noncommutative spectral geometry: a guided tour

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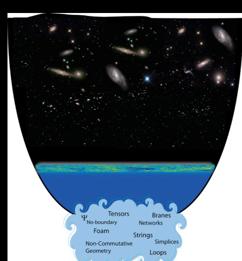

quantum gravity & quantum cosmology

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king's college london
university of london

outline

- motivation
- elements of noncommutative spectral geometry
- criticisms
- phenomenological consequences
- cosmological consequences
- conclusions

motivation

cosmology

EU models tested with

- accurate astrophysical data (CMB)
- high energy experiments (LHC)

despite the golden era of cosmology, a number of questions:

- origin of DE / DM
- search for natural and well-motivated inflationary model (alternatives...)

...

are still awaiting for a definite answer main approaches:

- string theory
- LQC, SF, WdW, CDT, CS,...

- noncommutative spectral geometry

$$\mathcal{S}^{\mathrm{E}} = \int \Big(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^\star R^\star + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha} + \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \Big) \sqrt{g}\ d^4x\ ,$$

particle physics

at low energies:

$$\boxed{S_{\text{Einstein-Hilbert}} + S_{\text{Standard Model}}}$$

GR
diffeomorphism invariance

gauge symmetries
local gauge invariance

this difference may be responsible for difficulty in finding a unified theory of all interactions including gravity in addition:

- why G is $U(1) \times SU(2) \times SU(3)$?
- why fermions occupy the representations they do ?
- why 3 families / why 16 fundamental fermions per each ?
- what is the origin of Higgs mechanism and SSB ?
- what is the Higgs mass and how are explained all fermionic masses?

...

to be answered by the ultimate unified theory of all interactions

noncommutative spectral geometry

- **NCG** : bottom-up approach

guess small-scale structure of ST from knowledge at EW scale

QG $\Rightarrow$ ST is a wildly noncommutative manifold at a very high energy

at an intermediate scale (few orders below planck scale) the algebra of coordinates is only a mildly noncommutative algebra of matrix valued functions

suitably chosen $\Rightarrow$ SM coupled to gravity

- **string theory** : top-down approach

derive SM directly from planck scale physics

SM as a phenomenological model, which dictates geometry of ST, so that the maxwell-dirac action produces the SM

$\Downarrow$

geometric space defined by the product $\mathcal{M} \times \mathcal{F}$ of a continuum compact riemannian manifold $\mathcal{M}$ and a tiny discrete finite noncommutative space $\mathcal{F}$ composed of 2 points

geometry: tensor product of an internal (zero-dim) geometry for the SM and a continuous geometry for space-time

SM as a phenomenological model, which dictates geometry of ST,
so that the maxwell-dirac action produces the SM

geometric space defined by the product $\mathcal{M} \times \mathcal{F}$

almost commutative geometry

4-dim ST with an "internal" kaluza-klein space attached to each point

the "fifth" dim is a discrete, 0-dim space

NCSG based on 3 ansatz:

1. at some energy level, ST is the product $\mathcal{M} \times \mathcal{F}$

the axioms of spectral triples imply that the Dirac operator of the internal space is the ***fermionic mass matrix***

real spectral triple

$$\mathcal{F} = (\mathcal{A}_{\mathcal{F}}, \mathcal{H}_{\mathcal{F}}, D_{\mathcal{F}})$$

involutive algebra

self-adjoint operator

(nformation carried by metric)

focus on $D_{\mathcal{F}}$ instead of $g_{\mu\nu}$

complex Hilbert space carrying a representation of the algebra

Space X	Algebra $\mathcal{A}$
Real variable x^μ	Self-adjoint operator H
Infinitesimal dx	Compact operator ϵ
Integral of infinitesimal	$\int \epsilon =$ coefficient of $\log(\Lambda)$ in $\mathrm{Tr}_\Lambda(\epsilon)$
Line element $\sqrt{g_{\mu\nu}\, dx^\mu dx^\nu}$	$D^{-1} =$ Fermion propagator

NCSG based on 3 ansatz:

1. at some energy level, ST is the product $\mathcal{M} \times \mathcal{F}$

real spectral triple

$$\mathcal{F} = (\mathcal{A}_{\mathcal{F}}, \mathcal{H}_{\mathcal{F}}, D_{\mathcal{F}})$$

spectral geometry given by

$$\mathcal{A} = \underbrace{C^\infty(\mathcal{M}, C)}_{\text{algebra of smooth complex valued functions on } \mathcal{M}} \otimes \mathcal{A}_{\mathcal{F}}$$

$$\mathcal{H} = \underbrace{L^2(\mathcal{M}, S)}_{\text{space of square integrable Dirac spinors over } \mathcal{M}} \otimes \mathcal{H}_{\mathcal{F}}$$

$$D = \underbrace{D_{\mathcal{M}}}_{\text{Dirac operator on } \mathcal{M}} \otimes 1 + \gamma_5 \otimes D_{\mathcal{F}}$$

$$\partial\!\!\!/_{\mathcal{M}} = \sqrt{-1}\gamma^\mu \nabla^s_\mu$$

II. the finite dimensional algebra $\mathcal{A}_{\mathcal{F}}$ is (main input):

left-right symmetric algebra $\mathbb{C} \oplus \mathbb{H}_L \oplus \mathbb{H}_R \oplus M_3(\mathbb{C})$

algebra of quaternions $\mathbb{H} \subset M_2(\mathbb{C})$ $\quad \mathbb{H} = \left\{ \begin{pmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{pmatrix} \; ; \; \alpha, \beta \in \mathbb{C} \right\}$

however

- model should account for massive neutrinos & neutrino oscillations

$\Rightarrow$ cannot be left-right symmetric model

- NCG imposes constraints on algebras of operators in Hilbert space
- avoid fermion doubling

11. the finite dimensional algebra $\mathcal{A}_{\mathcal{F}}$ is (main input):

$$\mathcal{A}_{\mathcal{F}} = M_a(\mathbb{H}) \oplus M_k(\mathbb{C})$$

$k = 2a$

algebra of quaternions

algebra of complex $k \times k$ matrices

$k = 4$ first value that produces correct # of fermions per generation

prediction

- the number of fermions is the square of an even integer
- the existence of 3 generations is a physical input

III. spectral action principal:

$\mathcal{D}_A = \mathcal{D}_{\mathcal{F}} + A + \epsilon' J A J^{-1}$ $\quad$ $A = \sum_j a_j [\mathcal{D}_{\mathcal{F}}, b_j]$, $a_j, b_j \in \mathcal{A}_{\mathcal{F}}$ $\quad$ $J^2 = \varepsilon$, $J\gamma = \varepsilon'' \gamma J$

the action functional depends only on the <u>spectrum</u> of the generalised Dirac operator and is of the form:

$$Tr(f(D_A/\Lambda))$$

bosonic part

cut-off function

fixes the energy scale

a positive function that falls to zero at large values of its argument ⇒

$$\int_0^\infty f(u)u\,du \;, \quad \int_0^\infty f(u)\,du$$

finite

III. spectral action principal:

the action functional depends only on the spectrum of the generalised Dirac operator and is of the form:

$$Tr(f(D_A/\Lambda)) + \frac{1}{2}\langle J\Psi, \mathcal{D}_A\Psi\rangle \ , \ \ \Psi \in \mathcal{H}_{\mathcal{F}}^{+}$$

III. spectral action principal:

the action functional depends only on the spectrum of the generalised Dirac operator and is of the form:

$$Tr(f(D_A/\Lambda))$$

action sums up eigenvalues of $\mathcal{D}_A$ which are smaller than Λ

evaluate trace with heat kernel techniques:

$$\sum_{n=0}^{\infty} F_{4-n}\Lambda^{4-n}a_n \quad \text{where} \quad F(\mathcal{D}_A^2) = f(\mathcal{D}_A)$$

f : cut-off function ⇒ its taylor expansion at zero vanishes

⇒ the asymptotic expansion of the trace reduces to:

$$\mathrm{Tr}\left(f\left(\frac{D_A}{\Lambda}\right)\right) \sim 2\Lambda^4 f_4 a_0 + 2\Lambda^2 f_2 a_2 + f_0 a_4$$

f plays a rôle through its momenta f_0, f_2, f_4

real parameters related to the coupling constants at unification, the gravitational constant, and the cosmological constant the full lagrangian of SM, minimally coupled to gravity in euclidean form,* obtained as the asymptotic expansion (in inverse powers of Λ) of the spectral action for product ST:

chamseddine, connes, marcolli (2007)

* the discussion of phenomenological aspects relies on a wick rotation to imaginary time

$$\mathcal{L}_{SM} = -\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4} g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu -$$
$$\frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu (W^+_\mu \partial_\nu W^-_\mu -$$
$$W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - igs_w(\partial_\nu A_\mu (W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu (W^+_\mu \partial_\nu W^-_\mu -$$
$$W^-_\mu \partial_\nu W^+_\mu) + A_\mu (W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \frac{1}{2} g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2} g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu +$$
$$g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w (A_\mu Z^0_\nu (W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) \ldots$$

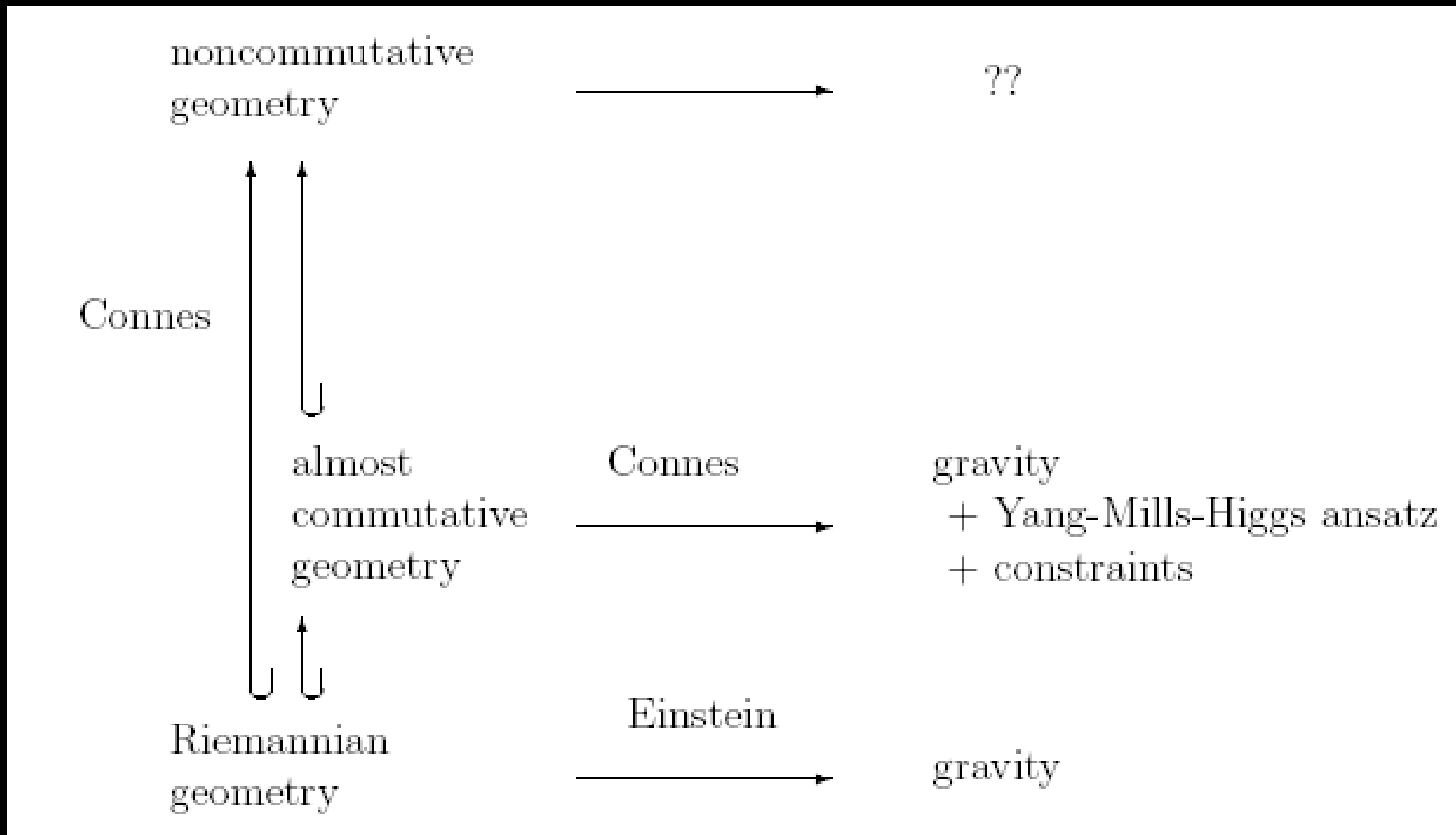
NCSG offers a geometric interpretation of SM

$$\beta_h \left(\frac{2M^2}{g^2} \ldots \right) -$$
$$\frac{1}{8} g^2 \alpha_h (H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2) - gMW^+_\mu W^-_\mu H -$$
$$\frac{1}{2} g \frac{M}{c_w^2} Z^0_\mu Z^0_\mu H - \frac{1}{2} ig (W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)) +$$
$$\frac{1}{2} g (W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)) + \frac{1}{2} g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) +$$
$$M (\frac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+) - ig\frac{s_w^2}{c_w} M Z^0_\mu (W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_w M A_\mu (W^+_\mu\phi^- - W^-_\mu\phi^+) -$$
$$ig\frac{1-2c_w^2}{2c_w} Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4} g^2 W^+_\mu W^-_\mu (H^2 + (\phi^0)^2 + 2\phi^+\phi^-) -$$
$$\frac{1}{8} g^2 \frac{1}{c_w^2} Z^0_\mu Z^0_\mu (H^2 + (\phi^0)^2 + 2(2s_w^2-1)^2\phi^+\phi^-) - \frac{1}{2} g^2 \frac{s_w^2}{c_w} Z^0_\mu \phi^0 (W^+_\mu\phi^- + W^-_\mu\phi^+) -$$
$$\frac{1}{2} ig^2 \frac{s_w^2}{c_w} Z^0_\mu H (W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac{1}{2} g^2 s_w A_\mu \phi^0 (W^+_\mu\phi^- + W^-_\mu\phi^+) + \frac{1}{2} ig^2 s_w A_\mu H (W^+_\mu\phi^- - W^-_\mu\phi^+) -$$
$$g^2\frac{s_w}{c_w}(2c_w^2-1) Z^0_\mu A_\mu \phi^+\phi^- - g^2 s_w^2 A_\mu A_\mu \phi^+\phi^- + \frac{1}{2} ig_s \lambda^a_{ij}(\bar q^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu - \bar e^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar\nu^\lambda(\gamma\partial +$$
$$m^\lambda_\nu)\nu^\lambda - \bar u^\lambda_j(\gamma\partial + m^\lambda_u)u^\lambda_j - \bar d^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_w A_\mu (-(\bar e^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar u^\lambda_j\gamma^\mu u^\lambda_j) - \frac{1}{3}(\bar d^\lambda_j\gamma^\mu d^\lambda_j)) +$$
$$\frac{ig}{4c_w} Z^0_\mu \{(\bar\nu^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar e^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar d^\lambda_j\gamma^\mu(\frac{4}{3} s_w^2 - 1 - \gamma^5)d^\lambda_j) + (\bar u^\lambda_j\gamma^\mu(1 - \frac{8}{3} s_w^2 +$$
$$\gamma^5)u^\lambda_j)\} + \frac{ig}{2\sqrt{2}} W^+_\mu ((\bar\nu^\lambda\gamma^\mu(1+\gamma^5){U^{lep}}_{\lambda\kappa} e^\kappa) + (\bar u^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)) +$$
$$\frac{ig}{2\sqrt{2}} W^-_\mu \left((\bar e^\kappa {U^{lep}}^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar d^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) +$$
$$\frac{ig}{2M\sqrt{2}}\phi^+ (-m^\kappa_e(\bar\nu^\lambda {U^{lep}}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar\nu^\lambda {U^{lep}}_{\lambda\kappa}(1+\gamma^5)e^\kappa) +$$
$$\frac{ig}{2M\sqrt{2}}\phi^- \left(m^\lambda_e(\bar e^\lambda {U^{lep}}^\dagger_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar e^\lambda {U^{lep}}^\dagger_{\lambda\kappa}(1-\gamma^5)\nu^\kappa)\right) - \frac{g}{2}\frac{m^\lambda_\nu}{M}H(\bar\nu^\lambda\nu^\lambda) - \frac{g}{2} \frac{m^\lambda_e}{M}H(\bar e^\lambda e^\lambda) +$$
$$\frac{ig}{2}\frac{m^\lambda_\nu}{M}\phi^0(\bar\nu^\lambda\gamma^5\nu^\lambda) - \frac{ig}{2}\frac{m^\lambda_e}{M}\phi^0(\bar e^\lambda\gamma^5 e^\lambda) - \frac{1}{4} \bar\nu_\lambda M^R_{\lambda\kappa} (1-\gamma_5)\hat\nu_\kappa - \frac{1}{4}\overline{\bar\nu_\lambda M^R_{\lambda\kappa} (1-\gamma_5)\hat\nu_\kappa} +$$
$$\frac{ig}{2M\sqrt{2}}\phi^+ (-m^\kappa_d(\bar u^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar u^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j) +$$
$$\frac{ig}{2M\sqrt{2}}\phi^- \left(m^\lambda_d(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j)\right) - \frac{g}{2}\frac{m^\lambda_u}{M}H(\bar u^\lambda_j u^\lambda_j) - \frac{g}{2}\frac{m^\lambda_d}{M}H(\bar d^\lambda_j d^\lambda_j) +$$
$$\frac{ig}{2}\frac{m^\lambda_u}{M}\phi^0(\bar u^\lambda_j\gamma^5 u^\lambda_j) - \frac{ig}{2}\frac{m^\lambda_d}{M}\phi^0(\bar d^\lambda_j\gamma^5 d^\lambda_j)$$

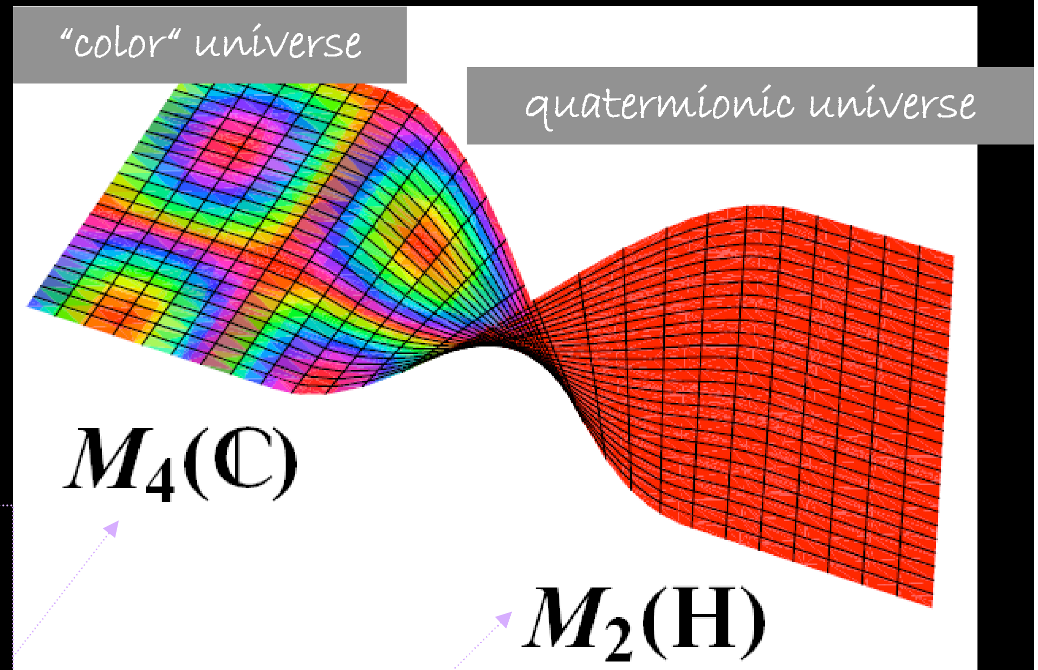
- full SM lagrangian
- majorana mass terms for right-handed neutrinos
- gravitational & cosmological terms coupled to matter

> - EH action with a cosmological term
> - topological term
> - conformal gravity term with the weyl curvature tensor
> - conformal coupling of higgs to gravity

the coefficients of the gravitational terms depend upon the yukawa parameters of the particle physics content


noncommutative geometry
??
Connes
almost commutative geometry
Connes
gravity
+ Yang-Mills-Higgs ansatz
+ constraints
Einstein
Riemannian geometry
gravity

ST is a parallel universe;
each copy is a 4dim manifold



algebra of 4x4 complex matrices,
decomposed into 1x1 and 3x3 matrices

→ split between leptons & quarks

algebra of 2x2 quatermionic matrices, broken by chirality operator to $(\mathbb{C} \oplus \mathbb{C}')_R \oplus \mathbb{H}_L$

fermions: live on both universes

higgs doublet: connects right to left sectors in quaternionic universe

→ this joining gives mass to quarks and leptons

$$S = \frac{1}{\pi^2}\int\left(\frac{1}{2\kappa_0^2}R + \alpha_0 C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^* R^* + \frac{1}{4}G^i_{\mu\nu}G^{\mu\nu i} + \frac{1}{4}F^{\alpha}_{\mu\nu}F^{\mu\nu\alpha}\right.$$
$$\left. + \frac{1}{4}B^{\mu\nu}B_{\mu\nu} + \frac{1}{2}|D_\mu \mathbf{H}|^2 - \mu_0^2|\mathbf{H}|^2 - \xi_0 R|\mathbf{H}|^2 + \lambda_0|\mathbf{H}|^4\right)\sqrt{g}\, d^4x\,,$$

bosonic

bare action a la wislon

$$\kappa_0^2 = \frac{12\pi^2}{96 f_2\Lambda^2 - f_0\mathfrak{c}}\,,$$
$$\alpha_0 = -\frac{3 f_0}{10\pi^2}\,,$$
$$\gamma_0 = \frac{1}{\pi^2}\left(48 f_4\Lambda^4 - f_2\Lambda^2\mathfrak{c} + \frac{f_0}{4}\mathfrak{d}\right),$$
$$\tau_0 = \frac{11 f_0}{60\pi^2}\,,$$
$$\mu_0^2 = 2\Lambda^2\frac{f_2}{f_0} - \frac{\mathfrak{e}}{\mathfrak{a}}\,,$$
$$\xi_0 = \frac{1}{12}\,,$$
$$\lambda_0 = \frac{\pi^2\mathfrak{b}}{2 f_0\mathfrak{a}^2}\,,$$

$$\mathbf{H} = (\sqrt{a f_0}/\pi)\phi$$

$\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}$ describe possible choices of $D_{\mathcal{F}}$

yukawa parameters and majorana terms for ν_{R}

$$\begin{aligned}
\mathfrak{a} &= \mathrm{Tr}\left(Y^\star_{(\uparrow 1)}Y_{(\uparrow 1)} + Y^\star_{(\downarrow 1)}Y_{(\downarrow 1)} + 3\left(Y^\star_{(\uparrow 3)}Y_{(\uparrow 3)} + Y^\star_{(\downarrow 3)}Y_{(\downarrow 3)}\right)\right)\,,\\
\mathfrak{b} &= \mathrm{Tr}\left(\left(Y^\star_{(\uparrow 1)}Y_{(\uparrow 1)}\right)^2 + \left(Y^\star_{(\downarrow 1)}Y_{(\downarrow 1)}\right)^2 + 3\left(Y^\star_{(\uparrow 3)}Y_{(\uparrow 3)}\right)^2 + 3\left(Y^\star_{(\downarrow 3)}Y_{(\downarrow 3)}\right)^2\right)\\
\mathfrak{c} &= \mathrm{Tr}\left(Y^\star_R Y_R\right)\,,\\
\mathfrak{d} &= \mathrm{Tr}\left(\left(Y^\star_R Y_R\right)^2\right)\,,\\
\mathfrak{e} &= \mathrm{Tr}\left(Y^\star_R Y_R Y^\star_{(\uparrow 1)}Y_{(\uparrow 1)}\right)\,,
\end{aligned}$$

Y matrics give fermion and lepton masses, as well as lepton mixing criticisms

- simple almost commutative space

  extend to less trivial noncommutative geometries

- purely classical model

  it cannot be used within EU when QC cannot be neglected

- action functional obtained through perturbative approach in inverse powers of cut-off scale

  it ceases to be valid at lower energy scales (astrophysics)

- model developed in euclidean signature

  physical studies must be done in lorentzian signature

the doubling of the algebra is related to dissipation
and the gauge field structure

⇒ the two-sheeted geometry is <u>the construction</u> that can lead to the gauge fields required to explain the SM

dissipation, may lead to a quantum evolution

⇒ the NCSG classical construction carries in the doubling of the algebra the seeds of quantisation

sakellariadou, stabile, vitiello, PRD 84 (2011) 045026 the need to double the degrees of freedom is implicit even in the classical theory when considering the brownian motion

$$m\ddot{x}(t) + \gamma\dot{x}(t) = f(t)$$

x-system: open (dissipating) system

to set up a canonical formalism

constraint condition at classical level introduces new coordinate y

euler-lagrange eqs:

$$\frac{d}{dt}\frac{\partial L_f}{\partial \dot{y}} = \frac{\partial L_f}{\partial y}\ ; \quad \frac{d}{dt}\frac{\partial L_f}{\partial \dot{x}} = \frac{\partial L_f}{\partial x}$$

$$L_f(\dot{x},\dot{y},x,y) = m\dot{x}\dot{y} + \frac{\gamma}{2}(x\dot{y} - y\dot{x}) + fy$$

$$m\ddot{x} + \gamma\dot{x} = f\ , \quad m\ddot{y} - \gamma\dot{y} = 0$$

{x – y} is a closed system

canonical formalism for dissipative systems the two-sheeted space of NCSG is related to the gauge structure

$$m\ddot{x} + \gamma\dot{x} + kx = 0$$

$$m\ddot{y} - \gamma\dot{y} + ky = 0$$

$$x_1(t) = \frac{x(t)+y(t)}{\sqrt{2}} \ , \qquad x_2(t) = \frac{x(t)-y(t)}{\sqrt{2}}$$

$$\boxed{A_i = \frac{B}{2}\epsilon_{ij}x_j \quad (i,j=1,2)}$$

$$B \equiv \frac{c}{e}\gamma \ , \quad \epsilon_{ii} = 0 \ , \quad \epsilon_{12} = -\epsilon_{21} = 1$$

$$L = \frac{m}{2}(\dot{x}_1^2 - \dot{x}_2^2) + \frac{e}{2}(\dot{x}_1 A_1 + \dot{x}_2 A_2) - e\Phi$$

$$\Phi \equiv (k/2/e)({x_1}^2 - {x_2}^2)$$

- doubled coordinate, e.g. x_2 acts as gauge field component A_1 to which x_1 coordinate is coupled
- energy dissipated by one system is gained by the other one
- gauge field as *bath/reservoir* in which the system is embedded

in agreement with 't hooft's conjecture, loss of information (dissipation) in a regime of completely deterministic dynamics may be responsible of the system's QM evolution

$$m\ddot{x} + \gamma\dot{x} + kx = 0$$

$$m\ddot{y} - \gamma\dot{y} + ky = 0$$

$$\boxed{H = \sum_{i=1}^{2} p_i\, f_i(q)}$$

$$H = H_{\mathrm{I}} - H_{\mathrm{II}}$$

impose constraint

$$H_{\mathrm{II}}|\psi\rangle = 0$$

to define physical states and guarantie that H is bounded from below

this constraint introduces information loss

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physical states are invariant under time reversal and periodical (τ)

$${}_H\langle\psi(\tau)|\psi(0)\rangle_H = e^{i\phi} = e^{i\alpha\pi}$$

dissipation term in H of a couple of classical damped-amplified oscillators manifests itself as a geometric phase

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$$_H\langle\psi(\tau)|\psi(0)\rangle_H = e^{i\phi} = e^{i\alpha\pi}$$

$$\Omega = \sqrt{\frac{1}{m}(k - \frac{\gamma^2}{4m})}$$

$$\langle\psi_n(\tau)|H|\psi_n(\tau)\rangle = \hbar\Omega(n + \frac{\alpha}{2}) = \hbar\Omega n + E_0$$

due to interaction with environment

dissipation term in H of classical damped-amplified oscillators manifests itself as geometric phase and leads to zero point energy

phenomenology

chamseddine, connes, marcolli (2007)

- algebra $\mathcal{A}_{\mathcal{F}}$ of the discrete space $\mathcal{F}$: $M_2(\mathbb{H}) \oplus M_4(\mathbb{C})$

$\Downarrow$

$4^2 = 16$ fermions (# of states on Hilbert space) per family

- gauge bosons: inner fluctuations along continuous directions

- Higgs doublet: inner fluctuations along discrete directions

- mass of the Higgs doublet with –tive sign and a quartic term with a + sign $\Longrightarrow$ mechanism for SSB of EW symmetry

- assuming f is approximated by cut-off function:

normalisation of kinetic terms:

$$\frac{g_3^2 f_0}{2\pi^2} = \frac{1}{4}$$

$$g_2^2 = g_3^2 = \frac{5}{3} g_1^2$$

coincide with those obtained in GUTs

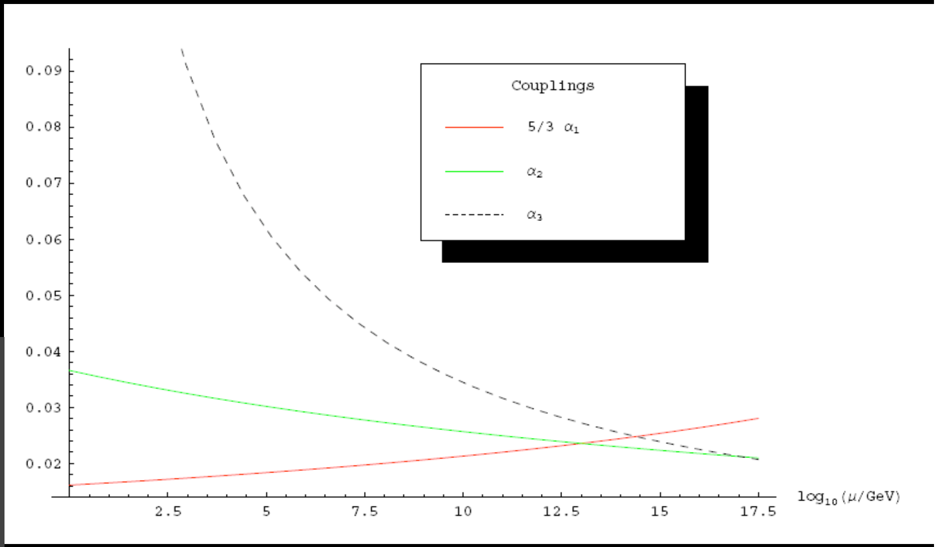
$$\Rightarrow \quad \sin^2 \theta_W = \frac{3}{8}$$

a value also obtained in SU(5) and SO(10)

- assuming big desert hypothesis, the running of the couplings $\alpha_i = g_i^2/(4\pi)$, $i = 1, 2, 3$ up to 1-loop corrections:

$$\beta_i = \frac{1}{(4\pi)^2} b_i g_i^3 \quad \text{with} \quad b = \left(\frac{41}{5}, -\frac{19}{6}, -7\right)$$

not a unique unification energy



- big desert hypothesis approximately valid
- f can be approximated by the cut-off function but there are small deviations

- see-saw mechanism for m_ν with large $m_{\nu_{\text{right-handed}}}$

- constraint on yukawa couplings at unification scale:

$$\sum_\sigma (y_\nu^\sigma)^2 + (y_e^\sigma)^2 + 3(y_u^\sigma)^2 + 3(y_d^\sigma)^2 = 4g^2$$

- mass of top quark:

at unification scale $\Lambda \sim 1.1 \times 10^{17}\text{GeV}$, the $g \sim 0.517$
the RGE predicts $\boxed{m_{\text{top}} \sim 179 \text{ GeV}}$

- in zeroth order approximation: $\xcancel{m_{\text{higgs}} \sim 170\ \text{GeV}}$

o sensitive to the value of unification scale

o sensitive to deviations of spectral function from cut-off function

the higgs mass will be determined by considering higher order corrections and incorporating them to the appropriate RGE

there is a real scalar singlet associated with the majorana mass of right-handed neutrino; this field is nontrivally mixed with higgs

responsible for breakdown of symmetry of discrete space:

$$\mathbb{H} \oplus \mathbb{H} \oplus M_4(\mathbb{C}) \longrightarrow \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$$

connes, chamseddine (2012)

cosmological consequences

corrections to einstein's equations

nelson, sakellariadou, PRD 81 (2010) 085038 bosonic action in euclidean signature:

$$\mathcal{S}^{\mathrm{E}} = \int \Big(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^\star R^\star$$
$$+ \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha} + \frac{1}{4} B^{\mu\nu} B_{\mu\nu}$$
$$+ \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2$$
$$- \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \Big) \sqrt{g}\; d^4x \;,$$

Yang-Mills

scalar minimal coupling

EH term

Weyl curvature term

cosmological term

scalar mass term

topological, thus nondynamical

$$R^\star R^\star = \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} \epsilon_{\alpha\beta\gamma\delta} \; R^{\alpha\beta}_{\mu\nu} R^{\gamma\delta}_{\rho\sigma}$$

$$\mathbf{H} = \left(\sqrt{a f_0} / \pi\right) \phi$$

coupling gravity with matter

scalar quartic potential

gravitational & coupling between Higgs field and Ricci curvature

$\Longrightarrow$ equations of motion

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R + \frac{1}{\beta^2}\delta_{\rm cc}\left[2C^{\mu\lambda\nu\kappa}_{;\lambda;\kappa} + C^{\mu\lambda\nu\kappa}R_{\lambda\kappa}\right] = \kappa_0^2\delta_{\rm cc}T^{\mu\nu}_{\rm matter}$$

$$\beta^2 \equiv -\frac{1}{4\kappa_0^2\alpha_0}$$

$$\delta_{\rm cc} \equiv [1 - 2\kappa_0^2\xi_0\mathbf{H}^2]^{-1}$$

$$\alpha_0 = \frac{-3f_0}{10\pi^2}$$

neglect nonminimal coupling between geometry and higgs

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R + \frac{1}{\beta^2}\delta_{\rm cc}\left[2C^{\mu\lambda\nu\kappa}_{;\lambda;\kappa} + C^{\mu\lambda\nu\kappa}R_{\lambda\kappa}\right] = \kappa_0^2\delta_{\rm cc}T^{\mu\nu}_{\rm matter}$$

$$\beta^2 \equiv -\frac{1}{4\kappa_0^2\alpha_0}$$

$$\delta_{\rm cc} \equiv [1 - 2\kappa_0^2\xi_0\mathbf{H}^2]^{-1}$$

$$\delta_{\rm cc} = 1$$

FLRW:
weyl tensor vanishes, so NCSG corrections to einstein eq. vanish

corrections to einstein's eqs. will be apparent at leading order, only in anisotropic models

bianchi V

$$g_{\mu\nu} = \mathrm{diag}\left[-1, \{a_1(t)\}^2 e^{-2nz}, \{a_2(t)\}^2 e^{-2nz}, \{a_3(t)\}^2\right]$$

$$A_i(t) = \ln a_i(t)$$

$$\kappa_0^2 T_{00} =$$

$$-\dot{A}_3\left(\dot{A}_1+\dot{A}_2\right) - n^2 e^{-2A_3}\left(\dot{A}_1\dot{A}_2 - 3\right)$$

$$+\frac{8\alpha_0\kappa_0^2 n^2}{3} e^{-2A_3}\Big[5\left(\dot{A}_1\right)^2 + 5\left(\dot{A}_2\right)^2 - \left(\dot{A}_3\right)^2$$

$$-\dot{A}_1\dot{A}_2 - \dot{A}_2\dot{A}_3 - \dot{A}_3\dot{A}_1 - \ddot{A}_1 - \ddot{A}_2 - \ddot{A}_3 + 3\Big]$$

$$-\frac{4\alpha_0\kappa_0^2}{3}\sum_i \Big\{\dot{A}_1\dot{A}_2\dot{A}_3\dot{A}_i$$

$$+\dot{A}_i\dot{A}_{i+1}\left(\left(\dot{A}_i - \dot{A}_{i+1}\right)^2 - \dot{A}_i\dot{A}_{i+1}\right)$$

$$+\left(\ddot{A}_i + \left(\dot{A}_i\right)^2\right)\Big[-\ddot{A}_i - \left(\dot{A}_i\right)^2 + \frac{1}{2}\left(\ddot{A}_{i+1} + \ddot{A}_{i+2}\right)$$

$$+\frac{1}{2}\left(\left(\dot{A}_{i+1}\right)^2 + \left(\dot{A}_{i+2}\right)^2\right)\Big]$$

$$+\left[\dddot{A}_i + 3\dot{A}_i\ddot{A}_i - \left(\ddot{A}_i + \left(\dot{A}_i\right)^2\right)\left(\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\right)\right]$$

$$\times\left[2\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\right]\Big\}$$

$$\kappa_0^2 T_{00} =$$

$$-\dot{A}_3\left(\dot{A}_1+\dot{A}_2\right)-n^2e^{-2A_3}\left(\dot{A}_1\dot{A}_2-3\right)$$

$$+\frac{8\alpha_0\kappa_0^2n^2}{3}e^{-2A_3}\left[5\left(\dot{A}_1\right)^2+5\left(\dot{A}_2\right)^2-\left(\dot{A}_3\right)^2\right.$$

$$\left.-\dot{A}_1\dot{A}_2-\dot{A}_2\dot{A}_3-\dot{A}_3\dot{A}_1-\ddot{A}_1-\ddot{A}_2-\ddot{A}_3+3\right]$$

neglecting nonminimal coupling between geometry and higgs field, NCSG corrections to einstein's eqs. are present only in inhomogeneous and anisotropic spacetimes

$$A_i(t) = \ln a_i(t)$$

$$+\left(\ddot{A}_i+\left(\dot{A}_i\right)^2\right)\left[-\ddot{A}_i-\left(\dot{A}_i\right)^2+\frac{1}{2}\left(\ddot{A}_{i+1}+\ddot{A}_{i+2}\right)\right.$$

$$\left.+\frac{1}{2}\left(\left(\dot{A}_{i+1}\right)^2+\left(\dot{A}_{i+2}\right)^2\right)\right]$$

$$+\left[\dddot{A}_i+3\dot{A}_i\ddot{A}_i-\left(\ddot{A}_i+\left(\dot{A}_i\right)^2\right)\left(\dot{A}_i-\dot{A}_{i+1}-\dot{A}_{i+2}\right)\right]$$

$$\times\left[2\dot{A}_i-\dot{A}_{i+1}-\dot{A}_{i+2}\right]\Bigg\}$$

at energies approaching higgs scale, the nonminimal coupling of higgs field to curvature cannot be neglected

equation of motion:

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \kappa_0^2 \left[\frac{1}{1 - \kappa_0^2 |\mathbf{H}|^2/6} \right] T^{\mu\nu}_{\mathrm{matter}}$$

the effect of a nonzero higgs field is to create an effective gravitational constant alternatively, consider the effect on e.o.m. for the higgs field

action for pure higgs field:

$$\mathcal{L}_{|\mathbf{H}|} = -\frac{R}{12}|\mathbf{H}|^2 + \frac{1}{2}|D^\alpha \mathbf{H}||D^\beta \mathbf{H}|g_{\alpha\beta} - \mu_0|\mathbf{H}|^2 + \lambda_0|\mathbf{H}|^4$$

the self interaction of the higgs field is increased:

$$-\mu_0|\mathbf{H}|^2 \rightarrow -\left(\mu_0 + \frac{R}{12}\right)|\mathbf{H}|^2$$

the nominimal coupling of the higgs field to the curvature increases the higgs mass redefine higgs:

$$\tilde{\phi} = -\ln\left(|\mathbf{H}|/(2\sqrt{3})\right)$$

rewrite higgs lagrangian in terms of 4dim dilatonic gravity

$$\mathcal{L}_{|\mathbf{H}|} = -\frac{R}{12}|\mathbf{H}|^2 + \frac{1}{2}|D^\alpha \mathbf{H}||D^\beta \mathbf{H}|\, g_{\alpha\beta} - \mu_0 |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4$$

$$\mathcal{L}_{\tilde{\phi}} = e^{-2\tilde{\phi}}\left[-R + 6D^\alpha\tilde{\phi}D^\beta\tilde{\phi}g_{\alpha\beta} - 12\left(\mu_0 - 12\lambda_0 e^{-2\tilde{\phi}}\right)\right]$$

link with compactified string models

chameleon models

scalar field with nonminimal coupling to standard matter

NCSG

scalar field (higgs) with nonzero coupling to bckg geometry

mass & dynamics of higgs dependent on local matter content

link with chameleon cosmology

gravitational waves in NCSG

nelson, ochoa, sakellariadou, RD 82 (2010) 085021

nelson, ochoa, sakellariadou, PRL <u>105</u> (2010) 101602 linear perturbations around minkowski background in synchronous gauge:

$$g_{\mu\nu} = \mathrm{diag}\left(\{a(t)\}^2 \left[-1, \left(\delta_{ij} + h_{ij}\left(x\right)\right)\right]\right)$$

$$a(t) = 1$$

$$\nabla_i h^{ij} = 0$$

linearised eqs. of motion from NCSG for such perturbations:

$$\left(\Box - \beta^2\right)\Box h^{\mu\nu} = \beta^2 \frac{16\pi G}{c^4} T^{\mu\nu}_{\mathrm{matter}}$$

with conservation eqs: $\frac{\partial}{\partial x^\mu} T^\mu{}_\nu = 0$

$$\beta^2 = -\frac{1}{32\pi G\alpha_0}$$

β^2 plays the role of a mass, so it must be positive $\Longrightarrow \alpha_0 < 0$

$$\alpha_0 = \frac{-3f_0}{10\pi^2}$$

$$\frac{g_3^2 f_0}{2\pi^2} = \frac{1}{4} \qquad g_3^2 = g_2^2 = \frac{5}{3} g_1^2$$

constraint on curvature squared terms (of different form but of the same order to the weyl term) from orbital precession of mercury

$$\boxed{\beta > 3.2 \times 10^{-9} \mathrm{m}^{-1}}$$

stelle (1978)

energy lost to gravitational radiation by orbiting binaries:

$$\boxed{-\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} \approx \frac{c^2}{20G}|\mathbf{r}|^2 \dot{h}_{ij}\dot{h}^{ij}}$$

strong deviations from GR at frequency scale

$$2\omega_c \equiv \beta c \sim (f_0 G)^{-1/2} c$$

set by the moments of the test function f

scale at which NCSG effects become dominant

Binary	Distance (pc)	Orbital Period (hr)	Eccentricity	GR (%)
PSR J0737-3039	~ 500	2.454	0.088	0.2
PSR J1012-5307	~ 840	14.5	$< 10^{-6}$	10
PSR J1141-6545	> 3700	4.74	0.17	6
PSR B1916+16	~ 6400	7.752	0.617	0.1
PSR B1534+12	~ 1100	10.1	?	1
PSR B2127+11C	~ 9980	8.045	0.68	3

restrict β by requiring that the magnitude of deviations from GR must by less than the uncertainty

Binary	Distance (pc)	Orbital Period (hr)	Eccentricity	GR (%)
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approximate accuracy to which the rate of change of orbital period agrees with predictions of GR

$$\omega < \omega_c \quad \text{i.e.} \quad \beta > 2\omega/c$$

PSR J0737-3039	$\beta > 7.55 \times 10^{-13}$ m^{-1}
PSR J1012-5307	$\beta > 7.94 \times 10^{-14}$ m^{-1}
PSR J1141-6545	$\beta > 3.90 \times 10^{-13}$ m^{-1}
PSR B1913+16	$\beta > 2.39 \times 10^{-13}$ m^{-1}
PSR B1534+12	$\beta > 1.83 \times 10^{-13}$ m^{-1}
PSR B2127+11C	$\beta > 2.30 \times 10^{-13}$ m^{-1}

future observations of rapidly orbiting binaries, relatively close to the earth, could improve this constraint by many orders of magnitude

amplitude of effects is proportional $(1 - 2\omega/c\beta)^{-1}$

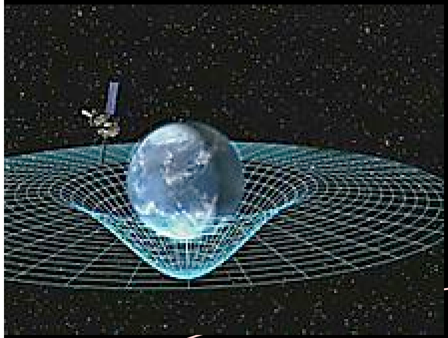
gravitational field in NCSG

gravity probe B

lambiase, sakellariadou, stabile, arXiv:1302.2336 [gr-qc]

gravity probe B

the satellite contains a set of gyroscopes in low circular polar orbit with altitude h=650 km



geodesic precession in the orbital plane

Lense-Thirring precession in the plane of earth equator

GR

Effect	Measured	Predicted
Geodesic precession	6602 ± 18	6606
Lense-Thirring precession	37.2 ± 7.2	39.2

milliarcsec/yr

e.o.m. for gyro spin 3 vector S:

$$\frac{d\mathbf{S}}{dt} = \left.\frac{d\mathbf{S}}{dt}\right|_{\mathrm{G}} + \left.\frac{d\mathbf{S}}{dt}\right|_{\mathrm{LT}}$$

metric:

$$ds^2 = -(1+2\Phi)dt^2 + 2\mathbf{A}\cdot d\mathbf{x}dt + (1+2\Psi)d\mathbf{x}^2$$

instantaneous geodesic precession

$$\left.\frac{d\mathbf{S}}{dt}\right|_{\mathrm{G}} = \mathbf{\Omega}_{\mathrm{G}} \wedge \mathbf{S} \quad \text{with} \quad \mathbf{\Omega}_{\mathrm{G}} = \frac{1}{2}[\nabla(\Phi - 2\Psi)] \wedge \mathbf{v}$$

instantaneous Lense-Thirring precession:

$$\left.\frac{d\mathbf{S}}{dt}\right|_{\mathrm{LT}} = \mathbf{\Omega}_{\mathrm{LT}} \wedge \mathbf{S} \quad \text{with} \quad \mathbf{\Omega}_{\mathrm{LT}} = \frac{1}{2}\nabla \wedge \mathbf{A}$$

geodesic precession

$$\Omega_{\rm G} = \Omega_{\rm G(GR)} + \Omega_{\rm G(NCG)}$$

$$\begin{aligned} \boldsymbol{\Omega}_{\rm G(GR)} &= \frac{3GM}{2r^3}(\mathbf{r} \wedge \mathbf{v})\,, \\ \boldsymbol{\Omega}_{\rm G(NCSG)} &= -\frac{20}{27}(1+\beta r)\, e^{-\beta r}\, \boldsymbol{\Omega}_{\rm G(GR)} \end{aligned}$$

$$\begin{aligned} \Omega_{\rm LT(GR)} &= -\frac{2G}{r^3}\mathbf{J}\,, \\ \Omega_{\rm LT(NCSG)} &= -e^{-\beta r}(1+\beta r+\beta^2 r^2)\Omega_{\rm LT(GR)} \end{aligned}$$

Lense-Thirring precession

$$\boldsymbol{\Omega}_{\rm LT} = \boldsymbol{\Omega}_{\rm LT(GR)} + \boldsymbol{\Omega}_{\rm LT(NCSG)}$$

$$r = R_\oplus + h$$

$$\Omega_{\rm G(GR)} = 6606 \ \ {\rm mas/y}$$

require $|\Omega_{\rm G(NCSG)}| \lesssim \delta\Omega_{\rm G}$ with $\delta\Omega_{\rm G} = 18 \ {\rm mas/y}$

$\Rightarrow$ $\beta \gtrsim 1.1 \times 10^{-6} {\rm m}^{-1}$

inflation through the nonminimal coupling between the geometry and the higgs field

nelson, sakellariadou, PLB 680 (2009) 263

buck, fairbairn, sakellariadou, PRD 82 (2010) 043509 proposal: the higgs field, could play the rôle of the inflaton

but

GR: to get the amplitude of density perturbations, the higgs mass would have to be 11 orders of magnitude **higher**

re-examine the validity of this statement within NCSG

$$S^{\mathrm{L}}_{\mathrm{GH}} = \int \Big[\frac{1 - 2\kappa_0^2\xi_0 H^2}{2\kappa_0^2} R - \frac{1}{2}(\nabla H)^2 - V(H)\Big]\sqrt{-g}\; d^4x$$

$$V(H) = \lambda_0 H^4 - \mu_0^2 H^2$$

subject to radiative corrections as a function of energy

$$f_0 = \pi^2/(2g^2)$$

$$\kappa_0^2 = \frac{12\pi^2}{96 f_2 \Lambda^2 - f_0 \mathfrak{c}}$$

$$\xi_0 = \frac{1}{12}$$

$$\lambda_0 = \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2}$$

$$\mu_0 = 2\Lambda^2 \frac{f_2}{f_0}$$

a priori unconstrained

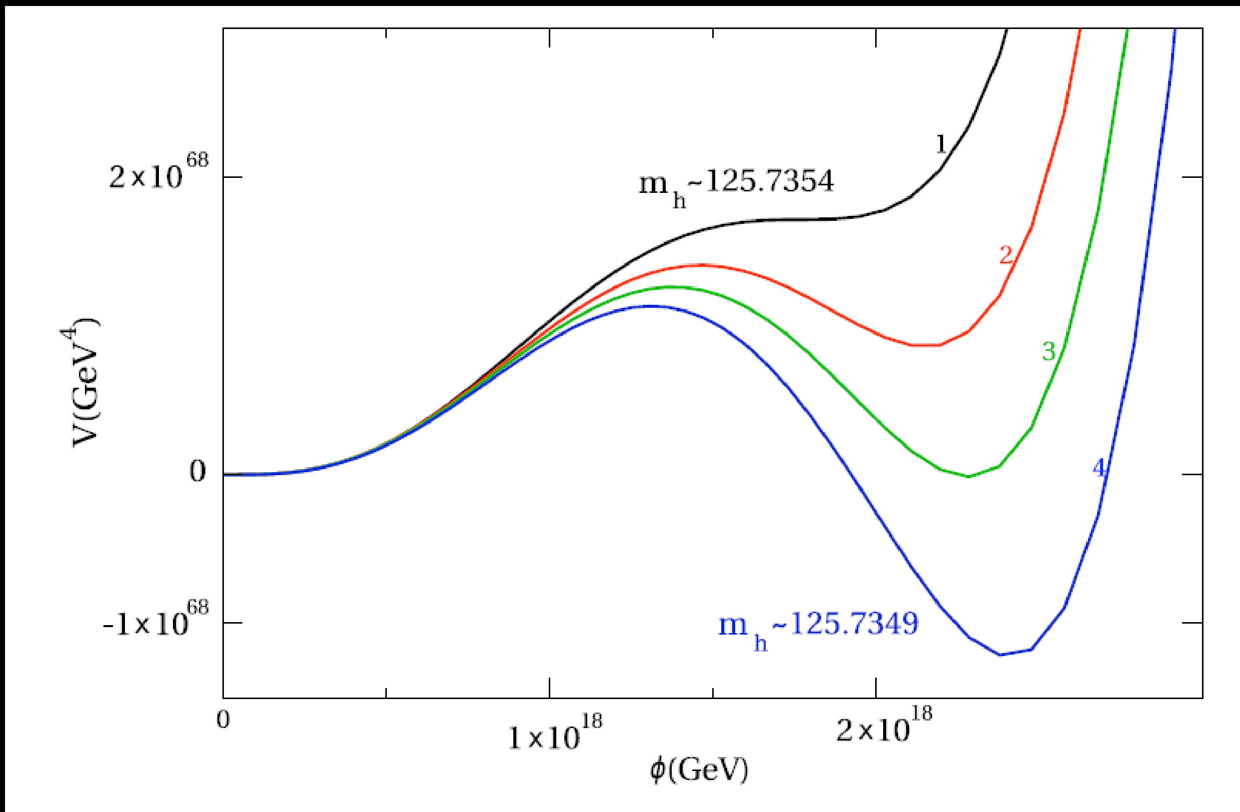
yukawa and majorana parameters subject to RGE aim: flat potential through 2-loop quantum corrections of SM

classical potential: $\boxed{V(H) = \lambda_0 H^4 - \mu_0^2 H^2}$

for very large values of the field $\mathbf{H}$, one needs to calculate the normalised value of the parameters λ_0 and μ_0

effective potential at high energies:

$$V(H) = \lambda(H) H^4$$



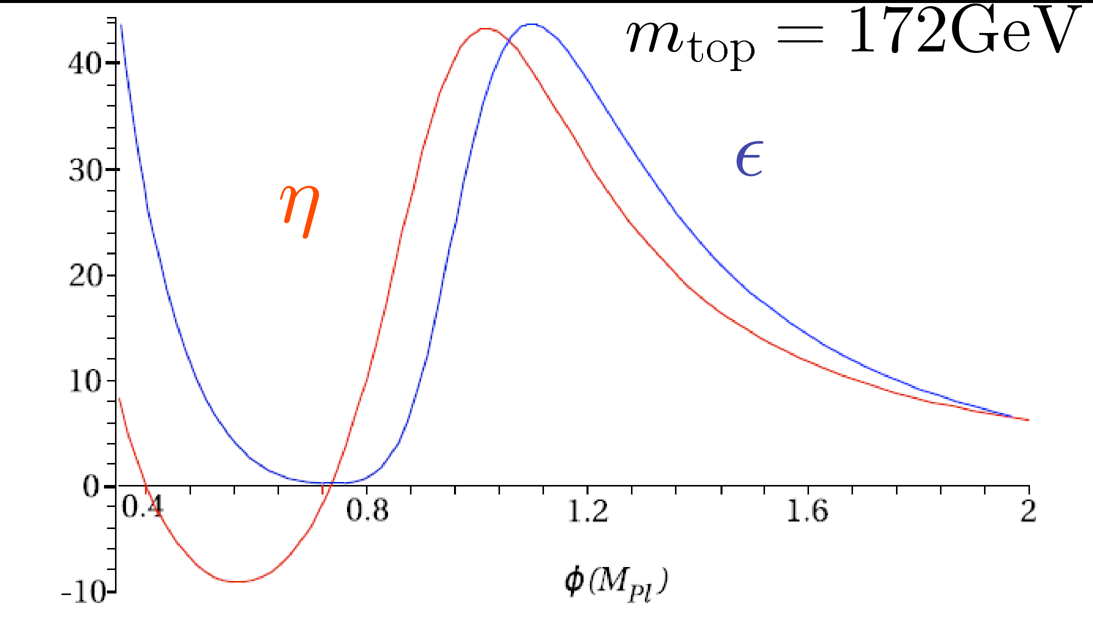
for each value of $m_{\rm top}$ there is a value of $m_{\rm higgs}$ where $V_{\rm eff}$ is on the verge of developing a metastable minimum at large values of $\mathbf{H}$ and $V_{\rm higgs}$ is locally flattened

approach

- calculate renormalisation of higgs self-coupling

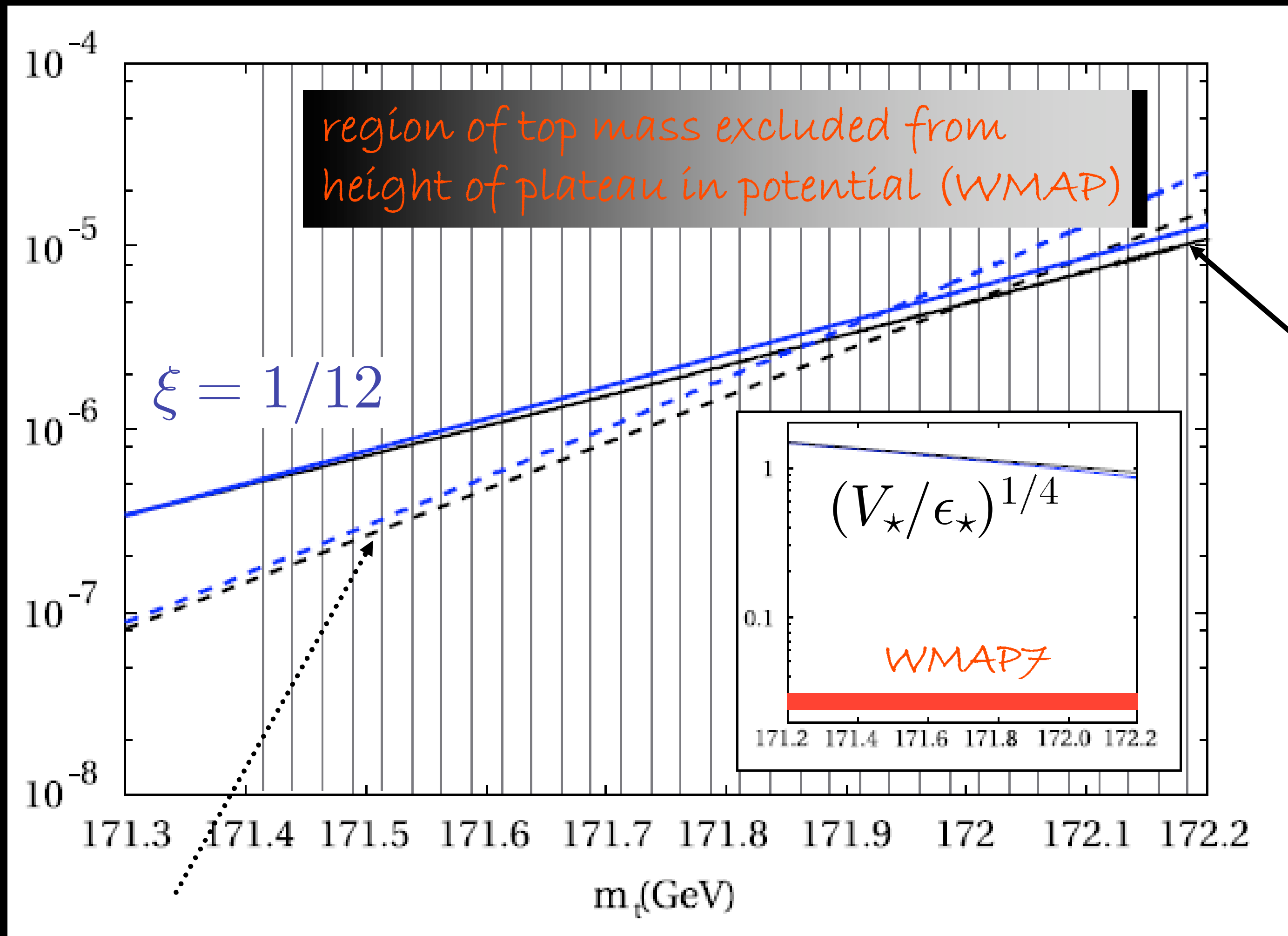
- construct effective potential which fits the RG improved potential around flat region for inflation to occur via the higgs field, the top quark mass fixes the higgs masss extremely accurately

region where **V** becomes flat is narrow, so slow-roll must be very slow



$$N \sim \epsilon^{-1/2} \mathrm{d}\phi$$

ϵ needs to be too small to allow for sufficient e-folds, and then $(V_\star/\epsilon_\star)^{1/4}$ becomes too large to fit the CMB constraint



potential in units of κ^{-4} for minimal coupling

maximum value of the first slow-roll parameter at horizon crossing for minimal coupling while the higgs field potential can lead to the slow-roll conditions being satisfied once the running of the self-coupling at two-loops is included, the constraints imposed from the CMB data make the predictions incompatible with the measured value of the top quark

running of the gravitational constant and corrections by considering the de sitter background do not favour the realisation of a successful inflationary era can we have inflation without introducing a scalar field?

the arbitrary mass scale in the spectral action for the Dirac operator can be made dynamical by introducing a dilaton field,

$$\mathcal{D}/\Lambda \to e^{-\Phi/2}\mathcal{D}e^{-\Phi/2}$$

[redacted] $+G^{\mu\nu} D_{\mu} H'^{*} D_{\nu} H' - V_0 \left(H'^{*} H'\right)\Big] d^4x$

f: dilaton decay constant

$$\boxed{\Phi = (1/f)\tilde{\sigma}}$$

dilaton

scalar field

could this dilaton field play the rôle of the inflaton?

conclusions

how can we construct a quantum theory of gravity coupled to matter ?

- purely gravitational theory without matter

or

- gravity-matter interaction is the most important aspect of dynamics

below planck scale: continuum fields and an effective action

NCSG:

SM fields and gravity packaged into geometry and matter on a kaluza-klein noncommutative space

alain connes' formulation of NCG:

mathematical/physical notions described in terms of spectral properties of operators

aim: differential geometry $\xleftrightarrow{\text{mapping}}$ algebraic terms

topology of space described in terms of algebras

NCSG depends crucially on choice of algebra $\mathcal{A}_{\mathcal{F}}$ represented on a Hilbert space $\mathcal{H}_{\mathcal{F}}$ and the Dirac operator $D_{\mathcal{F}}$

physical picture of the discrete space

- left/right-handed fermions are placed on two different sheets
- Higgs fields: the gauge fields in the discrete dimensions
- inverse of separation between the two sheets: EW energy scale

picture similar to the randall-sundrum scenario

4dim brane embedded into 5dim manifold as 3dim brane placed at $x_5 = 0$, $x_5 = \pi r_{\text{compactification}}$

NCSG extends notion of commutative spaces, using data encoded in a spectral triple on a space composed by $\mathcal{M} \times \mathcal{F}$

- geometric explanation for SM phenomenology

- framework for early universe cosmology