"Is the Wheeler de Witt Equation the way to quantum cosmology?"

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DeWitt, B.S., "Quantum Theory of Gravity. I. The Canonical Theory", *Phys. Rev.*, 160, 1113-1148, (1967)
Wheeler, J.A., "Superspace and the nature of quantum geometrodynamics", in DeWitt, C., and Wheeler, J.A., eds., *Battelle Rencontres: 1967 Lectures in Mathematics and Physics*, (W.A. Benjamin, New York, U.S.A., 1968)

Is there good reason to expect that the early universe will obey the Wheeler de Witt equation (and that equation alone)?

Problematic issues arising as regards this proposal:

1. Time parameter invariance
2. Problem of time
3. Trace free (unimodular) equations
4. Untestability
5. Continuous foundations
6. Does it make any sense?
7. Only unitary dynamics

Halliwell: quantum cosmology [arXiv 0909.2566]

The central object of interest in quantum cosmology is the wave function of a closed universe,

$$\Psi[h_{ij}(x), \Phi(x), B] \quad (1.1)$$

This is the amplitude that the universe contains a three-surface B on which the three-metric is hij(x) and the matter field configuration is $\Phi(x)$.

From such an amplitude one would hope to extract various predictions concerning the outcome of large scale observations.

•

To fix the amplitude (1.1), one first needs a theory of dynamics, such as general relativity.

From this one can derive an equation analagous to the Schrodinger equation, called the Wheeler-DeWitt equation, which the wave function of the universe must satisfy

The Wheeler-DeWitt equation will have many solutions, so in order to have any predictive power, it is necessary to propose a law of initial or boundary conditions to single out just one solution.

And finally, one needs some kind of scheme to interpret the wave function

The Hamiltonian form of the action is given by

$$S = \int dt\, [\dot{\alpha}\, \pi_\alpha + \dot{\varphi}\, \pi_\varphi - NH] \qquad (2.11)$$

where φ is a scalar field and e^α the scale factor. This form of the action exposes the fact that the lapse function N is a Lagrange multiplier which enforces the constraint

$$H = 0 \qquad (2.12)$$

This is just the phase-space form of the constraint. The constraint indicates the presence of a symmetry, in this case

Proceeding naively, we quantize this system by introducing a wave function $\Psi(\alpha, \varphi, t)$ and asking that it satisfy a time-dependent Schrodinger equation constructed from the canonical Hamiltonian (2.10):

$$i\, \partial\Psi/\partial t = H_c \Psi \qquad (2.13),$$

To ensure that the symmetry corresponding to the constraint (2.12) be imposed at the quantum level, we will also ask that the wave function is annihilated by the operator version of (2.12):

$$H\Psi = 1/2\ e^{-3\alpha} [\partial^2/\partial\alpha^2 - \partial^2/\partial\varphi^2 + e^{6\alpha}V(\varphi) - e^{4\alpha}]\ \Psi = 0 \quad (2.14)$$

where the momenta in (2.12) have been replaced by operators using the usual substitutions.

However, since H_c = NH, it follows from (2.13) and (2.14) that the wave function is independent of t;

thus the entire dynamics of the wave function is in fact contained in (2.14) with

$$\Psi = \Psi(\alpha, \varphi).$$

The fact that the wave function does not depend on the time parameter t explicitly is actually characteristic of parametrized theories such as general relativity.

(2.14) is called the Wheeler-DeWitt equation and is the central equation of interest in quantum cosmology.

Technical problems

- Euclidianisation,
- definition of measure
- Divergences

A Peres: “Critique of the Wheeler-DeWitt equation”
arXiv gr-qc/9704061

N P Landsman “Against the Wheeler - DeWitt equation”
CQG 12 (1995) L119

D Wiltshire: AN INTRODUCTION TO QUANTUM COSMOLOGY
gr-qc/0101003

Hermann Nicolai!

1: Time Parameter Invariance

What about the time parameter invariance of GR, basic in the ADM variational formalism?

- GR is NOT time parameter invariant!

Example: Einstein-de Sitter universe

$ds^2 = - dt^2 + a^2(t)\,[dx^2 + dy^2 + dz^2]$

Pressure free matter: $a(t) = t^{2/3}$

$\Rightarrow da/dt = 2/3\ t^{-1/3},\ d^2a/dt^2 = -2/9\ t^{-4/3} < 0$:

The universe is decelerating

Choose different time parameter (as advertised):

$T = 2/3 \ln t \Rightarrow a(t) = \exp(T) \Rightarrow d^2a(T)/dT^2 > 0$:

The universe is accelerating!

Local physics does indeed have a preferred time parameter: e.g. for a Simple Harmonic Oscillator using standard time t, $q(\tau) = A\cos(\omega t)$; these cycles measure physical time t like a metronome (which is why SHO's are used as clocks).

It is perverse to use any other time parameter for local physics

It applies equally to all local physics: each involves time t:

- Newton's laws of motion
- Maxwell's equations
- Schroedinger equation
- Dirac equation
- Einstein's equations

This parameter t is just proper time τ measured along relevant world lines, which provides a preferred time parameter in general relativity theory (defined up to affine transformations):

- ties GR into all local physics

Time parameter invariance

What about the time parameter invariance of General Relativity, as made manifest in the ADM variational formalism?

• The gravitational side of the ADM equations may be time-parameter invariant, but the matter side is not;

• In particular when $L = T - V = \frac{1}{2} m u^2 - V(r)$, rescaling time changes the kinetic energy $T(u)$ while leaving the potential energy $V(r)$ unchanged: will give different orbits.

• Change $t \rightarrow T = f(t)$ leaves L invariant iff $u'^2 = u^2 \Rightarrow f(t) = t + c$.

Hence any solutions with matter present (i.e. all realistic solutions) will not be time parameter invariant

This is part of the ongoing tension between the geometric and matter sides of the Einstein Field Equations

2: Problem of time

Can WdeW be correct in view of the problem of time it leads to?

The WdeW equation says the wave function is independent of time

This means probabilities are unchanging

Nothing happens

But things do happen!

Julian Barbour: *The End of Time*

There is no time: the entire universe and everything in it is static and unchanging.

Why? The Wheeler-de Witt equation

$$\partial\Psi / \partial t = H\,\Psi;$$

General relativity

$$H\,\Psi = 0 \;\rightarrow\; \partial\Psi / \partial t = 0$$

Time is an illusion! Applies to everything!

So how do we get the illusion of change?

The mind reads records.

A comoving proper time ADM formulation avoids this problem, as does a unimodular approach to gravity;

There are preferred time parameters in GR

[proper time along world lines]

Solutions are not time parameter invariant.

Ellis G F R and Goswami R. “ Space time and the passage of time” arXiv:1208.2611

The metric evolution: So if the metric tensor determines proper time, what determines the metric tensor?

The Einstein field equations, of course! Following ADM, the first fundamental form (the metric) is represented as

$$ds^2 = (-N^2 + N_i N^i)dt^2 + N_i dx^j dt + g_{ij} dx^i dx^j$$

where i, j = 1, 2, 3. The lapse function $N(x)$ and shift vector $N^i(x)$ represent coordinate choices, and can be chosen arbitrarily; $g_{ij}(x)$ is the metric of the 3-spaces {t = const}.

The second fundamental form is

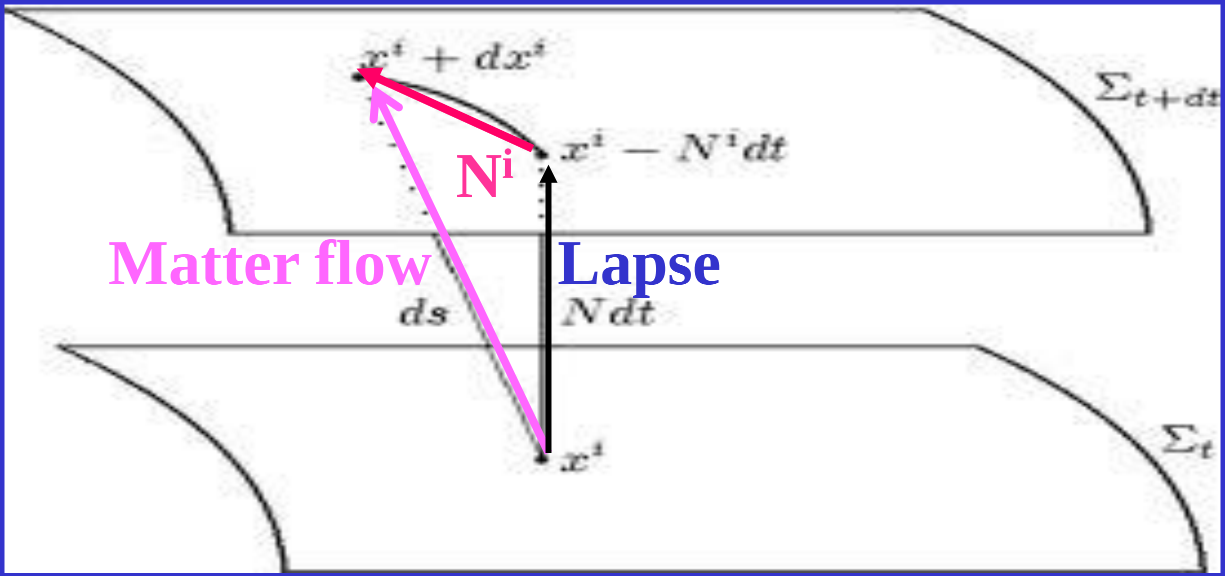
$$\pi_{ij} = n_{i;j}$$

where the normal to the surfaces {t = const} is $n_i = \delta^0{}_i$;

the matter flow lines have tangent vector $u^i = \delta^i{}_0$

(which differs from $n^i = g^{ij}nj$ whenever $N^i \neq 0$)

ADM coordinates:



Shift Vector $N^i(x^j)$ gives the change of the matter lines relative to the normal to the chosen time surfaces

Lapse function $N(x^i)$ gives the relation between coordinate time and proper time along the normal lines

The field equations for $g_{ij}(x^k)$ are as follows (where 3-dimensional quantities have the prefix (3)): four constraint equations

$$^{(3)}R + \pi^2 - \pi_{ij}\pi^{ij} = 16\pi\, \rho_H, \quad \text{(C1)}$$

$$R^\mu := -2\, \pi^{\mu j}{}_{|j} = 16\pi\, T^\mu{}_0 \qquad \text{(C2)}$$

where "$|j$" represents the covariant derivative in the 3-surfaces, and twelve evolution equations

$$\partial_t\, g_{ij} = 2Ng^{-1/2}(\pi_{ij} - 1/2 g_{ij}\pi) + N_{i|j} + N_{j|i}, \quad \text{(T1)}$$

$$\partial_t \pi_{ij} = -Ng^{-1/2}(^{(3)}R_{ij} - 1/2 g_{ij}{}^{(3)}R) + 1/2 Ng^{-1/2} g_{ij}(\pi_{mn}\pi^{mn} - 1/2\pi^2)$$
$$- 2Ng^{-1/2}(\pi_{im}\pi^m{}_j - 1/2\pi\pi_{ij}) + \sqrt{g}(N_{|ij} - g_{ij}N^{|m}{}_{|m}) + (\pi_{ij}N^m)_{|m}$$
$$- N_{i|m}\pi^m{}_j - N_{j|m}\pi^m{}_i + 16\pi\, {}^{(3)}T_{ij}. \qquad \text{(T2)}$$

Equations of state for matter terms in (C1), (C2), (T2) must be added, and the matter conservation equations $T^{ab}{}_{;b} = 0$ satisfied

This can be worked out using any time surfaces (that is the merit of the ADM formalism); in particular one can choose a unique gauge by specialising the time surfaces and flow lines to those defined above

1. We choose the flow lines to be Ricci Eigenlines:

$$T^{\mu}{}_{0} = 0 \Rightarrow R^{\mu} = -\,2\,\pi^{\mu j}{}_{|j} = 0 \qquad \text{(G1)}$$

which algebraically determines the shift vector $N^i(x^j)$, thereby solving the constraint equations (C1);

2. We determine the lapse function $N(x^i)$ by the condition that the time parameter t measures proper time τ along the fundamental flow lines:

$$ds^2 = -\,d\tau^2 \;\rightarrow\; N^2 = 1 + N_i N^i \qquad \text{(G2)}$$

These conditions uniquely determine the lapse and shift. Then,

• given the equations of state and dynamical equations for the matter, equations (T1), (T2) determine the time evolution of the metric in terms of proper time along the fundamental flow lines;

• the constraints (C1), (C2) are conserved because of energy-momentum conservation.

The development of spacetime with time takes place just as is the case for other physical fields, with the relevant time parameter being proper time τ along the fundamental flow lines.

There is no problem with either the existence or the rate of flow of time. Time flows at rate of one second per second, as determined by the metric tensor locally at each event.

The spacetime develops accordingly via (T1), (T2).

Arnowitt, Deser and Misner write of the Hamiltonian formalism as follows:

"Since the relation between q_{M+1} and $\boldsymbol{\tau}$ is undetermined, we are free to specify it explicitly, i.e., impose a "coordinate condition". If, in particular, this relation is chosen to be $q_{M+1} = \boldsymbol{\tau}$ (a condition which also determines N), the action (2.4) then reduces [to] (2.5) with the notational change $q_{M+1} \rightarrow \boldsymbol{\tau}$; the non-vanishing Hamiltonian [only] arises as a result of this process."

- This is the choice made above;
- the corresponding Hamiltonian will be non-zero as indicated in this quote,
- so WdeW will not hold: $\partial\Psi / \partial t \neq 0$.

[as is also the case for unimodular gravity].

3: Trace free (unimodular) equations

A unimodular approach also solves the problem of time

W. G. Unruh and R.M.Wald (1989) ``Time And The Interpretation Of Canonical Quantum Gravity". Phys.Rev. D40:2598.

In fact it should be a spin 2 quantum field equation if we believe the graviton is a spin 2 field; Then the resulting effective equations must be trace free.

G F. R. Ellis, H van Elst, J Murugan, J-P Uzan "On the Trace-Free Einstein Equations as a Viable Alternative to General Relativity" arXiv:1008.1196

The trace free EFE as an alternative to the EFE

- Problem of vacuum energy: QFT vacuum energy suggests Λ huge, discrepant with GR if vacuum gravitates

* MAJOR PROBLEM *

- Vacuum does not gravitate if we use TFE plus separate conservation equations ("unimodular gravity")

- Solves profound contradiction arising between WFT and EFE is we join them in the obvious way
- Then vacuum does not gravitate
- Also solves problem of time in quantum cosmology

  $H \neq 0$ and WdeW does not apply

$R_{ab} - ½ R g_{ab} + \Lambda g_{ab} = \kappa T_{ab}$ (1)

(10 equations) implies

$T^{ab}{}_{;b} = 0$ (2)

Instead, take trace free part:

$R_{<ab>} - ½ R g_{<ab>} + \Lambda g_{<ab>} = \kappa T_{<ab>}$

which is

$R_{ab} - 1/4 R g_{ab} = \kappa (T_{ab} - 1/4 R g_{ab})$ (3)

(9 equations) and assume (2) separately

Recovers (1): but now Λ is a constant of integration and has nothing to do with vacuum energy: which does not gravitate [Weinberg 1989]

Einstein tried this in 1919: but used wrong form

4 possibilities:

$G_{ab} = \kappa T_{ab}$ (a)

$G_{<ab>} = \kappa T_{ab}$ (b)

$G_{ab} = \kappa T_{<ab>}$ (c)

$G_{<ab>} = \kappa T_{<ab>}$ (d)

Only first and last OK

Last solves GR $\Longleftrightarrow$ QFT incompatibility!

Cosmology ok: even though only inertial mass density in EFE; Ok at junction with stars [arXiv:1008.1196]

- Related to Unimodular gravity [Finkelstein, Unruh]
- Variation principle? [Alvarez arXiv:1204.6162]

What does QFT version of gravity say?

- **[Feynman, Deser, Weinberg, Zee]**

- Should also give trace free version!
- Because graviton is symmetric trace free

- Needs to be revisited
- Assume energy momentum conservation separate from gravity equations
- Should get only trace free equations as the graviton can't get a handle in trace equation
- E.g. $L = T^{ab} h_{ab} = T^{ab} h_{\langle ab \rangle} = T^{\langle ab \rangle} h_{\langle ab \rangle}$

- **Should necessarily give Trace Free version of EFE**
- **- these have a good claim to be the correct equations**

4: Untestable

We can test Newton, Schrodinger, Dirac

Can we test WdeW?

Are there lab or collider tests of to prove it actually is valid?

There seems to be no prospect of testing it in any context whatever (so it is a major untested extrapolation from known and tested physics); it only applies in the case of cosmology

Then the results depend on assumed boundary conditions, solution method; one gets an indirect test of the results if it predicts CMB anisotropies or structure formation

- But this is far from unique. A variety of inflation models will work without WdeW.

What else? Why do we perceive time?

Barbour claims there exist records of events that our brains read sequentially, and so create a false illusion of the passage of time. Thus brain processes are responsible for illusion of change.

The prevalent view of present day neuroscience is that mental states $\boldsymbol{\Phi}$ are functions of brain states $\boldsymbol{B}$ which are based in the underlying neuronal states $\boldsymbol{b_i}$, determined by genetics and interactions in the brain, taking place in the overall environment $\boldsymbol{E}$.

$$\boldsymbol{\Phi} = \boldsymbol{\Phi}(\boldsymbol{B}) = \boldsymbol{\Phi}(\boldsymbol{b_i}, \boldsymbol{E}).$$

If time does not flow in microphysics, in an unchanging environment

$$\{d\boldsymbol{b_i}/dt = 0,\ d\boldsymbol{E}/dt = 0\} \Rightarrow d\boldsymbol{\Phi}/dt = 0:$$

Mental states cannot evolve!

Daily life proves this theory is wrong!

We do know is that time does flow in our experience. Hence the assumption that time does not flow in the underlying physics cannot be true: the data proves it to be wrong.

The implication runs the other way: Taking everyday life seriously, and comparing the claim ‘time is an illusion’ with the evidence from mental life, the contradiction between them is proof the WdeW equation does not apply to the universe as a whole at the present time, as proposed by Barbour.

The great merit of Barbour’s book is that it takes the Wheeler de Witt equation seriously, and pursues the implications to their logical conclusion; the evidence from daily life then shows it to be wrong

This argument applies equally to all claims that time is an illusion

- The experience of the flow of time is based in brain physics

5: Not discrete

In its usual form it is not based in discrete foundations,

as one might expect should be the case for any viable theory of quantum gravity, thereby avoiding infinities that are almost certainly unphysical.

other approaches have considered effects of spacetime discreteness on cosmology;

I regard this as a fundamental requirement to avoid unphysical infinities in the theory

Issue: The claimed existence of *physically existing infinities*

- infinity is an unattainable state rather than a number

(David Hilbert: “the infinite is nowhere to be found in reality, no matter what experiences, observations, and knowledge are appealed to.”**)**

not a scientific statement – if science involves testability by either observation or experiment.

One of the worst infinities in physics is the claim of uncountable infinities of physically existing points between points 10cm apart

This has to be wrong!!

Some kind of discrete bottom level structure

Apparent continuity and Lorentz invariance emergent

Coarse grain and the fine detail disappears

(cf. the air in this room)

Other approaches have considered

- discrete spacetime structure :
  causal set theory: Henson, Sorkin, Dowker, et al
  gr-qc/0601121

- effects of spacetime discreteness on cosmology;
  e.g. loop quantum cosmology: but not foundationally discrete

Discrete versions of WdeW have been proposed but have not caught on:

Barrett, J.W., and Crane, L., "An algebraic interpretation of the Wheeler-DeWitt equation",
Class. Quantum Grav., 14, 2113-2121, (1997).:
arXiv:gr-qc/9609030.

Mäkelä, J., "Simplicial Wheeler-DeWitt equation in 2+1 spacetime dimensions",
Phys. Rev. D, 48, 1679-1686, (1993)

Something like this is the way to go!

6: DOES IT MAKE SENSE?

What does the wave function of the universe mean?

How do we realise the meaning of the wave function?

If WdeW alone: only unitary transformations

BUT: If we don’t get outcomes the probabilities don’t apply to anything

The wave function has no physical meaning because does not apply to anything

Taking quantum theory seriously:

Unitary Evolution is not all that happens:

Real QM is non-unitary and irreversible when wave function projection takes place

This is the core of the flow of time:

The indefinite future becomes the definite past

This happens all the time everywhere

It does not need to relate to an experiment.

Quantum physics and Measurement

Schroedinger evolution is unitary and time reversible:

$$\Psi_2 = U_{21}\,\Psi_1$$

But this Is not all that happens!

Collapse of the wave function is where the indeterminateness of the future gives way to the definite state of the past. Things happen in quantum physics!

The outcome is unpredictable from past data: and this is where real dynamic change takes place.

Quantum physics is not time reversible!

Claims that it is (e.g. referring only to Hamiltonian dynamics) ignore measurements – a crucial feature of the theory

- It then has no specific outcomes

If a measurement of an observable ***A*** takes place at time $t = t^*$,

initially the wave function $\psi(x)$ is a linear combination of eigenfunctions $u_n(x)$ of the operator $\tilde{\boldsymbol{A}}$ that represents ***A***:
for $t < t^*$, the wave function is

$$\psi_1(x) = \Sigma_n \psi_n u_n(x). \qquad (1)$$

But immediately after the measurement has taken place,
the wave function is an eigenfunction of $\tilde{\boldsymbol{A}}$:

$$\psi_2(x) = a_N u_N(x) \qquad (2)$$

for some specific value N. The data for $t < t^*$ do not determine the index N; they just determine a probability for the choice N. One can think of this as due to the probabilistic time-irreversible collapse of the wave function.

The initial state (1) does not uniquely determine the final state (2); and this is not due to lack of data, it is due to the foundational nature of quantum interactions.

You can predict the statistics of what is likely to happen but not the unique actual physical outcome, which unfolds in an unpredictable way as time progresses; you can only find out what this outcome is after it has happened.

The data for $t < t^*$ do not determine either N or c_N; they merely determine a probability for each possible outcome, labelled by N, through the fundamental equation

$$p_N = c_N{}^2 = \langle e_N | \psi_1{}^2 \rangle$$

This is where the wave function gets its meaning!

Does the idea of a wave function of the universe as a whole makes any sense?

- rather than localised wave functions that exist everywhere and describe the evolution of local domains in the universe, the behaviour of the whole emerging from the behaviour of the parts;

Why should any quantum description apply to the unuiverse as a whole?

G F. R. Ellis "On the limits of quantum theory: contextuality and the quantum-classical cut"

arXiv:1108.5261

More fundamentally:

- The way physics works is that univeral laws apply at the lowest level of the hierarchy of complexity;
- The effective laws at each higher level need to be deduced from these lower level laws by suitable coarse graining procedures.
- In general the next higher level laws will be different from the lower level laws.

- Thus quantum phyicis applies eveywhere at all times on the lower levels. It will only hold at higher levels if proved to be so.
- Hence there is no a priori reason to believe the WdEW equation will hold globally: it has to be shown to be so.
- And there are good reasons to believe it will not be so (because collapse of the wave function takes place locally).

For detailed argument: see arxiv:1108.526

7 ONLY UNITARY DYNAMICS

If the idea does make sense, with the WdeW equation the only dynamics in operation,

then no wave function reduction to eigenstates takes place,

so no classical universe emerges.

In what way does it makes sense in foundational terms: how are these probabilities realised ?

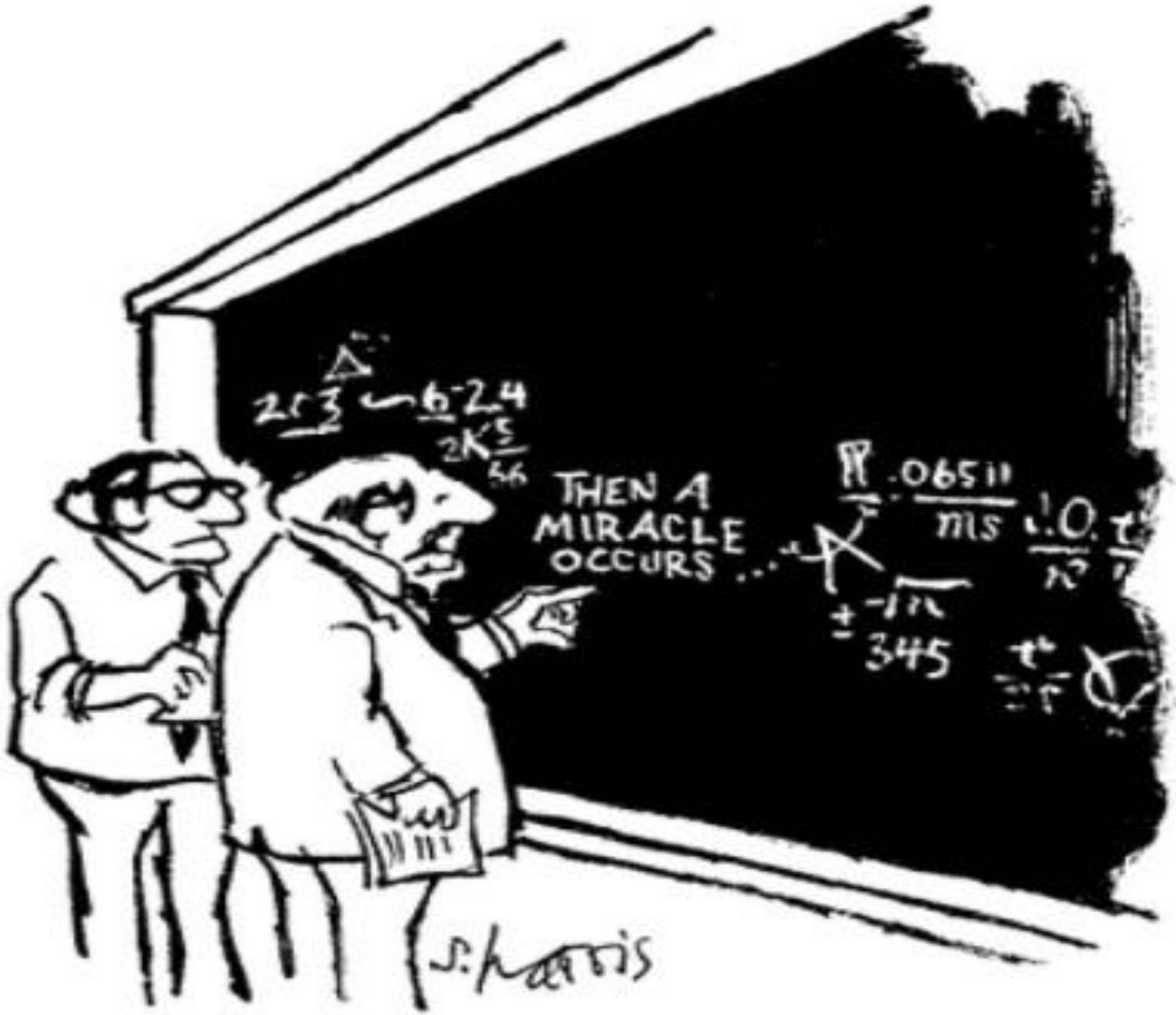
One must have a mechanism whereby this collapse happens or the wave function means nothing: the probabilities it is supposed to represent are never realised.

This is particularly problematic as regards the claimed emergence of classical perturbations from an early universe quantum state, that later lead to the existence of large scale structure in the universe and hence testable cosmological predictions.

This does not make sense without a mechanism whereby classical outcomes emerge

Decoherence does NOT solve it: does not give specific outcomes

An ensemble does not exist if individual members don’t exist



"I think you should be more explicit here in step two."

- *CSL Wave Function Collapse Model as a Mechanism for the Emergence of Cosmological Asymmetries in Inflation*
- Pedro Canate, Philip Pearle, and Daniel Sudarsky arXiv 1211.3463

- Tthe inflationary account for the emergence of the seeds of cosmic structure falls short of actually explaining the generation of primordial anisotropies and inhomogeneities. This description starts from a symmetric background, and invokes symmetric dynamics, so it cannot explain asymmetries. To generate asymmetries, we present an application of the Continuous Spontaneous Localization (CSL) model of wave function collapse[2] in the context of inflation.
- This modification of quantum dynamics introduces a stochastic non-unitary component to the evolution of the inflaton field perturbations. This leads to passage from a homogeneous and isotropicstage to another, where the quantum uncertainties in the initial state of inflation transmute into the primordial inhomogeneities and anisotropies. We examine requirements for, and show how to achieve, compatibility with the precise observations of the cosmic microwave background (CMB) radiation.

Yes I know one can propose the Everett many branch idea as a solution;

- this is also not testable
- apart from the problem of not making clear when wave function branching takes place,
- how often it takes place,
- the problem of getting the Born rule out,
- and above all causing Ockham to spin in his grave particularly on the Deutsch view requiring uncountable infinities of fungible particles for every event.

Here's a question for proponents: how many times has this splitting taken place since the start of the universe?

Conclusion

1: The WdW equation needs to be replaced by a version where $H \neq 0$ so that time flows

2. It needs to be converted to a discrete version

3: It needs to be supplemented by a mechanism for collapse of the wave function so that it’s not the only dynamics in operation

4: The latter is a need for all quantum gravity theories: what process are you proposing leads to classical emergence, given the decoherence won’t do the job?

"Is the Wheeler de Witt Equation the way to quantum cosmology?"

No!

The problems associated with the Euclidean approach to quantum gravity are considerable, however. Firstly, unlike ordinary field theories• such as Yang-Mills theory the gravitational action is not positive-definite, and thus the path integral does not converge if one restricts the sum to real Euclidean-signature metrics.

To make the path integral converge it is necessary to include complex metrics in the sum. However, there is no unique contour to integrate along in superspace and the result one obtains for the path integral may depend crucially on the contour that is chosen.

Furthermore, the measure is ill-defined.

[Wiltshire: AN INTRODUCTION TO QUANTUM COSMOLOGY gr-qc/0101003]

In practice to work with the infinite dimensions of the full superspace is not possible, at least with the techniques that have been developed to date. #

One useful approximation therefore is to truncate the infinite degrees of freedom to a finite number, thereby obtaining some particular minisuperspace model. An easy way to achieve this is by considering homogeneous metrics, since for each point $x \in \Sigma$ there are a finite number of degrees of freedom in superspace. The results we shall obtain by this approach do appear to have some predictive power.

However, the truncation to minisuperspace has not as yet been made part of a rigorous approximation scheme to full superspace quantum cosmology.

As they are currently formulated minisuperspace models should therefore be viewed as toy models, which we nonetheless hope will capture some of the essence of quantum cosmology.

Since we are simultaneously setting most of the field modes and their conjugate momenta to zero, which violates the uncertainty principle, this approach might seem rather ad hoc.

However, in classical cosmology homogeneity and isotropy are important simplifying assumptions which do have a sound observational basis. Therefore it is not entirely unreasonable to hope that a consistent truncation to particular minisuperspace models with particular symmetries might be found in future

Given a solution, Ψ, to the Wheeler-DeWitt equation it is necessary to construct a probability measure in order to make predictions. One central question in quantum cosmology is how one should construct such a measure.

One alternative to the question of the probability measure is to use $|\Psi|2$ directly as a probability measure, by defining the probability of the universe being in a region, A, of superspace by

$$P(A) \propto \int A\, |\Psi|^{2} *1 \quad (3.43)$$

where $*1$ is the volume-element on superspace, $*$ being the Hodge dual in the supermetric.

This definition of a necessarily positive-definite probability density works very well for homogeneous minisuperspaces, for which the volume form $*1$ is independent of $x \in \Sigma$.

This is perhaps not surprising since the assumption of homogeneity reduces the problem to one of quantum mechanics, and $|\Psi|^2$ is of course the probability density in conventional quantum mechanics.

Problems with the definition (3.43) do arise since even in some simple examples the wavefunction is not normalisable, but instead

$$<\Psi|\Psi> = \infty.$$