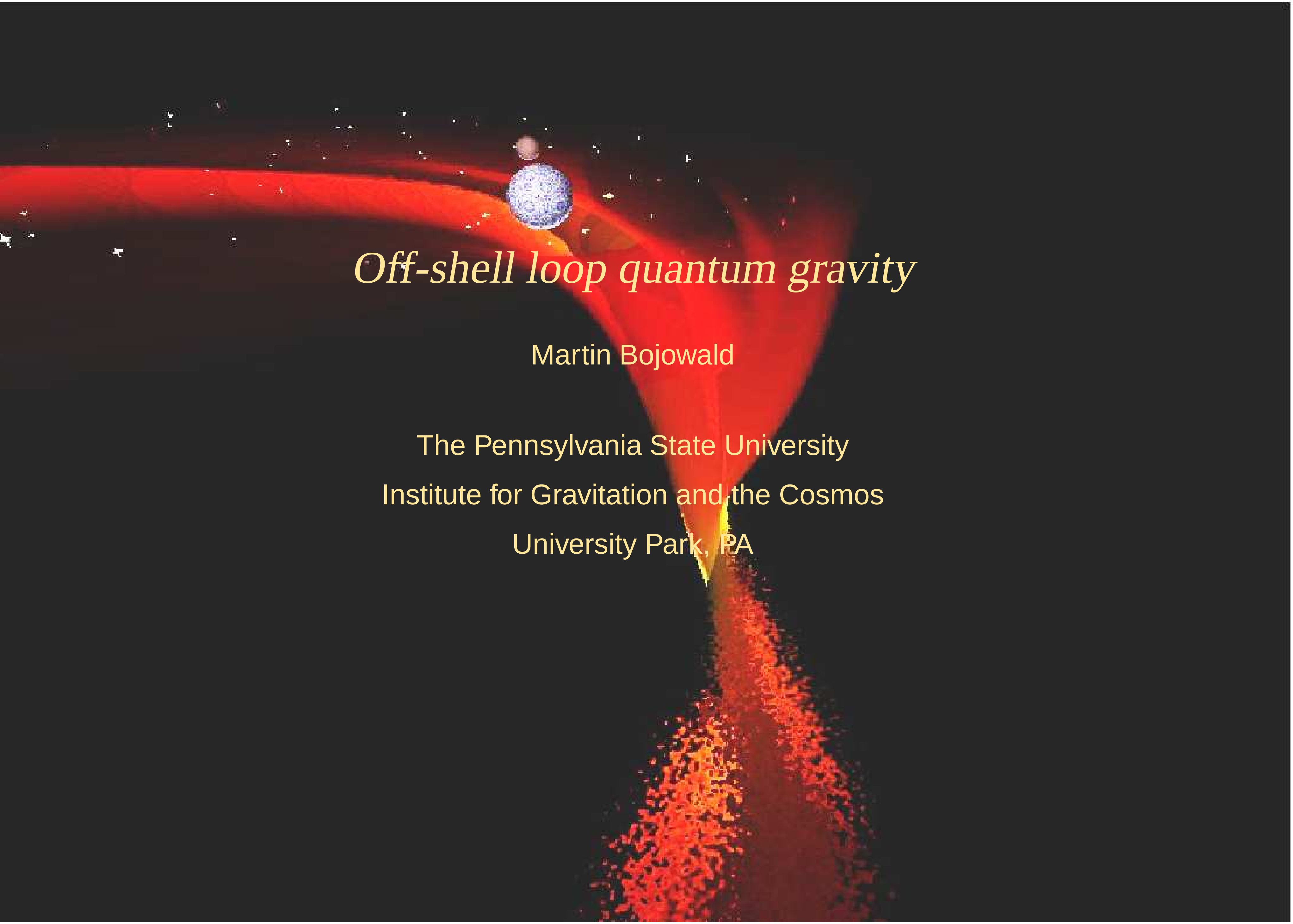

Off-shell loop quantum gravity

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Quantum gravity and the early universe

Expect large curvature: Higher-curvature effective action.

Quantum corrections in gravitational dynamics.

$$S = \frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{|\det g|}\left(R+\alpha R^2+\beta R_{ab}R^{ab}+\cdots\right)$$

All possible *covariant* combinations of metric derivatives should be present, with suitable coefficients.

Expected to be significant only at Planckian curvature/density. Difficult to test observationally.

Gravity closely related to space-time structure:
Not just R may receive quantum corrections but also "d^4x" or nature of *covariance and space-time tensors.*

Space-time structure

Usual arguments for higher-curvature corrections assume standard space-time tensor calculus. May not be complete.

- → Correct form to be derived from given candidate of quantum gravity.
- → Additional corrections may result, potentially more significant than curvature terms.
- → Interesting candidates: background-independent ones.

Here: Loop quantum gravity
(although status of background independence unclear)

Canonically quantized and fully constrained:
Generalize standard methods for derivation of effective actions.

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Effective canonical dynamics

Parameterize state by expectation values $\langle\hat{q}\rangle$ and $\langle\hat{p}\rangle$ of basic operators and moments

$$G^{a,n} = \langle(\hat{q}-\langle\hat{q}\rangle)^a(\hat{p}-\langle\hat{p}\rangle)^{n-a}\rangle_{\rm symm}$$

(Includes mixed states.)

Commutator of operators determines Poisson bracket of moments. (Symplectic space, but not after restriction to finite n.)

Expectation value of Hamiltonian, interpreted as function of $\langle\hat{q}\rangle$, $\langle\hat{p}\rangle$ and $G^{a,n}$, provides canonical equations of motion.

Effective equations: Correct classical equations for $\langle\hat{q}\rangle$, $\langle\hat{p}\rangle$ by quantum back-reaction from moments.

[with A Skirzewski: math-ph/0511043]

Quantum equations of motion

Anharmonic oscillator:

$$
\begin{aligned}
\dot{q} &= \frac{p}{m} \\
\dot{p} &= -m\omega^2 q - U'(q) - \sum_n \frac{1}{n!}\left(\frac{\hbar}{m\omega}\right)^{n/2} U^{(n+1)}(q)\tilde{G}^{0,n} \\
\dot{\tilde{G}}^{a,n} &= -a\omega\tilde{G}^{a-1,n} + (n-a)\omega\tilde{G}^{a+1,n} - a\frac{U''(q)}{m\omega}\tilde{G}^{a-1,n} \\
&\quad + \frac{\sqrt{\hbar}aU'''(q)}{2(m\omega)^{\frac{3}{2}}}\tilde{G}^{a-1,n-1}\tilde{G}^{0,2} + \frac{\hbar aU''''(q)}{3!(m\omega)^2}\tilde{G}^{a-1,n-1}\tilde{G}^{0,3} \\
&\quad - \frac{a}{2}\left(\frac{\sqrt{\hbar}U'''(q)}{(m\omega)^{\frac{3}{2}}}\tilde{G}^{a-1,n+1} + \frac{\hbar U''''(q)}{3(m\omega)^2}\tilde{G}^{a-1,n+2}\right) + \cdots
\end{aligned}
$$

∞ly many coupled equations for ∞ly many variables.

Low energy effective action

To second adiabatic order, as second order equation of motion:

$$\left(m+\frac{\hbar U'''(q)^2}{32m^2\omega^5\left(1+\frac{U''(q)}{m\omega^2}\right)^{\frac{5}{2}}}\right)\ddot{q}$$

$$+\frac{\hbar\dot{q}^2\left(4m\omega^2U'''(q)U''''(q)\left(1+\frac{U''(q)}{m\omega^2}\right)-5U'''(q)^3\right)}{128m^3\omega^7\left(1+\frac{U''(q)}{m\omega^2}\right)^{\frac{7}{2}}}$$

$$+m\omega^2q+U'(q)+\frac{\hbar U'''(q)}{4m\omega\left(1+\frac{U''(q)}{m\omega^2}\right)^{\frac{1}{2}}}=0\,.$$

as it results from

$$\Gamma_{\mathrm{eff}}[q(t)] = \int \mathrm{d}t\left(\frac{1}{2}\left(m+\frac{\hbar U'''(q)^2}{2^5m^2\left(\omega^2+m^{-1}U''(q)\right)^{\frac{5}{2}}}\right)\dot{q}^2 - \frac{1}{2}m\omega^2q^2-U(q)-\frac{\hbar\omega}{2}\left(1+\frac{U''(q)}{m\omega^2}\right)^{\frac{1}{2}}\right)$$

Higher time derivatives

Higher adiabatic order:

[with S Brahma, E Nelson: arXiv:1208.1242]

$$
\begin{aligned}
\ddot{q} &= -\omega^2 q - U'(q)/m \\
&\quad -\frac{\hbar}{2m^2\omega} U'''(q) \left(f(q,\dot{q}) + f_1(q,\dot{q})\ddot{q} + f_2(q)\ddot{q}^2 + f_3(q,\dot{q})\dddot{q} + f_4(q)\ddddot{q} \right) \\
&\quad + \cdots
\end{aligned}
$$

where

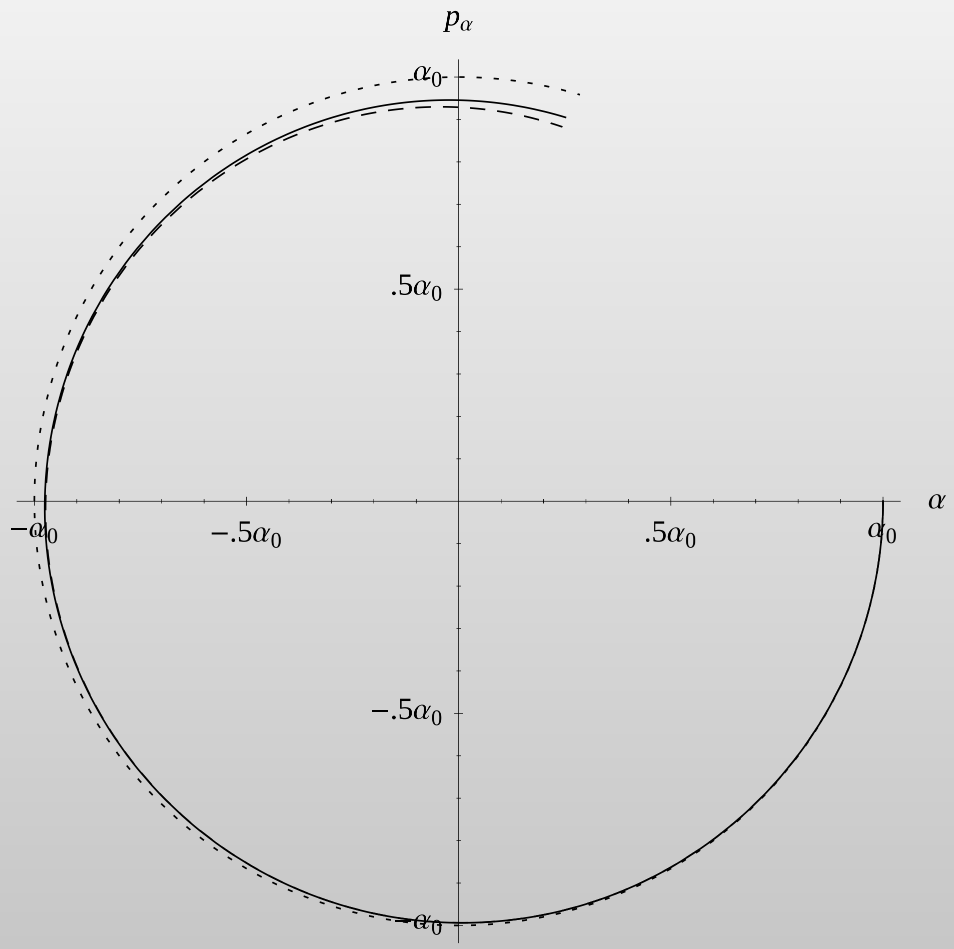
$$
\begin{aligned}
f(q,\dot{q}) &= \tfrac{1}{2}\left(1+\frac{U''(q)}{m\omega^2}\right)^{-1/2} + \frac{U''''(q)\dot{q}^2}{16m\omega^4}\left(1+\frac{U''(q)}{m\omega^2}\right)^{-5/2} - \frac{5(U'''(q))^2\dot{q}^2}{64m^2\omega^6}\Bigg(1+ \\
&\quad -\frac{U''''''(q)\dot{q}^4}{64m\omega^6}\left(1+\frac{U''(q)}{m\omega^2}\right)^{-7/2} + \frac{21(U''''(q))^2\dot{q}^4}{256m^2\omega^8}\left(1+\frac{U''(q)}{m\omega^2}\right)^{-9/2} \\
&\quad +\frac{7U'''''(q)U'''(q)\dot{q}^4}{64m^2\omega^8}\left(1+\frac{U''(q)}{m\omega^2}\right)^{-9/2} - \frac{231U''''(q)(U'''(q))^2\dot{q}^4}{512m^3\omega^{10}}\Bigg(1+\frac{U''(q}{m\omega^2} \\
&\quad +\frac{1155(U'''(q))^4\dot{q}^4}{4096m^4\omega^{12}}\left(1+\frac{U''(q)}{m\omega^2}\right)^{-13/2}
\end{aligned}
$$

Effective constraints

Dynamics by effective Hamiltonian $\langle\hat{H}\rangle(\langle\cdot\rangle,\Delta(\cdot))$ or effective constraints $\langle\widehat{\mathrm{pol}}\hat{C}\rangle(\langle\cdot\rangle,\Delta(\cdot))$.

→ Captures dynamics of physical states.

→ Allows local internal times.
[arXiv:1009.5953]



[Figure from arXiv:0911.4950]

[Cosmological model in P Höhn, E Kubalova, A Tsobanjan: arXiv:1111.5193]

Effective constraint algebras

Systems with several classical constraints C_I:

- → Effective constraints $C_{I,\mathrm{pol}} = \langle \widehat{\mathrm{pol}} \hat{C}_I \rangle (\langle \cdot \rangle, \Delta(\cdot))$ with $\widehat{\mathrm{pol}}$ polynomial in basic operators.
- → Compute effective constraint algebra by Poisson brackets. Easier than commutators of operators. Parameterizable.
- → Check *off-shell consistency* of first-class constraints.

Example: Hamiltonian and diffeomorphism constraints obey hypersurface-deformation algebra of classical space-time.

$$\begin{aligned}
[S(\vec{w}_1), S(\vec{w}_2)] &= -S(\mathcal{L}_{\vec{w}_2} \vec{w}_1) \\
[T(N), S(\vec{w})] &= -T(\vec{w} \cdot \vec{\nabla} N) \\
[T(N_1), T(N_2)] &= S(N_1 \vec{\nabla} N_2 - N_2 \vec{\nabla} N_1)
\end{aligned}$$

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Relevance of effective methods

- → "Big-bang state" largely unknown.
  Consider *generic states* in effective parameterization.

- → *Local internal times:* Do not require artificial matter contents such as free, massless scalar or dust.

- → No gauge-fixing or deparameterization required before quantization:

  Check *off-shell consistency* of constraint algebra and corresponding space-time structure.

Canonical gravity

Describe space-time geometry by su(2)-valued triad $\vec{E}_i$ and connection $\underset{\rightarrow}{A}_i$ (canonically conjugate).

Triad: determines spatial distances/angles by three orthonormal vectors $\vec{E}_i$, $i = 1, 2, 3$, at each point in space.

Connection: $\underset{\rightarrow}{A}_i$ combination of different measures of curvature of space. Ashtekar–Barbero connection.

Natural smearing of $\underset{\rightarrow}{A}_i$ along curves (holonomies) and $\vec{E}_i$ over surfaces (fluxes). Creation operators to construct state space.

Spatial background independence.

In what follows, use U(1)-connection $\underset{\rightarrow}{A}$ for simplicity.

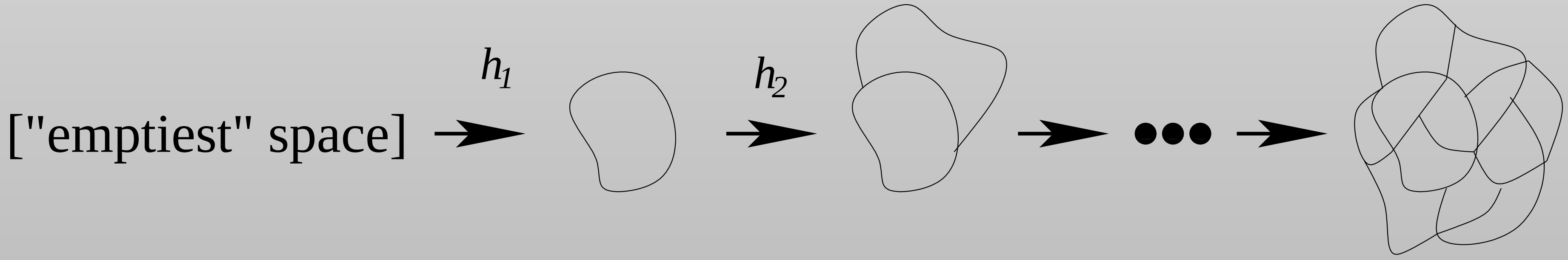
Loop quantum gravity

Holonomies $h_e = \exp(i\int_e \mathrm{d}\lambda \underset{\to}{A}\cdot\vec{t}_e)$ along curves e in space, tangent $\vec{t}_e$.

Start with basic state ψ_0 by $\psi_0(\underset{\to}{A}) = 1$.

Excited states:

$$
\begin{aligned}
\psi_{e_1,k_1;\ldots;e_i,k_i}(\underset{\to}{A}) &= \hat{h}_{e_1}^{k_1}\cdots\hat{h}_{e_i}^{k_i}\psi_0(\underset{\to}{A}) \\
&= \prod_e h_e(\underset{\to}{A})^{k_e} = \prod_e \exp(ik_e \int_e \mathrm{d}\lambda \underset{\to}{A}\cdot\vec{t}_e)
\end{aligned}
$$



Discrete Geometry

Derivative operator: Flux $\int_S \mathrm{d}^2 y \underset{\rightarrow}{n} \cdot \hat{\vec{E}}$ for surfaces S in space.

$$\int_S \mathrm{d}^2 y \underset{\rightarrow}{n} \cdot \hat{\vec{E}} \psi_{g,k} = \frac{8\pi G\hbar}{i} \int_S \mathrm{d}^2 y \underset{\rightarrow}{n} \cdot \frac{\delta \psi_{g,k}}{\delta \underset{\rightarrow}{A}(y)} = 8\pi \ell_{\mathrm{Pl}}^2 \sum_{e \in g} k_e \mathrm{Int}(S,e) \psi_{g,k}$$

with intersection number $\mathrm{Int}(S,e)$, Planck length $\ell_{\mathrm{Pl}} = \sqrt{G\hbar}$.

Discrete geometry: for gravity, flux represents spatial metric.
Scale $\sim \ell_{\mathrm{Pl}} \sqrt{k_e}$ state-dependent.

Spatially diffeomorphism invariant: sum over all deformed graphs.

More tricky: Space-*time* covariance, Hamiltonian constraint.
Background independence often forsaken at this stage.

Hamiltonian

Yang–Mills theory on Minkowski space-time:

$$H = \kappa \int \mathrm{d}^3x (|\vec{E}_i|^2 + |\vec{B}_i|^2)$$

for $\vec{B}_i = \underset{\to}{\nabla} \times \underset{\to}{A}_i + C_{ijk} \underset{\to}{A}_j \times \underset{\to}{A}_k$ (structure constants C_{ijk})

Gravity on any space-time:

$$H[N] = \frac{1}{16\pi G} \int \mathrm{d}^3x N \frac{\sum_{ijk} \epsilon_{ijk} (\vec{B}_i \times \vec{E}_j) \cdot \vec{E}_k}{\sqrt{\frac{1}{6} |\sum_{ijk} \epsilon_{ijk} (\vec{E}_i \times \vec{E}_j) \cdot \vec{E}_k|}} + \cdots$$

with $C_{ijk} = \epsilon_{ijk}$.

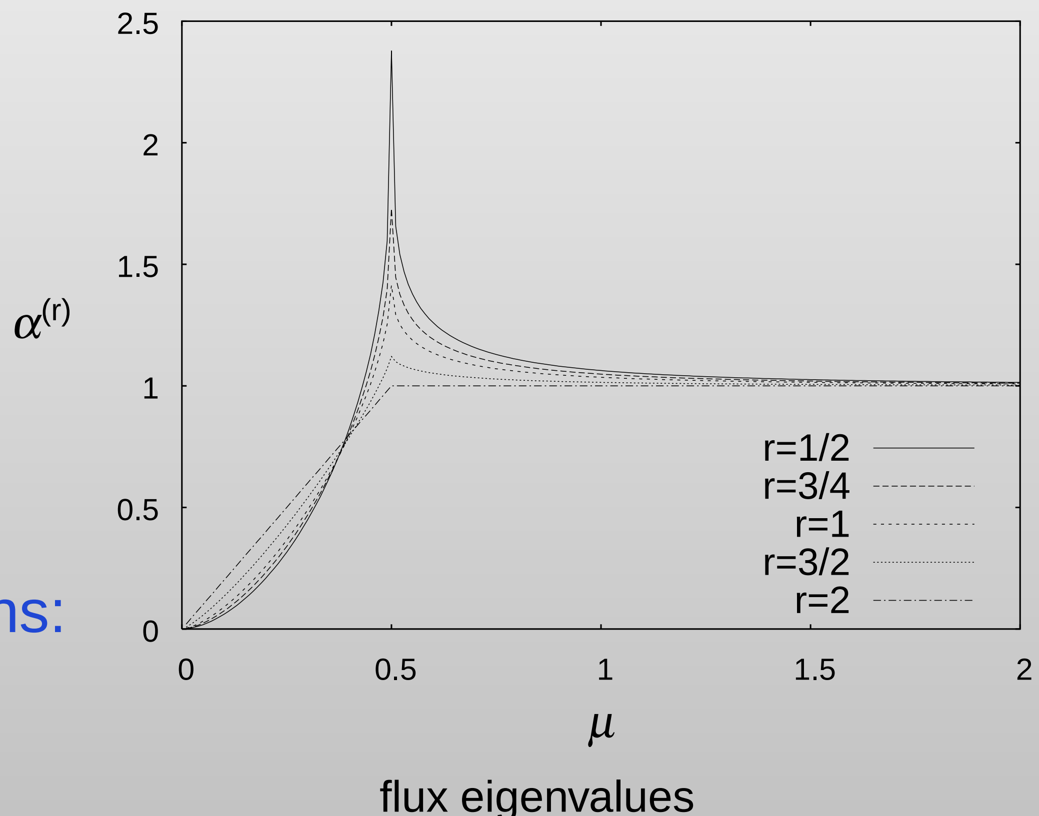
Implies characteristic corrections when quantized.

Quantum corrections

$\longrightarrow$ Inverse-triad corrections from quantizing [T Thiemann 1996]

$$\left\{\underset{\rightarrow}{A}^i, \int \sqrt{|\det E|}\mathrm{d}^3x\right\} = 2\pi G\epsilon^{ijk}\frac{\vec{E}_j \times \vec{E}_k}{\sqrt{|\det E|}}$$

Automatic cut-off of $1/E$-divergences.



$\longrightarrow$ Higher-order corrections: holonomies for $\vec{B}_i$

$\longrightarrow$ Quantum back-reaction

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Background independence?

Status of background independence in loop quantum gravity:

$(+)$ No spatial background metric used to define states and operators, summed over spatial diffeomorphisms. Uniqueness of representation.

$(-)$ Hamiltonian constraint quantized in much more messy way.

Modify classical expression by holonomy corrections (quantum corrections mixed with regulator).

$(-)$ Diffeomorphism constraint and Hamiltonian constraint contain curvature of $\underset{\to}{A}_i$, but treated differently.

$(-)$ Off-shell algebra of Hamiltonian constraints often ignored.

[see also H Nicolai, K Peeters, M Zamaklar: hep-th/0501114]

Can this procedure result in any consistent space-time picture?

Abhay’s curse

“In order to derive some physics, we must become less rigorous!”

Actually, we must become much more rigorous.

Off-shell closure (anomaly-freedom)

Consistency of *off-shell* constraint algebra has important *physical* implications:

Requires detailed balance of correction terms in constraints.

Making sure that there is a consistent off-shell system can have surprising consequences.

Overlooked when gauge fixed or time chosen in deparameterized models. Standard procedure:

- $\rightarrow$ Choose simple matter field ϕ as internal time, often added by hand as artificial matter ingredient.
- $\rightarrow$ Solve classical constraints for momentum p_ϕ.
- $\rightarrow$ Quantize $\hat{p}_\phi$ and solve quantum evolution with respect to ϕ.
- $\rightarrow$ Conveniently forget asking whether “predictions” depend on choice of time.

Background dependence

Using partial classical solutions, fixing the gauge or deparameterizing before quantization *could* lead to correct results, but unlikely.

→ Only a certain combination of classical constraints is quantized, the rest solved or eliminated classically.

→ Quantization complete only when one can show that predictions do not depend on chosen gauge fixing or internal time. Difficult!

→ Background independence often emphasized when it comes to space, but ignored when time is to be included.

→ Worry about energy conservation.

Energy conservation

[with M Kagan, G Hossain, C Tomlin: arXiv:1302.5695]

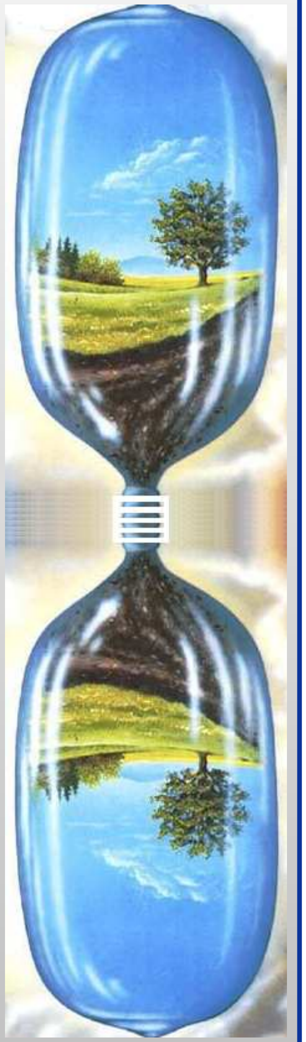
$$\begin{aligned} N\sqrt{\det h}\nabla_\mu T^\mu{}_0 &= -N\frac{\partial\mathcal{H}_{\rm matter}}{\partial t} - N^a\frac{\partial\mathcal{D}_a^{\rm matter}}{\partial t} \\ &\quad +\mathcal{L}_{\vec{N}}C_{\rm matter}[N,N^a] + \frac{\partial h_{ab}}{\partial t}\frac{\delta H_{\rm matter}}{\delta h_{ab}} \\ &\quad +\partial_b\left(N^2h^{ab}\mathcal{D}_a^{\rm matter} + 2N^ch^{ba}\frac{\delta H_{\rm matter}}{\delta h^{ac}}\right) \end{aligned}$$

Classical *off-shell* algebra: $\partial\mathcal{H}_{\rm matter}/\partial t = \{\mathcal{H}_{\rm matter}, H[N,N^a]\}$ cancels $\partial^a(N^2\mathcal{D}_a^{\rm matter})$, only one term from $\partial_b T^b{}_0$.

- $\rightarrow$ Deformed algebra cannot be taken care of by modified coefficients in $\nabla_\mu T^\mu{}_\nu = 0$.
- $\rightarrow$ No energy conservation if off-shell algebra broken (or unchecked in gauge-fixed/deparameterized models).

Big-bang singularity

Does loop quantum cosmology
"replace the big-bang singularity by a quantum bounce"?



Difference equation for wave function of the universe:

$$C_+(\mu)\psi_{\mu+1} - C_0(\mu)\psi_\mu + C_-(\mu)\psi_{\mu-1} = \hat{H}_{\mathrm{matter}}(\mu)\psi_\mu$$

Holonomy corrections: strong at nearly Planckian density.

Replace Hubble parameter $\mathcal{H}$ by $\sin(\ell\mathcal{H})/\ell$ in modified Friedmann equation

$$\frac{\sin^2(\ell\mathcal{H})}{\ell^2} = \frac{8\pi G}{3}\rho$$

Effective picture in simple models: bounce.
Exact for free, massless scalar in spatially flat isotropic universe.

High density

Holonomy corrections relevant near Planckian density if $\ell \sim \ell_{\rm P}$.

Higher-curvature corrections large in the same regime: $\rho/\rho_{\rm P}$.

Higher time derivatives contribute to homogeneous models, interfere with holonomy corrections.

Complete expansion

$$\sin^2(\ell\mathcal{H}) = \sum_{n=1}^{\infty} c_n(\ell\mathcal{H})^{2n}$$

used in bounce models.
All higher higher-curvature corrections (comparable to $n=2$-terms) are ignored.

[Details: arXiv:1209.3403]

Harmonic cosmology

- $\rightarrow$ Deparameterize free, massless scalar: holonomy-modified Hamiltonian linear in non-canonical variables V, $J = V\exp(i\ell\mathcal{H})$. *Linear algebra.*
- $\rightarrow$ Upon quantization, expectation values *do not couple* to fluctuations and higher moments. (Specific factor ordering.)
- $\rightarrow$ $\langle\hat{V}\rangle(\phi) \sim V_0\cosh(\phi-\phi_0)$, constant $\Delta V/\langle\hat{V}\rangle$.
  Volume fluctuations decrease exponentially toward bounce.
  Gaussian state: curvature fluctuations increase correspondingly.

[gr-qc/0608100; reproduced in arXiv:0710.3565 (Ashtekar, Corichi, Singh)]

Anharmonic: curvature fluctuations important for quantum back-reaction.

Complicated analysis required. Corrections depend on state. (Near-vacuum assumed for anharmonic oscillator.)

Quantum back-reaction in cosmology

Fluctuations important moments to leading order, but play role different from usual statistical one.

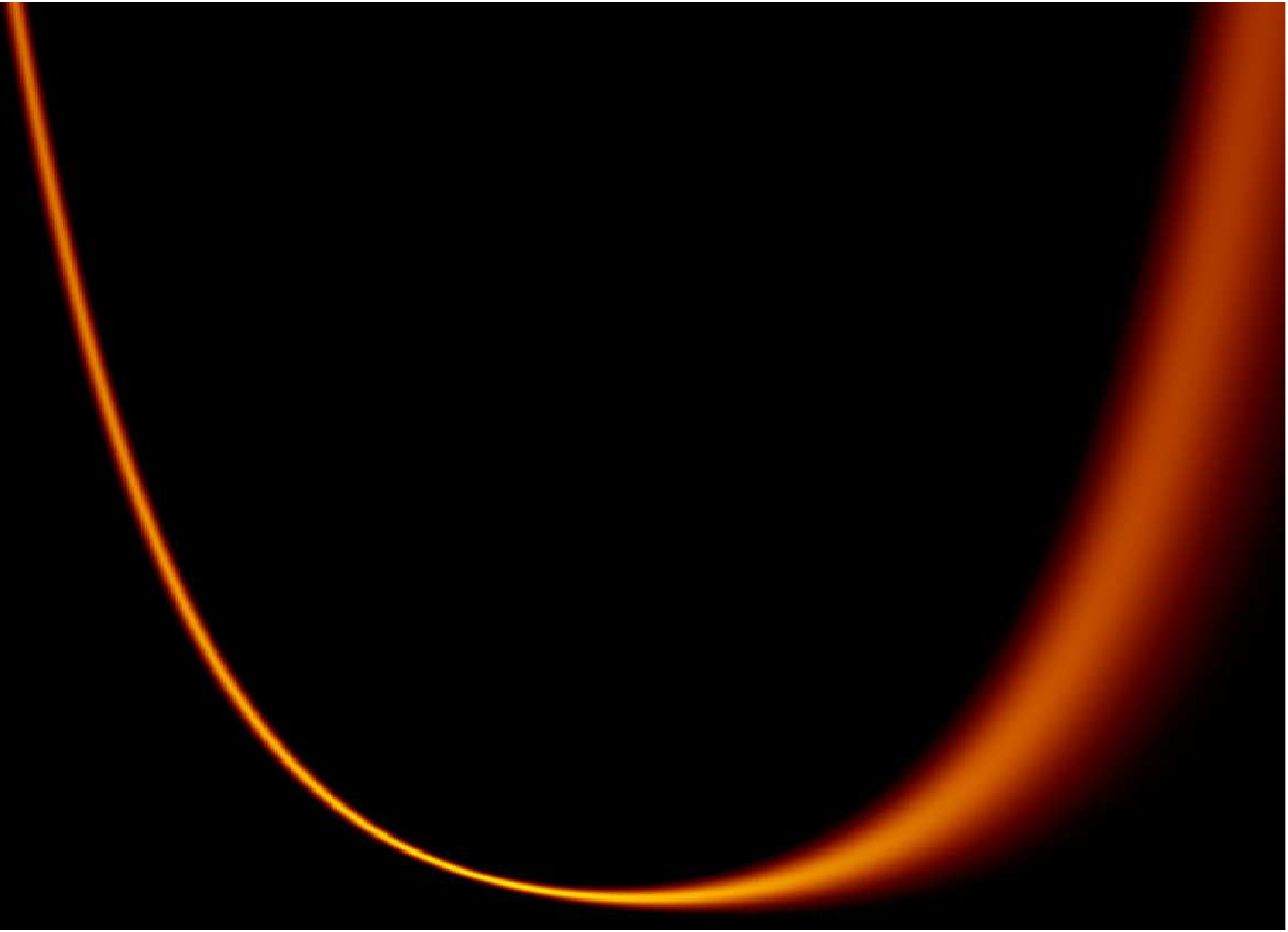
Cosmological constant in addition to free, massless scalar, $H = V\sqrt{\mathcal{H}^2 - \Lambda}$.

Linear in V, thus no ΔV in (WdW) effective equations

$$\frac{\mathrm{d}\langle\hat{\mathcal{H}}\rangle}{\mathrm{d}\phi} = -\sqrt{\langle\hat{\mathcal{H}}\rangle^2 - \Lambda} + \frac{1}{2}\Lambda\frac{(\Delta\mathcal{H})^2}{(\langle\hat{\mathcal{H}}\rangle^2 - \Lambda)^{3/2}} + \cdots$$

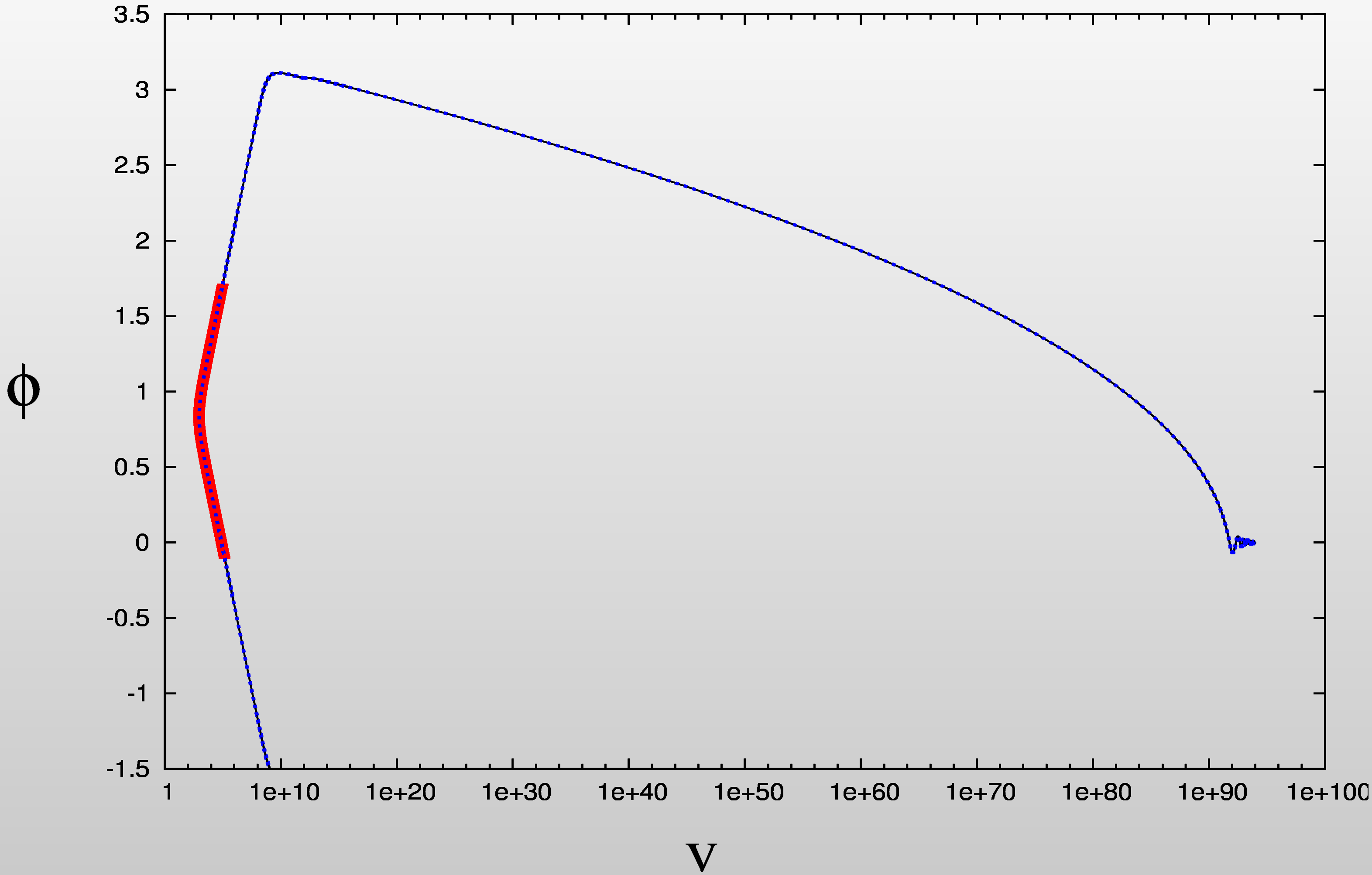
$$\frac{\mathrm{d}\langle\hat{V}\rangle}{\mathrm{d}\phi} = \frac{\langle\hat{V}\rangle\langle\hat{\mathcal{H}}\rangle}{\sqrt{\langle\hat{\mathcal{H}}\rangle^2 - \Lambda}} + \frac{3}{2}\Lambda\frac{\langle\hat{V}\rangle\langle\hat{\mathcal{H}}\rangle(\Delta\mathcal{H})^2}{(\langle\hat{\mathcal{H}}\rangle^2 - \Lambda)^{5/2}} - \Lambda\frac{\Delta(V\mathcal{H})}{(\langle\hat{\mathcal{H}}\rangle^2 - \Lambda)^{3/2}} + \cdots$$

Can make ΔV large without affecting dynamics much (until other moments increase).



Dispersion

Inflation



Expectations values and dispersions of $\hat{V}|_\phi$ for a massive inflaton ϕ with phenomenologically preferred parameters (AA, Pawlowski, Singh).

[Talk by Abhay Ashtekar in Erlangen, 2012]

Bounce?

No evidence for bounce in anything but the simplest models.

Restricted not just by symmetry but also, and crucially, by matter ingredients.

- → Usual matter choice (for deparameterization) eliminates quantum back-reaction, too restrictive.

- → Symmetry eliminates control on space-time structure.

  Homogeneous models trivialize the constraint algebra, crucial issues overlooked.

Need realistic matter and at least perturbative inhomogeneity to obtain propagation equations and see if structure evolves through high density (bounce or otherwise).

Deformed deformations

$\longrightarrow$ Inverse-triad corrections in Hamiltonian

$$\frac{1}{16\pi G}\int \mathrm{d}^3x N\alpha\frac{\epsilon_{ijk}F^i_{ab}E^a_jE^b_k}{\sqrt{|\det E|}}+\cdots$$

$\longrightarrow$ Poisson-bracket algebra modified.
Deform but do not violate covariance:

$$\begin{aligned}
[S(\vec{w}_1),S(\vec{w}_2)] &= -S(\mathcal{L}_{\vec{w}_2}\vec{w}_1)\\
[T(N),S(\vec{w})] &= -T(\vec{w}\cdot\vec{\nabla}N)\\
[T(N_1),T(N_2)] &= S(\alpha^2(N_1\vec{\nabla}N_2-N_2\vec{\nabla}N_1))
\end{aligned}$$

[with G Hossain, M Kagan, S Shankaranarayanan: arXiv:0806.3929]

Cosmological perturbation equations

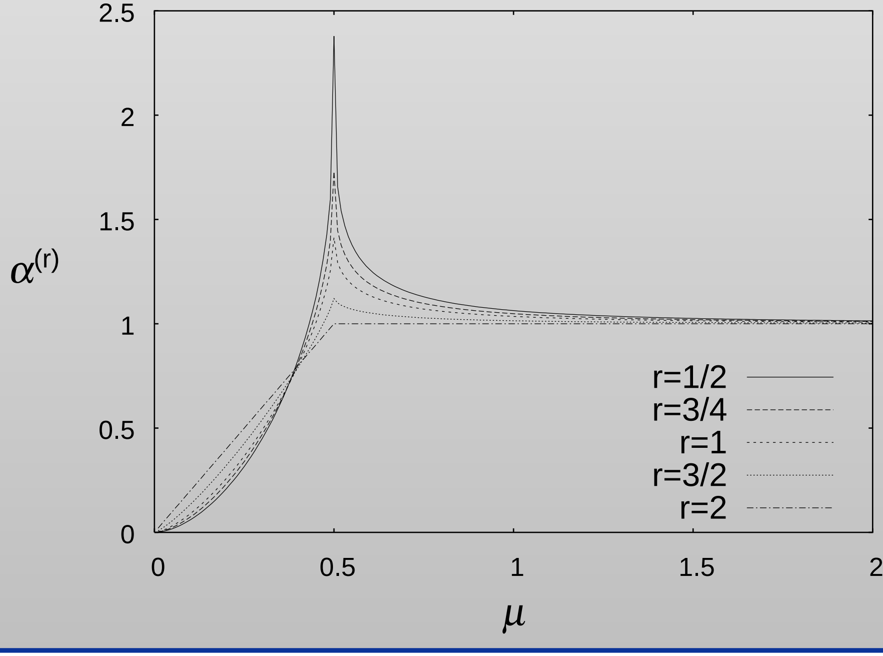
[with G Calcagni: arXiv:1011.2779]

Dynamics of density perturbations u, gravitational waves w:

$$-u'' + s(\alpha)^2 \Delta u + (\tilde{z}''/\tilde{z})u = 0$$

$$-w'' + \alpha^2 \Delta w + (\tilde{a}''/\tilde{a})w = 0$$

Different speeds for different modes: corrections to tensor-to-scalar ratio.



Do not need high density.

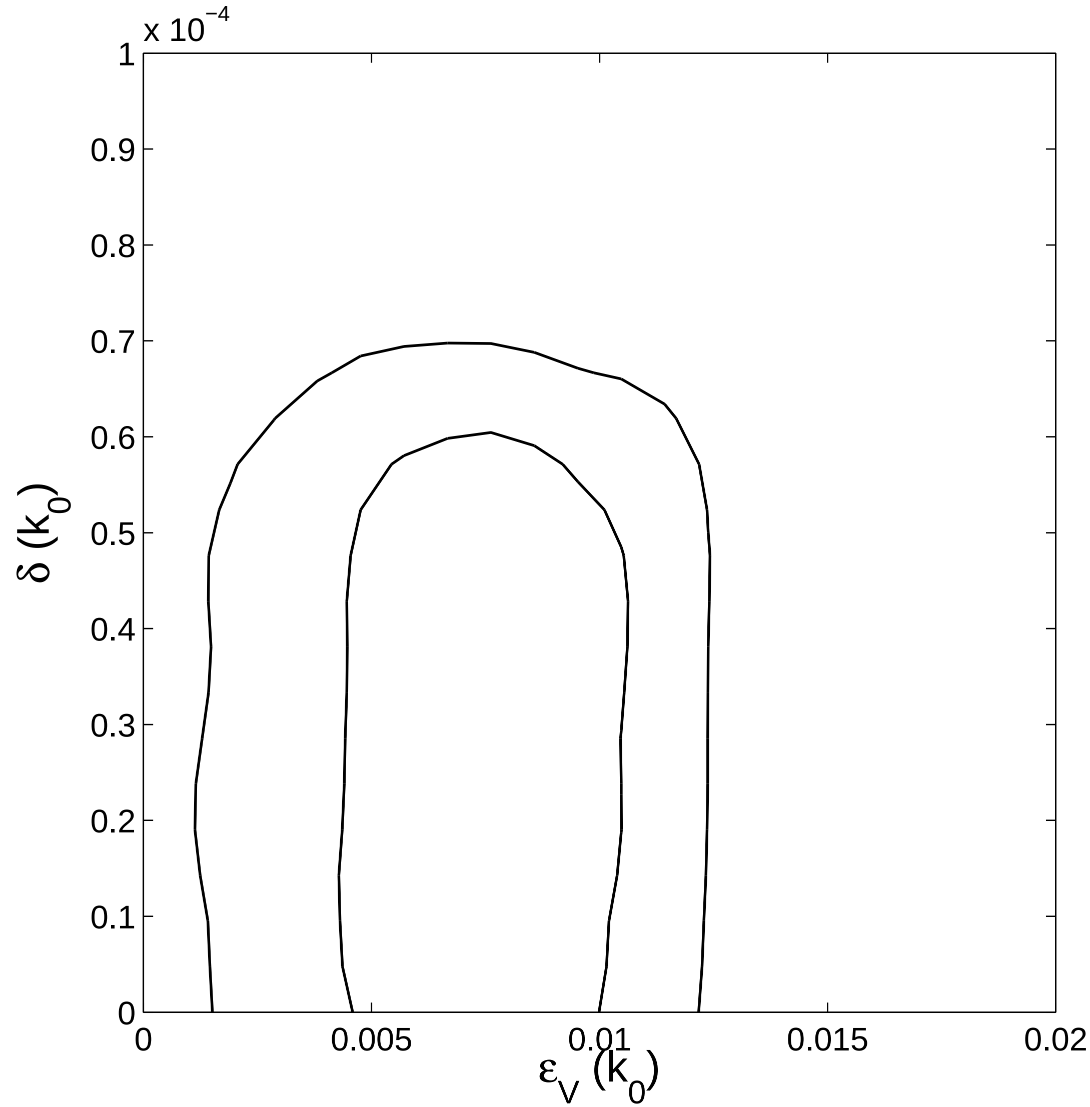
Crucial for falsifiability:
$\alpha - 1$ large for small lattice spacing.

Two-sided bounds on discreteness scale.

Scalar mode

$$-u'' + s(\alpha)^2 \Delta u + (\tilde{z}''/\tilde{z})u = 0$$

[with G Calcagni, S Tsujikawa 2011]



$$10^{-8} < \delta = \alpha - 1 < 10^{-4}$$

much closer than

$\ell_{\rm P}$ and $\ell_{\mathcal{H}}$

Holonomy corrections

Hypersurface-deformation algebra with pointwise holonomy corrections: $\mathcal{H} \longrightarrow \ell^{-1}\sin(\ell\mathcal{H})$ in perturbative cosmology.

Incomplete: Corrections in series expansion cannot be separated from higher space and time derivatives. ($R \sim \partial\Gamma + \Gamma^2$)

$$\begin{aligned}
[S(\vec{w}_1), S(\vec{w}_2)] &= -S(\mathcal{L}_{\vec{w}_2}\vec{w}_1) \\
[T(N), S(\vec{w})] &= -T(\vec{w}\cdot\vec{\nabla}N) \\
[T(N_1), T(N_2)] &= S(\beta(N_1\vec{\nabla}N_2 - N_2\vec{\nabla}N_1))
\end{aligned}$$

with $\beta = \cos(2\ell\mathcal{H}) < 0$ at high density.
$\beta = -1$ at maximal density (maximum of $\sin(\ell\mathcal{H})$, “bounce”).

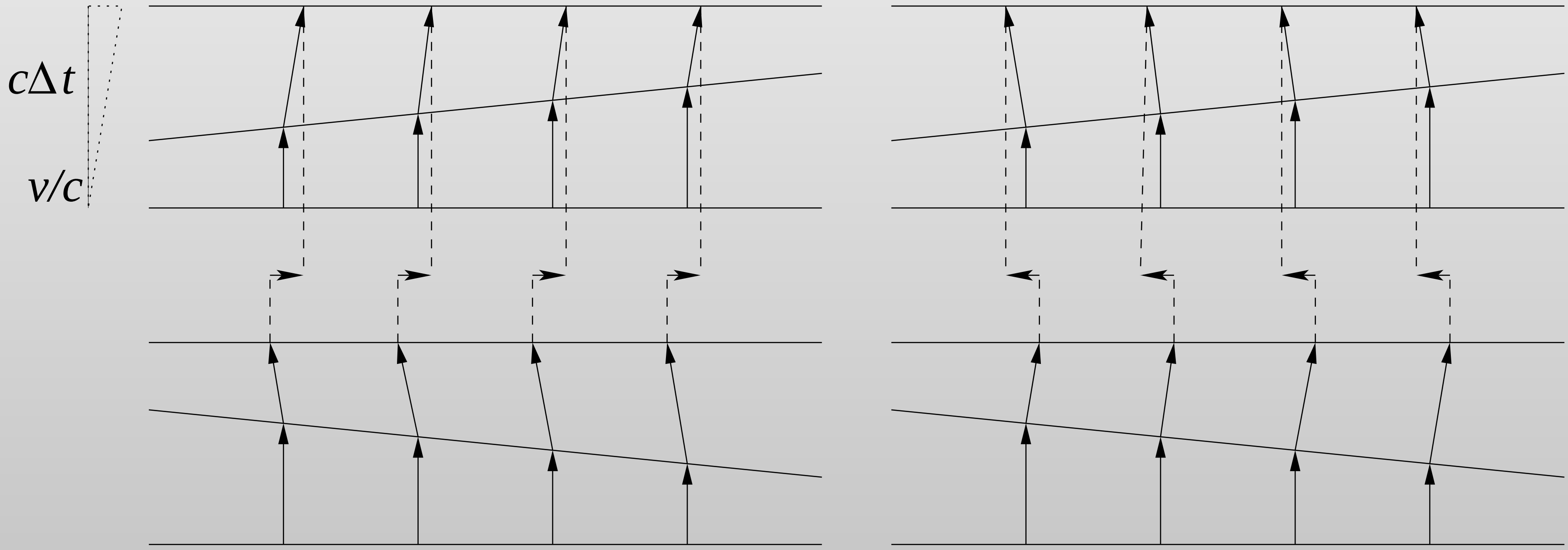
[J Reyes 2009; A Barrau, T Cailleteau, J Grain, J Mielczarek 2011]

Signature change

$\beta \approx -1$ at high density.

Space-time signature Euclidean.

[with G Paily: arXiv:1112.1899]



Minkowski geometry

Euclidean geometry

Ill-posed bounce

Bounce models require high density, where signature turns Euclidean.

- → Quantum space-time: no metric/line element, but deformed constraint algebra determines space(-time) structure.
- → Physical consequence: elliptic rather than hyperbolic partial differential equations for physical modes.

  No deterministic evolution. No initial-value problem.
- → Signature change not a consequence of small inhomogeneity:
  Inhomogeneity only used to probe space-time structure because homogeneous models are too restrictive.
- → Does not rely on subtleties of perturbation theory:
  Same effects in spherically symmetric models.

Conclusions

Big bounce blunder:

→ Wrong background evolution except for special cases, ignoring higher time derivatives.

→ In models where the unperturbed background seems to bounce, it does not evolve deterministically.

Off-shell constraint algebra is important.

Recent results for operators encouraging and fully consistent with effective calculations.

[A Perez, D Pranzetti: arXiv:1001.3292]

[A Henderson, A Laddha, C Tomlin: arXiv:1204.0211, arXiv:1210.3960]

[C Tomlin, M Varadarajan: arXiv:1210.6869]

Quantum corrections of covariance promising observationally.