A scenario for resolving a Big Crunch

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- "Universe Explosions" (R. Brustein, MSS), 1209.5222
- "A Braneworld puzzle about entropy bounds and a maximal temperature" (R. Brustein, D. Eichler, S. Foffa), Phys. Rev. D 71, 124015 (2005), th-0404320
- "A Bound on the effective gravitational coupling from semiclassical black holes" (R. Brustein, G. Dvali, G. Veneziano), JHEP 0910 085 (2009), 0907.5516

Introduction

What happens to a contracting universe?

- Singularity theorems $\Rightarrow$ Big Crunch
- Singularity resolved in quantum gravity?
- Entropy bounds improve singularity theorems.
- Attempt: Use Thermodynamics
- Saturation of entropy bounds $\Rightarrow$ Instability

Entropy bounds

Bekenstein: $S \leq ER$ (if $R > R_S = 2G_N E$)

Holography: $S \leq A l_P^{-2}$

Interpretation: Stability region of black holes

Consider theories with $N \gg 1$ light species (curvatures small)

Black hole vs. radiation

Consider a black hole of mass M, size $R_S = 2GM$, temperature $T_H = 1/R_S$

Compare with radiation

$$E_{BH} \sim \frac{R_S}{l_P^2} \qquad E_{rad} \sim NT^4R^3 = \frac{N}{R_S} \tag{1}$$

$$S_{BH} \sim \frac{R_S^2}{l_P^2} \qquad S_{rad} \sim NT^3R^3 = N \tag{2}$$

Equality reached when $R_S \sim \sqrt{N} l_P \Rightarrow$ Instability

Black hole decay

Assumption 1: Black body radiation

$$\frac{dM}{dt} = -NT_H^4 R_S^2 = -\frac{N}{R_S^2} \tag{3}$$

Assumption 2: 'Semiclassicality'

$$-\frac{1}{M}\frac{dM}{dt} < T_H \qquad -\frac{1}{M}\frac{dM}{dt} < \frac{1}{R_S} \tag{4}$$

Breakdown when $R_S = \sqrt{N} l_P$

Define Γ to be the inverse of the time until emission of a quantum. Then $\frac{dM}{dt} = -\Gamma T \Rightarrow \Gamma = \frac{N}{R_S}$
$\Gamma/M = 1$ implies $R_S = \sqrt{N} l_P$

'Quantum' computation

Semiclassical computation invalid when black hole is unstable

Treat black hole as large object decaying into many soft particles

Estimate decay width from phase space:

$$V_n(E) = \int \prod_{i=1}^{n} [d^4 p_i \delta(p_i^2) \theta(p_i^0)] \delta^4(P - \sum_{i=1}^{n} p_i) \sim E^{2n-4} \quad (5)$$

Microcanonical distribution!

Entropy: $S = \log V_n \sim 2n \log E$

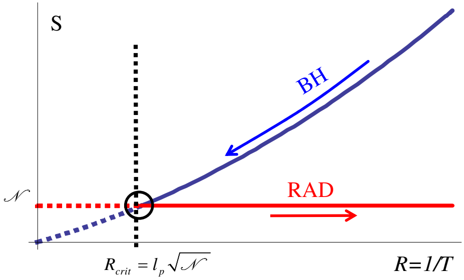
Temperature (from $1/T = dS/dE$): $T = E/2n$

Similar behaviour in string theory

Final moments of black hole

Conclusion: Characteristics of thermal decay

When black hole reaches the size $R_S \sim \sqrt{N} l_P$ it decays instantaneously



Cosmological entropy bounds

- Cosmological spacetimes have entropy
- Clearest example: de Sitter space
- Entropy associated with inaccessible region
- Assume: Entropy $\propto$ area for cosmological horizons
- Bekenstein: Entropy bounds $\Rightarrow$ Temperature bounded $\Rightarrow$ Singularity avoided
- Entropy bounds saturated when $H \sim \frac{M_P}{\sqrt{N}}$

Universe vs. radiation

Interpretation of entropy bounds: Saturation ⇒ Instability

Compare universe with radiation in equilibrium at $T = H$ in region of size $R = 1/H$

$$E_U = \rho V \sim \frac{H^2 R^3}{l_P^2} \sim \frac{1}{H l_P^2} \qquad E_{rad} \sim N T^4 R^3 = NH \quad (6)$$

$$S_U \sim \frac{R^2}{l_P^2} = \frac{1}{H^2 l_P^2} \qquad S_{rad} \sim N T^3 R^3 = N \quad (7)$$

Equality when $R = 1/H = \sqrt{N} l_P$

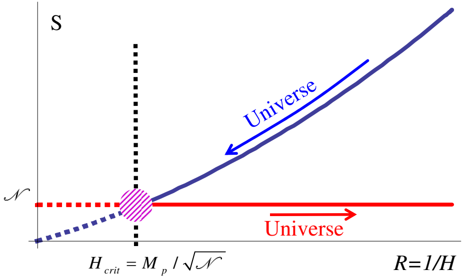
Proposal: universe transformed into radiation instantaneously

Evolution of universe

Proposal: universe transformed into radiation instantaneously

Highly excited quantum state created - no geometric description

Relaxation to lower density - semiclassical description valid



FRW brane in AdS Schwarzschild

Consider a RD brane universe in a black hole background - 'Mirage cosmology'

$$ds^2 = -H(R)dt^2 + \frac{dR^2}{H(R)} + R^2 d\Omega_3^2 \quad (8)$$

$$H(R) = 1 + \frac{R^2}{L^2} - \frac{b^4 L^2}{R^2} \quad (9)$$

$$b = \left(\frac{8 G_N^{(5)}}{3\pi} \frac{M}{L^2} \right)^{1/4} \gg 1 \quad (10)$$

Black hole large - brane universe flat

$$R_S : H(R_S) = 0 \qquad R_S \approx bL \qquad T_H = \frac{b}{L} \quad (11)$$

R_b: radial position of brane

$$\left(\frac{\dot{R}_b}{R_b}\right)^2 = \frac{b^4 L^2}{R_b^4} - \frac{1}{R_b^2} \tag{12}$$

Friedmann equation for RD universe; temperature $T = b/R_b$

Entropy bounds saturated when
$T = M_P/\sqrt{N} = 1/L \Rightarrow R_b = R_S$
i.e. when the brane crosses the horizon

energy of the brane: $E = \lambda R^3 = b^3 L^2 / G_N^{(5)}$
$E/M \sim 1/b \ll 1$

Time scales

Brane point of view:

'Infall' time: Integrate $\dot{R}_b = -\sqrt{\frac{b^4 L^2}{R_b^2} - 1}$ between $R_b = b^2 L$ and $R_b = bL$

Result: $\tau_{infall} = b^2 L$

At horizon crossing: $H = \dot{R}_b / R_b = 1/L$

$\Rightarrow$ Universe transformed into radiation very fast

Time scales

Bulk point of view:

Observer at $R = R_b$

$$\frac{1}{H(R)}\left(\frac{dR}{d\tau}\right)^2 - H(R)\left(\frac{dt}{d\tau}\right)^2 = -1 \quad (13)$$

$$t_{infall} = \frac{L}{4b}\log\frac{R_* - bL}{R_{min} - bL} \quad (14)$$

$$(\Delta t)_{U\to rad} = \frac{L}{4b}\log\frac{\Delta R}{R_{min} - bL} \quad (15)$$

$R_* \gg bL$, $\Delta R \ll R_* \Rightarrow$ Consistent with brane point of view

Bulk vs. brane point of view

Brane: Universe turns into radiation very fast

Bulk:

- Black hole swallows brane
- It becomes larger and hotter
- A shell of hotter Hawking radiation emitted
- Black hole returns to equilibrium state

Result: An outgoing shell of radiation - consistent with brane point of view

Compute emission time from $\frac{dM}{dt}|_{eff} = -\frac{(G_N^{(5)})^2}{L^6} ME$

$\Rightarrow t_{1/2} = \frac{L^3}{G_N^{(5)}} \frac{1}{b^3} \frac{L}{b}$

Collapsing universe made of dust

Consider spherically symmetric distribution of particles
Density increases due to gravitational interaction

$$\frac{d^2R(t)}{dt^2} = -\frac{GM(R(t))}{R(t)^2} \tag{16}$$

$$\frac{1}{2}\left(\frac{dR(t)}{dt}\right)^2 - \frac{GM}{R(t)} = e \tag{17}$$

Define $H = \dot{R}(t)/R(t)$
Instability when $H = 1/(l_P\sqrt{N}) \Rightarrow R_{min} = (Nl_P^2 R_S(R(t)))^2$
$\Rightarrow$ Transition to radiation

Summary

- Scenario for resolution of Big Crunch
- Use thermodynamics and entropy bounds
- Black hole/Universe turn into radiation