Cosmology from Group Field Theory

Daniele Oriti

Albert Einstein Institute

“Quantum Gravity and Fundamental Cosmology”
Albert Einstein Institute, Golm
07/03/2013

Plan of the talk and main message(s)

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Plan:

- intro to GFT formalism
- relation to LQG, spin foams and tensor models
- GFT states $\longleftrightarrow$ (approximate) continuum geometries
- examples of GFT condensates
- effective dynamics for GFT condensates (general)
- special case and approximate FRW equations
- conclusions and outlook

GFT basics

recent general introductions and reviews:

D. Oriti, arXiv: gr-qc/0607032

D. Oriti, arXiv: 0912.2441 [hep-th]

R. Gurau, J. Ryan, arXiv: 1109.4812 [hep-th]

D. Oriti, arXiv: 1111.5606 [hep-th]

V. Rivasseau, arXiv:1112.5104 [hep-th]

work by:

Baratin, Ben Geloun, Bonzom, Carrozza, De Pietri, Fairbairn, Freidel, Gielen, Girelli, Gurau, Livine, Louapre, Krajewski, Krasnov, Magnen, Noui, Oriti, Perez, Raasakka, Reisenberger, Rivasseau, Rovelli, Ryan, Sindoni, Smerlak, Tanasa, Vitale, ...

GFT basics (4d case): kinematics

Quantum field theory over group manifold $SL(2,\mathbb{C})^{\times 4}$ Lorentzian signature

(or corresponding Lie algebra) $Spin(4)^{\times 4}$ Riemannian signature

$$\varphi(g_1, g_2, g_3, g_4) \leftrightarrow \varphi(B_1, B_2, B_3, B_4) \to \mathbb{C}$$

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$$\varphi(g_1, g_2, g_3, g_4) \hookrightarrow \varphi(x_1, x_2, x_3, x_4) \qquad x_i \in X \subset G$$

classical phase space of reference:

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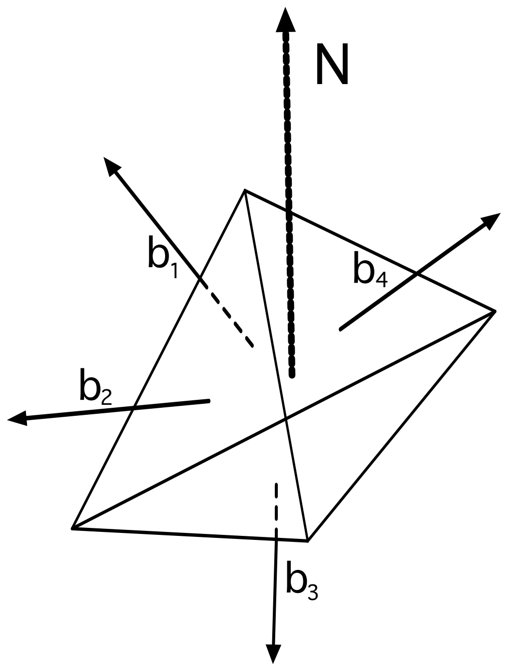
$$B_i^{IJ} \simeq N^I \wedge b_i^J$$



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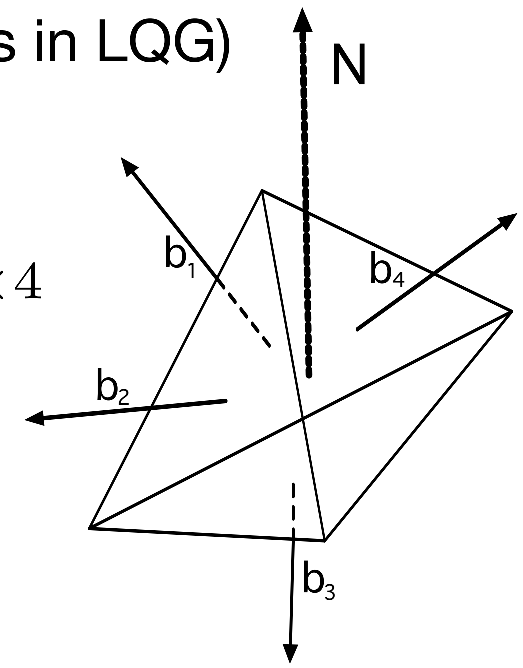

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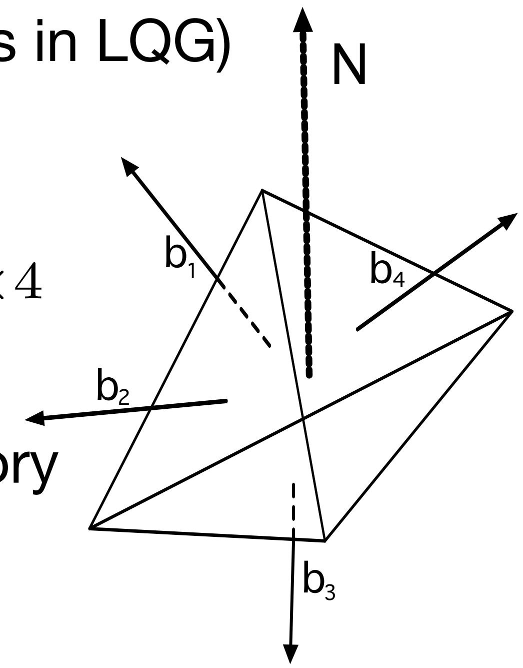

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$B_i^{IJ} \simeq N^I \wedge b_i^J$ also obtained from discretization of continuum theory (gravity = BF theory + constraints)

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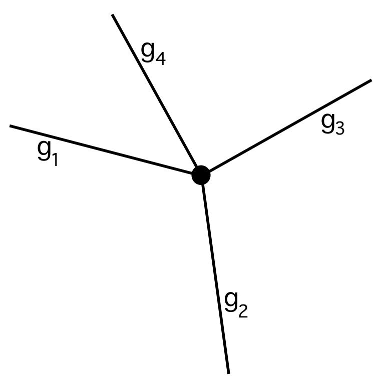

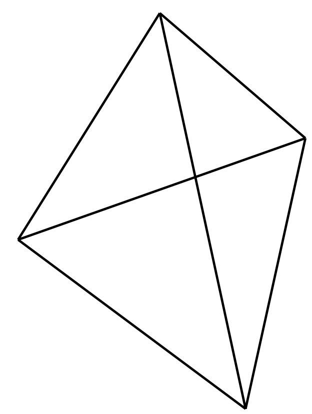

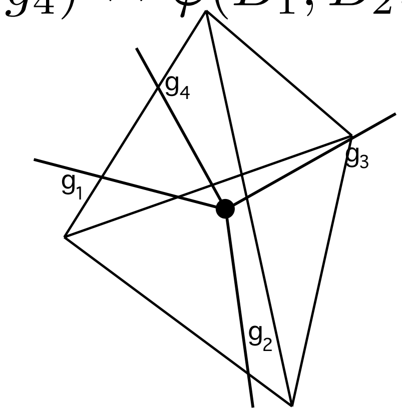

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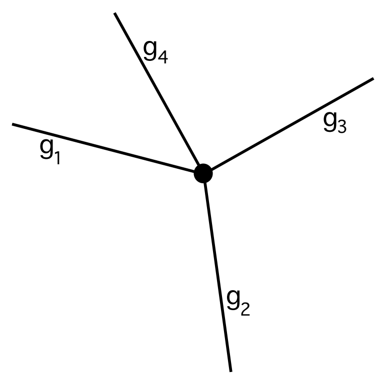

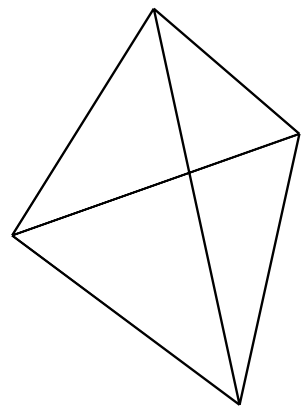

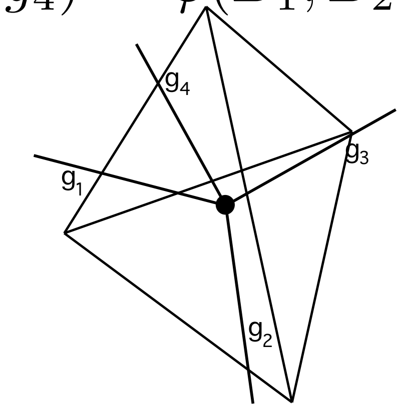

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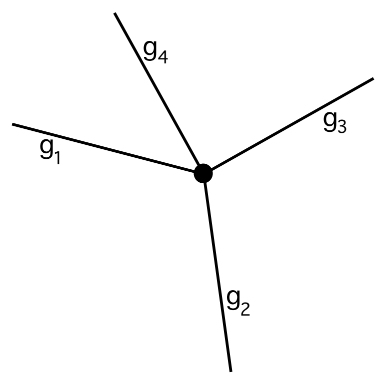

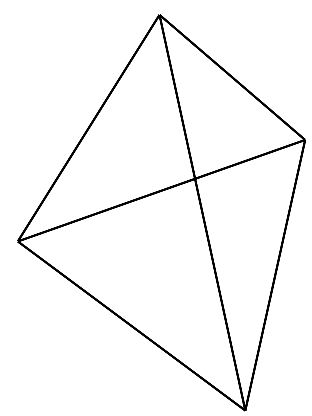

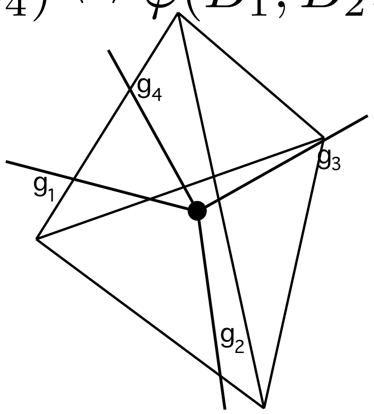

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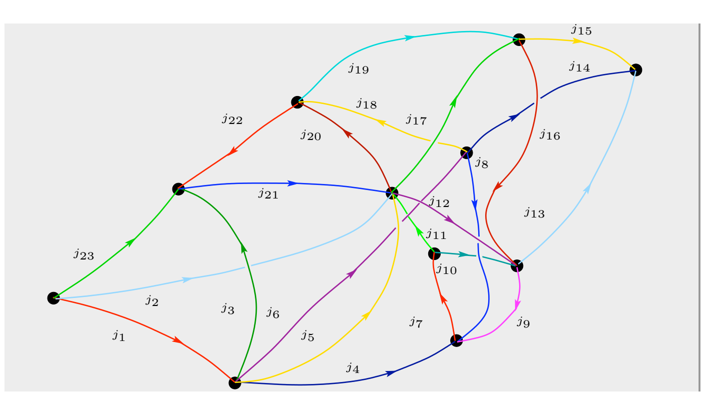

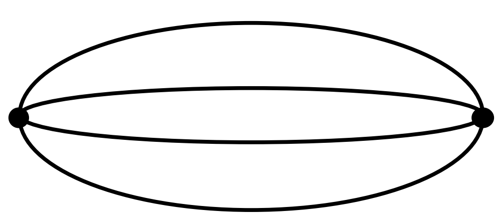

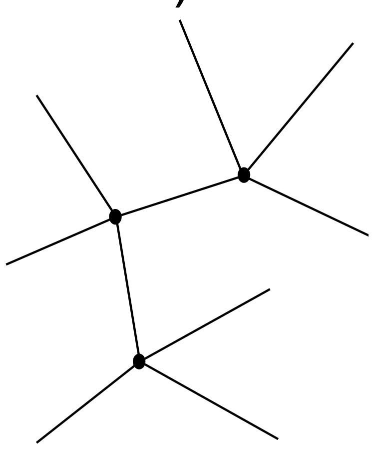

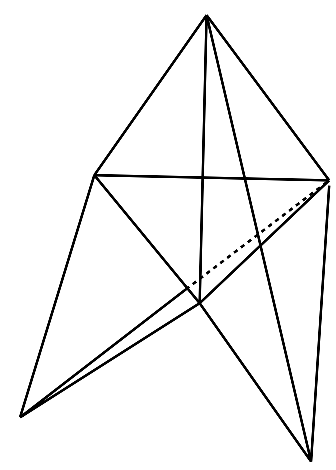

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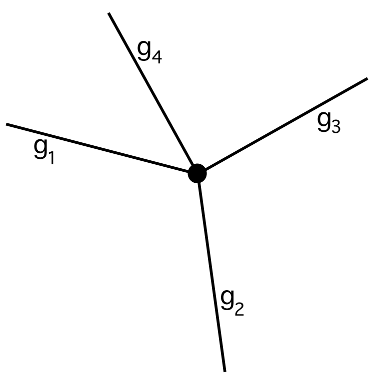

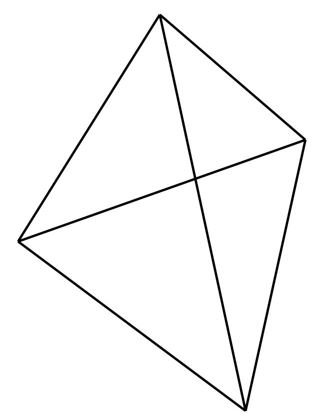

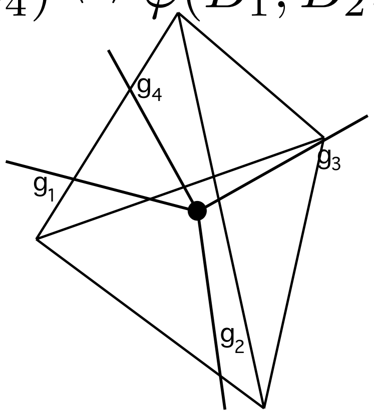

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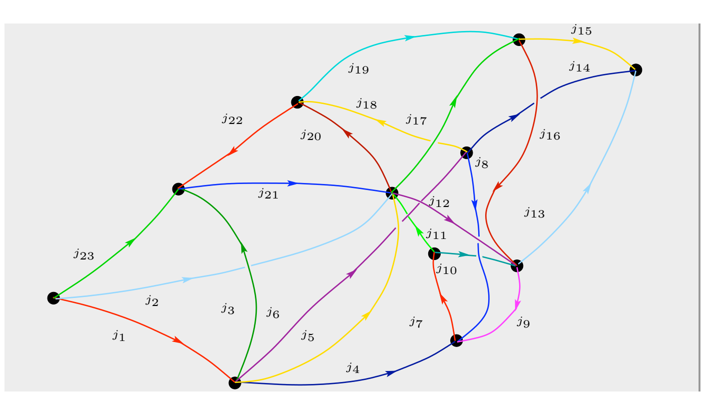

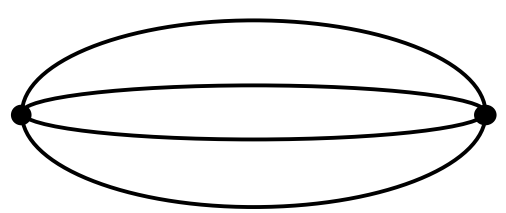

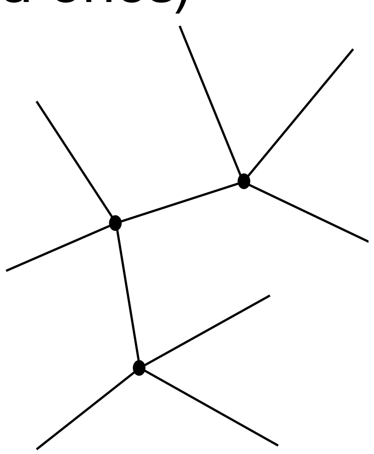

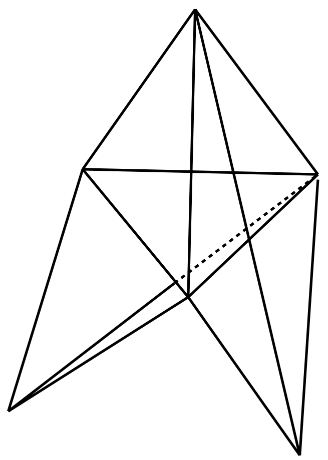

second quantized version of (generalized) LQG (adapted to simplicial context), but dynamics not derived from canonical quantization of GR

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classical action: kinetic (quadratic) term + (higher order) interaction (convolution of GFT fields)

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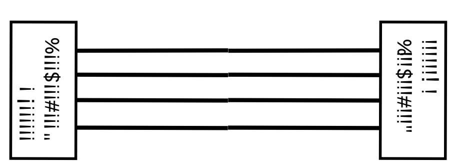

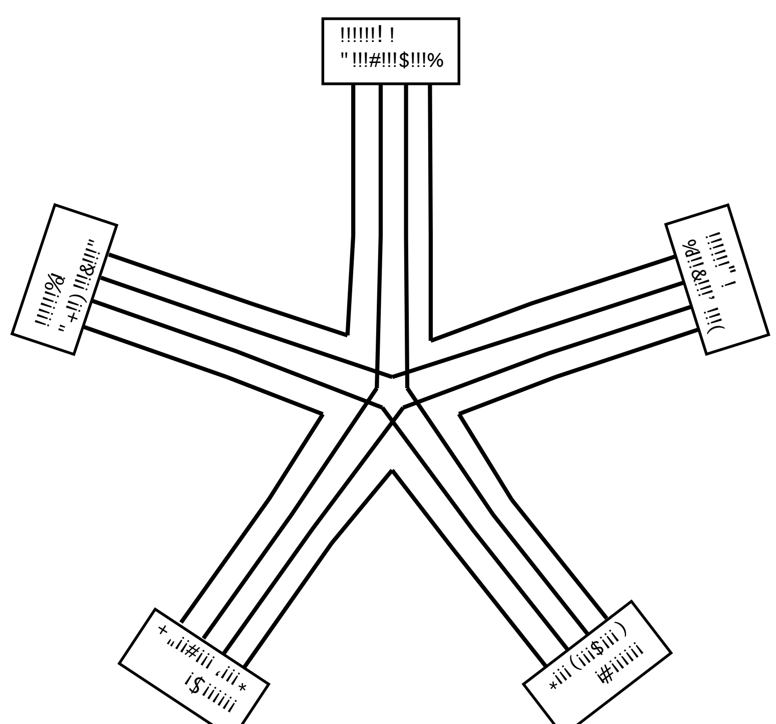

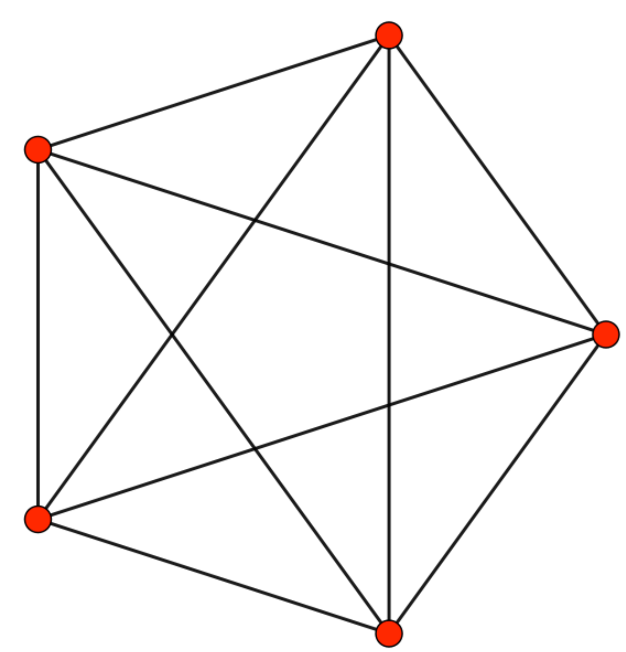

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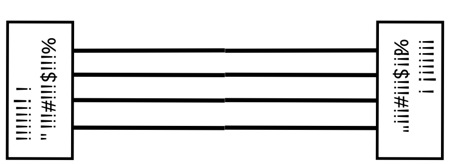
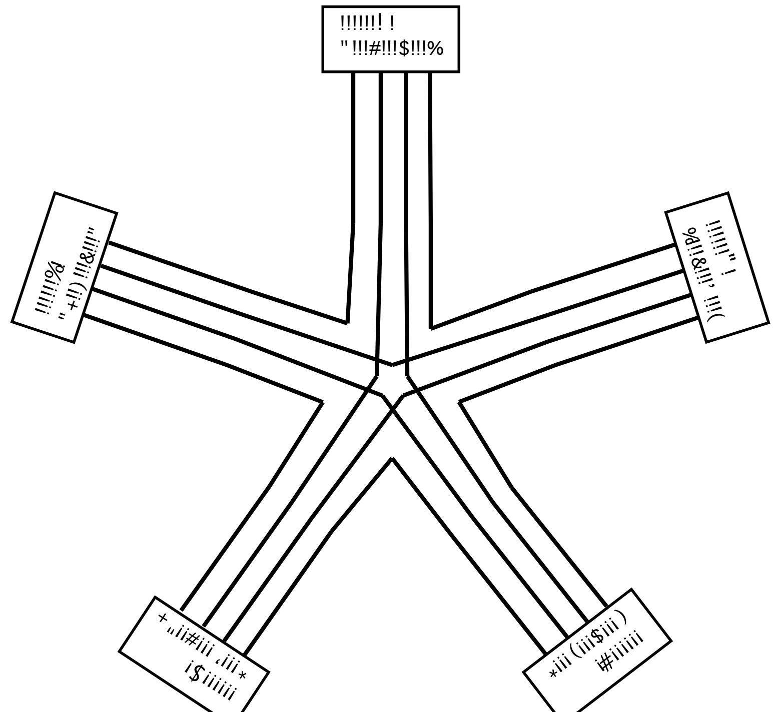
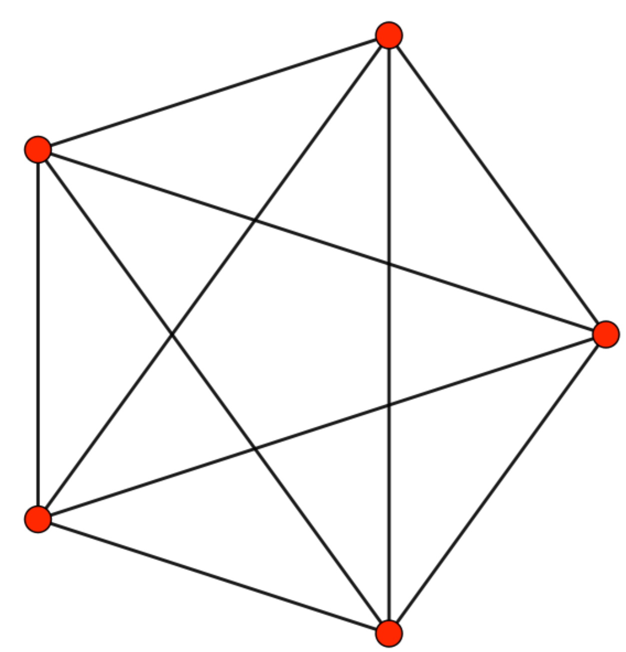


with fields constrained to satisfy "geometricity" conditions





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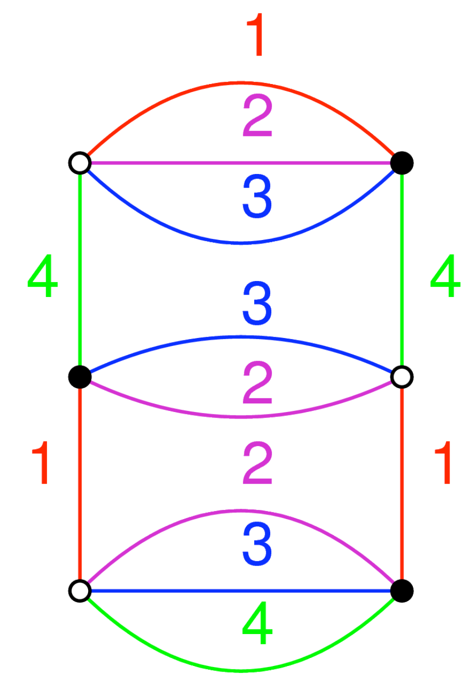
indexed by d-colored "bubbles"



example: $$\int [\mathrm{d}g_i]^{12} \varphi(g_1,g_2,g_3,g_4)\overline{\varphi}(g_1,g_2,g_3,g_5)\varphi(g_8,g_7,g_6,g_5)$$
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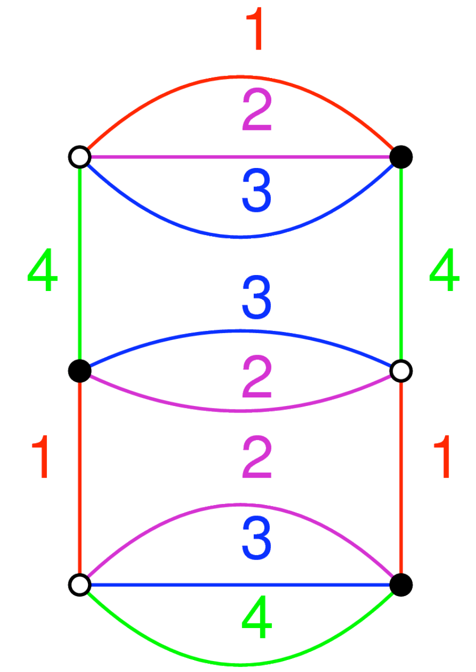

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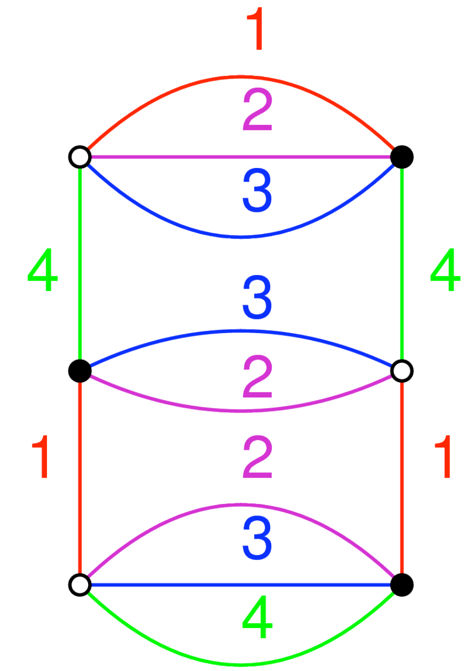

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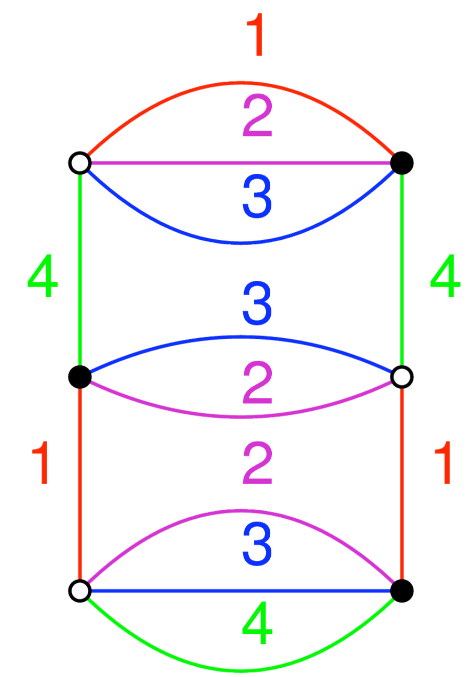
- interesting models exist with: nice simplicial geometry, direct links with discrete GR and simplicial path integrals, LQG-like Hilbert space, ...

GFT basics (4d case): dynamics

other possibility (motivated by tensor models and renormalization):
(tensor) invariant interactions



$$S(\varphi,\overline{\varphi}) = \sum_{b\in\mathcal{B}} t_b I_b(\varphi,\overline{\varphi})$$

indexed by d-colored "bubbles"

example: $$\int [\mathrm{d}g_i]^{12} \varphi(g_1,g_2,g_3,g_4)\overline{\varphi}(g_1,g_2,g_3,g_5)\varphi(g_8,g_7,g_6,g_5)$$
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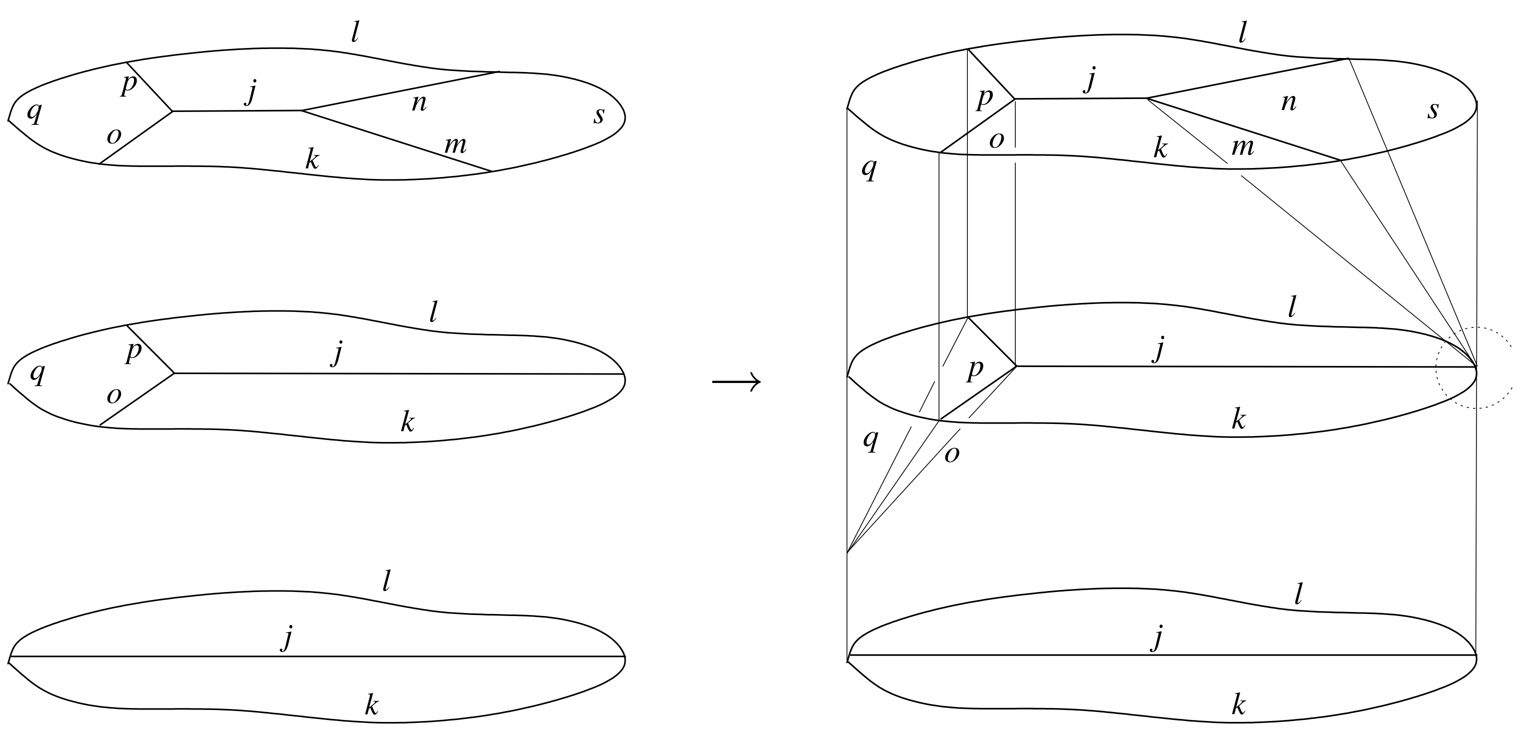

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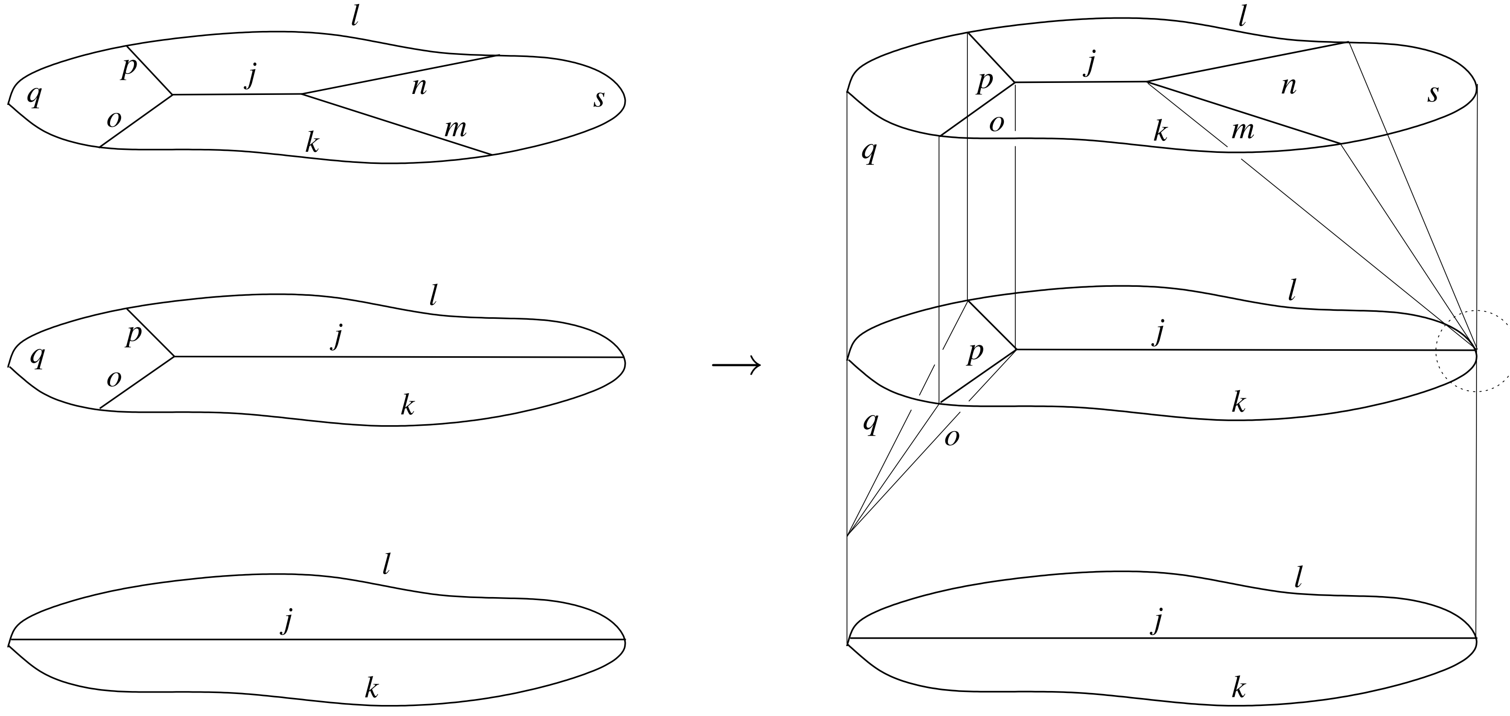

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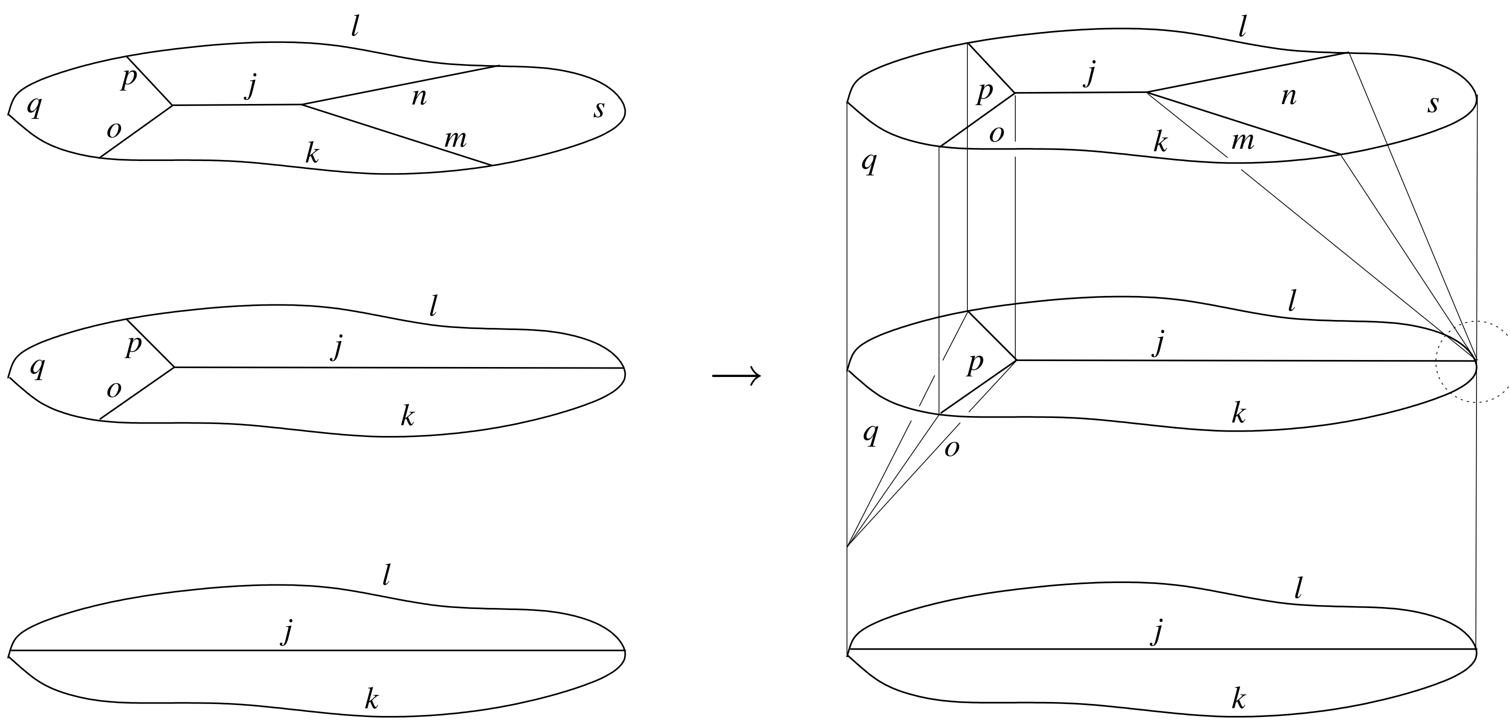

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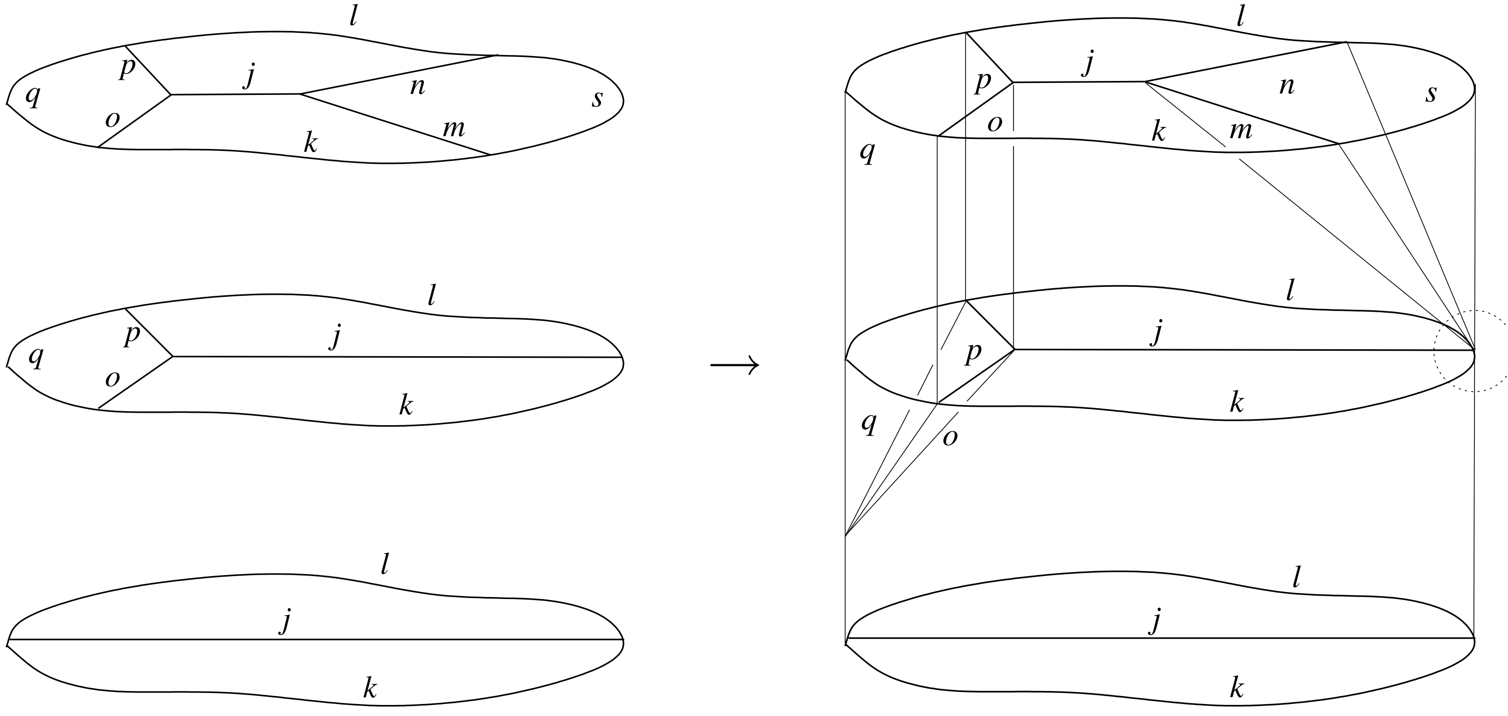

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Group Field Theory and Tensor Models

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Matrix models

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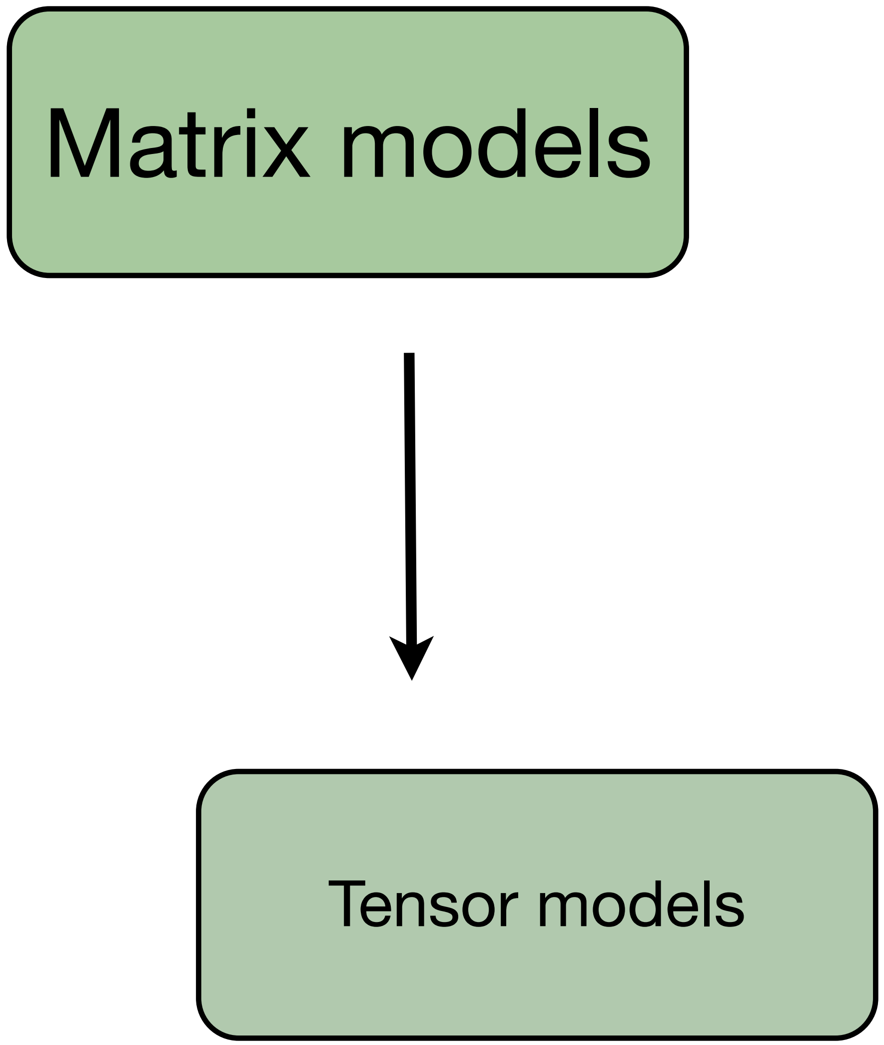
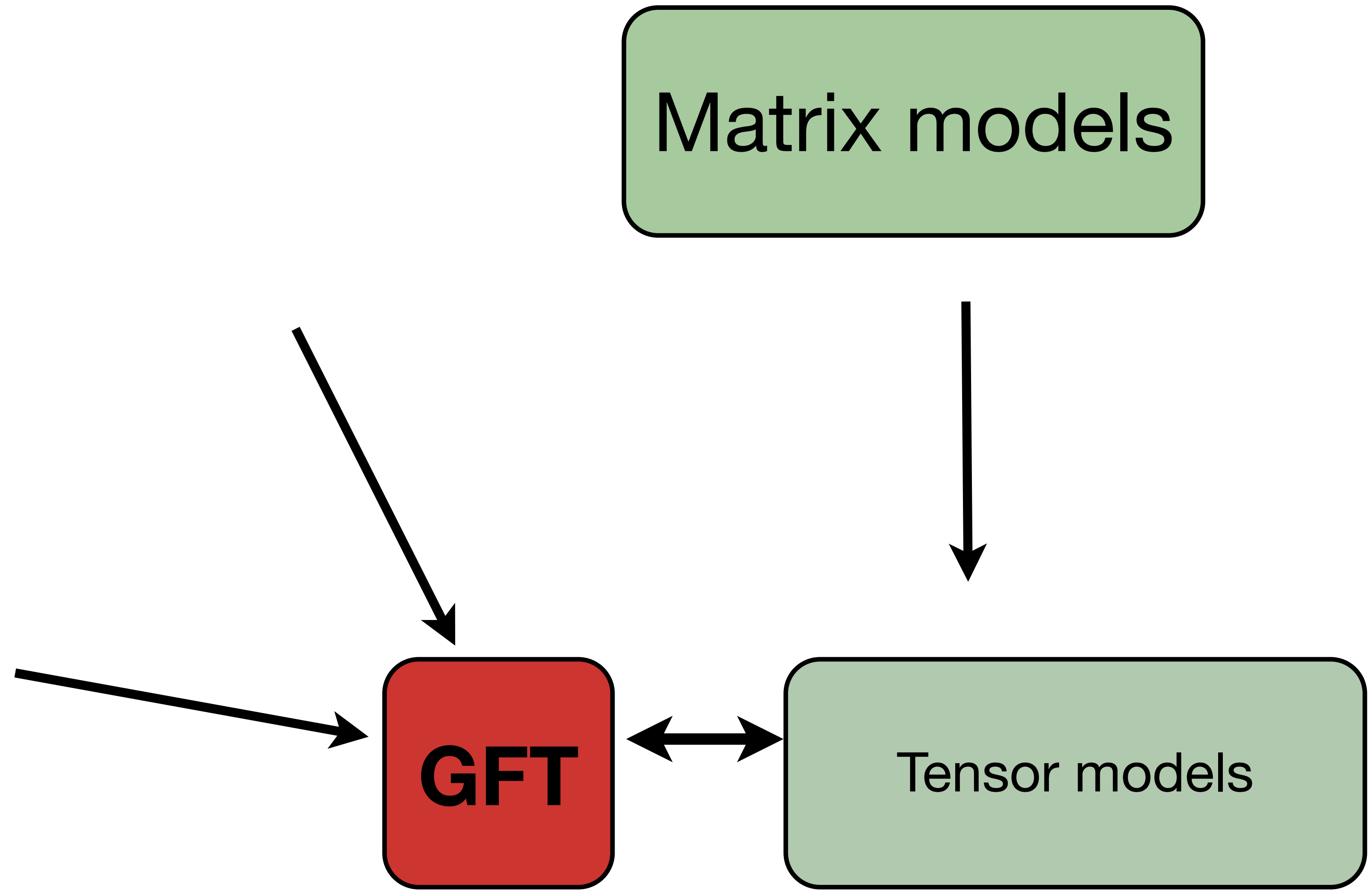


Group Field Theory and Tensor Models



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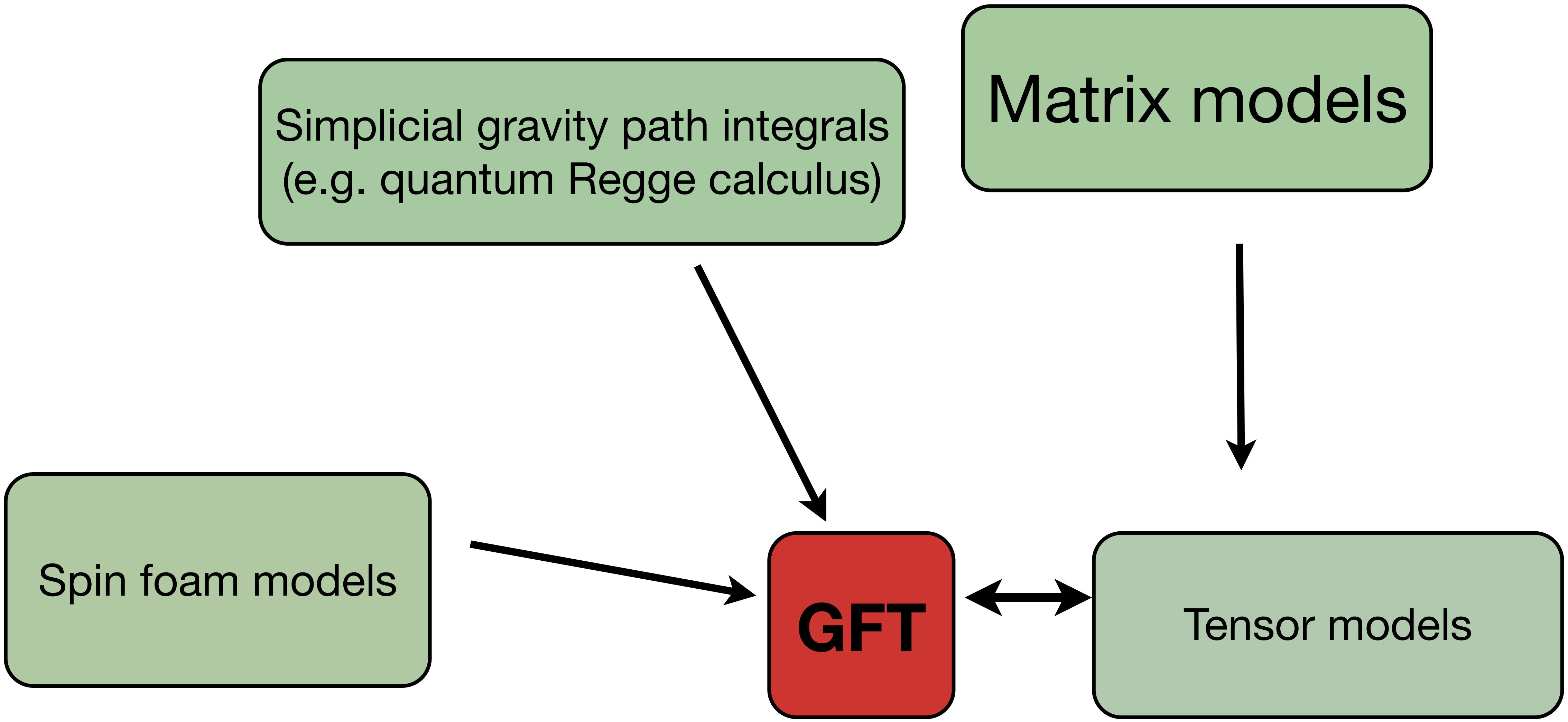

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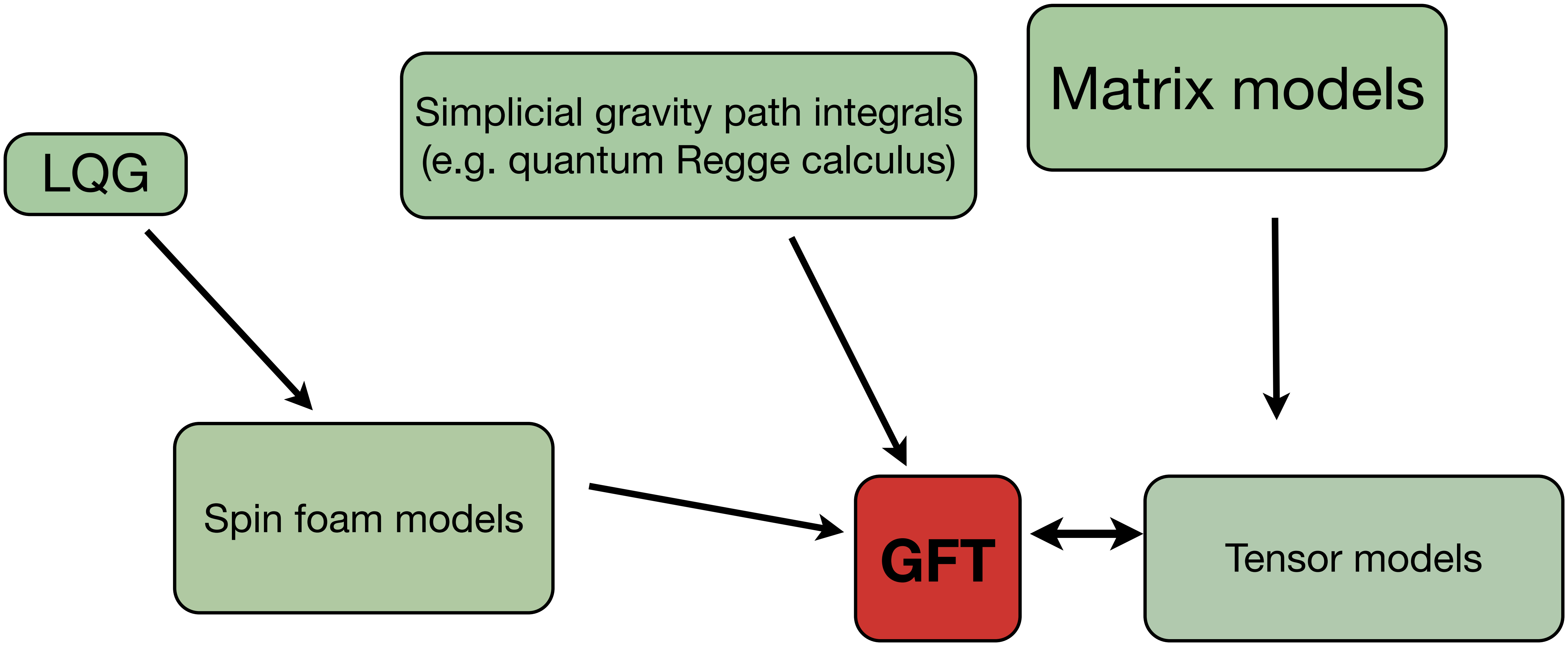

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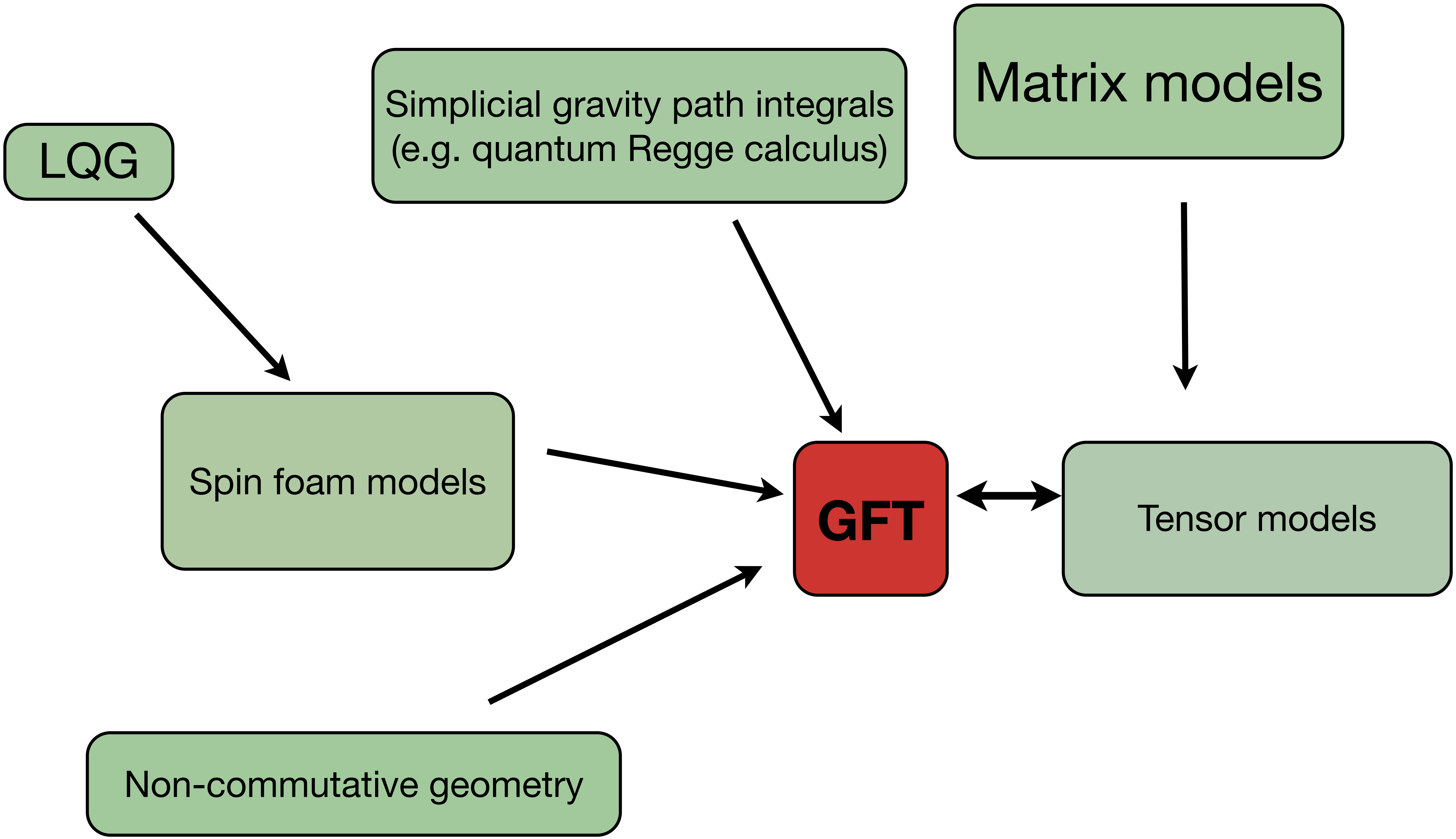

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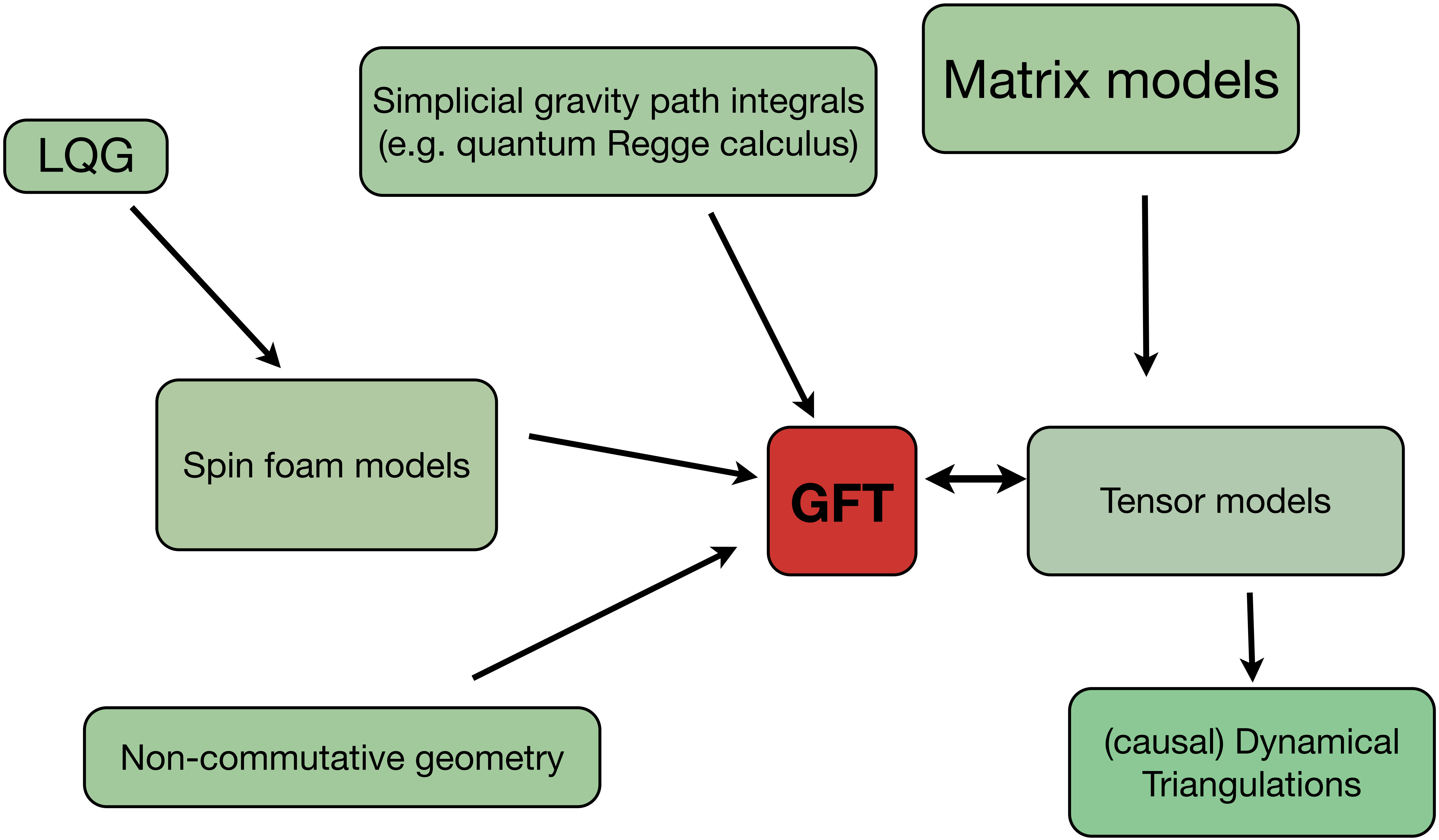

GFTs, spin foams, tensor models: many recent results

- construction of interesting 4d gravity models (inspired by LQG)
- encoding of simplicial geometry
- field theory symmetries
- understanding of combinatorial structures (GFT Feynman diagrams)
- large-N expansion
- GFT renormalization (various renormalizable models)
- critical behaviour (in tensor models)
- mean field expansion (emergent matter, effective QG dynamics,...)
- simplified models (for cosmology)
- ......

Continuum spacetime and geometry?
(physics?)

Continuum spacetime and geometry from GFT:

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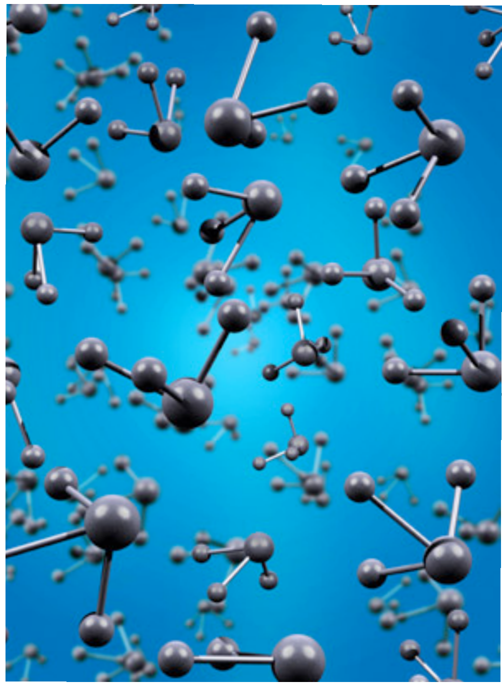
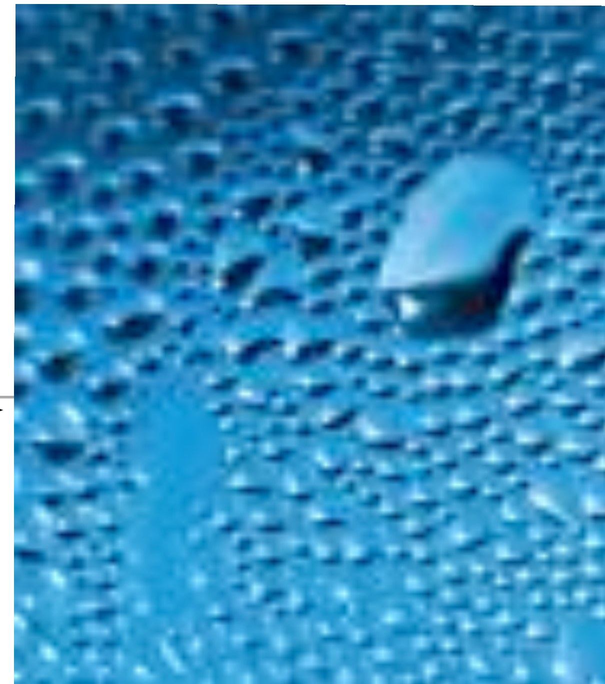
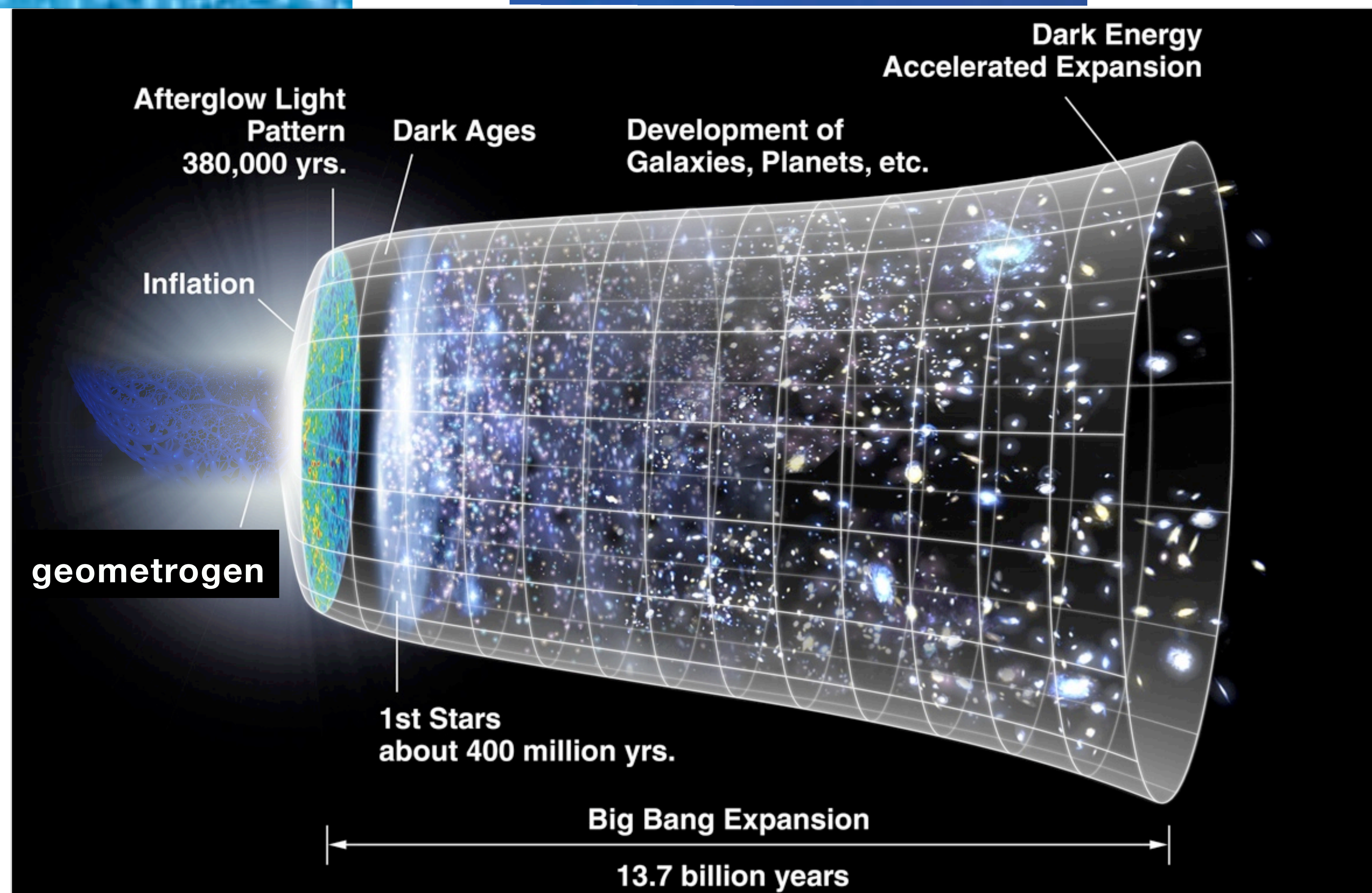
(Oriti ’07, ’11, ’13, Rivasseau ’11, ’12, Sindoni ’11)





spacetime as condensate of QG building blocks

Big Bang as phase transition (condensation)



Cosmology from GFT

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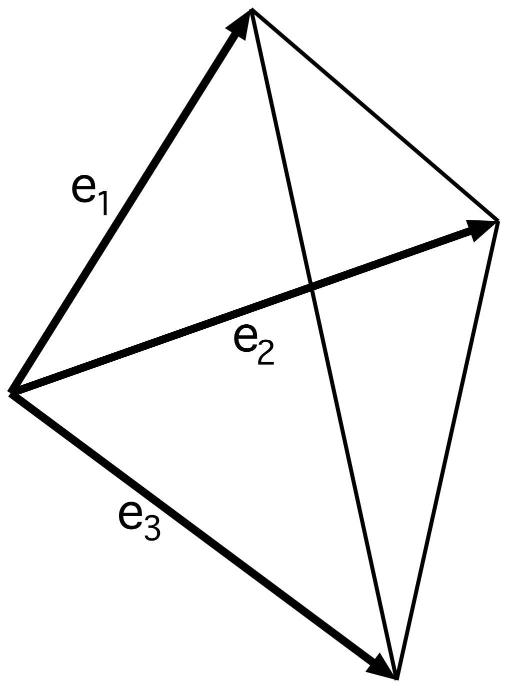
S. Gielen, DO, L. Sindoni,
AEI-2013-051, arXiv: 1303.XXXX

GFT states and approximate continuum geometries

- work with GFT with simplicial geometric interpretation (Riemannian SO(4) case for simplicity)

$$\varphi(g_1, g_2, g_3, g_4) \leftrightarrow \varphi(B_1, B_2, B_3, B_4) \to \mathbb{C}$$ describes geometric tetrahedron



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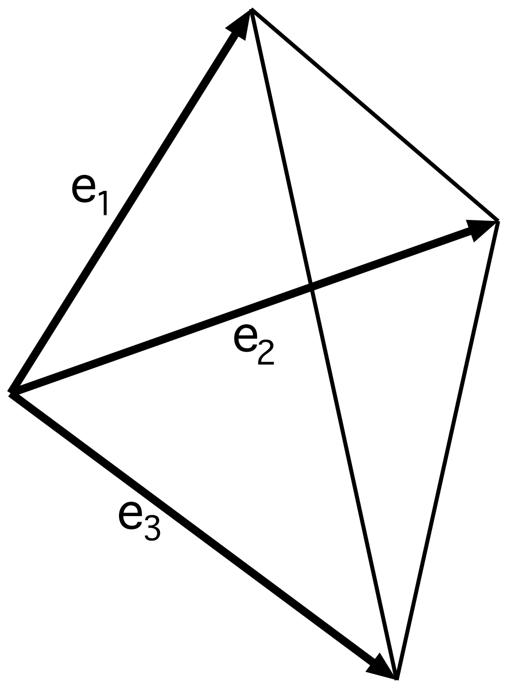

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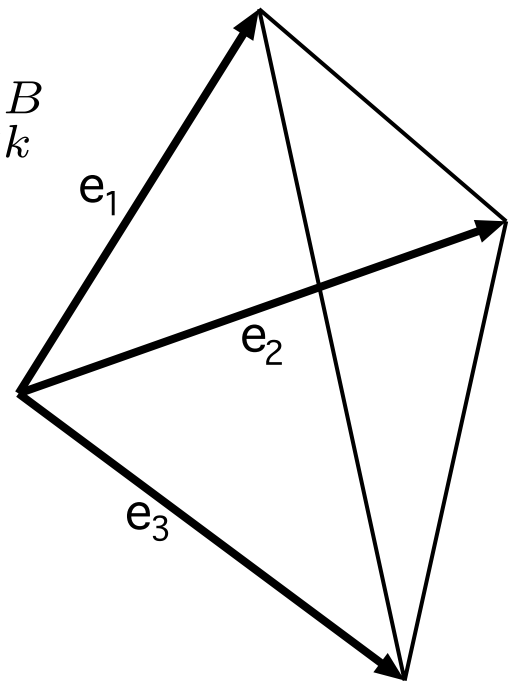

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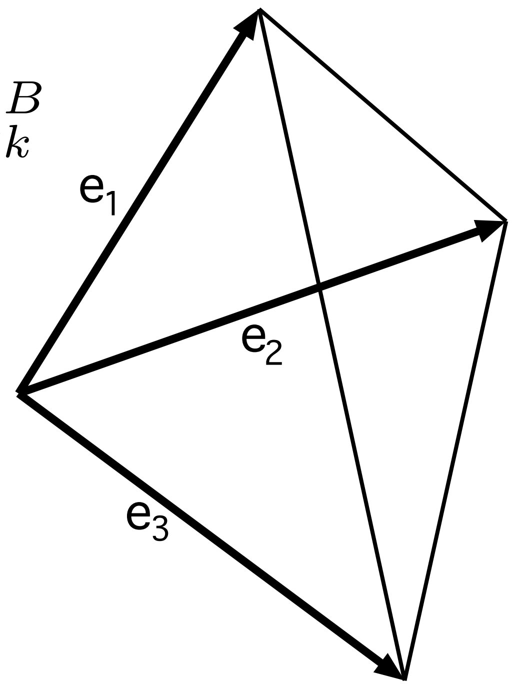

- generic N-particle GFT state (N geometric tetrahedra):

$$|B_{I(m)}\rangle := \prod_{m=1}^{N} \hat{\varphi}^\dagger(B_{1(m)}, \ldots, B_{4(m)})|0\rangle$$

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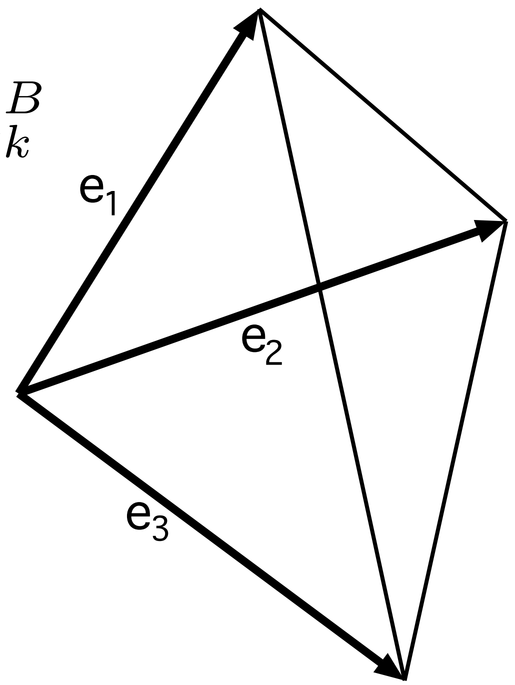

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- think of tetrahedra as embedded in symmetric 3-manifold (wrt group H) - implies choosing embedding point and 3 reference vectors:

$$\triangle_m \mapsto \left\{x_m \in \mathcal{M}, \left\{\mathbf{v}_{1(m)}, \mathbf{v}_{2(m)}, \mathbf{v}_{3(m)}\right\} \subset T_{x_m}\mathcal{M}\right\}$$

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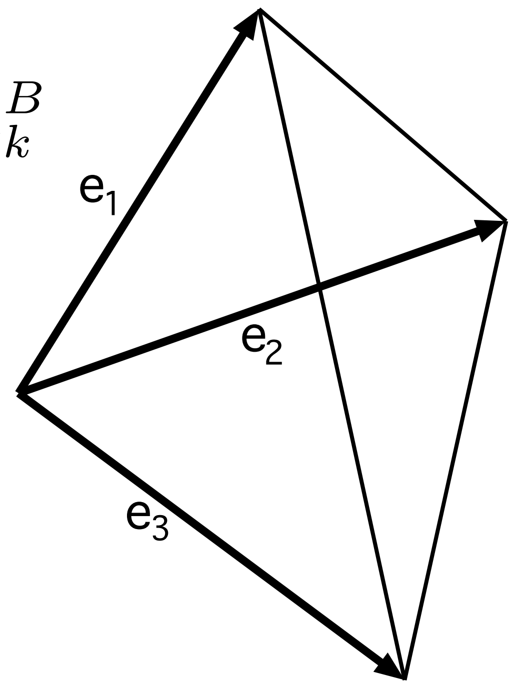

- generic N-particle GFT state (N geometric tetrahedra):

$$|B_{I(m)}\rangle := \prod_{m=1}^{N} \hat{\varphi}^\dagger(B_{1(m)}, \ldots, B_{4(m)})|0\rangle$$

- think of tetrahedra as embedded in symmetric 3-manifold (wrt group H) - implies choosing embedding point and 3 reference vectors:

$$\triangle_m \mapsto \left\{x_m \in \mathcal{M}, \left\{\mathbf{v}_{1(m)}, \mathbf{v}_{2(m)}, \mathbf{v}_{3(m)}\right\} \subset T_{x_m}\mathcal{M}\right\}$$

- choose embedding vectors to be aligned with left-invariant vector fields of H

GFT states and approximate continuum geometries

- interpret discrete triad variable in GFT state with physical triad field integrated along embedding vector

  requires: tetrahedra small enough and flat enough

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- if GFT state satisfy additional gauge invariance condition under SO(4) at every "point", then it can be put in 1-1 correspondence with such approximate continuum metric

$$B_{i(m)} \mapsto \left(h_{(m)}\right)^{-1} B_{i(m)} h_{(m)}\,, \quad e_{i(m)} \mapsto e_{i(m)} h_{(m)}$$

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$\Longrightarrow$ Continuum homogeneous spacetimes are quantum GFT condensates

similar constructions in LQG (Alesci, Cianfrani) and LQC (Bojowald, Wilson-Ewing, ...)

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two simple choices of quantum GFT condensate states
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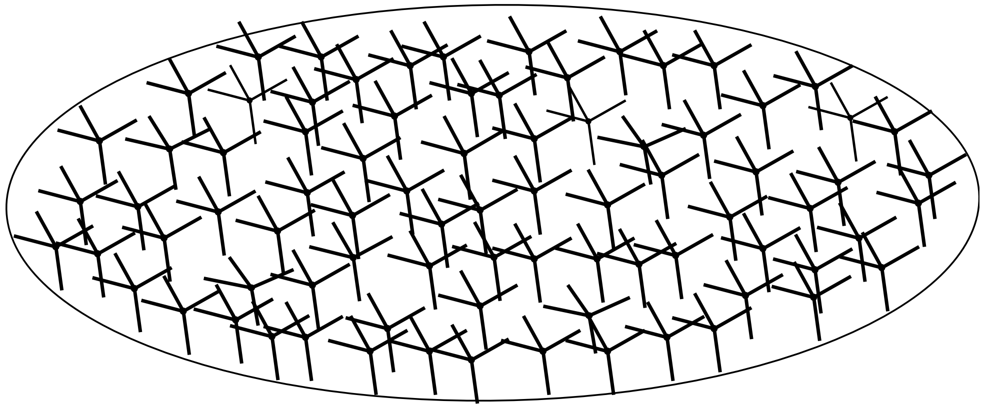

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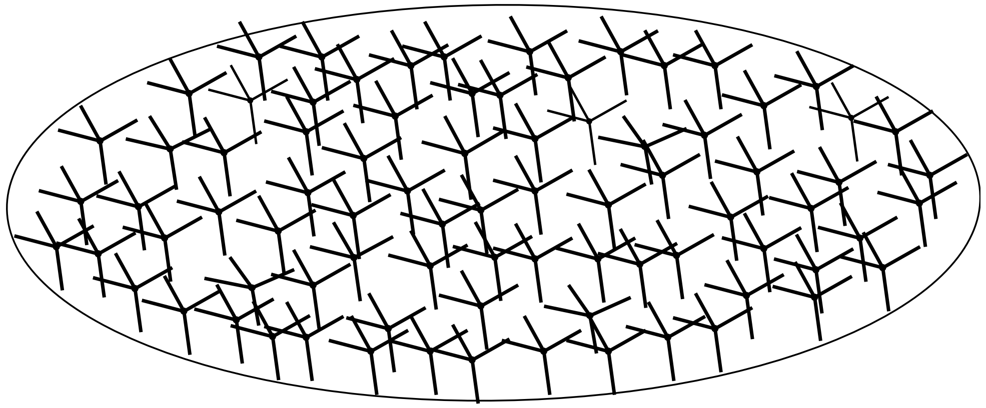

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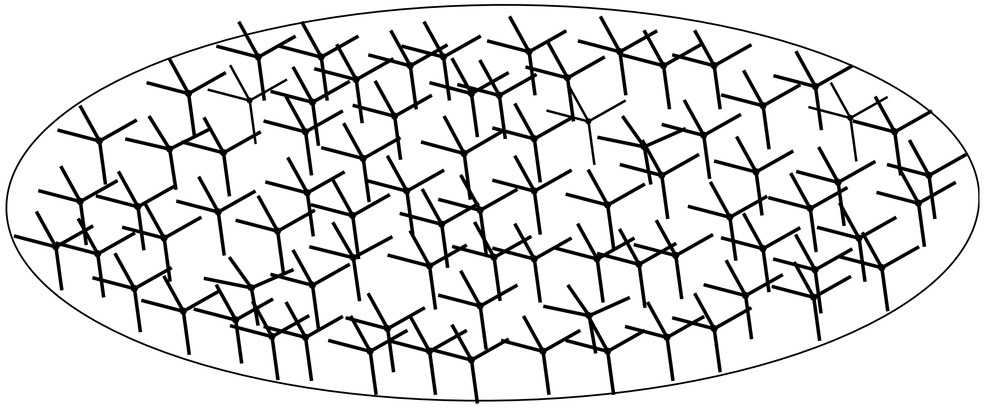

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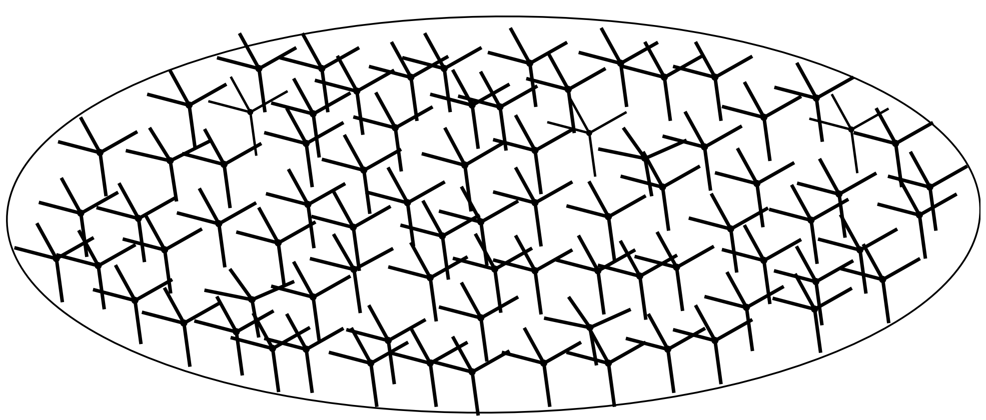

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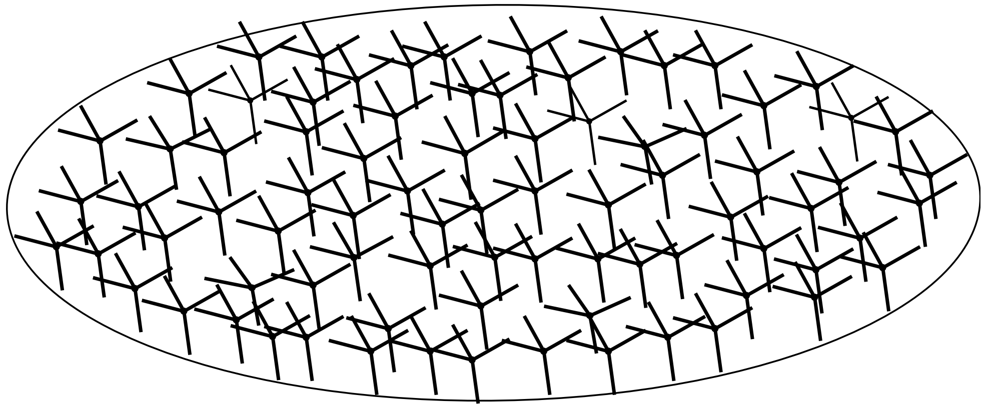

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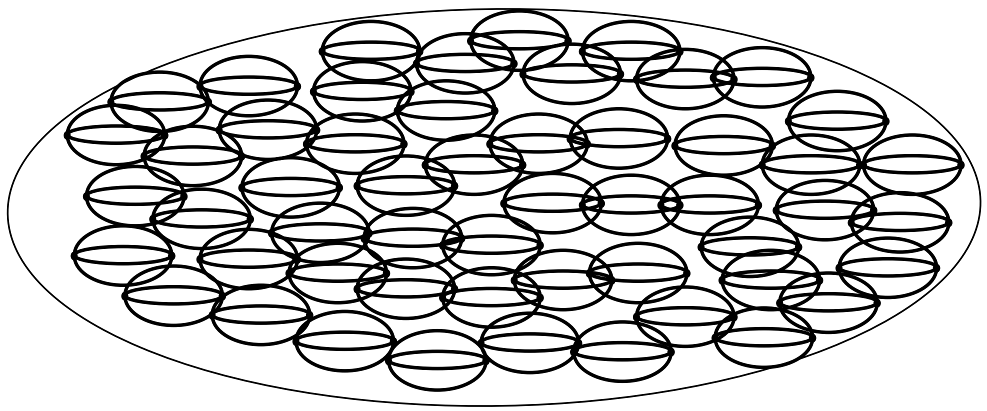

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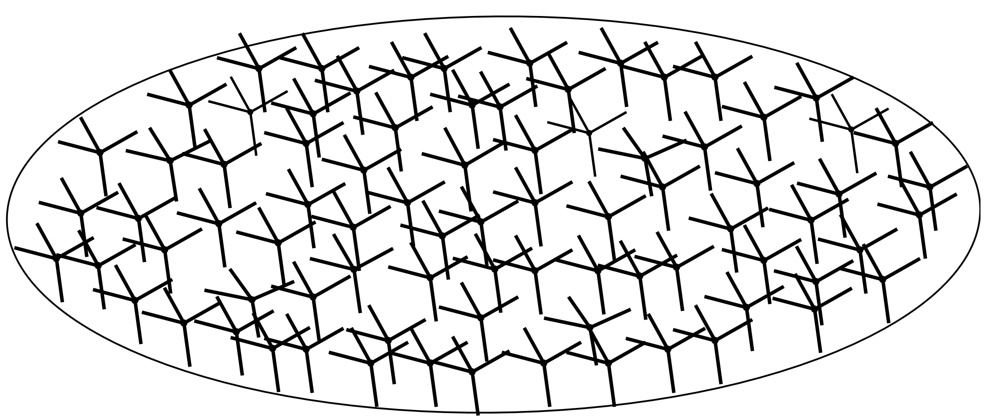

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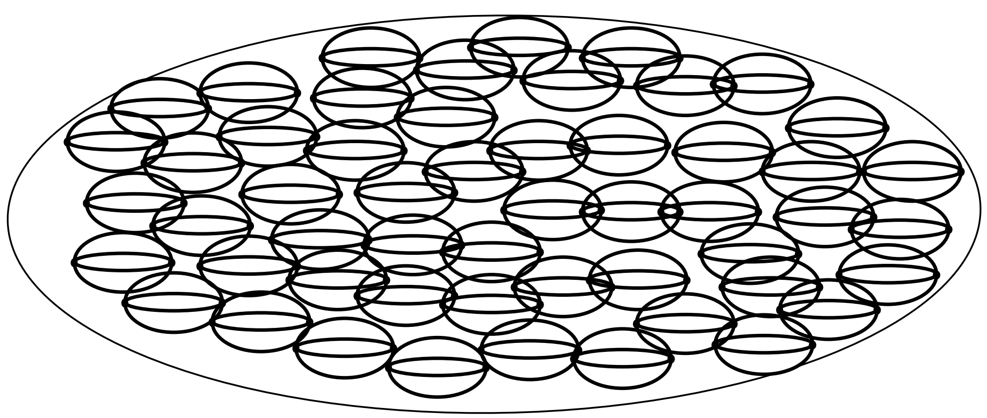

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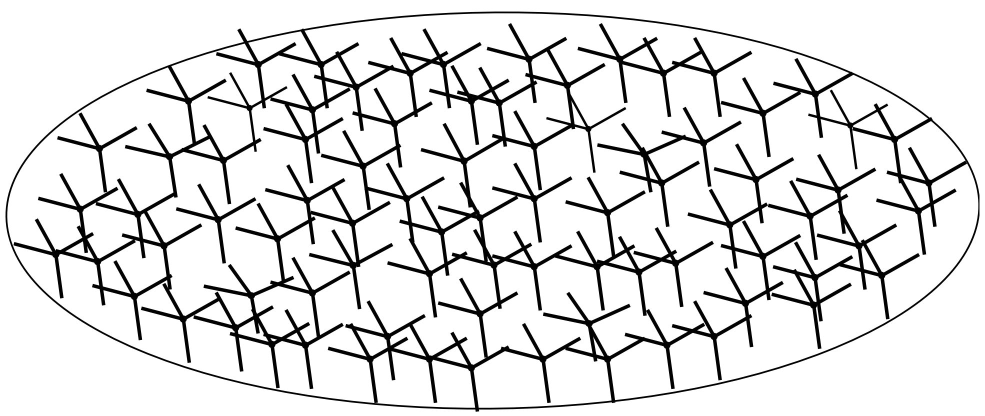

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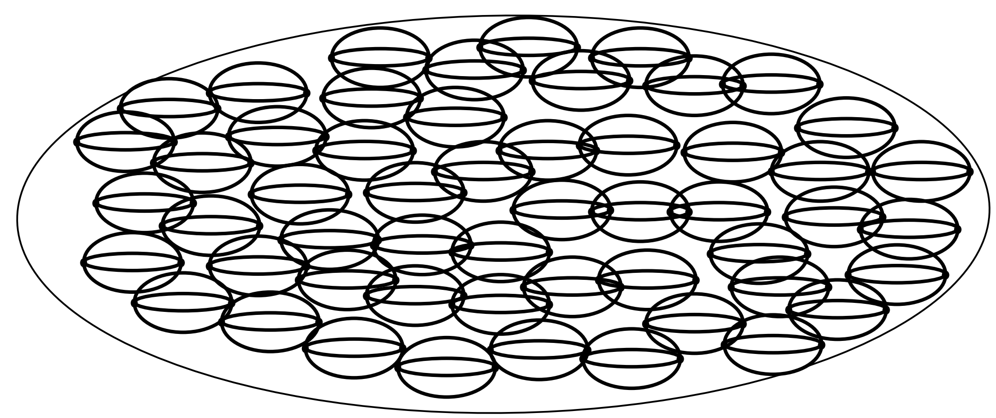

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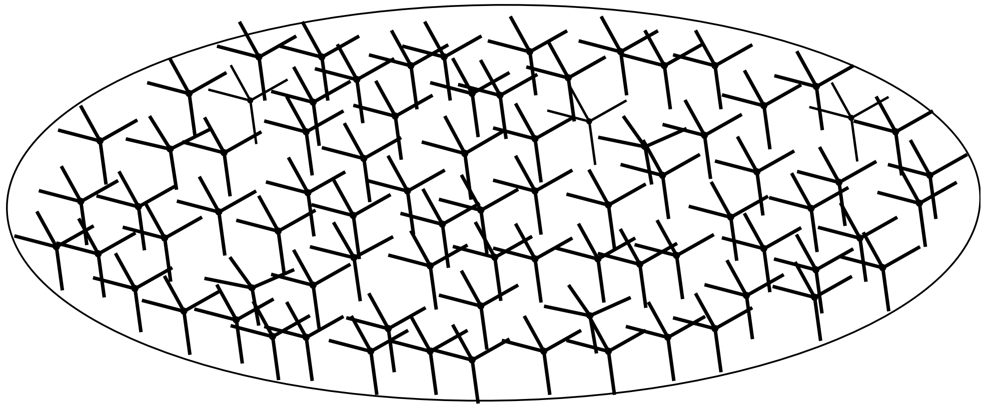

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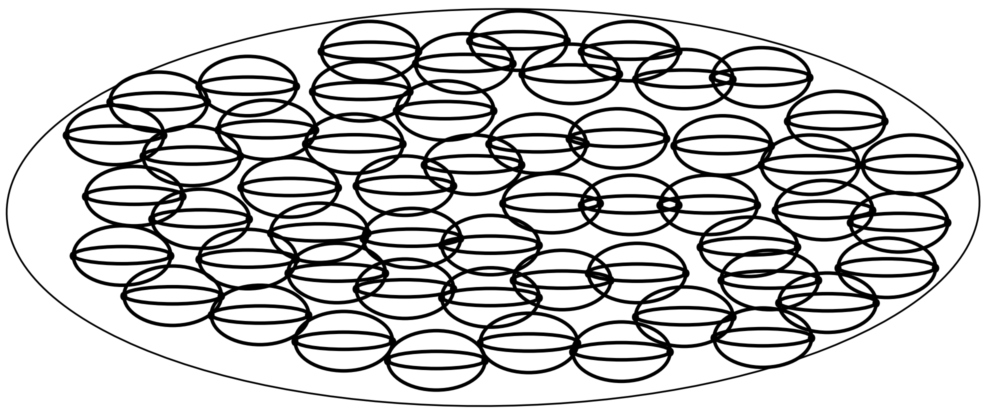

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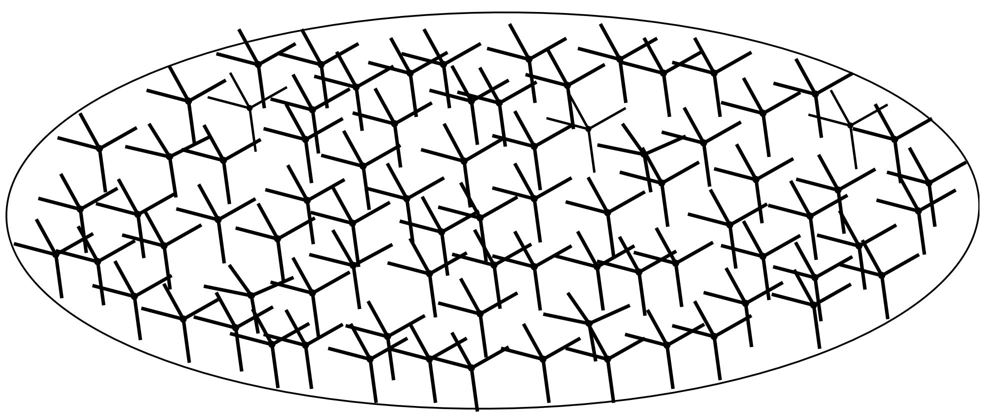

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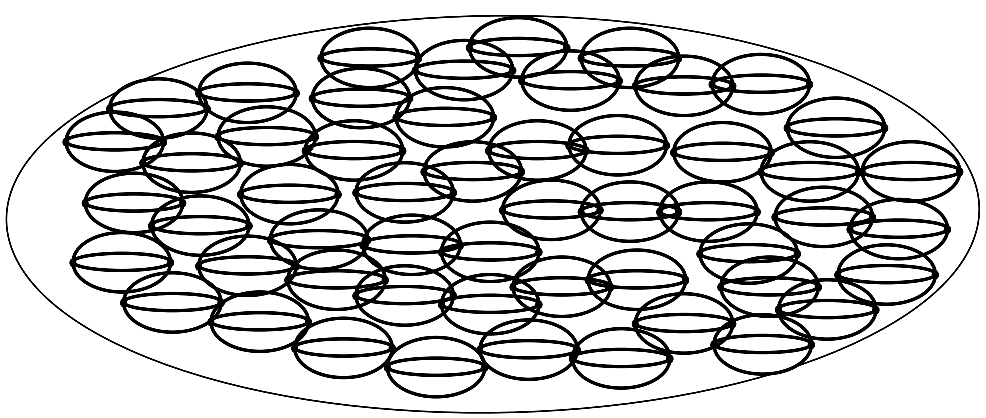

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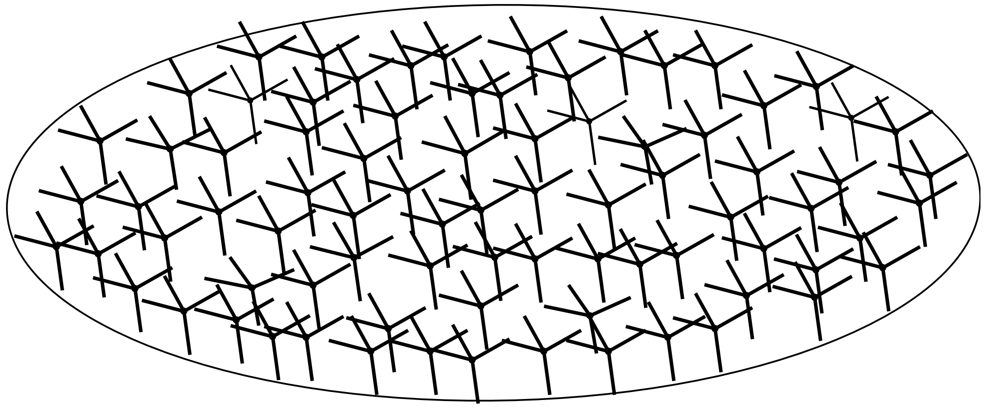

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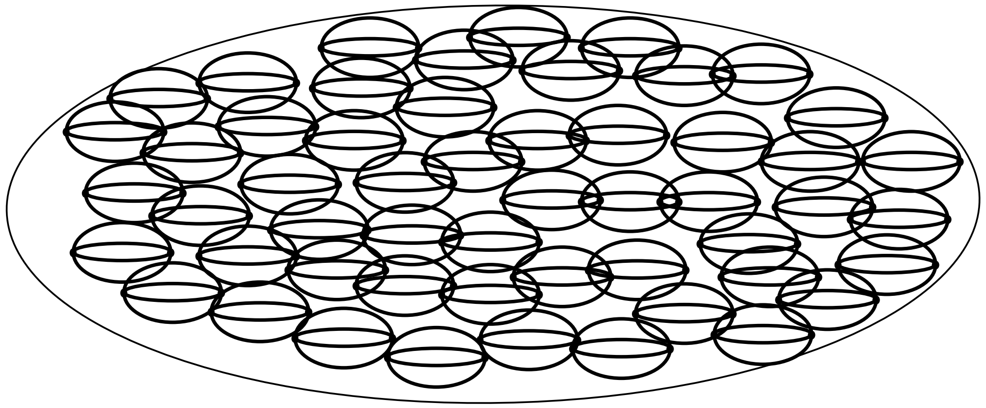

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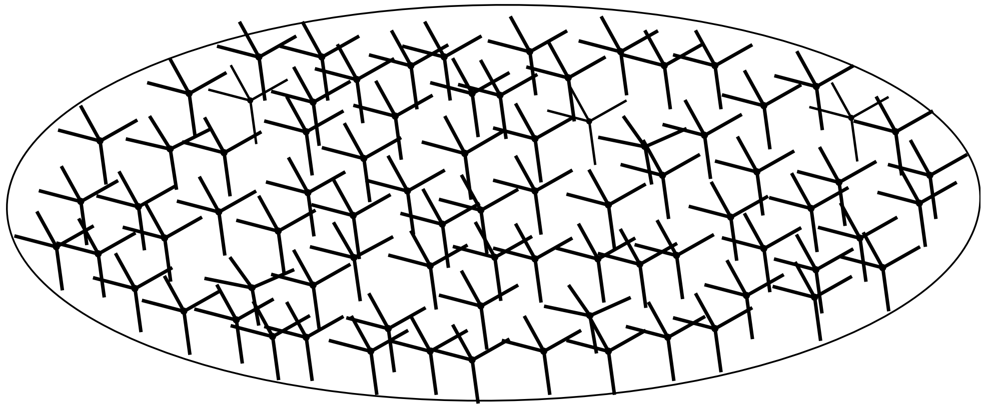

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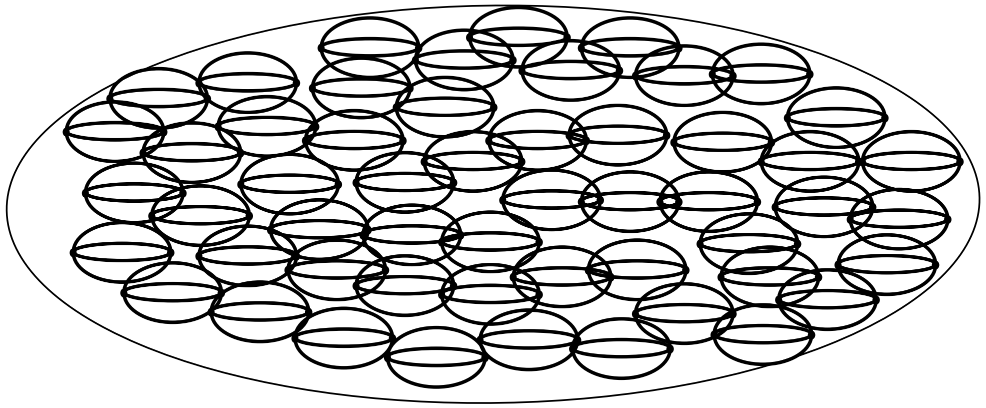

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with extra approximations required for consistent continuum geometric interpretation: GFT quanta "small enough" and "flat enough":

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non-linear and non-local extension of quantum cosmology-like equation for "collective wave function

QG (GFT) analogue of Gross-Pitaevskii hydrodynamic equation in BECs

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similar equations obtained in non-linear extension of LQC (Bojowald et al. '12)

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special case: (effective) kinetic term = Laplacian on SU(2)^4
(suggested by simplicial geometry, LQG, GFT renormalization,..): $\mathcal{K}(g_I, \tilde{g}_I) = \left(\sum_I \Delta_{g_I} + \mu\right)(g_I, \tilde{g}_I)$

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- using geometric interpretation of states and variables, we can identify:

$B_I = a_I^2 T_I \quad \pi_I = \dot{a}_I V_I \qquad I = 1, 2, 3$ a 's are scale factors

$B_4 = B_4(B_1, B_2, B_3) \quad \pi_4 = \pi_4(\pi_1, \pi_2, \pi_3)$

T, V = normalized dimensionless Lie algebra elements (state dependent)

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another way to extract effective classical equations from GFT hydrodynamics: take order parameter to be coherent state for mini-superspace (DO, L. Sindoni, '10)

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derivation of cosmology from full QG formalism!

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