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Aspects of the String-Black Hole Correspondence

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Outline

1. The string-black hole correspondence & stringholes
2. Stringhole production in high-energy gravitational scattering
3. Scattering off a stringhole and quantum hair
4. Stringholes and a model for the big bounce

The String-BH correspondence

Entropy of free string states

(FV, BM, 1969)

# of physical string states @ vanishing string coupling ($\alpha' M^2 = N$, C = central charge, c=1).

$$d(N) = N^{-p} e^{2\pi\sqrt{\frac{CN}{6}}} = M^{-2p} e^{2\pi\sqrt{\frac{\alpha' C}{6\hbar}}M}$$

Neglecting numerical factors this gives, at large M,

$$S_{st} \sim \frac{\alpha' M}{l_s} = \frac{M}{M_s} \; ; \; l_s = \sqrt{2\alpha'\hbar} \; ; \; M_s = \sqrt{\frac{\hbar}{2\alpha'}}$$

Physical interpretation of S_{st}: the number of "string bits" contained in the total length of the string, $L = \alpha' M$.

Semiclassical BH entropy

Bekenstein-Hawking formula for arbitrary D

$$S_{BH} = \frac{\mathcal{A}}{4 l_D^{D-2}} \;\; ; \;\; \mathcal{A} \sim R_S^{D-2} \sim (G_D M)^{\frac{D-2}{D-3}} \Rightarrow S_{BH} = \frac{M R_S}{\hbar}$$

$$l_D^{D-2} = G_D \hbar$$

can be compared with previous

$$S_{st} \sim \frac{\alpha' M}{l_s} = \frac{M}{M_s} \; ; \; l_s = \sqrt{2\alpha' \hbar} \; ; \; M_s = \sqrt{\frac{\hbar}{2\alpha'}}$$

The two entropies look very different but can we trust both results everywhere in parameter space?

Let's assume for the moment that we can.

The correspondence curve

S_{BH} grows faster than S_{st} but the latter starts higher at small M. Hence, the two entropies must meet at some finite value of M:

$$\frac{S_{BH}}{S_{st}} = \frac{MR_S/\hbar}{M/M_s} = \frac{M_s R_S}{\hbar} = \frac{R_S}{l_s}$$

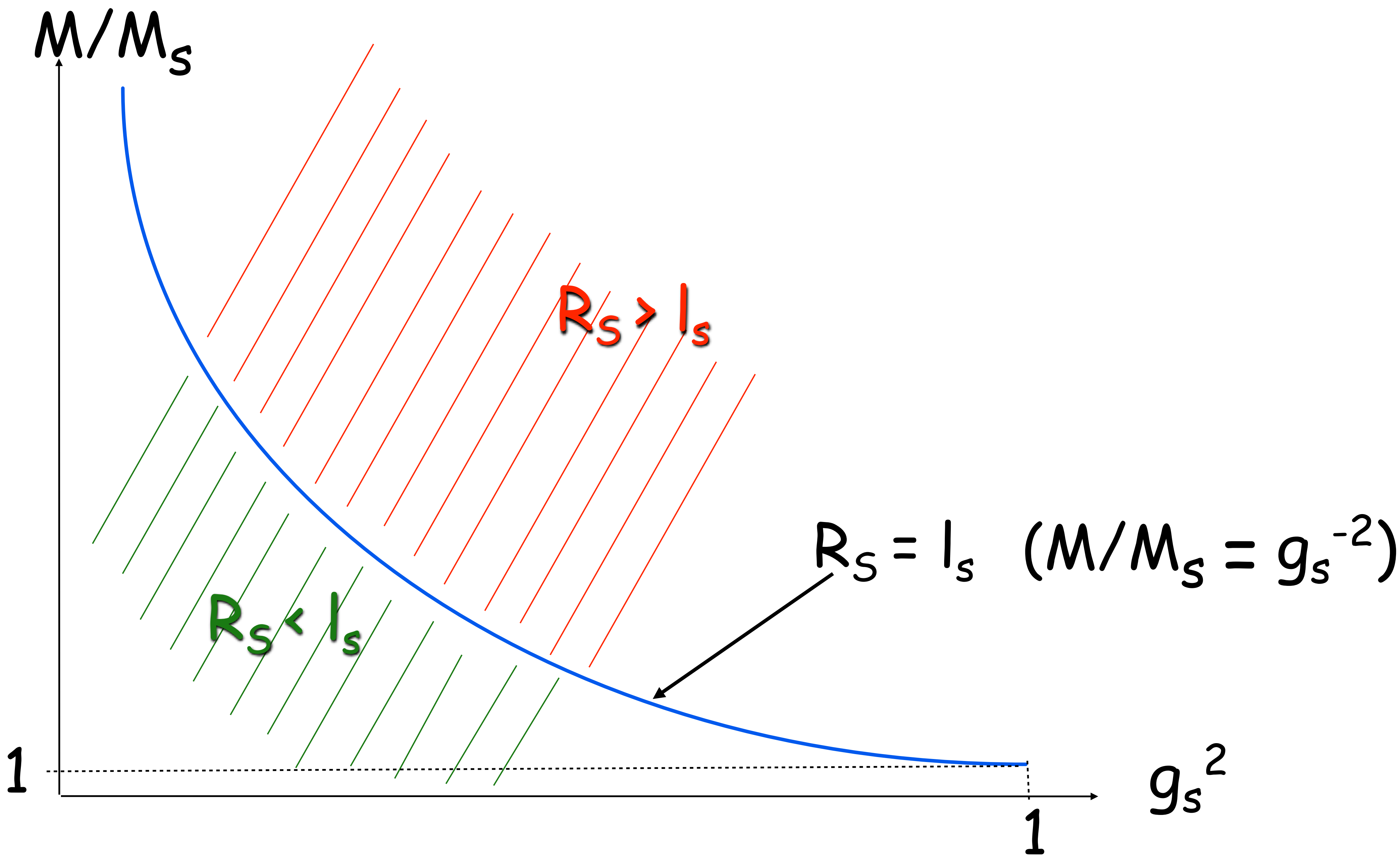
S_{BH} wins over S_{st} for $R > l_s$, the opposite is true for $R < l_s$. They coincide at $R = l_s$ and take the value:

$$S_{BH} = S_{st} = \frac{l_s^{D-2}}{l_D^{D-2}} = g_s^{-2} \gg 1 \Rightarrow M = M_* \equiv g_s^{-2} M_s$$

$S_{BH} = S_{st}$ defines a hyperbola in the (g_s, M) plane called the correspondence curve.

NB: at very small string coupling $M_* \gg M_P \gg M_s$

The correspondence curve



Below the correspondence curve

Below the correspondence curve (CC) the Schwarzschild radius of the string is smaller than the string length scale. The latter is believed to be the minimal size of any string.

Hence such strings are simply NOT BHs.

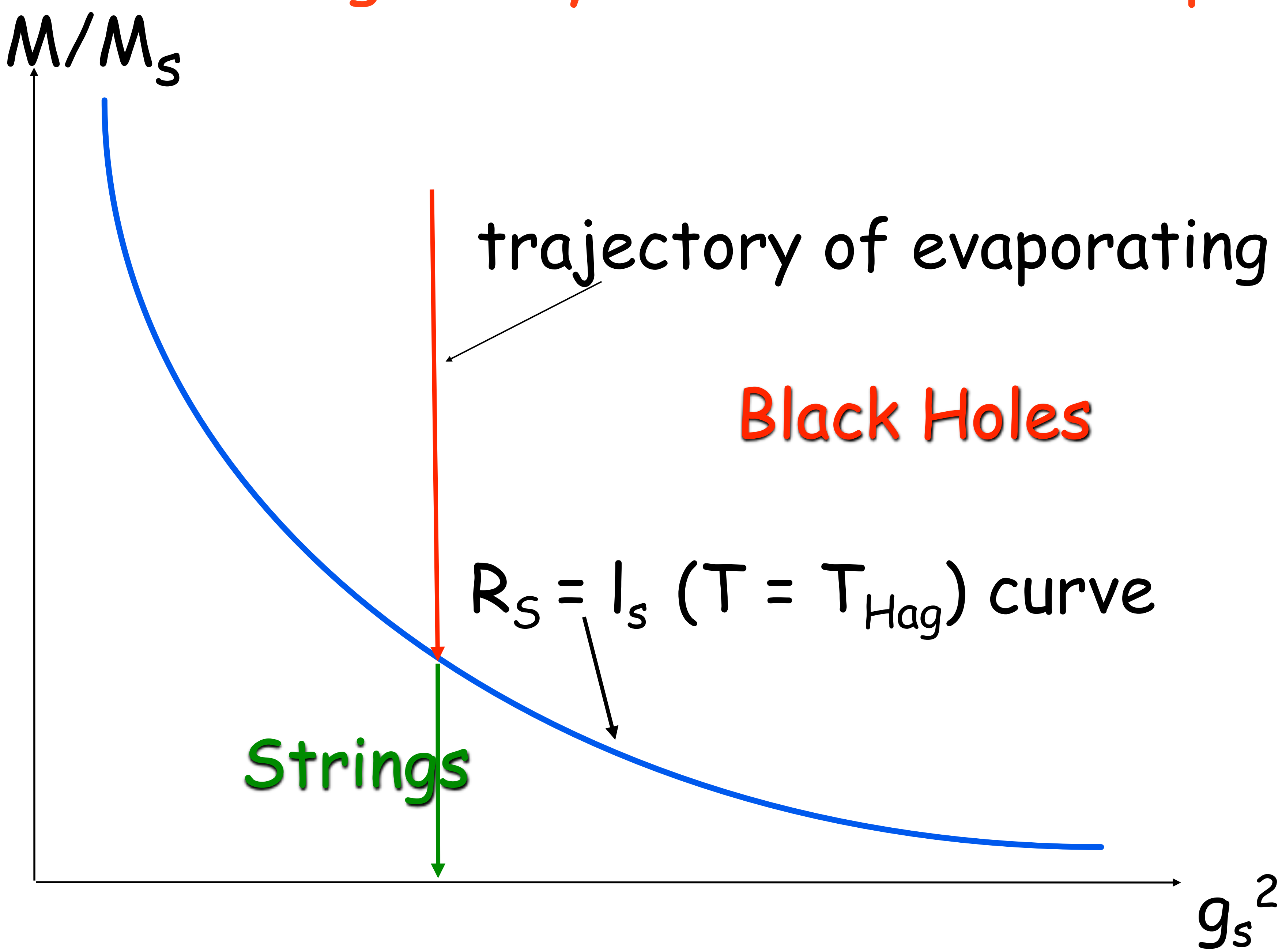
Interpretation: in QST there are no BHs whose R_S is smaller than l_s, i.e. whose Hawking temperature is higher than M_s. ($T = M_s$ is believed ST's maximal temperature)

So far, everything looks consistent!

It can even solve the problem of end-point of evaporation!

Evaporation of a BH at fixed g_s (Bowick et al. 1987)

Singularity at the end of evaporation avoided?



Approaching the correspondence curve: the random-walk puzzle

If we want to identify BH with FS above the CC, their properties should match as we approach the curve.

By definition the two entropies match (up to O(1) factors) but there is still a "random-walk puzzle".

S_{st} can be understood in terms of a "random walk" but then a string on the CC being much longer (heavier) than l_s (M_s), will have a typical size much bigger than its Schwarzschild radius l_s.

But then it has nothing to do with a BH!

Size distribution of free strings

The resolution of the RW puzzle is quite simple. One has to compute the distribution of the string sizes for a given M (NB: M fixes length not size!).

This was done by T. Damour & GV (2000). The entropy of strings of given M and size R is given by (c_1, c_2 are positive numbers O(1), calculation reliable for R > R_S):

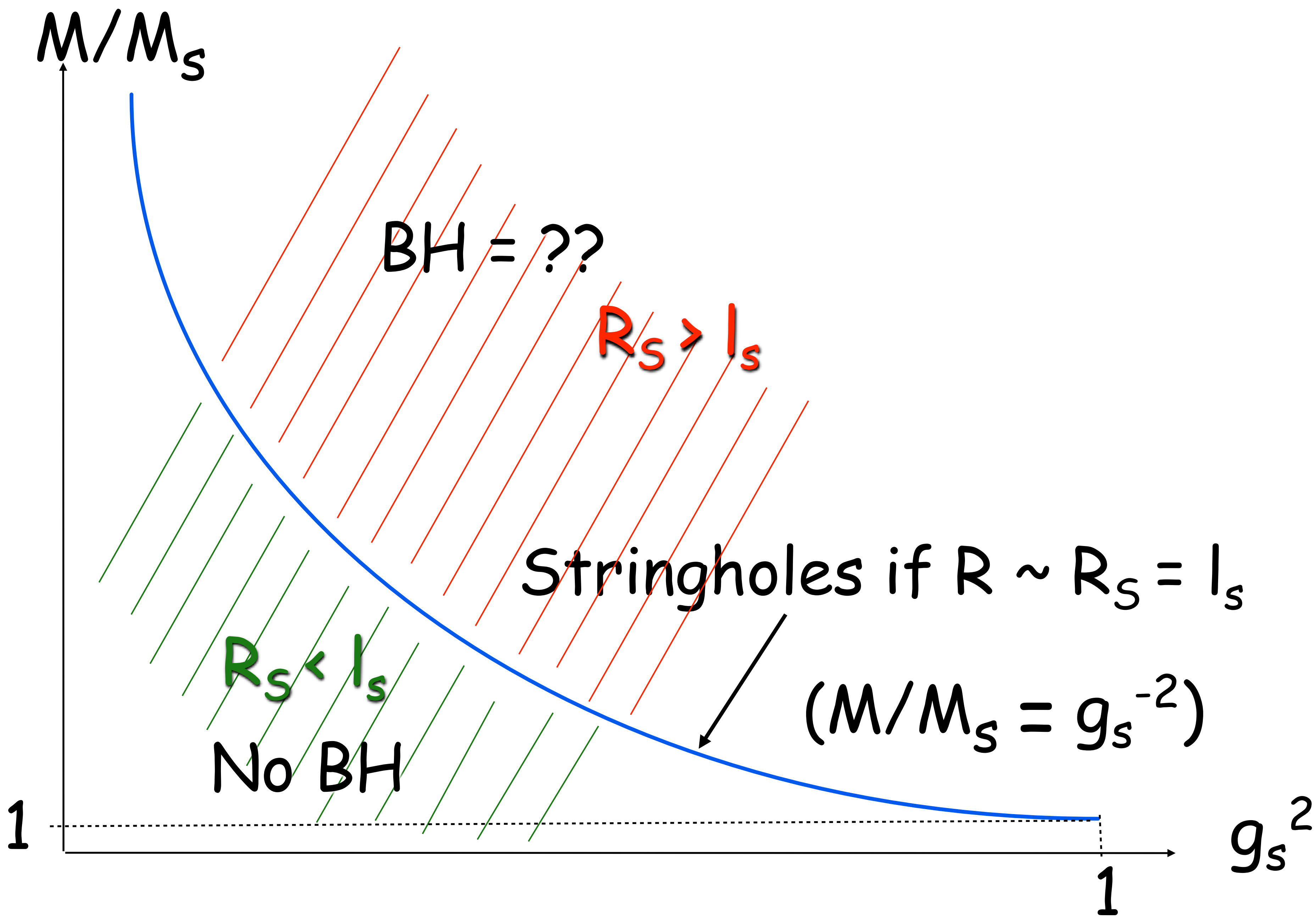
$$S(M,R) \equiv \log d(M,R) = a_0 \frac{M}{M_s} f\left(\frac{R}{l_s}, \frac{\alpha' M}{l_s}\right);$$

$$a_0 = 2\pi\sqrt{\frac{D-2}{6}}\ ;\ f\left(\frac{R}{l_s}, \frac{\alpha' M}{l_s}\right) = \left(1 - \frac{c_1 l_s^2}{R^2}\right)\left(1 - \frac{c_2 R^2}{(\alpha' M)^2}\right)$$

Entropy is maximized for: $\frac{R}{l_s} \sim \sqrt{\frac{M}{M_s}} = \text{random walk value}$

But there is still an S of order M/M_s in strings of size $O(l_s)$!
We shall call such strings lying on the CC "stringholes"

Stringholes



Above the correspondence curve

It is reassuring that the string-coupling corrections become of O(1) just when we can reproduce BH properties up to factors O(1).

As we go farther and farther above the CC the discrepancy between free-string and BH entropy becomes larger and larger.

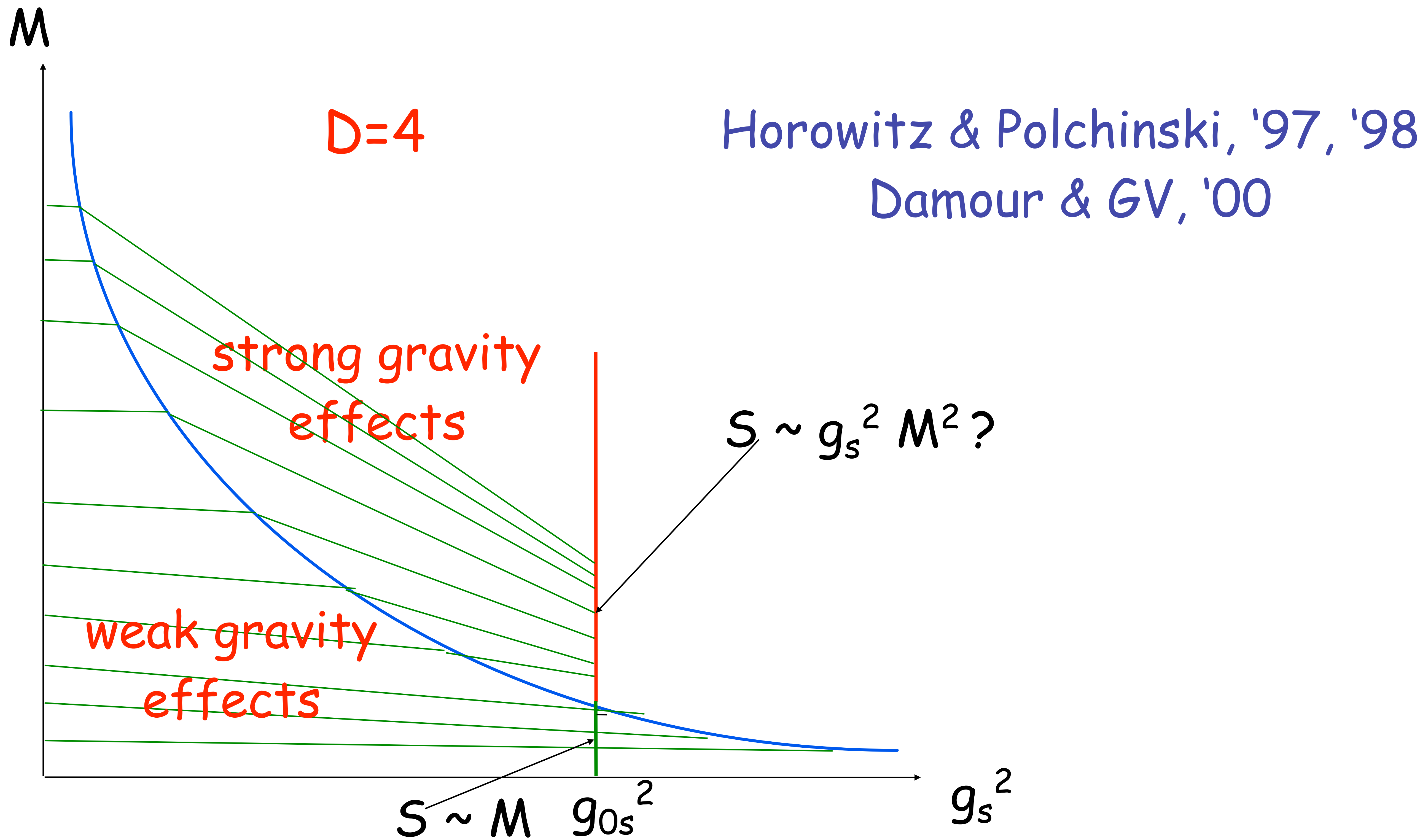
In order to see whether we can have agreement there we would have to compute the effect of interactions when they become non-perturbative.

This is a hard & unsolved problem.

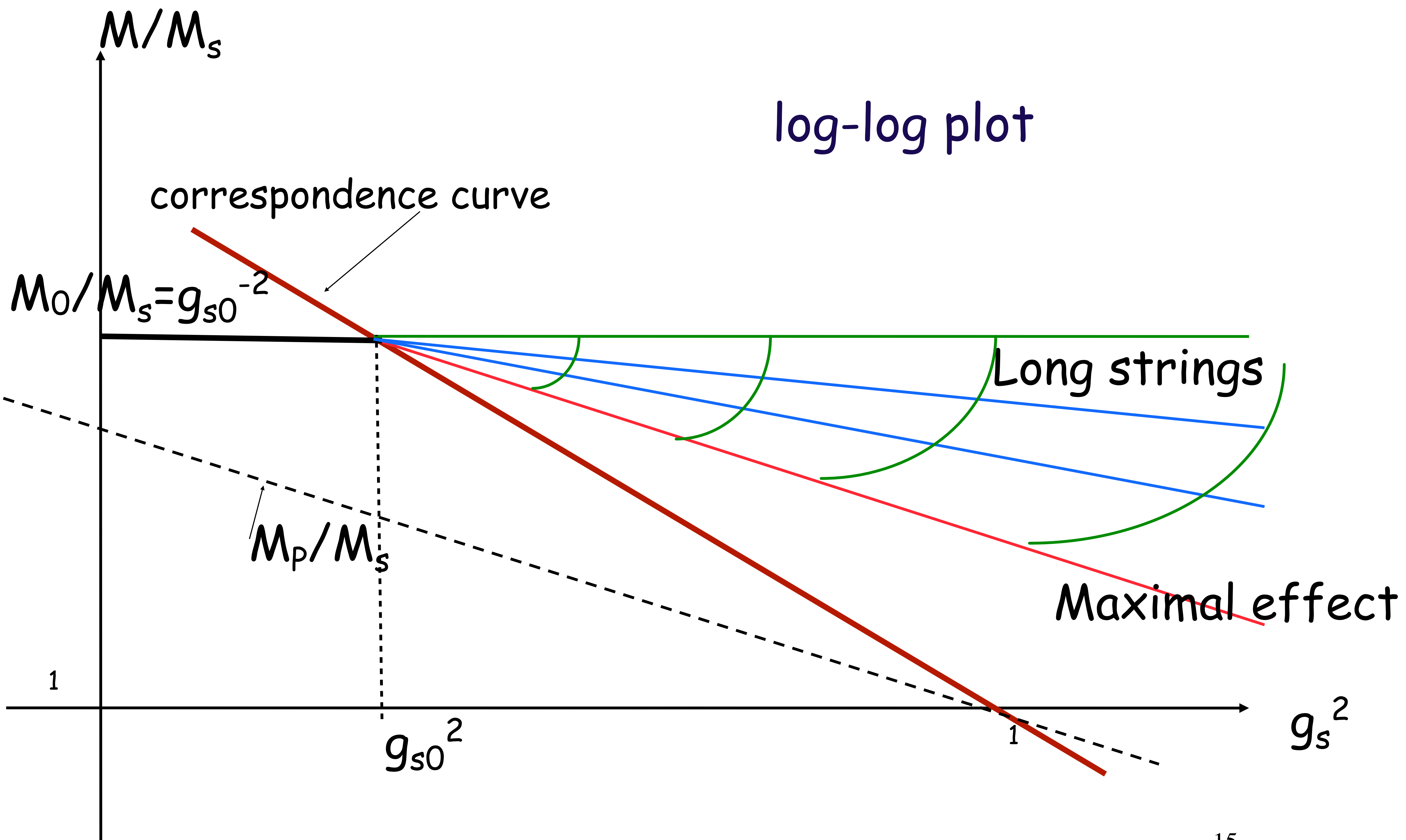
Here is an example of what could possibly do the job,

but looks very contrived (see below for a different hint)

Gravity-induced increase in density of states



Bound on self-gravity effects (D > 4)



Transplanckian-energy strings collisions: stringhole production

(GV: 0410.166 and references therein)

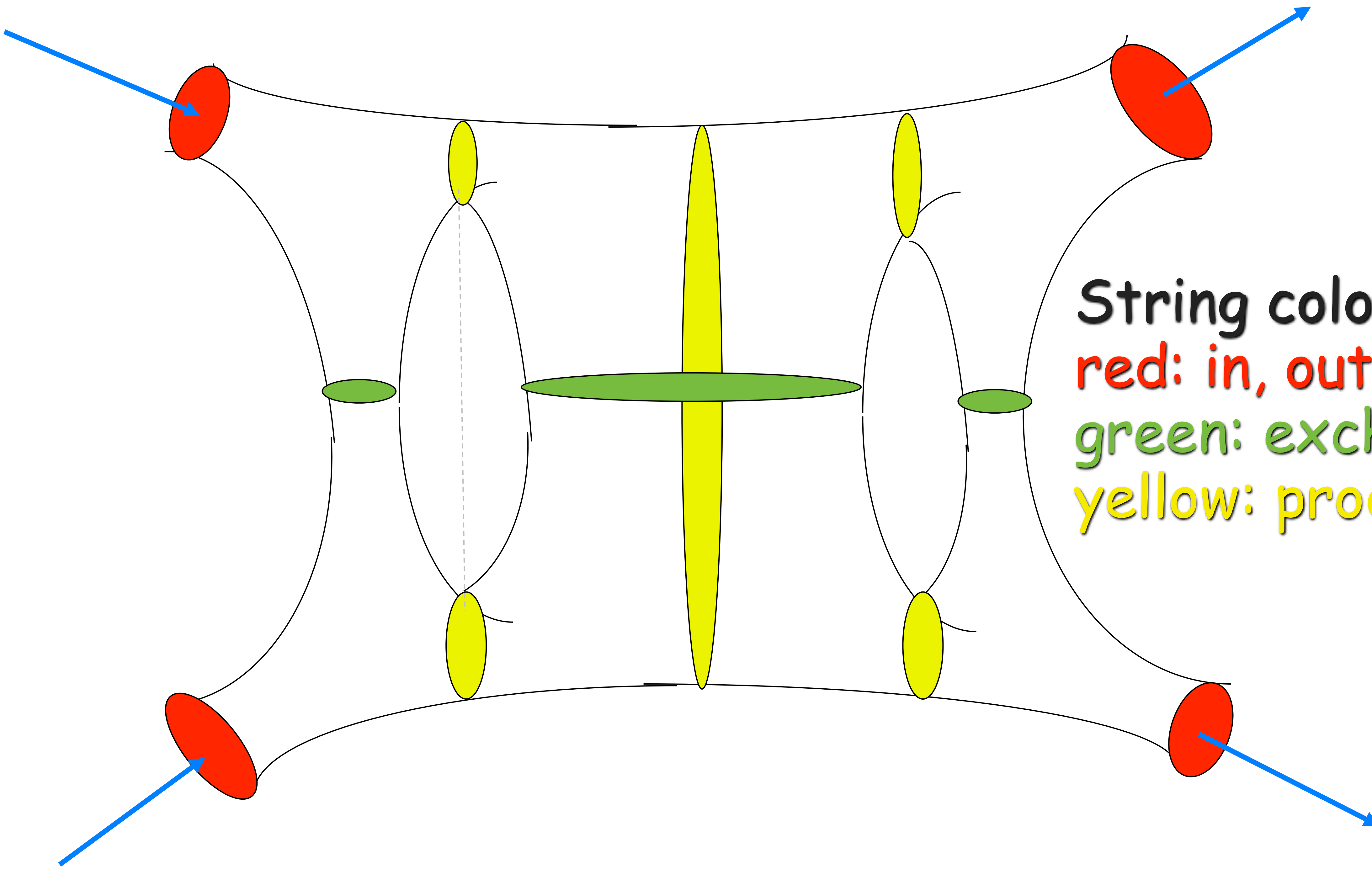
Trans-Planckian-Energy (TPE => $E \gg M_P c^2$, or $Gs/c^5 h \gg 1$) string collisions as a theoretical laboratory for studying deep questions about quantum gravity.

We can hardly imagine a simpler pure initial state that could lead to BH formation and whose unitary evolution we would like to understand/follow.

We do not assume a metric. Calculations are done in flat spacetime and D =10.

An effective metric will emerge at the end.

TPE (closed)string-string collisions
(a two-loop contribution)

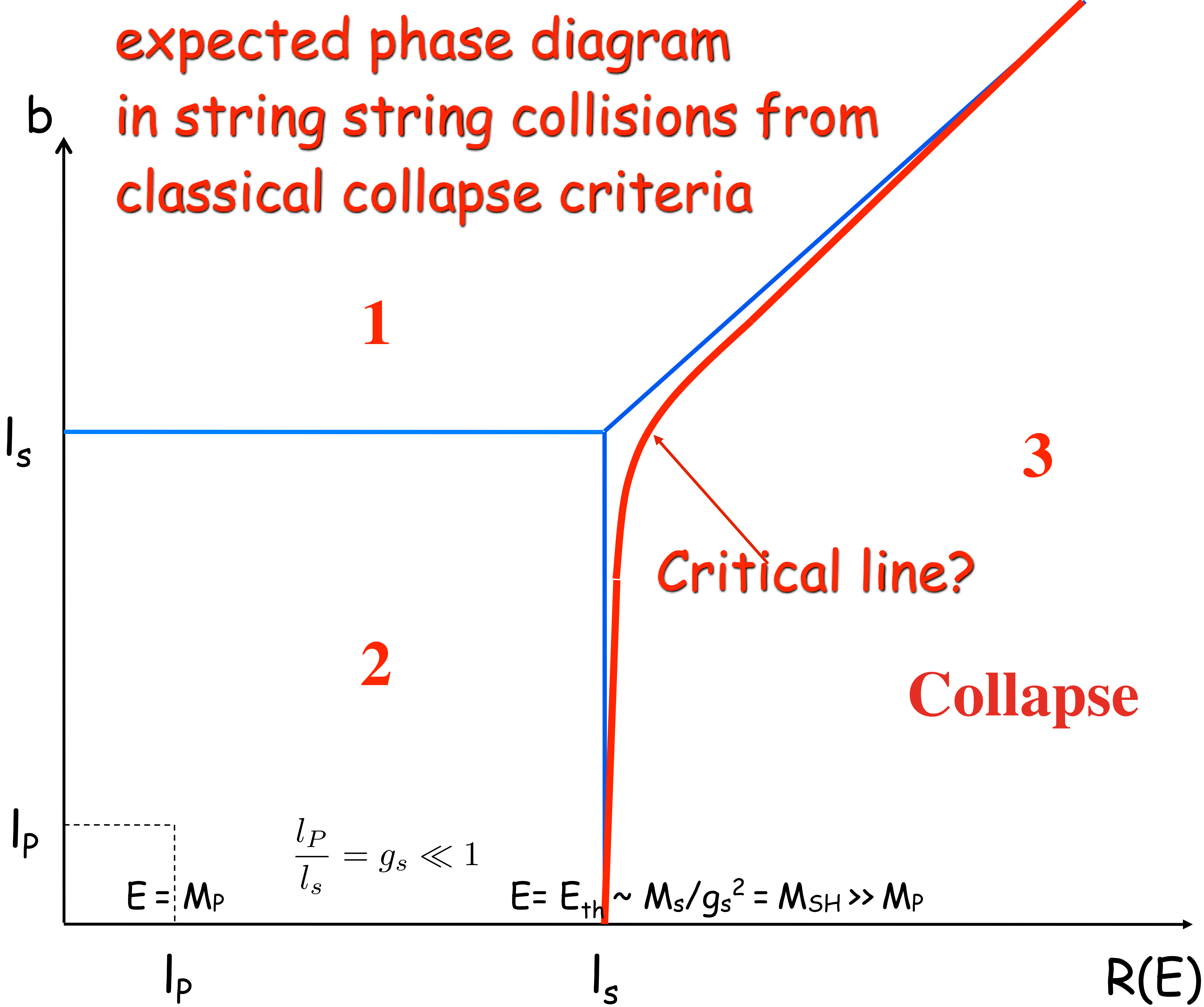


String colour code:
red: in, out
green: exchanged
yellow: produced

Parameter-space for high-energy string collisions

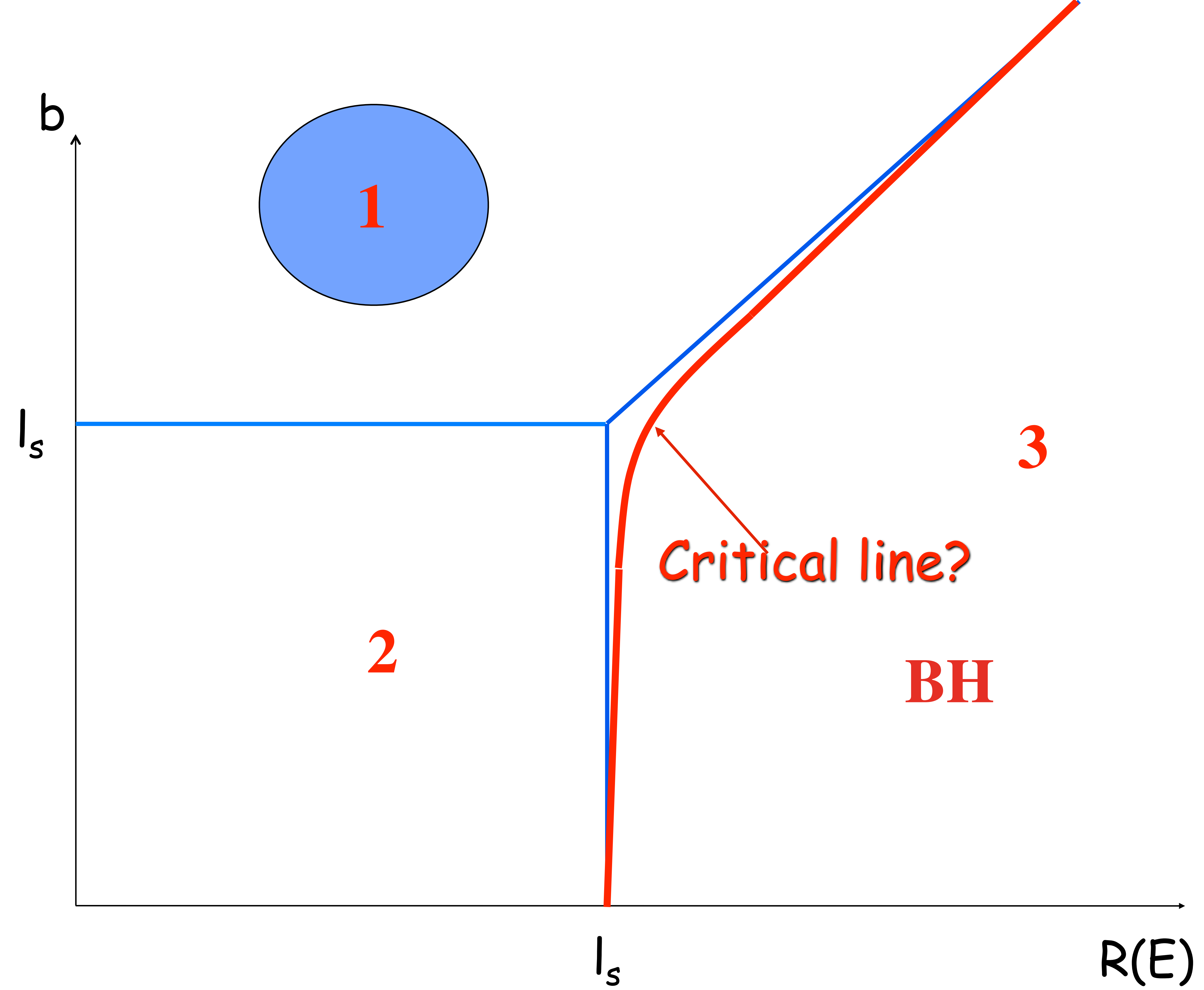
- 3 relevant length scales (neglecting l_P @ g_s << 1)
- Playing with s and g_s we can make R_D/l_s arbitrary

$$b \sim \frac{2J}{\sqrt{s}} \quad ; \quad R_D \sim \left(G\sqrt{s}\right)^{\frac{1}{D-3}} \quad ; \quad l_s \sim \sqrt{\alpha' \hbar} \quad ; \quad G\hbar = l_P^{D-2} \sim g_s^2 l_s^{D-2}$$



expected phase diagram
in string string collisions from
classical collapse criteria
b
l_s
l_P
1
2
3
Critical line?
Collapse
$\frac{l_P}{l_s} = g_s \ll 1$
E = M_P
E= E_{th} ~ M_s/g_s^2 = M_{SH} >> M_P
l_P
l_s
R(E)

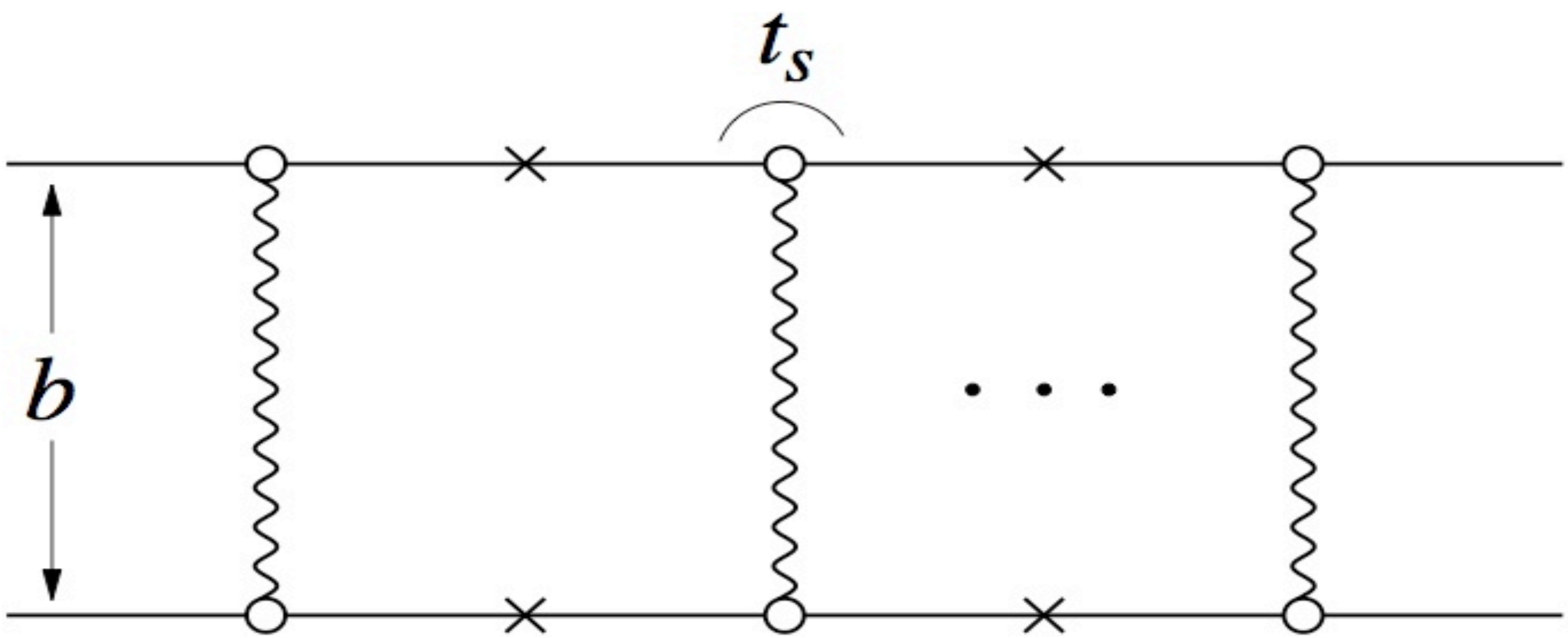
The weak-gravity regime



b
1
l_s
3
Critical line?
2
BH
l_s
R(E)

$$S(E,b) \sim exp\left(i\frac{A_{cl}}{\hbar}\right) \quad ; \quad \frac{A_{cl}}{\hbar} \sim \frac{Gs}{\hbar} c_D b^{4-D} \left(1 + O((R/b)^{2(D-3)}) + O(l_s^2/b^2) + O((l_P/b)^{D-2}) + \dots\right)$$

Leading eikonal diagrams (crossed ladders included)



Particle-particle scattering @ large b

$$S(E,b) \sim exp\left(i\frac{Gs}{\hbar}c_D b^{4-D}\right) \; ; \; S(E,q) = \int d^{D-2}b \; e^{-iqb} S(E,b) \quad ; \quad s = 4E^2 \; , \; q \sim \theta E$$

The integral is dominated by a saddle point at:

$$b_s^{D-3} \sim \frac{G\sqrt{s}}{\theta} \; ; \; \theta \sim \left(\frac{R_S}{b}\right)^{D-3} \; ; \; R_S^{D-3} \sim G\sqrt{s}$$

Generalization of Einstein's deflection formula to ultra-relativistic collisions and arbitrary D. It corresponds precisely to the relation between b and θ in the metric generated by a relativistic point-particle of energy E. This is an effective metric , NOT a class. one!

- At fixed θ, larger E probe larger b (i.e. the IR). How come?
- (Gs/h) b^{4-D} gives the average loop-number. The total q = θ E is shared among as many exchanged gravitons so that:

$$q_{ind} \sim \frac{\hbar q}{G \; s \; b^{4-D}} \sim \frac{\hbar \; \theta \; b^{D-4}}{R^{D-3}} \sim \frac{\hbar}{b_s}$$

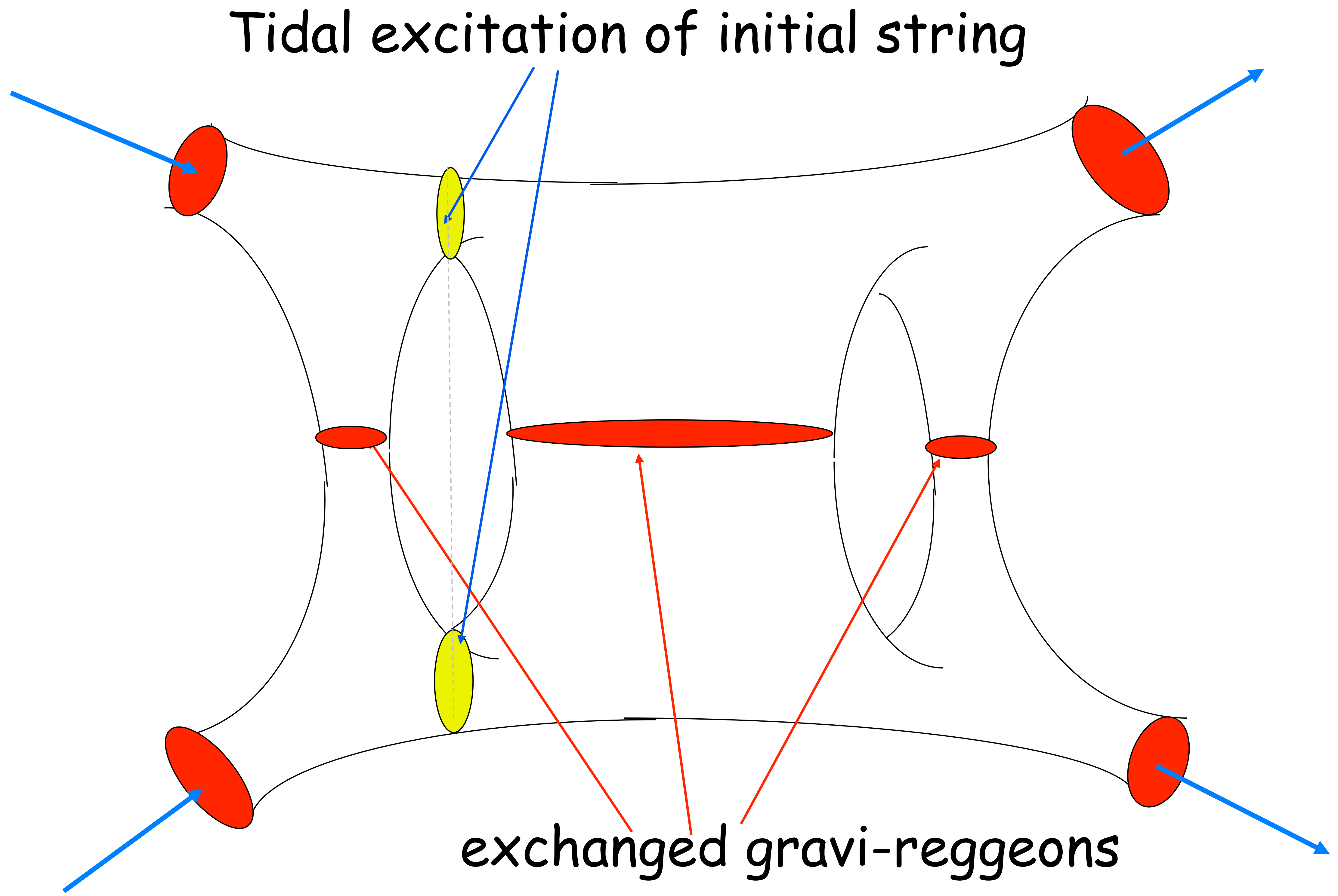
String-string scattering @ large b

(new effects because of imaginary part)

$$S(E,b) \sim exp\left(i\frac{A_{cl}}{\hbar}\right) \quad ; \quad \frac{A_{cl}}{\hbar} \sim \frac{Gs}{\hbar} c_D b^{4-D} \left(1 + O((R/b)^{2(D-3)}) + O(l_s^2/b^2) + O((l_P/b)^{D-2}) + \dots\right)$$

Graviton exchanges can excite one or both strings. Reason (Giddings '06): a string moving in a non-trivial metric feels tidal forces as a result of its finite size. A simple argument gives the critical impact parameter b_t below which the phenomenon kicks-in (as found by direct calculation by ACV). It is parametrically larger than l_s.

$$b_t \sim \left(\frac{Gsl_s^2}{\hbar}\right)^{\frac{1}{D-2}}$$

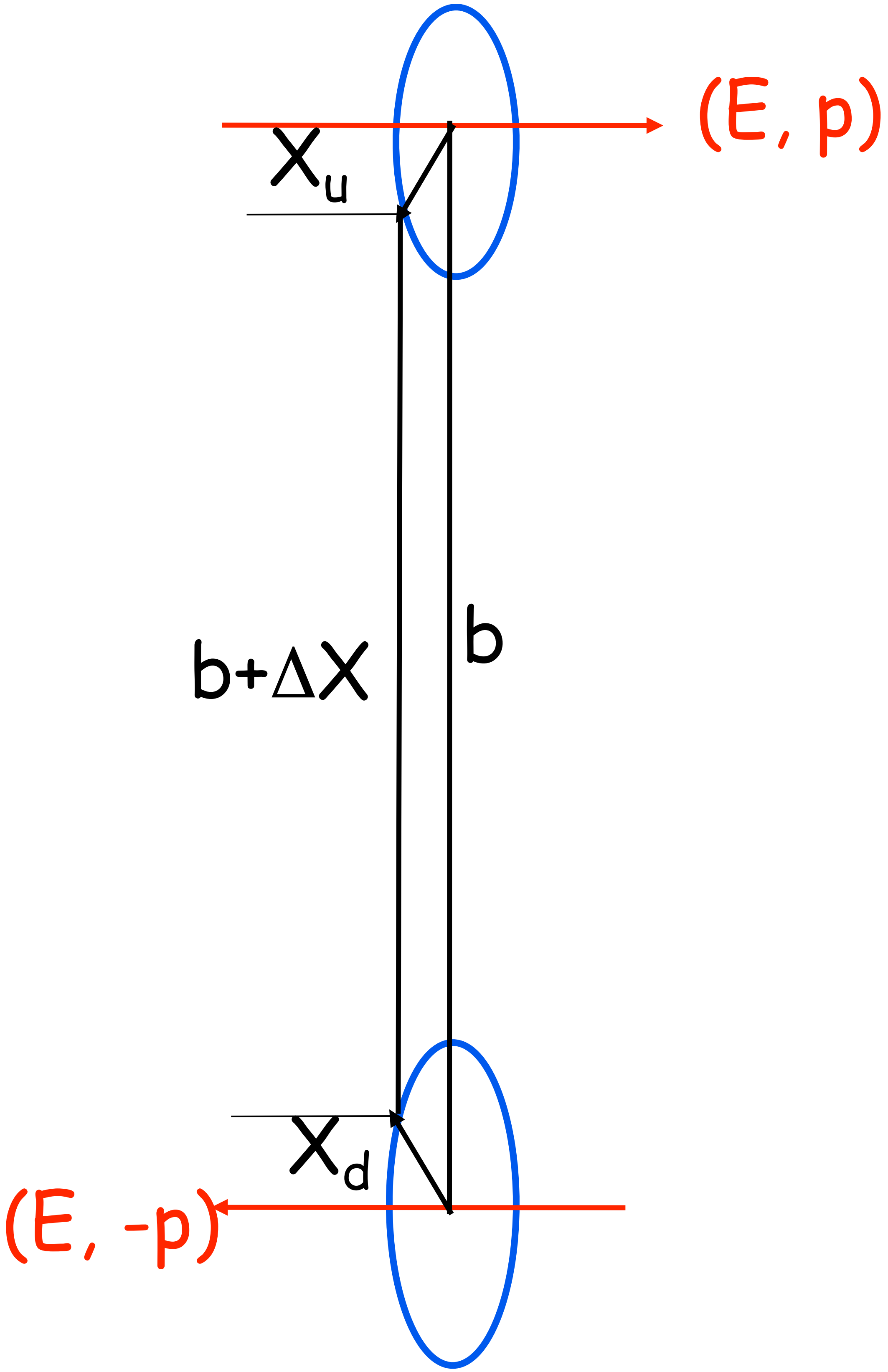


Tidal excitation of initial string
exchanged gravi-reggeons

These effects are neatly captured, at the leading eikonal level, by replacing the impact parameter **b** by a shifted impact parameter, displayed by each string's position operator (stripped of its zero modes) evaluated at $\tau = 0$ (= collision time) and averaged over σ.

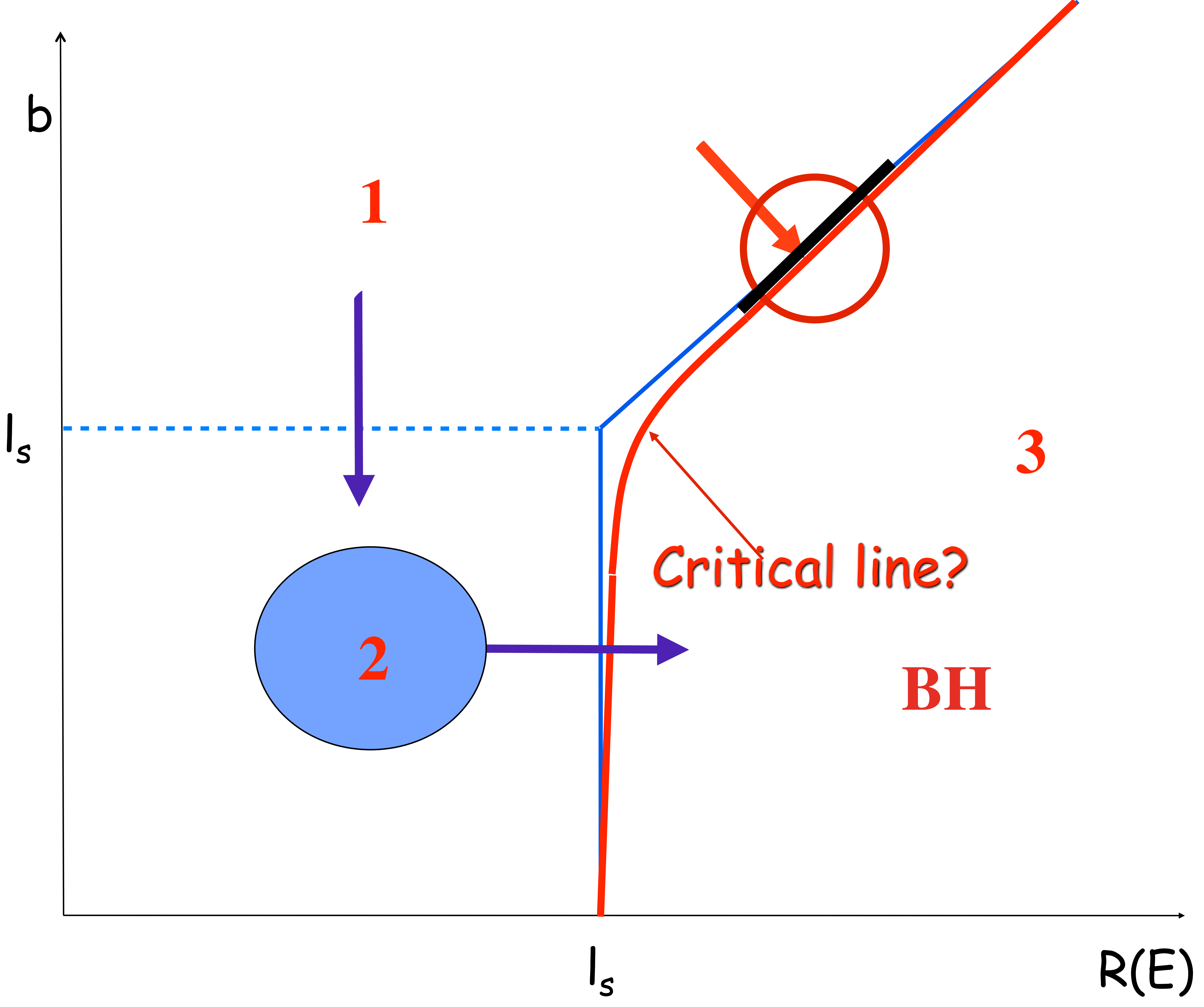
This leads to a (unitary) operator eikonal formula.

More details later...



(E, p)
X_u
b+ΔX
b
X_d
(E, -p)

The string-gravity regime: approaching stringhole production


b
1
l_s
3
Critical line?
2
BH
l_s
R(E)

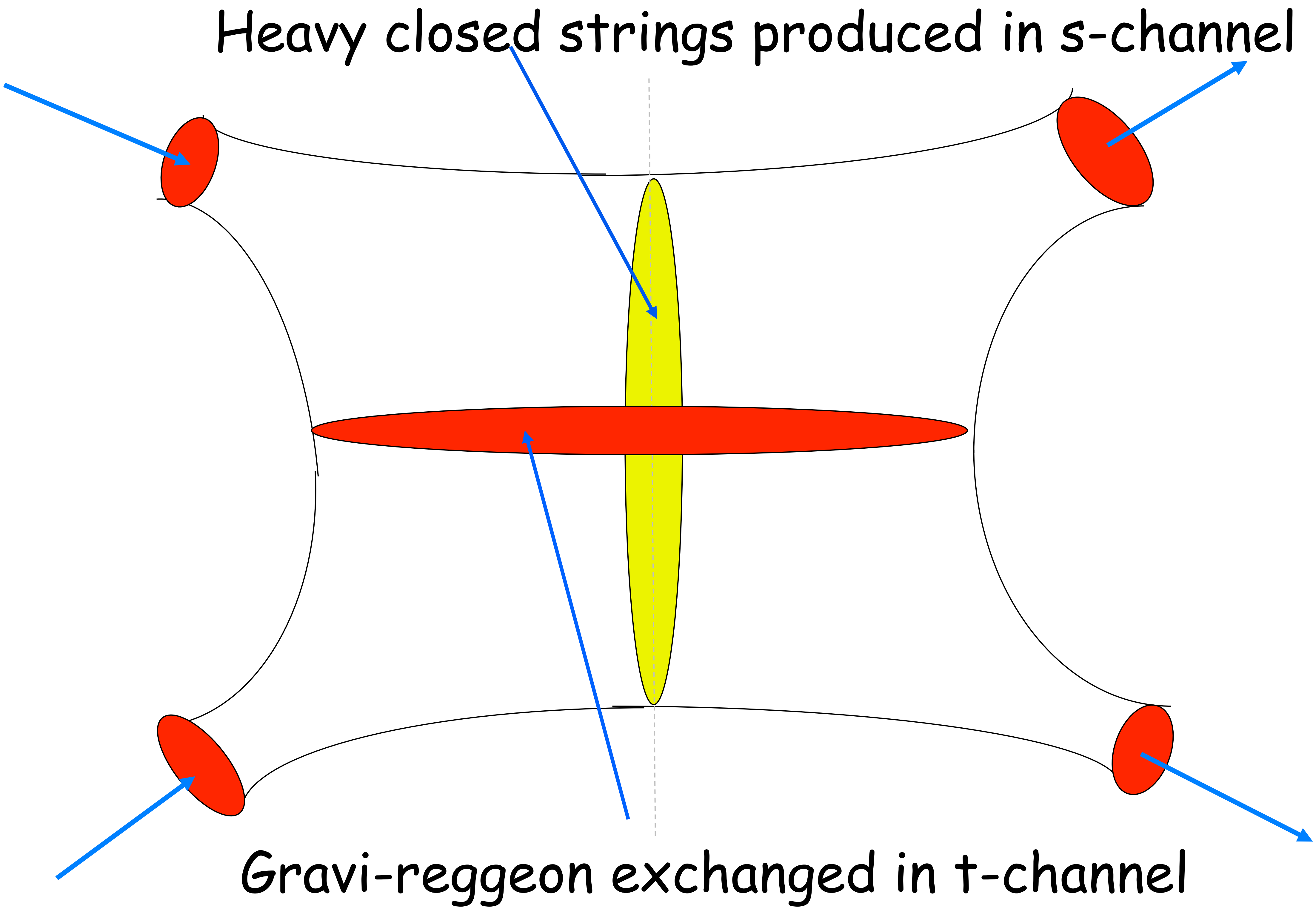
String-string scattering @ b,R < l_s

$$S(E,b) \sim exp\left(i\frac{A_{cl}}{\hbar}\right) \quad ; \quad \frac{A_{cl}}{\hbar} \sim \frac{Gs}{\hbar} c_D b^{4-D} \left(1 + O((R/b)^{2(D-3)}) + O(l_s^2/b^2) + O((l_P/b)^{D-2}) + \dots\right)$$

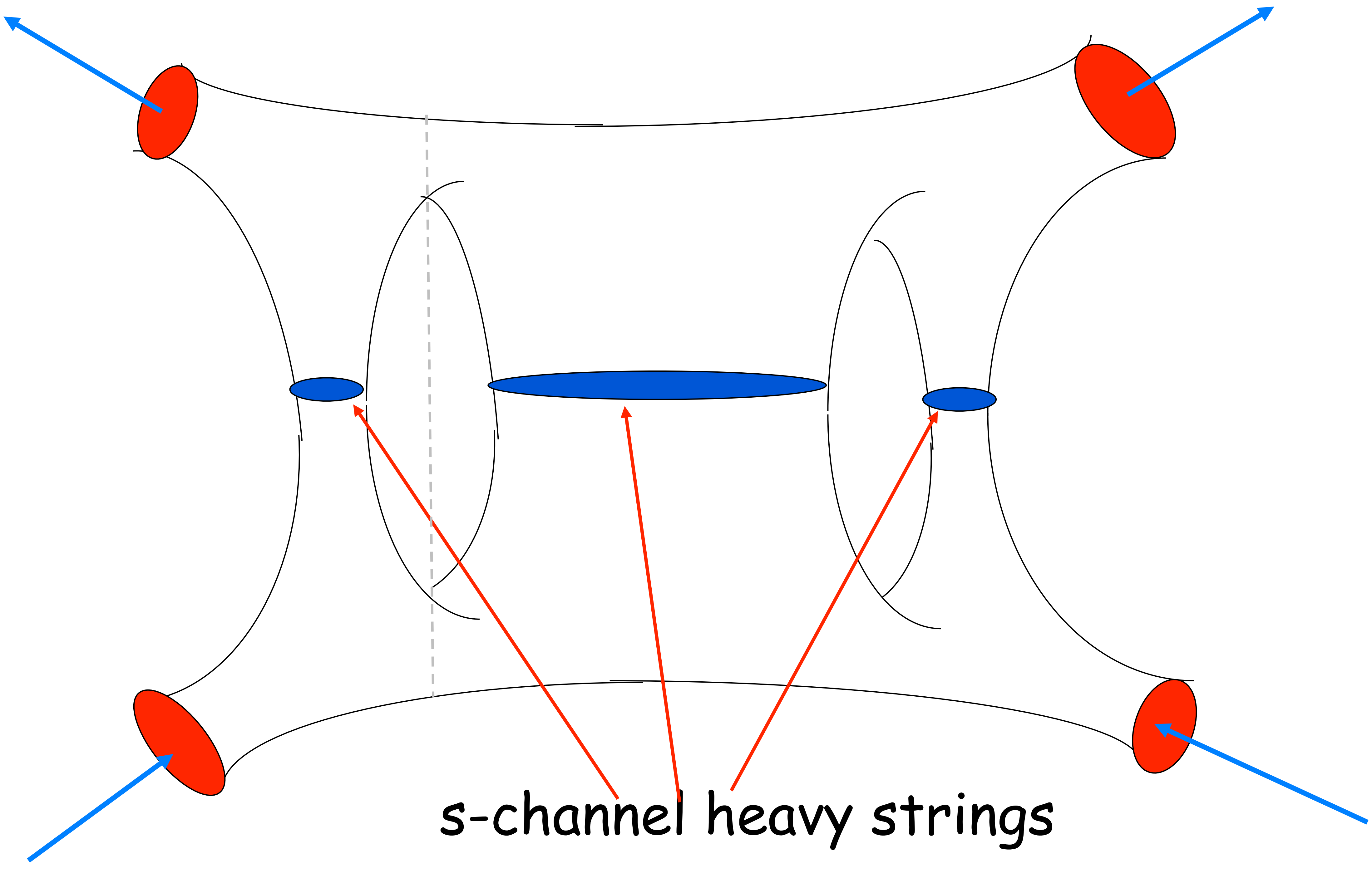
Because of (good old DHS) duality even single graviton exchange does not give a real scattering amplitude. The imaginary part is due to formation of closed-strings in the s-channel.

It is exponentially small at large impact parameter (hence irrelevant in region 1, important in region 2)

Im A is due to closed strings in s-channel (DHS duality)



Turning the previous diagram by 90°



$$\mathrm{Im} A_{cl}(E,b) \sim \frac{G\ s\ l_s^{4-D}}{\hbar} \exp\left(-\frac{b^2}{l_s^2\ \log s}\right)$$

As one goes to impact parameters below the string scale one starts producing more and more strings. The average number of produced strings grows like $Gs \sim E^2$ (Cf. # of exchanged strings) so that, above M_s/g, the average energy of each final string starts decreasing as the incoming energy is increased

$$\langle E_{final} \rangle \sim \frac{M_s^2}{g^2 \sqrt{s}} \to M_s \text{ at } \sqrt{s} = E_{th}$$

Similar to what we expect in BH physics!

Fast grow of <n> & consequent softening: an interesting signature even below the actual threshold of BH production!

A hint on the nature of BHs in String Theory?

If extrapolation to $R_S > l_s$ can be qualitatively trusted it would indicate that above the correspondence line it becomes entropically preferable to break up the heavy string/black hole into its massless decay products.

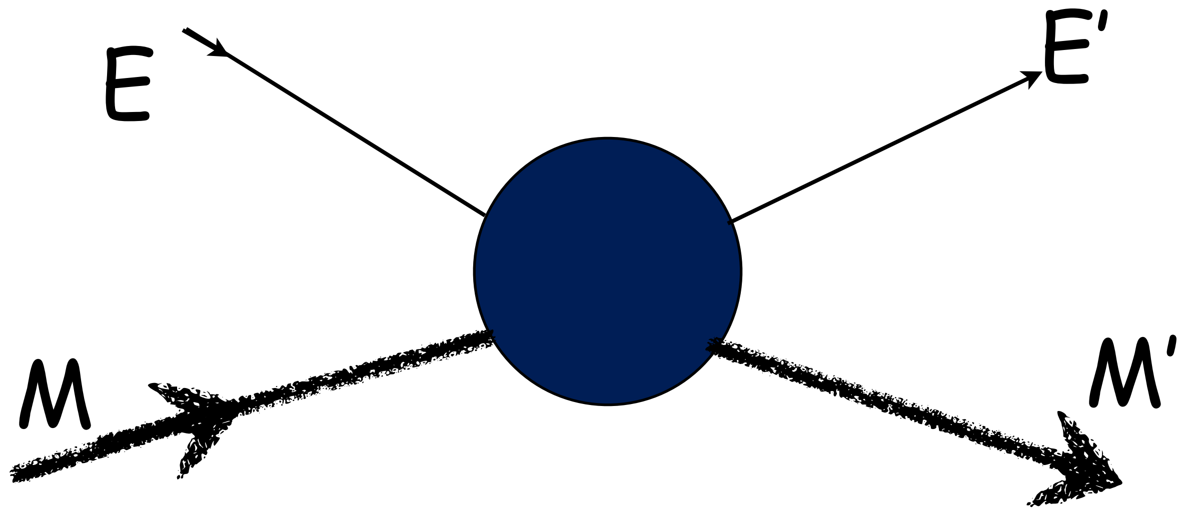
As argued by Dvali and Gomez the number of massless quanta ("gravitons") whose energies add up to the total mass M, and which can bind gravitationally in a region of size R_S, is of order $M R_S/h$, i.e. of order S_{BH}.

Our results appear to lend some credibility to their picture (not necessarily in its details).

Scattering off a stringhole and quantum hair

(GV: 1212.2606)

Scattering of a massless string on a heavy one



kinematical region:

$$M_s M \ll s - M^2 = -2p \cdot P = 2EM \ll M^2$$

Light string acting as a probe

Leading eikonal generalizing ACV-DDRV

$$S(E,M,b) \sim \exp(iS_{cl}) = \exp\left(i\frac{4GEM}{\hbar}c_D b^{4-D}\right) \equiv e^{2i\delta(E,M,b)} \quad ; \quad c_D = \Omega_{D-4}^{-1} \equiv \frac{\Gamma(\frac{D-4}{2})}{2\pi^{\frac{D-4}{2}}}$$

Check of deflection angle:

$$\theta = \frac{8\pi GM}{\Omega_{D-2}b^{D-3}} \sim \left(\frac{R}{b}\right)^{D-3} \ll 1 \quad ; \quad (GM)^{\frac{1}{D-3}} \sim R \ll b$$

Adding tidal excitation a la ACV

$$\delta(E,M,b) \to \hat{\delta}(E,M,b) = \langle \delta(b+\hat{X}_H-\hat{X}_L)^{4-D}\rangle = 2GEM\hbar^{-1}c_D\langle (b+\hat{X}_H-\hat{X}_L)^{4-D}\rangle$$

$$\langle (b+\hat{X}_H-\hat{X}_L)^{4-D}\rangle \equiv \frac{1}{4\pi^2}\int_0^{2\pi} d\sigma_L \int_0^{2\pi} d\sigma_H : \left(b+\hat{X}_H(\sigma_H,0)-\hat{X}_L(\sigma_L,0)\right)^{4-D} :$$

Expansion of phase shift operator in l_s/b:

$$2(\hat{\delta}-\delta)=\frac{2\pi GEM(D-2)}{\hbar\Omega_{D-2}b^{D-2}}\langle Q_H^{ij}+Q_L^{ij}\rangle\hat{b}_i\hat{b}_j$$

$$Q_H^{ij}=\hat{X}_H^i\hat{X}_H^j-\frac{\delta_{ij}}{D-2}\sum_{i=1}^{D-2}\hat{X}_H^i\hat{X}_H^i$$

$$Q_H^{ij}\hat{b}_i\hat{b}_j=\hat{X}_H^i\hat{X}_H^j\left(\hat{b}_i\hat{b}_j-\frac{\delta_{ij}}{D-2}\right)\equiv\Pi_{ij}\hat{X}_H^i\hat{X}_H^j$$

Lorentz-contracted, b-projected quadrupole operator!

Higher multipoles appear at higher orders.

$$\tilde{Q}^{ij}\ \hat{b}_i\hat{b}_j=\Pi_{ij}\ \sum_{n=1}^{\infty}\frac{1}{n}\left(a_n^{\dagger i}a_n^j+\tilde{a}_n^{\dagger i}\tilde{a}_n^j+a_n^i\tilde{a}_n^i+a_n^{\dagger j}\tilde{a}_n^{\dagger j}\right)$$

Using standard techniques we can get S

$$S(E,M,b) = \exp(2i\delta)\ \Sigma_L\ \Sigma_H\ ;\ \Sigma_{L,H} = \exp\left(i(D-2)\Delta\ \tilde{Q}^{ij}_{L,H}\ \hat{b}_i\hat{b}_j\right)$$

$$\Delta = \frac{2\pi GEM l_s^2}{\hbar\Omega_{D-2} b^{D-2}}\quad ;\quad \tilde{Q} = l_s^{-2} Q \qquad \tilde{Q}^{ij}\ \hat{b}_i\hat{b}_j = \Pi_{ij}\ \sum_{n=1}^{\infty}\frac{1}{n}\left(a_n^{\dagger i}a_n^j + \tilde{a}_n^{\dagger i}\tilde{a}_n^j + a_n^i\tilde{a}_n^i + a_n^{\dagger j}\tilde{a}_n^{\dagger j}\right)$$

$$\begin{aligned}
\Sigma_H &= \Sigma^{(univ)}\ \Sigma^{(hair)} \\
\Sigma^{(univ)} &= \Gamma(1+i\Delta)^{D-3}\ \Gamma(1-i(D-3)\Delta) \\
\Sigma^{(hair)} &= :\exp\left(\sum_{n=1}^{\infty}(a_n^{\dagger i}+\tilde{a}_n^i)(a_n^j+\tilde{a}_n^{\dagger j})\left[C_n(\Delta)(\delta_{ij}-\hat{b}_i\hat{b}_j)+\tilde{C}_n(\Delta)\hat{b}_i\hat{b}_j\right]\right): \\
C_n(\Delta) &= -\frac{i\Delta}{n+i\Delta}\ ;\ \tilde{C}_n(\Delta) = C_n(-(D-3)\Delta)
\end{aligned}$$

$$\Delta = \frac{GEMl_s^2}{\hbar\ b^{D-2}} \rightarrow \frac{El_s}{\hbar}\left(\frac{l_s}{b}\right)^{D-2} \qquad 1 \ll \frac{El_s}{\hbar} \ll g_s^{-2} \qquad \left(\frac{l_s}{b}\right)^{D-2} \sim \theta^{\frac{D-2}{D-3}} \ll 1$$

We finally take the heavy string to be a "stringhole"
The resulting S-matrix has many universal factors satisfying the no-hair idea for the SH but it also has terms that probe the quadrupole (and at higher order also other multipoles) of the SH.

This is like some quantum hair of the SH that can be "seen" via our thought experiment.

It turns out to be relatively large, possibly only a power of g_s^2 smaller than the no-hair terms.

If we apply the S-BH correspondence idea, we would conclude that also BHs should have such a large amount of quantum hair (Cf. again Dvali-Gomez's recent papers), but:

Q_1: Are SHs good representatives of BH?

Q_2: Can the situation suddenly change above the CC?

Stringholes and the Big Bounce

(GV: 0312.182)

Consider a contracting Universe classically doomed to end at a big crunch singularity.

It is well known that holographic cosmological entropy bounds get threatened (relaxed) in a contracting (expanding) Universe if $w < 1$.

Consider for instance the Hubble entropy bound

$$\sigma \equiv \frac{S}{V} \leq H l_P^{2-D} \equiv \sigma_{HB}$$

corresponding to one Hubble-size BH per Hubble volume.

In string theory the maximal value of this upper bound is reached for $H=1/l_s$ if we believe that such is the maximal curvature scale.

Using also Friedmann's equation we find (using $e^{\phi} = (M_s/M_P)^{D-2}$):

$$\sigma_{max} \sim l_s^{-1} l_P^{2-D} = e^{-\phi} M_s^{D-1} \quad ; \quad \rho_{max} \equiv \frac{E}{V} = l_s^{-2} l_P^{2-D} = e^{-\phi} M_s^{D}$$

In this case the Hubble-size BHs are, by definition, stringholes. Such a stringhole network has the highest possible entropy & energy density. It's natural to identify it as what replaces the BB singularity.

Interestingly, such a system can evolve adiabatically (i.e. conserve entropy in a fixed comoving volume) provided (in SF):

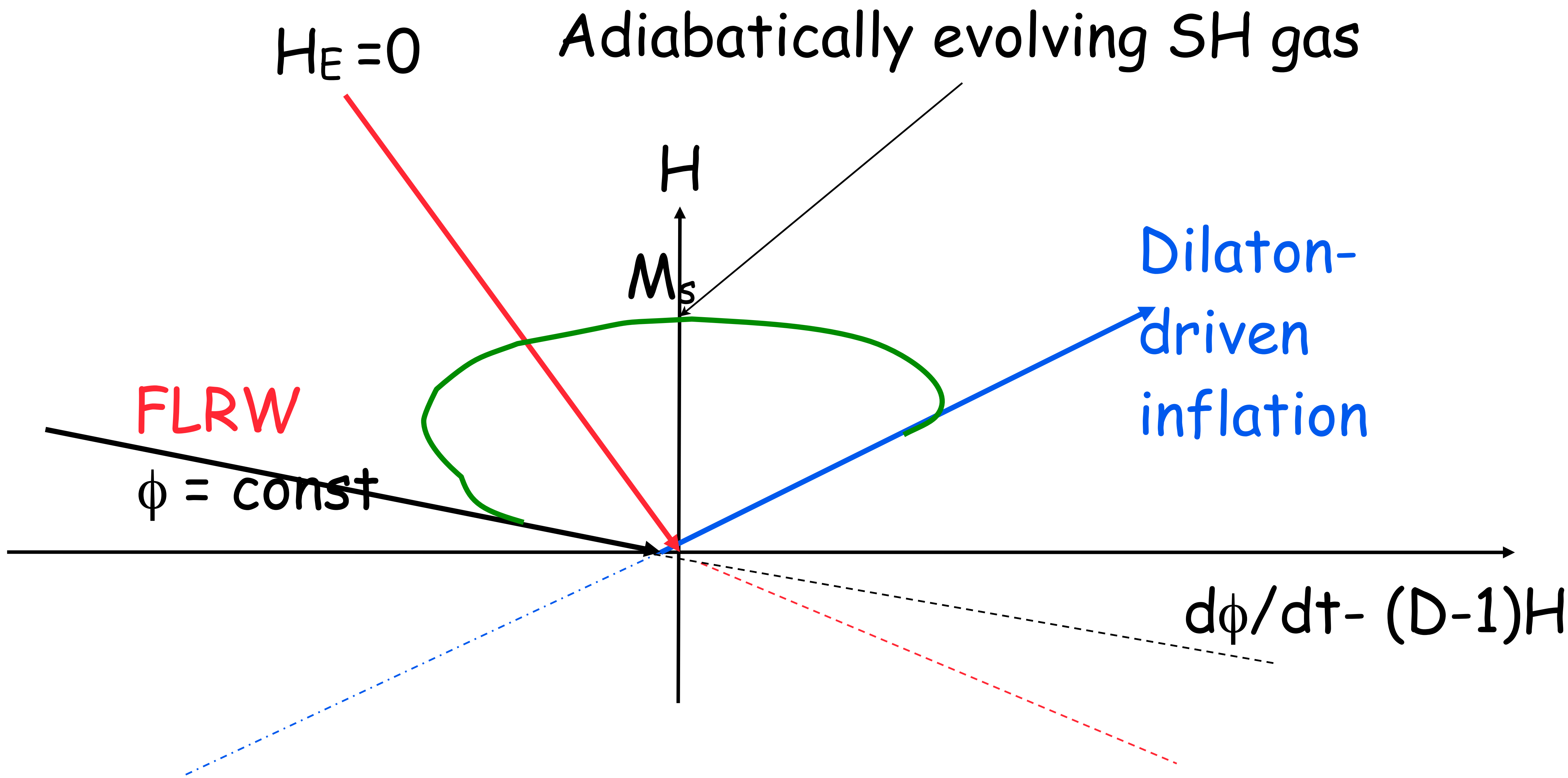
$$S(V) = \sigma V \sim e^{-\phi} V \sim const.$$

This also implies that energy in a fixed comoving volume is constant

$$E(V) = \rho V \sim e^{-\phi} V \sim const. \Rightarrow p = 0$$

In the SF, as the string coupling and the Universe grows the mass of each SH gets redistributed among several lighter SHs so that total mass is conserved and the SHs keep filling all the available space.

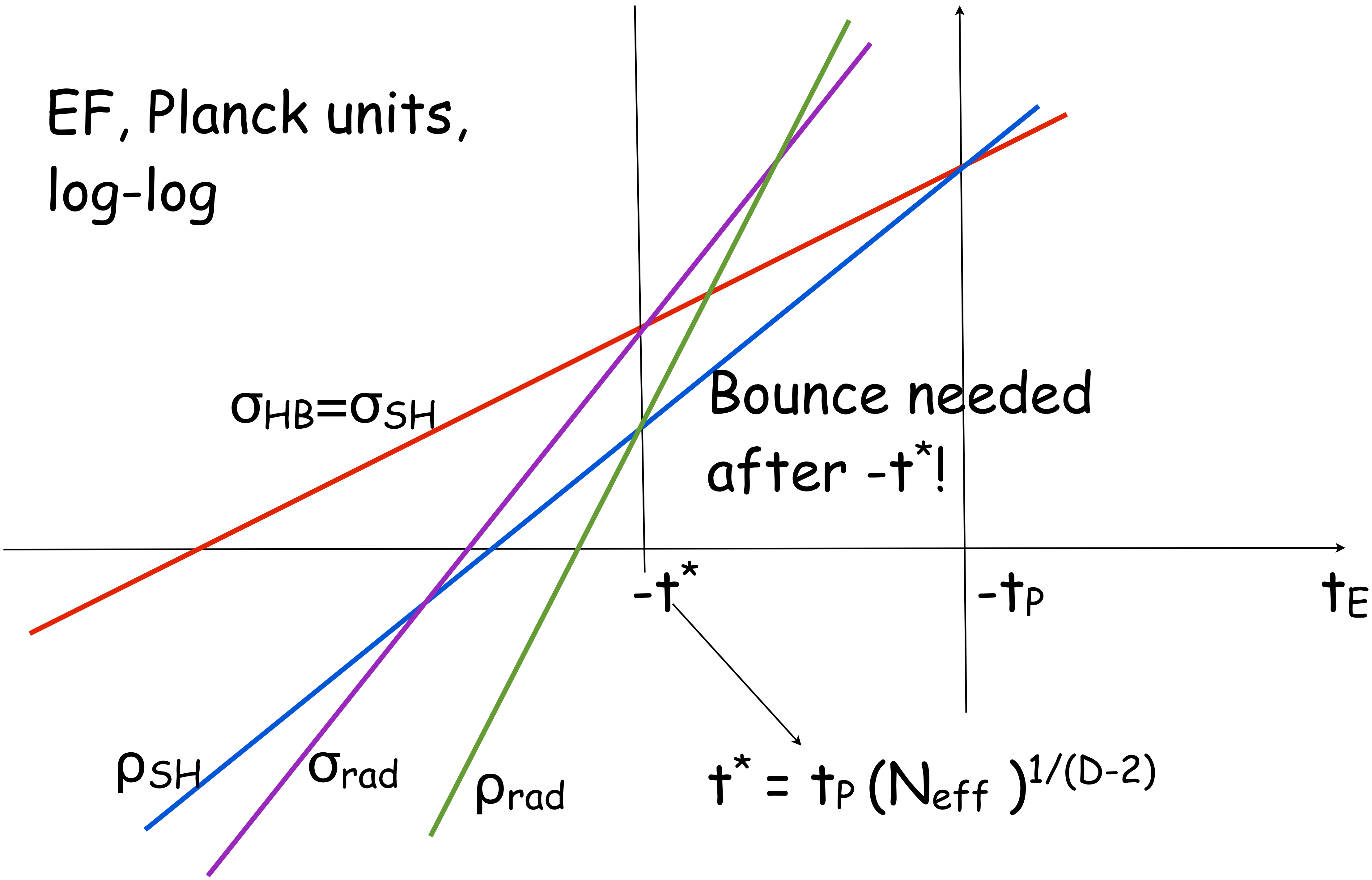
(In the EF the picture is somewhat different: the Universe shrinks and so does the Hubble radius and the SH size.
dE = - p dV is fulfilled thanks to a w=1 equation of state: cf. black crunch idea of Banks and Fischler)



When $(M_s/M_P)^{D-2}$ reaches a critical (maximal?) value $O(1/N_{eff})$ this (supposedly) maximal entropy becomes equal to the entropy of a radiation gas with N_{eff} species. It looks as if, beyond that limit, it is entropically favorable for the SHs to decay into massless strings and for $(M_s/M_P)^{D-2}$ to freeze. But then we MUST have an EF bounce!

Q: How did we achieve the bounce vis a vis energy conditions etc?

Bounce is necessary to avoid violation of entropy bounds!



Conclusion

The string-black hole correspondence (and stringholes) can be useful tools for testing quantum-string gravity ideas in a regime still (though barely?) under control.

And the game can be lots of fun!

Thank you!

String-string vs. string-brane scattering @ b, $R_s < l_s$

In string-string scattering:

$$\langle n_{closed} \rangle \sim \frac{E R_S}{\hbar} \left(\frac{R_S}{l_s} \right)^{D-4} \quad \Rightarrow \quad \langle E_{closed} \rangle \sim M_s \left(\frac{l_s}{R_S} \right)^{D-3} \sim \frac{M_s^2}{g_s^2 E}$$

If extrapolated to $R_S > l_s$ this gives only massless string modes (Hawking radiation?). Can it be trusted?

In string-brane scattering (DDRV, in progress):

$$\langle n_{open} \rangle \sim \frac{E l_s}{\hbar} \left(\frac{R_p}{l_s} \right)^{7-p} \quad \Rightarrow \quad \langle E_{open} \rangle \sim M_s \left(\frac{l_s}{R_p} \right)^{7-p} \sim M_s (g_s N)^{-1}$$

Now the calculation should be reliable even for $R_p > l_s$. This is where we should be able to make contact with a CFT living on the brane system.