



Cosmology from Group Field Theory

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"Quantum Gravity and Fundamental Cosmology" Albert Einstein Institute, Golm 07/03/2013





Main message(s)

• GFTs: interesting formalism for QG: strict relation to LQG & tensor models

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Plan:

•	intro to GFT formalism
•	relation to LQG, spin foams and tensor models
•	GFT states $\leftarrow \rightarrow$ (approximate) continuum geometries
•	examples of GFT condensates
•	effective dynamics for GFT condensates (general)
•	special case and approximate FRW equations
•	conclusions and outlook

GFT basics

recent general introductions and reviews:

D. Oriti, arXiv: gr-qc/0607032

D. Oriti, arXiv: 0912.2441 [hep-th]

R. Gurau, J. Ryan, arXiv: 1109.4812 [hep-th]

D. Oriti, arXiv: 1111.5606 [hep-th]

V. Rivasseau, arXiv:1112.5104 [hep-th]

work by:

Baratin, Ben Geloun, Bonzom, Carrozza, De Pietri, Fairbairn, Freidel, Gielen, Girelli, Gurau, Livine, Louapre, Krajewski, Krasnov, Magnen, Noui, Oriti, Perez, Raasakka, Reisenberger, Rivasseau, Rovelli, Ryan, Sindoni, Smerlak, Tanasa, Vitale,

GEAT Classific S(4(d classes); Kkieenatics

Quantum field theory over group manifold

(or corresponding Lie algebra)

 $Sp(A)^{4\times4}$ Lorentzian signature $SpSp(A)^{4\times4}$ Riemannian signature

 $\varphi(g_1, \mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_2, \mathfrak{g}_3; \mathfrak{g}_4) (B_1, \mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3; \mathfrak{g}_4) (B_1, \mathfrak{g}_2, \mathfrak{g}_3, \mathfrak{g}_4; \mathfrak{g}_3, \mathfrak{g}_4; \mathfrak{g}_3, \mathfrak{g}_4; \mathfrak$

$$\begin{split} \varphi(g_1,g_2,g_3,g_4) &\hookrightarrow \varphi(x_1,x_2,x_3,x_4) \qquad x_i \in X \subset G \\ \mathcal{T}^*SL(2,\mathbb{C})]^{\times 4} \quad or \quad [\mathcal{T}^*Spin(4)]^{\times 4} \quad \hookleftarrow \ [\mathcal{T}^*SU(2)]^{\times 4} \qquad \bigoplus_{a \in \mathcal{A}} \mathbb{C} \\ \downarrow b_a \quad \downarrow b_$$

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with ("simplicity") conditions enforced on the field or the dynamics to impose "geometricity" of simplicial structures

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classical phase space of reference:

$$[\mathcal{T}^*SL(2,\mathbb{C})]^{\times 4}$$
 or $[\mathcal{T}^*Spin(4)]^{\times 4} \hookrightarrow [\mathcal{T}^*SU(2)]^{\times 4}$

 $B_i^{IJ} \simeq N^I \wedge b_i^J$

group ~ elementary holonomy Lie algebra ~ "discretized triad"

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generic quantum state: arbitrary collection of spin network vertices (including glued ones) or tetrahedra (including glued ones)

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second quantized version of (generalized) LQG (adapted to simplicial context), but dynamics not derived from canonical quantization of GR

classical action: kinetic (quadratic) term + (higher order) interaction (convolution of GFT fields)

$$S(\varphi,\overline{\varphi}) = \frac{1}{2} \int [dg_i] \overline{\varphi(g_i)} \mathcal{K}(g_i) \varphi(g_i) + \frac{\lambda}{D!} \int [dg_{ia}] \varphi(g_{i1}) \dots \varphi(\overline{g}_{iD}) \mathcal{V}(g_{ia},\overline{g}_{iD}) + c.c.$$

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other possibility (motivated by tensor models and renormalization): (tensor) invariant interactions



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- easier to extract physics and geometry

Matrix models













GFTs, spin foams, tensor models: many recent results

- construction of interesting 4d gravity models (inspired by LQG)
- encoding of simplicial geometry
- field theory symmetries
- understanding of combinatorial structures (GFT Feynman diagrams)
- large-N expansion
- GFT renormalization (various renormalizable models)
- critical behaviour (in tensor models)
- mean field expansion (emergent matter, effective QG dynamics,...)
- simplified models (for cosmology)

....

Continuum spacetime and geometry? (physics?)

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- phase transition leading to spacetime and geometry (GFT condensation) is what replaces Big Bang singularity (geometrogenesis)
- cosmology as "relaxation to equilibrium condensate"

- GFT analogous to QFT for atoms in condensed matter system
- continuum spacetime (with GR-like dynamics) emerges from collective behaviour of large numbers of GFT building blocks (spin nets, simplices)
- requires (GFT analogue of) thermodynamic limit, macroscopic approximation, appropriate phase

- more specific hypothesis: continuum spacetime is GFT condensate
 - GR-like dynamics from GFT hydrodynamics
- phase transition leading to spacetime and geometry (GFT condensation) is what replaces Big Bang singularity (geometrogenesis)
- cosmology as "relaxation to equilibrium condensate"

(Oriti '07, '11, '13, Rivasseau '11, '12, Sindoni '11)



spacetime as condensate of QG building blocks

Big Bang as phase transition (condensation)



Cosmology from GFT

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S. Gielen, DO, L. Sindoni, AEI-2013-051, arXiv: 1303.XXXX THE CONTRACTION OF THE TWO IN THE PARTY PA GIT STRUCTOR GIT STORE OF THE VEL LICES ALL THREE' # By det Been Big asserbing (of Deratist Real or field teneroties eneity, we propose agalass of quantum states as diving than a for a by platting integral, whose action macroscopic homogeneous; geometries einethe of signete southeatrone Feynmanlex pansion condition of the philip of the southeatrone theory (GET) approach to quantum gravity estro while with a function of the constant of the solution of of many the here and the second to the seco guration with the pateral description he discription province the second strate in the second strate in the second strate in the second nogeneity in this context. We see that the exist vectors of the the sectors of a a the city of the provision of the second converted a second converted to the second converted to th propriaces a single diduct a diamon and within the propried spectral propried by it direction of the grav use the eight at the formed to have been all the stand of $dynamics of such states. While the resulting <math>B_{B}$ is that of the by the the substant of the bit of the bit of the bit of the transmitted to the the solution of the bit of of which gives a fine a feation (BPT he programmit) to indentify the selding hat care of by requiring Ton the condensare hedrow is sportifiethiby giving the location of the SO(4) invariant qua reduces to the Hamilton date in at ongen where eas a content by the deficition of the discrete geometry is the Fock vacuum, as a discrete geometry $g_{1}, g_{2}, g_{3}, g_{4} = \varphi(g_{1}, h_{1}, g_{2}, h_{2}, g_{3}, h_{3}, g_{4}, h_{4}) \forall x \in VO(3)$ 5) of N directions of the three edges enangering is reply to ise Mertersur copies of $S^3 \sim SU(2)$ 'faces. Assuming that the closure and simplicity constraints hold, we can parametrize (7) by ond quantized formalism, we will interpre-se $\{B_{i(m)}\}$ (i = 1, ..., 3, m = 1, ..., N) and assume that catlon of the t t space, such as cation of the tetrahedra on $^{\mathsf{S}}$ all B[n_n) \mathbf{p} and \mathbf{p} in the stand \mathbf{p} in the standard standard \mathbf{p} is the standard \mathbf{p} in the standard standard \mathbf{p} is the sta the location of the other three vertices $|B_{I(m)}\rangle := 1$ $\hat{\tilde{\varphi}}^{\dagger}(B_{\mathbf{L}(m)}, \dots, B_{4(m)})|0\rangle$,

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GFT states and approximate continuum geometries

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GFT states and approximate continuum geometries

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- from the B's (or the e's) construct:

$$g_{ij} = \frac{1}{8\operatorname{tr}(B_1 B_2 B_3)} \epsilon_i{}^{kl} \epsilon_j{}^{mn} \tilde{B}_{km} \tilde{B}_{ln} \qquad \qquad \tilde{B}_{ij} := B_i^{AB} B_{jAB}$$

or:
$$g_{ij(m)} = e^{A}_{i(m)} e_{Aj(m)}$$

nnihilations pretations is symmetry of gravity, corresponding $(g_I)_{0}$ are islem than of rot draving of is advantageous h_{0} reduce $i_{(m)}h_{(m)}$, $e_{i(m)} \mapsto e_{i(m)}h_{(m)}$ \hat{a}_{ν} are preinary appropriation space. For each time gravity, correspondent of the closer end o GFPErseivernazefeitevorh, entitle is an patients for the patients for the second station. It is advantageous to red field $G_{i(m)}$ to the gauge-invariant configuration space. For e field $G_{i(m)}$ for the field $G_{i(m)}$ for the state with physical field integrated field integrated field integrated and may be a state of the sector. Space is six-dimensional and may l-as a tetrahedron with geometry ortsDefining property of gravity, courses the standard of the construction of the construction of the standard of the standard of the construction of the standard of the sta I understation the estremation space. For each $e_{i(m)} = e_{i(m)}^A e_{Aj(m)}$. mutative g_{ij} \tilde{B}_{ij} as carpbe verified $\mathcal{F}_{\mathcal{F}}^{\mathcal{A}}$ (9) $\int_{0}^{4} g \prod_{i \in \mathcal{I}} \frac{1}{2} \sum_{j \in \mathcal{I}} \frac{1$ tion on spatial hypers interestionate Lie group of whose action of spatial hypersection of spatial hypersections of the hypersection $\tilde{B}_{ij} = \epsilon_i{}^{kl}\epsilon_j{}^{mn}g_{km}g_{ln}$. of platte two possible choices for G given by the Bianchi clas-slatsification \tilde{g} (**Barise marked and the state of t** $\begin{array}{c} \begin{array}{c} \begin{array}{c} \begin{array}{c} \end{array} \\ \end{array} \end{array}$ ordinary bosonic annihit are expressed aver gritter in a for the quantities $g_{ij(m)}$ to be computed with the subscription of the subscription

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Continuum homogeneous spacetimes are quantum GFT condensates

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Continuum homogeneous spacetimes are quantum GFT condensates

similar constructions in LQG (Alesci, Cianfrani) and LQC (Bojowald, Wilson-Ewing,)

Quantum GFT condensates

two simple choices of quantum GFT condensate states (homogeneous continuum quantum spacetimes)

Quantum GFT condensates

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single-particle condensate

(Gross-Pitaevskii approximation)

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We then consider two types of candidate states for macroscopic, homogeneous configurations of tetrahedra:

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$$V_{i(m)} = \mathbf{e}_{i}(x_{m}),$$
 (14)
single-particle GFT condensate:
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 σ := exp(σ) by fields on \overline{G} . f
left-invariant vector fields on \overline{G} . f
of the physical metric now reads

$$g(x_m)(\mathbf{e}_i(x_m), \mathbf{e}_j(x_m)), \qquad (15)$$

a homogeneous metric will be one ients. We can then say that a distetrahedra, specified by the data with spatial homogeneity if

$$\forall ij(k) \quad \forall k, m = 1, \dots, N.$$
 (16)

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follow closely procedure used in real BECs $\hat{\sigma} := \int d^4g \ \sigma(g_I) \hat{\varphi}^{\dagger}(g_I) \tag{17}$

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A second possibility is to use a two-particle operator which automatically has the required gauge invariance:

$$\hat{\xi} := \frac{1}{2} \int d^4g \ d^4h \ \xi(g_I h_I^{-1}) \hat{\varphi}^{\dagger}(g_I) \hat{\varphi}^{\dagger}(h_I), \qquad (18)$$

where due to (1) and $[\hat{\varphi}^{\dagger}(g_I), \hat{\varphi}^{\dagger}(h_I)] = 0$ the function ξ can be taken to satisfy $\xi(g_I) = \xi(kg_Ik')$ for all k, k' in SU(2) and $\xi(g_I) = \xi(g_I^{-1})$. ξ is a function on the gaugeinvariant configuration space of a single tetrahedron.

We then consider two types of candidate states for macroscopic, homogeneous configurations of tetrahedra:

$$|\sigma\rangle := \exp(\hat{\sigma}) |0\rangle, \quad |\xi\rangle := \exp(\hat{\xi}) |0\rangle.$$
 (19)

 $|\sigma\rangle$ corresponds to the simplest case of single-particle con-

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$$\forall ij(k) \quad \forall k, m = 1, \dots, N.$$
 (16)

es intrinsic geometric data and does embedding information apart from is a very natural notion of spatial iscrete context.

ry compatible with spatial homon compatible with spatial isotropy $\int [\vec{a}g_2^1] \int \mathcal{K}(g_i^4, g_i^4, g_i^4) \notin (g_i^4, h_I^{-1}) \hat{\varphi}_i^{\dagger}(g_I) \hat{\varphi}^{\dagger}(h_I) = 0 \quad (18)$

where due to (1) and $[\hat{\varphi}^{\dagger}(g_I), \hat{\varphi}^{\dagger}(h_I)] = 0$ the function ξ can be taken to satisfy $\xi(g_I) = \xi(kg_Ik')$ for all k, k' in SU(2) and $\xi(g_I) = \xi(g_I^{-1})$. ξ is a function on the gaugeinvariant configuration space of a single tetrahedron.

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$$\hat{\sigma} := \int d^4g \ \sigma(g_I) \hat{\varphi}^{\dagger}(g_I) \qquad (17)$$

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 $v_{i(m)} = \mathbf{e}_i(x_m),$ (14) single-particle GFT condensate:

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We then consider two types of candidate states for macroscopic, homogeneous configurations of tetrahedra: Thursday, March 7, 2013

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derivation of cosmological equations from GFT quantum dynamics very general it rests on:

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similar equations obtained in non-linear extension of LQC (Bojowald et al. '12)

special case: (effective) kinetic term = Laplacian on SU(2)^4 (suggested by simplicial geometry, LQG, GFT renormalization,..): $\mathcal{K}(g_I, \tilde{g}_I) = \left(\sum_{I} \Delta_{g_I} + \mu\right) (g_I, \tilde{g}_I)$

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take order parameter to be of the form: $\Psi(g_I) = A(g_I)e^{\frac{i}{\kappa}S(g_I)}$

and consider (formal) eikonal WKB approximation $\kappa \to 0$

ametrization for SU(2) given by g = 1 approximation scale might be contained in th number of quanta per unit volume, there is no ≤ 1 , where σ^i are the Paulis $|\vec{\pi}|$ place-Beltrami $\stackrel{\Delta g J}{\text{operator on }} \stackrel{\pi}{\text{su}} \stackrel{\sigma}{=} \stackrel{\sigma}{\stackrel{\sigma}{=}} \stackrel{\pi}{\stackrel{\sigma}{=}} \stackrel{\pi}{\text{su}} \stackrel{\sigma}{=} \stackrel{\pi}{\stackrel{\sigma}{=}} \stackrel{\pi}{\text{su}} \stackrel{\sigma}{=} \stackrel{\pi}{\text{su}} \stackrel{\sigma}{=} \stackrel{\sigma}{\stackrel{\sigma}{=}} \stackrel{\pi}{\text{su}} \stackrel{\sigma}{=} \stackrel{\sigma}{\text{su}} \stackrel{\sigma}{=} \stackrel{\pi}{\text{su}} \stackrel{\sigma}{=} \stackrel{\sigma}{\text{su}} \stackrel{\sigma}{=} \stackrel{\sigma}{=} \stackrel{\sigma}{\text{su}} \stackrel{\sigma}{=} \stackrel{\sigma}{=} \stackrel{\sigma}{\text{su}} \stackrel{\sigma}{=} \stackrel{\sigma$ triangulation associated to it, since $\begin{aligned} & \sum_{I} \left[g_{I} \right] = A[\pi_{I}] \left[\exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking the (formal)}^{\mathrm{moving takes}} \left[\exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking the (formal)}^{\mathrm{moving takes}} \left[\exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking the (formal)}^{\mathrm{moving takes}} \left[\exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking the (formal)}^{\mathrm{moving takes}} \left[\exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking the (formal)}^{\mathrm{moving takes}} \left[\exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking the (formal)}^{\mathrm{moving takes}} \left[\exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking the (formal)}^{\mathrm{moving takes}} \left[\exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking the (formal)}^{\mathrm{moving takes}} \left[\exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking the (formal)}^{\mathrm{moving takes}} \left[\exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking the (formal)}^{\mathrm{moving takes}} \left[\exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking the (formal)}^{\mathrm{moving takes}} \left[\exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking the (formal)}^{\mathrm{moving takes}} \left[\exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking the (formal)}^{\mathrm{moving takes}} \left[\exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking the (formal)}^{\mathrm{moving takes}} \left[\exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking the (formal)}^{\mathrm{moving takes}} \left[\exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking the (formal)}^{\mathrm{moving takes}} \left[\exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking the (formal)}^{\mathrm{moving takes}} \left[\exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking the (formal)}^{\mathrm{moving takes}} \left[\exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking takes} \left[\exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking takes} \begin{bmatrix} \exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking takes} \begin{bmatrix} \exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking takes} \begin{bmatrix} \exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking takes} \begin{bmatrix} \exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking takes} \begin{bmatrix} \exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking takes} \begin{bmatrix} \exp(\mathrm{i}S[\pi_{I}]/\kappa) \right] & \text{and taking takes} \begin{bmatrix} \exp(\mathrm{i}S[\pi_{I}]/\kappa \right] & \text{and takes} \begin{bmatrix} \exp(\mathrm{i}S[\pi_{I}]/\kappa \right] \\ & \exp(\mathrm{i}S[\pi_{I}]/\kappa \right] & \text{and takes} \begin{bmatrix} \exp(\mathrm{i}S[\pi_{I}]/\kappa \right] & \text{and takes} \begin{bmatrix} \exp(\mathrm{i}S[\pi_{I}]/\kappa \right] \\ & \exp(\mathrm{i}S[\pi_{I}]/\kappa \right] & \text{and takes} \begin{bmatrix} \exp(\mathrm{i}S[\pi_{I}]/\kappa \right] & \text{and takes} \begin{bmatrix} \exp(\mathrm{i}S[\pi_{I}]/\kappa \right] \\ & \exp(\mathrm{i}S[\pi_{I}]/\kappa \right] \\ & \exp(\mathrm{i}S[\pi_{I}]/\kappa \right] \\ & \exp(\mathrm{i}S[\pi_{I}]/\kappa \right] & \exp(\mathrm{i}S[\pi_{I}]/\kappa \right] \\ & \exp(\mathrm{i}S[\pi_{I}]/\kappa \right$ s spectronal limit for the state in the state in the state in the state in the state in the state in the state in the state in the state in the state in the state of the stat 0, this equation reduces $I_{to}B_I - 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another way to extract effective classical equations from GFT hydrodynamics: take order parameter to be coherent state for mini-superspace (DO, L. Sindoni, '10)

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derivation of cosmology from full QG formalism!

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Thank you for your attention!