence for EFTs.

Two General Notions of Emergence:

(a) *Emergence as descriptive of the ontology (entities, properties) associated with a physical system with respect to another.*

- To say phenomena associated with an EFT are emergent is to say the entities or properties described by the EFT emerge from those described by a high-energy theory.

(b) *Emergence as a relation between theories.*

- To say phenomena associated with an EFT are emergent is to say the EFT stands in a certain relation to a high-energy theory.

4. A Concept of Emergence for EFTs.

- *Note*: An EFT does not stand in a precise mathematical relation to a high-energy theory.
  - Choice of cutoff and choice of low-energy DOF are context-specific.
  - Identification of symmetries is context-dependent.

- *Suggests*: Emergence in EFTs is not a formal characteristic of theories; but rather an interpretation-dependent characteristic.

4. A Concept of Emergence for EFTs.

Desiderata (Mainwood 2006)

(i) Emergence should involve *microphysicalism*: The emergent system should ultimately be composed of microphysical systems that comprise the fundamental system and that obey the fundamental system's laws.

(ii) Emergence should involve *novelty*: The properties of the emergent system should not be deducible from the properties of the fundamental system.

- (i) and (ii) are underwritten in the EFT context by the particular type of elimination of DOF involved in the construction of an EFT.

4. A Concept of Emergence for EFTs.

How the properties of a system described by $\mathcal{L}_{eff}$ *emerge from a fundamental system described by* $\mathcal{L}$*:*

(i) *Microphysicalism*: High-energy DOF are integrated out of $\mathcal{L}$, which entails that the DOF of $\mathcal{L}_{eff}$ are exactly the low-energy DOF of $\mathcal{L}$.

(ii) *Novelty*:

- $\mathcal{L}_{eff}$ is *dynamically distinct* from $\mathcal{L}$ in the sense of a failure of law-like deducibility from $\mathcal{L}$ of the properties described by $\mathcal{L}_{eff}$.
- $\mathcal{L}_{eff}$ is *ontologically distinct* from $\mathcal{L}$ in the sense of being a functional of field variables that do not appear in $\mathcal{L}$.

4. A Concept of Emergence for EFTs.

How relativistic spatiotemporal structure emerges in the low-energy sector of a fundamental condensate:

(i) Relativistic spatiotemporal entities or properties are composed of the microphysical entities or properties of a fundamental condensate (*microphysicalism*).

(ii) Relativistic spatiotemporal entities or properties cannot be deduced from the entities or properties of the fundamental condensate alone (*novelty*).

5. Conclusion.

I. *Two ways relativistic spatiotemporal structure can be said to emerge in the low-energy sector of a fundamental condensate*:

(a) It can emerge from a fundamental condensate with non-relativistic metrical structure.

(b) It can emerge from a fundamental condensate with topological and differentiable, but not metrical, structure.

- If metrical structure is a necessary characteristic of spatio-temporal structure, then:
  - In (a), relativistic spatiotemporal structure emerges from a more fundamental *non-relativistic spatiotemporal* reality.
  - In (b), relativistic spatiotemporal structure emerges from a more fundamental *non-spatiotemporal* reality.

5. Conclusion.

II. *Emergence in this context can be characterized by the elimination of high-energy DOF from the theory describing the condensate.*

- This results in an EFT that can be interpreted as describing *novel* entities or properties in the sense of being dynamically independent of, and thus not deducible from, the entities or properties associated with the condensate.

- These novel entities or properties can be said to ultimately be composed of the entities or properties that are constitutive of the condensate, insofar as the DOF exhibited by the former are exactly the low-energy DOF exhibited by the latter.

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