Quantum gravity and cosmology: will they ever meet?

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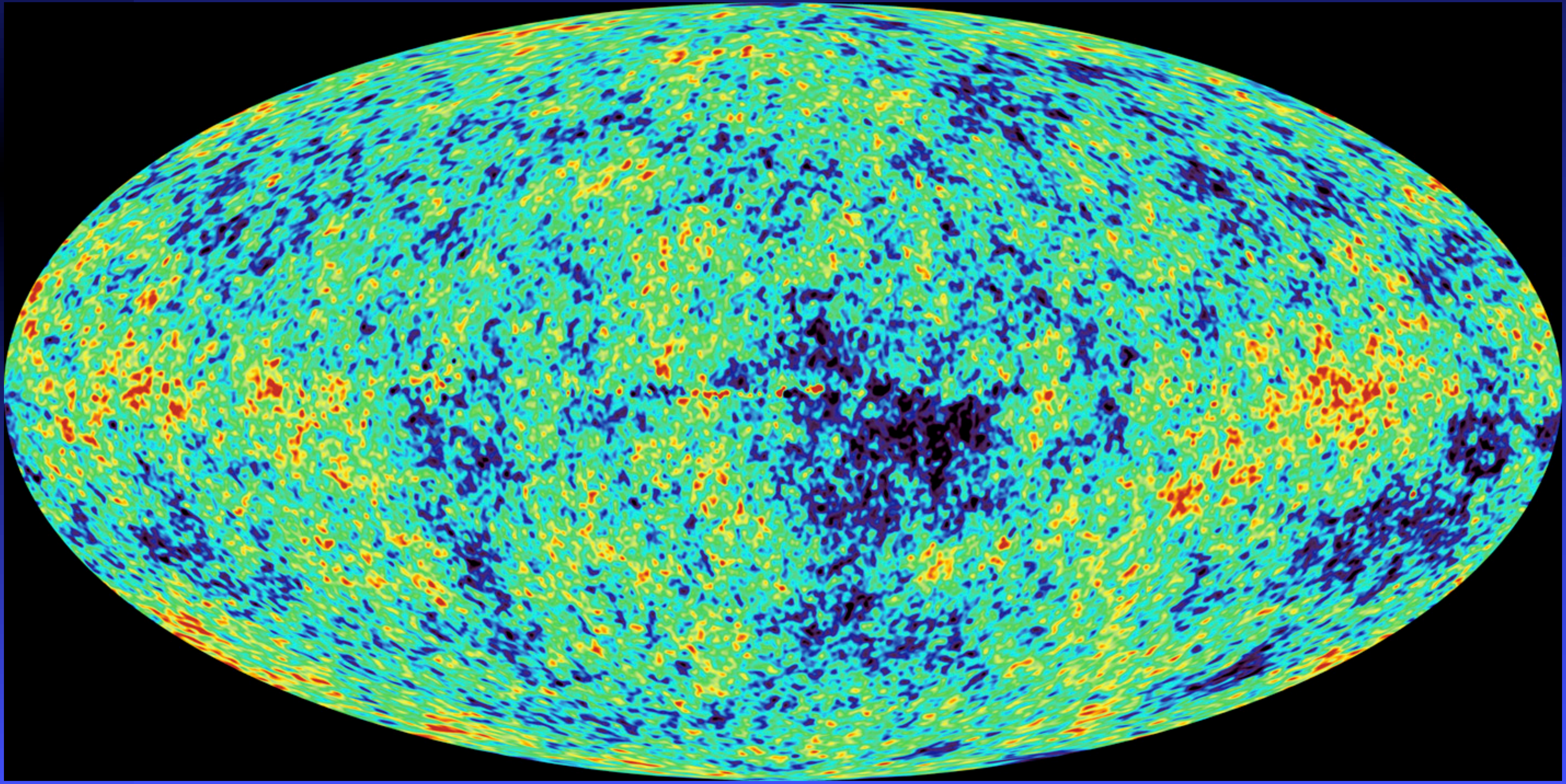
Probably not! Judging from past record…

- The quantum gravity community is either:
  - obsessed with its mathematical navel and treats data and the real world as a venereal disease.
  - or, shows a distinct lack of sociological balls and tries to force contact with mainstream cosmology: i.e. inflation.

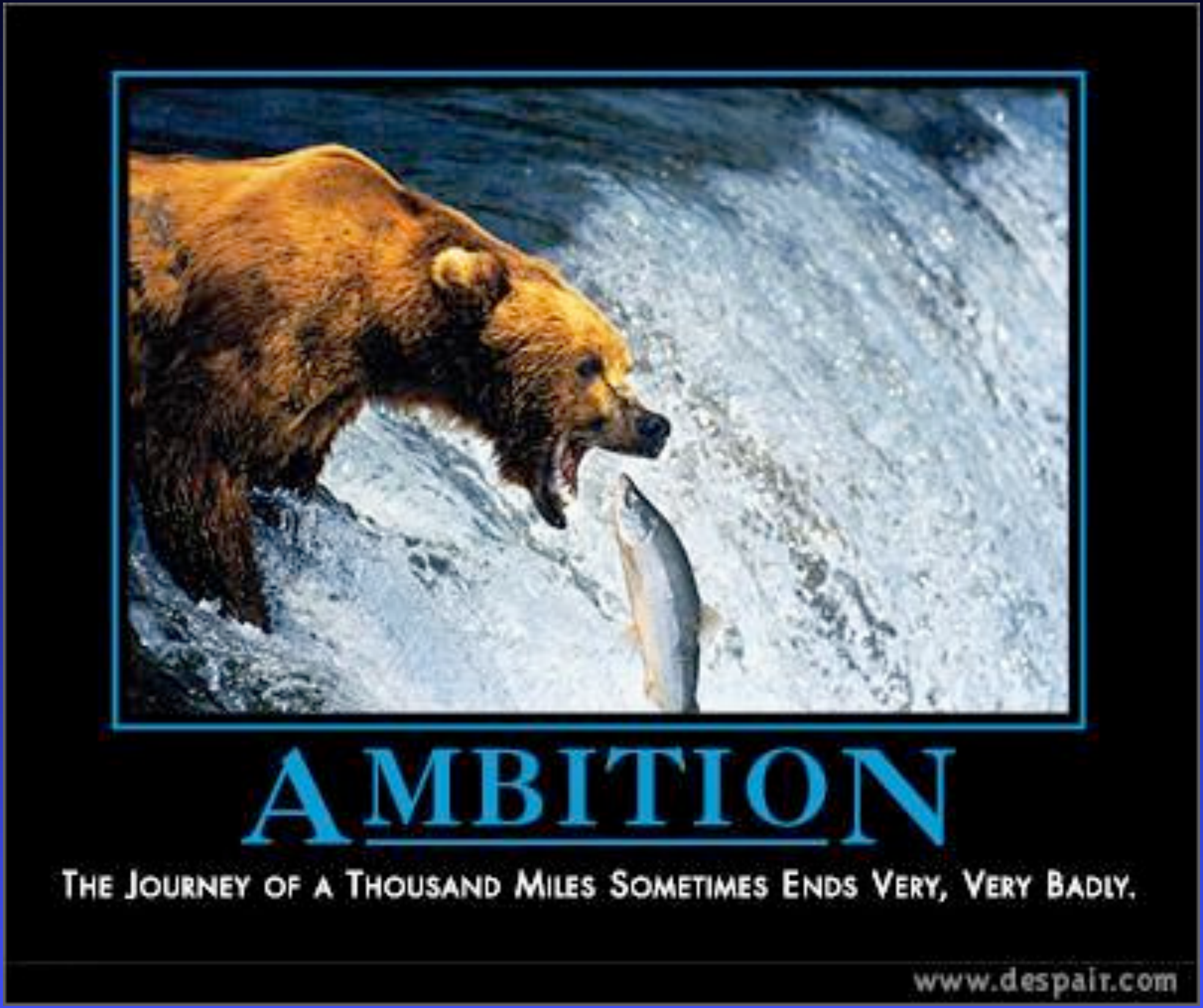
This could be a most inappropriate pairing:

Cosmology has become massively data driven:



A possibility not to be discarded:

- ALL cosmological models available are not needed.
- Quantum gravity explains what they explain DIRECTLY.
- E.g.: Whatever caused geometrogenesis (a transition to semi-classical space-time) explains the initial value problem of cosmology


AMBITION
THE JOURNEY OF A THOUSAND MILES SOMETIMES ENDS VERY, VERY BADLY.
www.despair.com

Examples:

- Hollands and Wald (has problems, but…)
- No boundary proposal
- Holography and thermal fluctuations

$$\mathcal{P}_\Phi(k) = 2\left(\frac{T_c}{T_{Pl}}\right)^2 \left[1 - \left(\frac{l_0 k}{a_c}\right)^{\frac{1}{\gamma}}\right]^2$$

My personal view: too many “conjectures” in this type of work

It could also be that quantum gravity connects better with "off-mainstream" cosmology

- Locality and causality are an afterthought in quantum gravity.
- Lorentz invariance is emergent.
- Varying speed of light models might be a simplistic ("effective") way to capture this feature

Varying c theories

c(x,t)

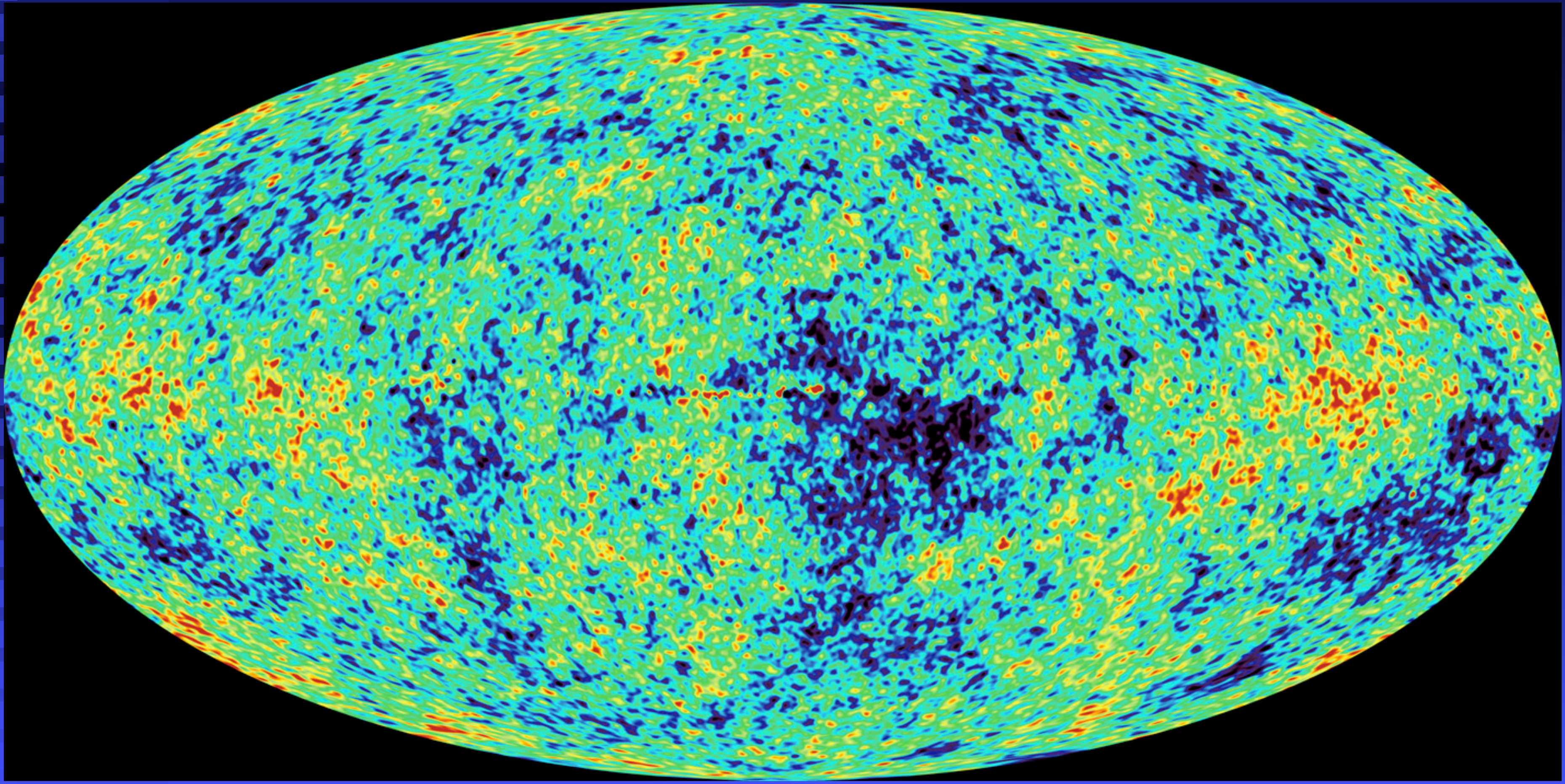
- Covariant and Lorentz invariant [Moffat,Magueijo, etc, etc]
- Bimetric theories [Moffat, Clayton, Drummond, etc, etc]
- Preferred frame [Albrecht, Magueijo,Barrow,etc,etc]

c(E)

- Deformed dispersion relations

[Amelino-Camelia, Mavromatos, Magueijo & Smolin, etc, etc]

The conundrum, part I



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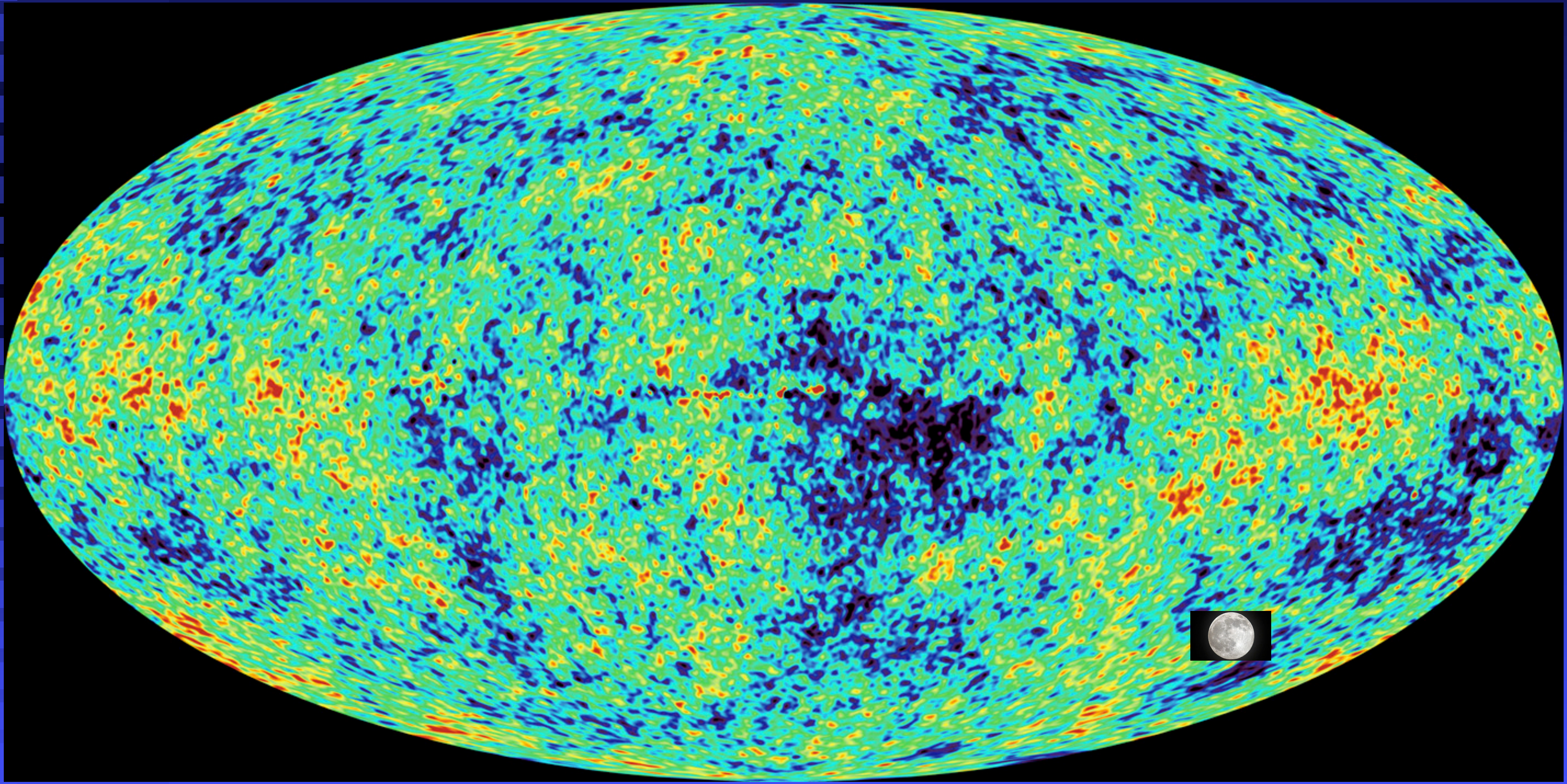
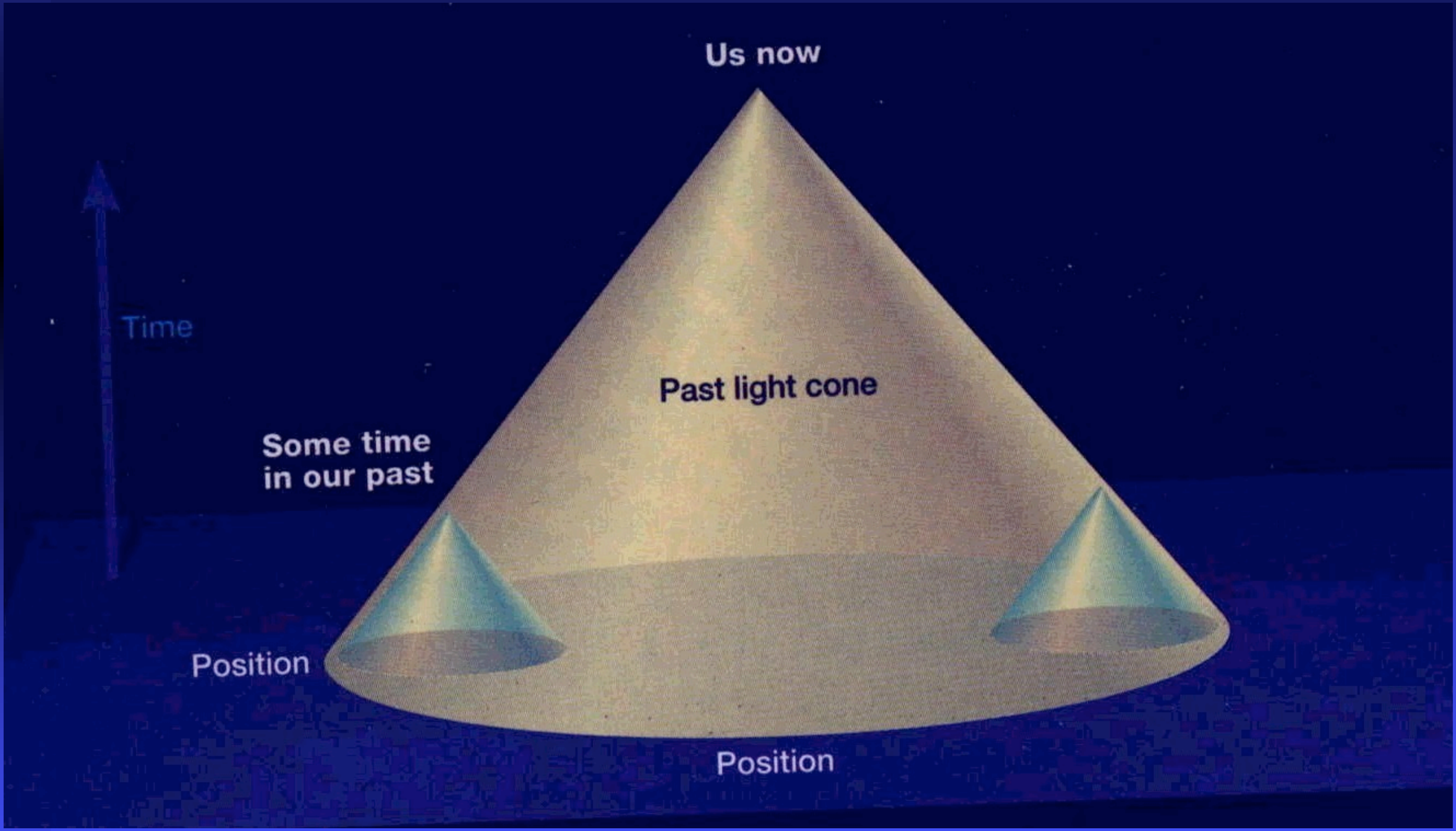
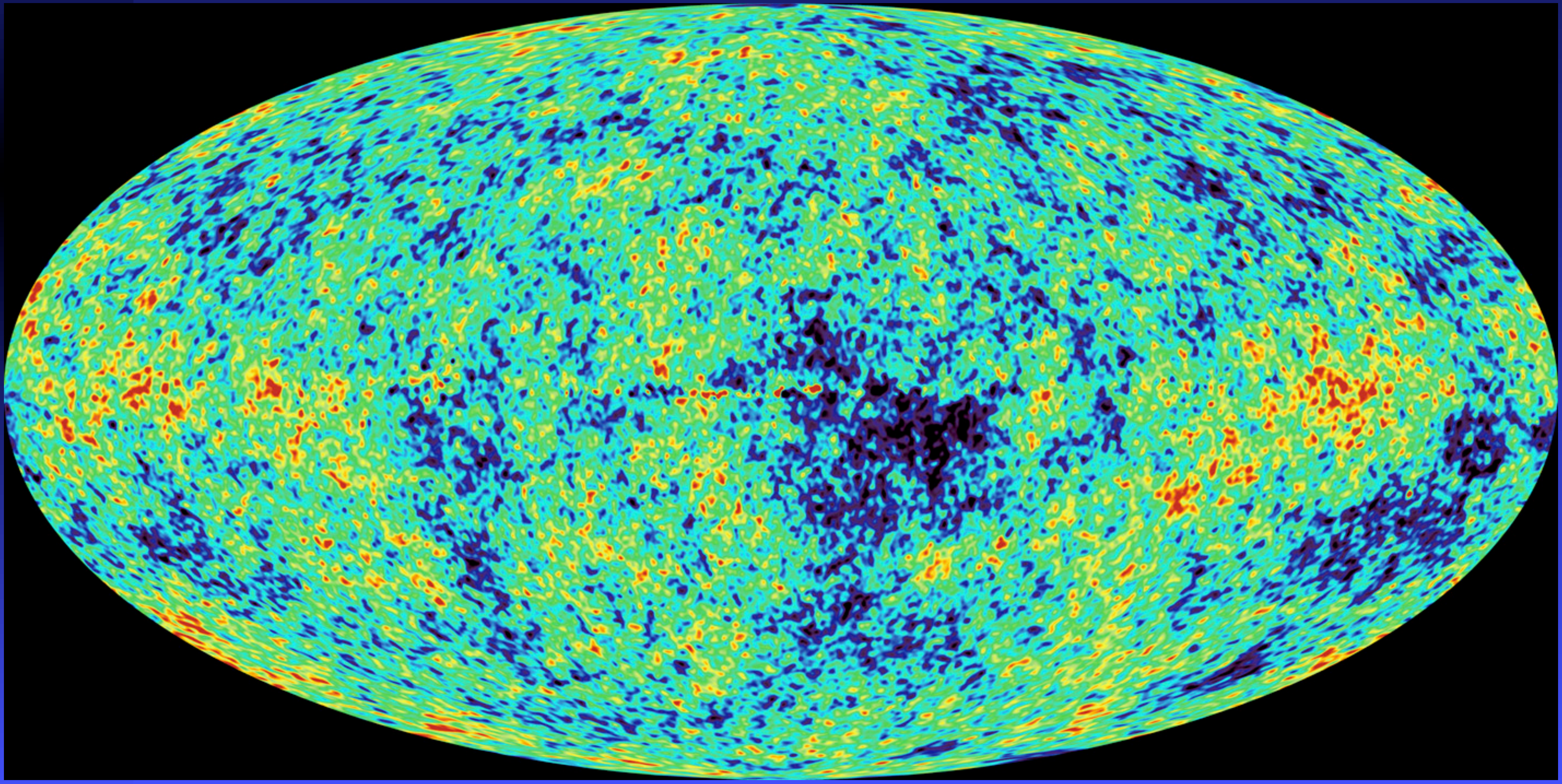


A non-inflationary solution to the horizon problem



But who cares about the horizon problem… Here's the real problem:



The comoving curvature fluctuation

- Take a bunch of cosmological observers (a perturbed Hubble flow)
- Integrate their world-lines into orthogonal surfaces
- Work out the extrinsic curvature ζ and Fourier transform.
- Note that: $[\zeta(k)] = L^{3/2}$

A scale-invariant spectrum

Purely on dimensional grounds:

$$k^3|\zeta(k)|^2 = A^2\left(\frac{k}{k_c}\right)^{n_S-1}$$

A scale-invariant spectrum (Harrisson Zeldovich) must have:

$$n_s = 1$$

The zero-th order "holy grail" of cosmology:

$$k^3|\zeta(k)|^2 = A^2\left(\frac{k}{k_c}\right)^{n_S-1}$$

- Near scale-invariance

$$n_S \sim 1$$

- Amplitude

$$A \sim 10^{-5}$$

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Bimetric theories

A metric for gravity (Einstein frame):

$$g_{\mu\nu} \xrightarrow{gravity} S = \int dx^4 \sqrt{-g}\, R$$

A metric for matter (matter frame):

$$\hat{g}_{\mu\nu} \xrightarrow{matter} S_m = \int dx^4 \sqrt{-\hat{g}}\, L(\hat{g}_{\mu\nu}, \Psi, etc)$$

This is a rather conservative thing to do…

- If the two metrics are conformal, we have a varying-G (Brans-Dicke) theory

$$\hat{g}_{\mu\nu} = e^{\phi} g_{\mu\nu}$$

- If they are disformal we have a VSL theory

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + B\partial_\mu\phi\partial_\nu\phi$$

- The speed of light differs from the speed of

$$+ - - -$$

We can avoid causality paradoxes and have “faster than light travel”

- Always do Lorentz transformations with respect to the appropriate metric (i.e. use 2 copies of SO(3,1), one for gravity one for matter).
- No anti-telephones, etc…

The minimal bimetric VSL theory

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + B\partial_\mu\phi\partial_\nu\phi$$

$$B = B(\phi) = const$$

$$S = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}\,R[g_{\mu\nu}] + \int d^4x\sqrt{-\hat{g}}\,\mathcal{L}_m[\hat{g}_{\mu\nu},\Phi_{Matt}] + S_\phi$$

$$S_\phi = ???$$

What sort of fluctuations come out of these theories?

- If we project onto the Einstein frame, we end up with the same formalism usually used for inflation, but…
- including a varying speed of sound.
- This is the so-called K-inflation (an inflaton with non-quadratic kinetic terms).

The tools of (K-essence) varying speed of sound:

$$\mathcal{L} = K(X) - V(\phi)$$

$$X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

$$\begin{aligned} p &= K - V \\ \rho &= 2XK_{,X} - K + V \end{aligned}$$

$$c_s^2 = \frac{K_{,X}}{K_{,X} + 2XK_{,XX}}$$

Check formulae with inflation, cuscaton, etc…

How to compute fluctuations:

$$\left\{ \zeta = \frac{v}{z} \qquad z = \frac{a}{c_s} \right\}$$

$$v'' + \left[c_s^2 k^2 - \frac{z''}{z} \right] v = 0$$

I *II*

How to compute fluctuations:

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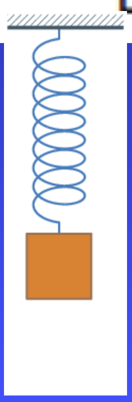

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II

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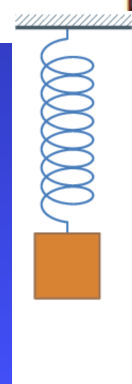

I

II

Why the horizon problem leads to a

- If $c_s = const$
- If $1 + 3w > 0$ (with $w = \frac{p}{\rho}$)

$$v'' + \left[c_s^2 k^2 - \frac{z''}{z}\right] v = 0$$

$$\propto \frac{1}{\eta^2}$$

$$z = \frac{a}{c_s}$$

Dominates
at late times

How inflation solves the problem:

- With $1 + 3w < 0$ $\eta < 0$

$$v'' + \left[c_s^2 k^2 - \frac{z''}{z} \right] v = 0$$

Dominates earlier

Dominates later $\propto \frac{1}{\eta^2}$

- But why do we get scale invariance?

Follow up vacuum quantum

- Consider first the regime $k|\eta| \gg 1$

$$v = \frac{e^{-ik\eta}}{\sqrt{2k}}$$

- With this normalization when we second quantize the amplitudes become creation/ annihilation operators

$$v = \frac{e^{-ik\eta}}{\sqrt{2k}} a$$

- A miracle happens near deSitter (w=-1)

$$v = \frac{e^{-ik\eta}}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right)a$$

- Compute the vacuum expectation value

$$< 0|\hat{v}^2|0 >= v^2 < 0|a^\dagger a + \frac{1}{2}|0 >$$

- In the limit'"' $k|\eta| \ll 1$ we get:

$$< 0|\hat{v}^2|0 > \propto k^{-3}$$

How a varying speed of light solves

- With $1+3w>0$ but $c_s \propto \eta^\beta$ with $\beta < -1$ we still get:

$$v'' + \left[c_s^2 k^2 - \frac{z''}{z}\right] v = 0$$

Dominates earlier

Dominates later

$$\propto \frac{1}{\eta^2}$$

But could this lead to scale-

- Consider first the regime $k|\eta| \gg 1$

$$v = \frac{e^{-ik\eta}}{\sqrt{2k}} \quad \longrightarrow \quad v \sim \frac{e^{ik\int c_s d\eta}}{\sqrt{2c_s k}}$$

- With this normalization when we second quantize the amplitudes become creation/annihilation operators

- Can solve for a generic w and c_s

$$v = \sqrt{\beta\eta}(AJ_{\nu}(\beta c_s k\eta) + BJ_{-\nu}(\beta c_s k\eta))$$

- Compute the vacuum expectation value

$$< 0|\hat{v}^2|0 >= v^2 < 0|a^{\dagger}a + \frac{1}{2}|0 >$$

- take the limit"" $k|\eta| \ll 1$ and see when we get:

$$< 0|\hat{v}^2|0 >\propto k^{-3}$$

A remarkable result (!!!!!!!!!!!!!!)

- For ALL equations of state

$$c_s \propto \rho \implies n_s = 1$$

This scaling law for c seems to be uniquely associated with scale invariance.

(For experts only; cf. k-essence)

- This can be understood from:

$$k^3\zeta^2 \sim \frac{(5+3w)^2}{1+w}\frac{\rho}{M_{Pl}^4 c_s}$$

Where does the amplitude come from?

- Obviously the variations in c must be cut off at low energies:

$$c_s = c\left(1 + \frac{\rho}{\rho_\star}\right)$$

- The cut-off scale fixes the amplitude:

$$k^3\zeta^2 \sim \frac{(5+3w)^2}{1+w}\frac{\rho_\star}{M_{Pl}^4} \sim 10^{-10}$$

The minimal bimetric VSL theory

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + B\partial_\mu\phi\partial_\nu\phi$$

$$B = B(\phi) = const$$

$$S = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}\,R[g_{\mu\nu}] + \int d^4x\sqrt{-\hat{g}}\,\mathcal{L}_m[\hat{g}_{\mu\nu},\Phi_{Matt}] + S_\phi$$

A subtlety with the variational calculus problem:

$$S_\phi = ???$$

The KG Lagrangian in the matter frame does NOT give the KG equation.

Something truly cool…

$$S^1_\phi = \int d^4x\sqrt{-\hat{g}}(-2\hat{\Lambda})$$

Gives a Klein-Gordon equation in matter frame

$$\hat{g} = g(1+2BX)$$

$$X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

$$S^1_\phi = \int d^4x\sqrt{-g}\sqrt{1+2BX}(-2\hat{\Lambda})$$

A cosmological constant in the matter frame leads to the (anti)DBI action

$+ - - -$

$$\mathcal{L} = -\frac{1}{f(\phi)}\sqrt{1-2f(\phi)X} + \frac{1}{f(\phi)} - V(\phi)$$

Specifically need a positive Lambda in the Einstein frame balanced by a negative lambda in the matter frame, to get the right low-energy limit:

$$S_\phi = \int d^4x\sqrt{-\hat{g}}\frac{1}{B} - \int d^4x\sqrt{-g}\frac{1}{B}$$

with $f = -B < 0$.

Recall our K-essence toolbox

$$\mathcal{L} = K(X) - V(\phi)$$

$$X = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi$$

$$\begin{aligned} p &= K - V \\ \rho &= 2XK_{,X} - K + V \end{aligned}$$

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$$p = K - V$$
$$\rho = 2XK_{,X} - K + V$$

Constant w solutions for mass potentials

$$c_s^2 = \frac{K_{,X}}{K_{,X} + 2XK_{,XX}}$$

Apply to (anti)DBI to find that…

$$c_s = c\left(1 + \frac{\rho}{\rho_\star}\right)$$

So our remarkable result is even

- Not only is it possible to identify a universal varying speed of sound law associated with scale invariance…
- but this law can be realized by an anti-DBI model (in the Einstein frame), which…
- turns out to be the minimal dynamics associated with a bimetric VSL

What about thermal fluctuations?

- Implicit in all previous "power spectra" is the multiplicative factor:

$$\ldots \times \langle 0|a^\dagger a + \frac{1}{2}|0\rangle = \ldots \times \frac{1}{2}$$

- But what if the state is a thermal state?

$$\ldots \times \langle T|a^\dagger a + \frac{1}{2}|T\rangle = \ldots \times n(k,T)$$

What speed of sound profile would lead to thermal scale-invariance?

- For ALL equations of state we find that we need a sudden phase transition in c_s ($\beta \gg 1$)
- Amplitude of the fluctuation is now fixed by the temperature at which the phase transition occurs:

$$A^2 \sim \left(\frac{T_\star}{M_P}\right)^3$$

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- Deformed dispersion relations (DDRs)

[Amelino-Camelia, Mavromatos, Magueijo & Smolin, etc, etc]

c(x,t)

c(E)

Varying c theories

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  [Moffat,Mag…
- Bimet… …, Drummond, etc, etc]
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c(E)

What we did with bimetric VSL can

- Deformed dispersion relations can give a frequency dependent speed of light

$$E^2 - g^2 p^2 = m^2$$

- The speed of light/sound would then also vary in time, by proxy, via expansion:

$$\omega = kg(\lambda k/a)$$

$$c = \frac{d\omega}{dk} = (\gamma + 1)\frac{\omega}{k} \propto \left(\frac{\lambda k}{a}\right)^{\gamma}$$

Also in this context scale-invariance

$$v'' + \left[\omega^2 - \frac{z''}{z}\right] v = 0$$

$$\omega^2 - k^2(1 + (\lambda k)^2)^2 = m^2$$

$$\lambda \sim 10^5 L_{Pl}.$$

- Cf. Horava-Lifschitz.

Beyond the "zeroth order" holy grail

- If the relation between the two metrics is

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + B\partial_\mu\phi\partial_\nu\phi \qquad B = B(\phi) \propto \phi^\alpha$$

then we obtain a tilted spectrum

$$n_S = f(\alpha)$$

Is this then another “theory of anything”? No!

- No gravity waves, but a possibility for a “consistency relation” is to look into the bispectrum (3-point function):

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)P_\zeta^{\ 2}\frac{1}{\Pi_j k_j^3}\mathcal{A}\,.$$

For scale-invariant varying c_s we obtain an equilateral bispectrum

$$\mathcal{A}_{c_s\to\infty} = -\frac{1}{8}\sum_i k_i^3 + \frac{1}{K}\sum_{i<j} k_i^2 k_j^2 - \frac{1}{2K^2}\sum_{i\neq j} k_i^2 k_j^3 .$$

(where $K \equiv k_1 + k_2 + k_3$), just like in DBI inflation, but now with a different amplitude:

$$\mathcal{A}_{\epsilon\to 0} = \left(1 - \frac{1}{c_s^2}\right)\mathcal{A}_{c_s\to\infty} \; + \; \mathcal{O}(n_s - 1)$$

Summary in terms of f_{NL} (if you really must!)

$$f_{\rm NL} = 30 \frac{\mathcal{A}_{k_1=k_2=k_3}}{K^3}$$

$$k_1 = k_2 = k_3 = K/3$$

Standard inflation $\quad f_{NL} \sim \epsilon \sim 0.1$

VSL $\quad f_{NL} \sim 1 > 0$

DBI inflation $\quad f_{NL} \sim -100$

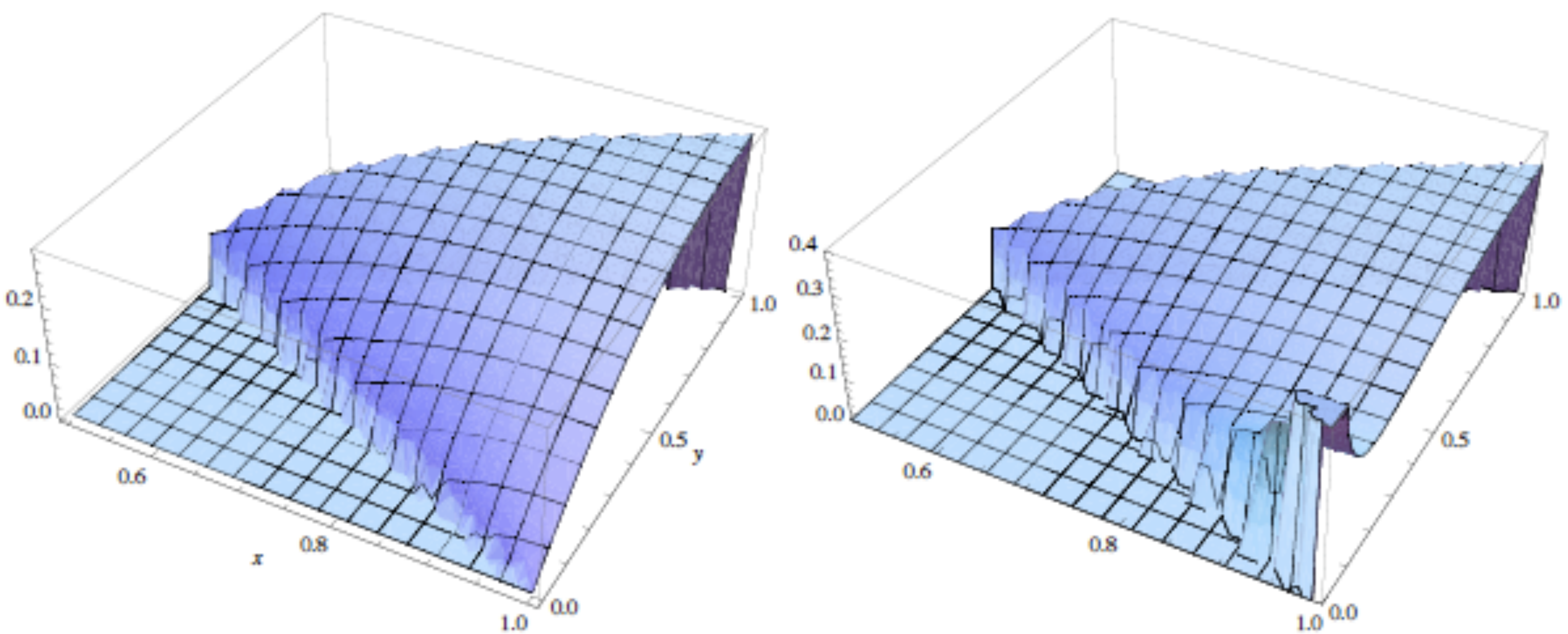
However if we depart from scale-invariance with varying c_s we obtain:

$$
\begin{aligned}
\mathcal{A} = \left(\frac{k_1k_2k_3}{2K^3}\right)^{n_s-1} \Bigg[& -\frac{1}{8}\sum_i k_i^3 + \frac{1}{K}\sum_{i<j} k_i^2k_j^2 - \frac{1}{2K^2}\sum_{i\neq j} k_i^2k_j^3 \\
& + (n_s-1)\left(-\frac{1}{8}\sum_i k_i^3 - \frac{1}{8}\sum_{i\neq j} k_ik_j^2 + \frac{1}{8}k_1k_2k_3 + \frac{1}{2K}\sum_{i<j} k_i^2k_j^2 - \frac{1}{2K^2}\sum_{i\neq j} k_i^2k_j^3\right) \\
& + \mathcal{O}\left(\frac{1}{c_s^2}\right)\Bigg] , \qquad (4.
\end{aligned}
$$

Specifically note the small "collapsed component":

$$
\mathcal{A}_{k_1\ll k_2,k_3} \approx -\frac{1}{2}(n_s-1)\left(\frac{k_1}{k_2}\right)^{n_s-1}
$$

Is this then another “theory of anything”? No.



(a) $-\mathcal{A}(1, x_2, x_3)/(x_2 x_3)$ for $n_s = 1$ (b) $-\mathcal{A}(1, x_2, x_3)/(x_2 x_3)$ for $n_s = 0.96$

Is this then another “theory of anything”? No.

The failure of quantum gravity and cosmology to meet IS an embarrassment:

Between the madhouse of quantum gravity and cosmology there might be a:

Worries of modern cosmology

- The trans-Planckian menace…
- Do we really know the vacuum state?
- The perception that “inflation is insulated from quantum gravity” is merely a dogma, or at best “wishful thinking”

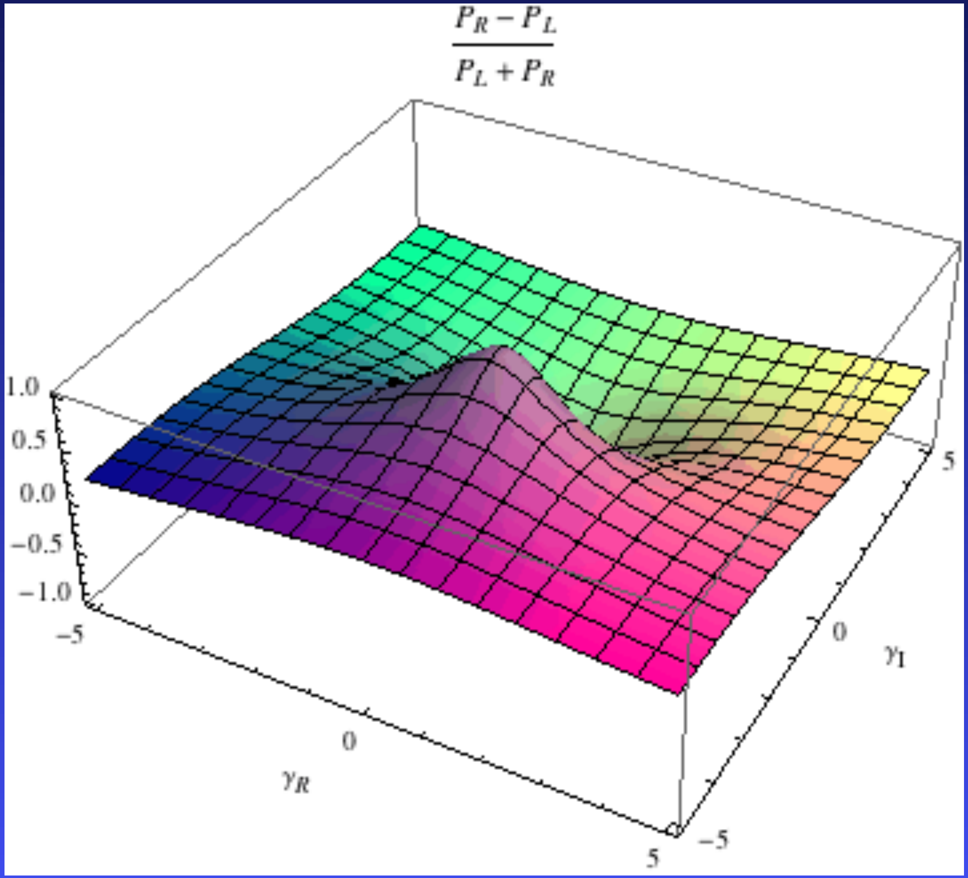
Quantum gravity does correct the

- Scale invariant tensor fluctuations are left outside the horizon, but they are chiral:

$$\frac{P_R - P_L}{P_R + P_L} = \frac{2i\gamma}{1-\gamma^2}$$

- The chirality depends on the Barbero-

What if the Immirzi parameter is

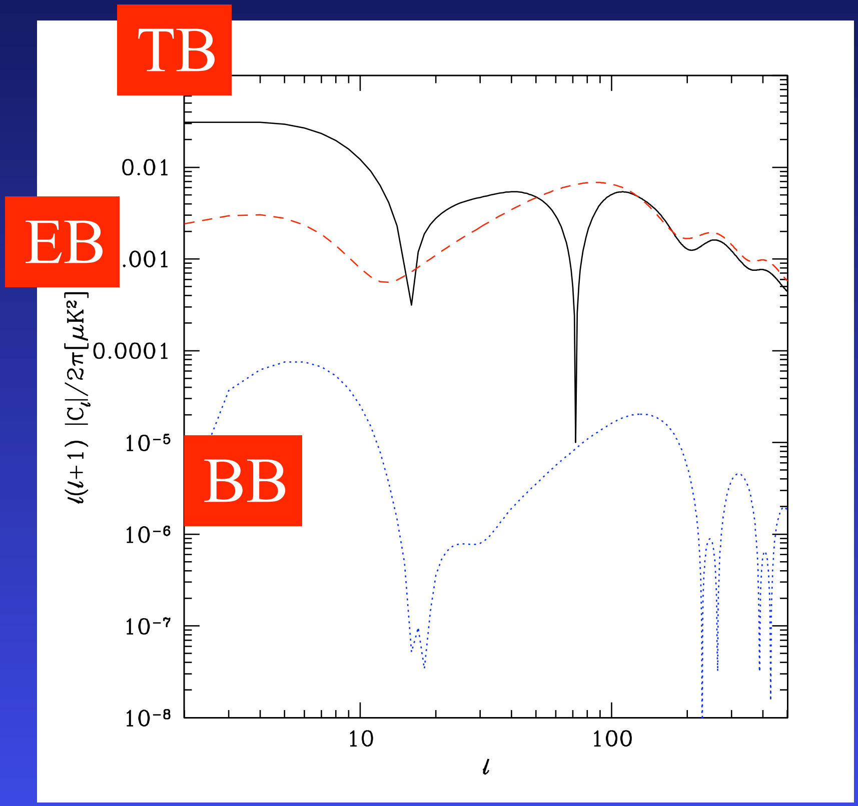


Therefore……….

- SO MUCH FOR INFLATION BEING BLIND TO QUANTUM GRAVITY
- SO MUCH FOR THE BUNCH DAVIES VACUUM BEING THE SELF-EVIDENT GOD'S CHOICE

We now find a unique prediction of

PRL101141101,2008 (Contaldi, JM, Smolin)

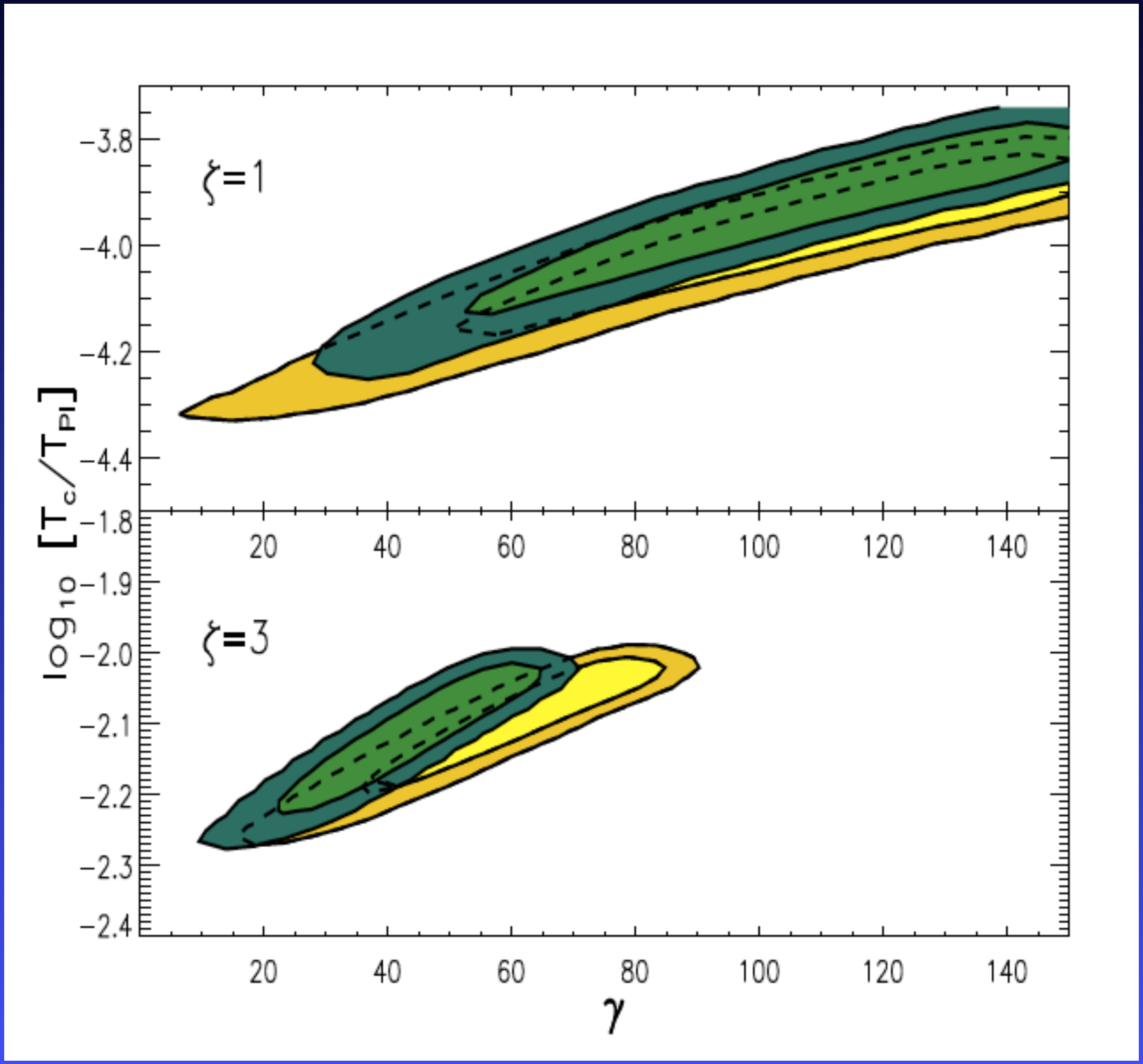


The signature in TB (and EB) is typically much larger than in BB

Killing two pigeons with one stone

- Obviously it may be that there are no tensor modes.
- But if they do exist they will be easier to detect via chirality (TB) for a wide range of Immirzi parameters:

$$\frac{1}{800} < |Im\gamma| < 800$$



ζ=1
ζ=3
log10 [Tc/TPl]
−3.8
−4.0
−4.2
−4.4
−1.8
−1.9
−2.0
−2.1
−2.2
−2.3
−2.4
20
40
60
80
100
120
140
γ