Emergent Space-Time, The Random Geometry Perspective

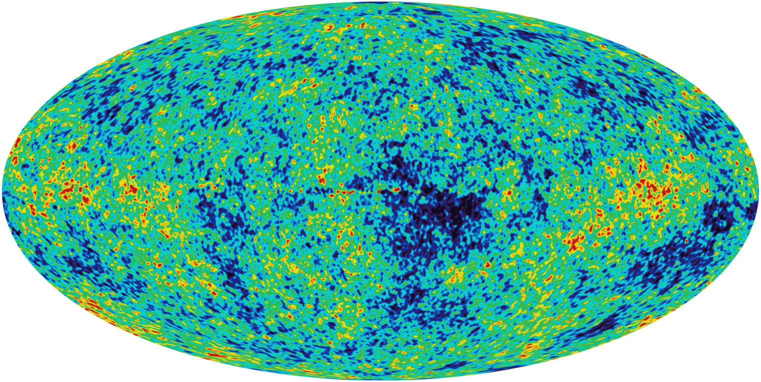
Vincent Rivasseau

Laboratoire de Physique théorique
Université Paris-Sud
and Perimeter Institute

Albert Einstein Institute,
March 6, 2013

The data to explain

Good Questions for a theory of quantum gravity



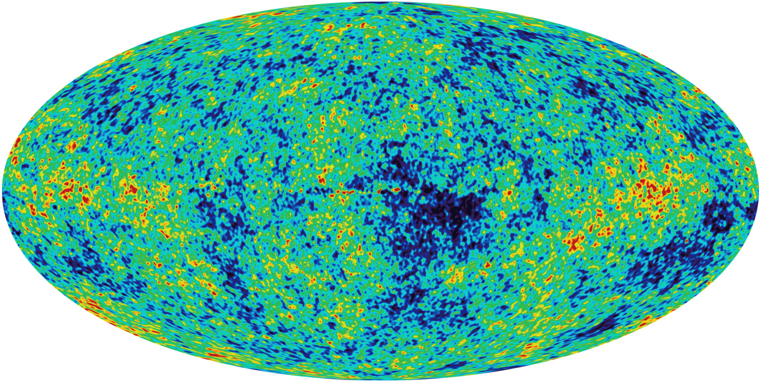
How did this came into being?

$$\Lambda \simeq 10^{-52} m^{-2} \simeq 10^{-122} \quad \text{(in Planck units)}$$

Why is this so small but not zero?

The data to explain

Good Questions for a theory of quantum gravity



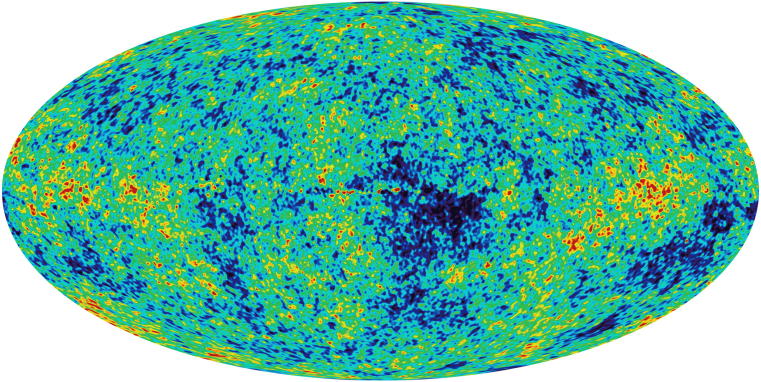
How did this came into being?

$$\Lambda \simeq 10^{-52} m^{-2} \simeq 10^{-122} \quad \text{(in Planck units)}$$

Why is this so small but not zero?

The data to explain

Good Questions for a theory of quantum gravity



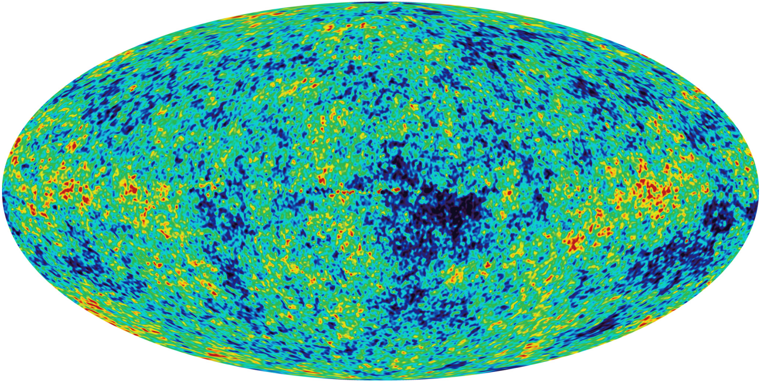
How did this came into being?

$$\Lambda \simeq 10^{-52} m^{-2} \simeq 10^{-122} \quad \text{(in Planck units)}$$

Why is this so small but not zero?

The data to explain

Good Questions for a theory of quantum gravity



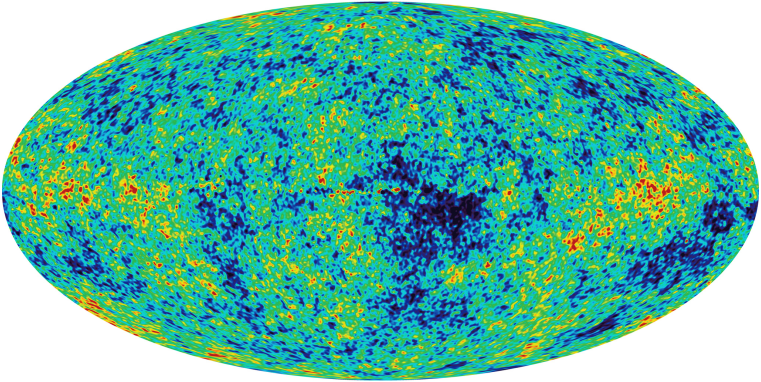
How did this came into being?

$$\Lambda \simeq 10^{-52} m^{-2} \simeq 10^{-122} \quad \text{(in Planck units)}$$

Why is this so small but not zero?

The data to explain

Good Questions for a theory of quantum gravity



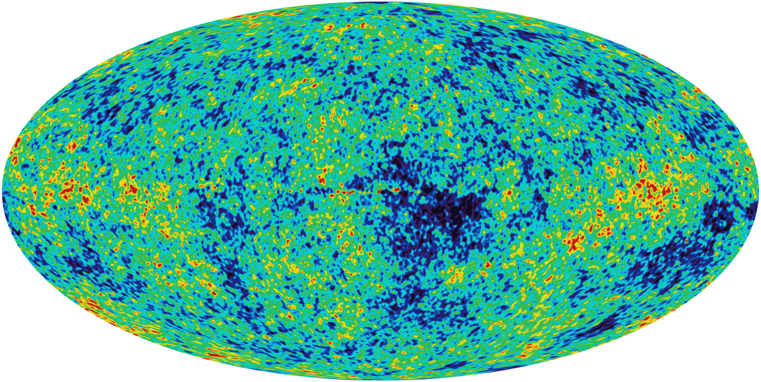
How did this came into being?

$$\Lambda \simeq 10^{-52} m^{-2} \simeq 10^{-122} \quad \text{(in Planck units)}$$

Why is this so small but not zero?

The data to explain

Good Questions for a theory of quantum gravity



How did this came into being?

$$\Lambda \simeq 10^{-52} m^{-2} \simeq 10^{-122} \quad \text{(in Planck units)}$$

Why is this so small but not zero?

Quantum Gravity as (Large) Random Geometry

Quantizing Gravity ≃ Randomizing Geometry ?

$$Z \simeq \int Dg \quad e^{\int S_{EH}(g)}$$

But what is the measure Dg? On which underlying space-time? Should one also sum on topologies? How to relate quantum gravity to classical space and time as we know them?

Quantum Gravity as (Large) Random Geometry

Quantizing Gravity $\simeq$ Randomizing Geometry ?

$$Z \simeq \int Dg \quad e^{\int S_{EH}(g)}$$

But what is the measure Dg? On which underlying space-time? Should one also sum on topologies? How to relate quantum gravity to classical space and time as we know them?

Quantum Gravity as (Large) Random Geometry

Quantizing Gravity $\simeq$ Randomizing Geometry ?

$$Z \simeq \int Dg \quad e^{\int S_{EH}(g)}$$

But what is the measure Dg? On which underlying space-time? Should one also sum on topologies? How to relate quantum gravity to classical space and time as we know them?

Quantum Gravity as (Large) Random Geometry

Quantizing Gravity $\simeq$ Randomizing Geometry ?

$$Z \simeq \int Dg \quad e^{\int S_{EH}(g)}$$

But what is the measure Dg? On which underlying space-time? Should one also sum on topologies? How to relate quantum gravity to classical space and time as we know them?

Quantum Gravity as (Large) Random Geometry

Quantizing Gravity $\simeq$ Randomizing Geometry ?

$$Z \simeq \int Dg \quad e^{\int S_{EH}(g)}$$

But what is the measure Dg? On which underlying space-time? Should one also sum on topologies? How to relate quantum gravity to classical space and time as we know them?

Quantum Gravity as (Large) Random Geometry

Quantizing Gravity $\simeq$ Randomizing Geometry ?

$$Z \simeq \int Dg \quad e^{\int S_{EH}(g)}$$

But what is the measure Dg? On which underlying space-time? Should one also sum on topologies? How to relate quantum gravity to classical space and time as we know them?

Quantum Gravity as (Large) Random Geometry

Quantizing Gravity $\simeq$ Randomizing Geometry ?

$$Z \simeq \int Dg \quad e^{\int S_{EH}(g)}$$

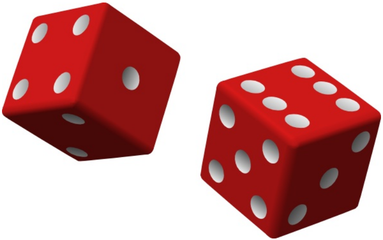
But what is the measure Dg? On which underlying space-time? Should one also sum on topologies? How to relate quantum gravity to classical space and time as we know them?

Probability and Enumerative Combinatorics

The tensor track is an ab initio approach to quantum gravity. best described as an extension of random matrix theory and a reformulation of dynamical triangulations.

It proposes to use mathematics from quantum field theory, geometry, probability theory and enumerative combinatorics.

In probability theory careful counting is critical.

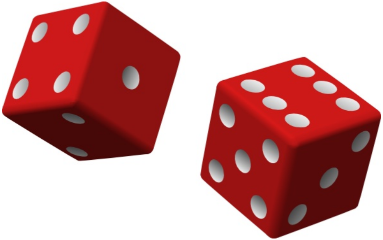


Probability and Enumerative Combinatorics

The tensor track is an ab initio approach to quantum gravity. best described as an extension of random matrix theory and a reformulation of dynamical triangulations.

It proposes to use mathematics from quantum field theory, geometry, probability theory and enumerative combinatorics.

In probability theory careful counting is critical.

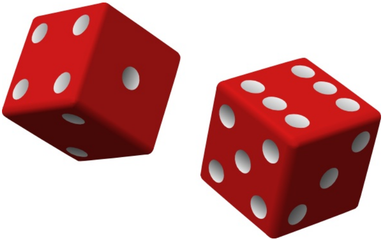


Probability and Enumerative Combinatorics

The tensor track is an ab initio approach to quantum gravity. best described as an extension of random matrix theory and a reformulation of dynamical triangulations.

It proposes to use mathematics from quantum field theory, geometry, probability theory and enumerative combinatorics.

In probability theory careful counting is critical.

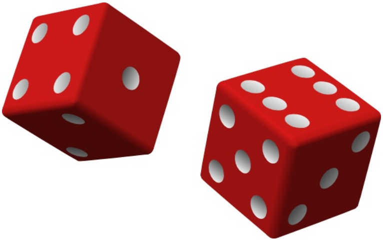


Probability and Enumerative Combinatorics

The tensor track is an ab initio approach to quantum gravity. best described as an extension of random matrix theory and a reformulation of dynamical triangulations.

It proposes to use mathematics from quantum field theory, geometry, probability theory and enumerative combinatorics.

In probability theory careful counting is critical.

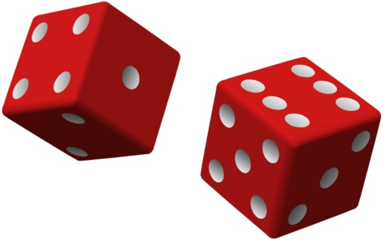


Probability and Enumerative Combinatorics

The tensor track is an ab initio approach to quantum gravity. best described as an extension of random matrix theory and a reformulation of dynamical triangulations.

It proposes to use mathematics from quantum field theory, geometry, probability theory and enumerative combinatorics.

In probability theory careful counting is critical.

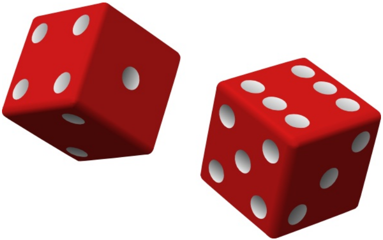


Probability and Enumerative Combinatorics

The tensor track is an ab initio approach to quantum gravity. best described as an extension of random matrix theory and a reformulation of dynamical triangulations.

It proposes to use mathematics from quantum field theory, geometry, probability theory and enumerative combinatorics.

In probability theory careful counting is critical.



Random Spaces

Right now (2013) probabilists know only two universal non-trivial large random spaces:

- The Continuous Random Tree (Aldous, $\simeq$ 1990
- The Brownian Sphere (come also as Brownian plane or as fixed genus Riemann surface version)

The second space can be thought of as made of a non trivial "quantum field" living on the first.

Random Spaces

Right now (2013) probabilists know only two universal non-trivial large random spaces:

- The Continuous Random Tree (Aldous, $\simeq$ 1990
- The Brownian Sphere (come also as Brownian plane or as fixed genus Riemann surface version)

The second space can be thought of as made of a non trivial "quantum field" living on the first.

Random Spaces

Right now (2013) probabilists know only two universal non-trivial large random spaces:

- The Continuous Random Tree (Aldous, $\simeq$ 1990
- The Brownian Sphere (come also as Brownian plane or as fixed genus Riemann surface version)

The second space can be thought of as made of a non trivial "quantum field" living on the first.

Random Spaces

Right now (2013) probabilists know only two universal non-trivial large random spaces:

- The Continuous Random Tree (Aldous, $\simeq$ 1990
- The Brownian Sphere (come also as Brownian plane or as fixed genus Riemann surface version)

The second space can be thought of as made of a non trivial "quantum field" living on the first.

Random Spaces

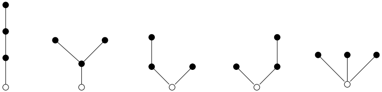
Right now (2013) probabilists know only two universal non-trivial large random spaces:

- The Continuous Random Tree (Aldous, $\simeq$ 1990
- The Brownian Sphere (come also as Brownian plane or as fixed genus Riemann surface version)

The second space can be thought of as made of a non trivial "quantum field" living on the first.

The Continuous Random Tree

Plane trees are quite universal. The number of plane trees with n edges is the Catalan number $C_n = \frac{1}{n+1}\binom{2n}{n}$.

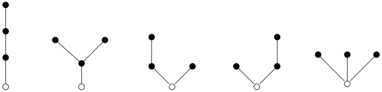


The 5 plane trees at $n = 3$

The equidistributed measure on plane trees converges (in Gromov-Hausdorff sense) to a universal object as $n \to \infty$, namely the CRT.

The Continuous Random Tree

Plane trees are quite universal. The number of plane trees with n edges is the Catalan number $C_n = \frac{1}{n+1}\binom{2n}{n}$.

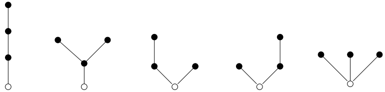


The 5 plane trees at $n = 3$

The equidistributed measure on plane trees converges (in Gromov-Hausdorff sense) to a universal object as $n \to \infty$, namely the CRT.

The Continuous Random Tree

Plane trees are quite universal. The number of plane trees with n edges is the Catalan number $C_n = \frac{1}{n+1}\binom{2n}{n}$.

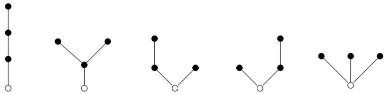


The 5 plane trees at $n = 3$

The equidistributed measure on plane trees converges (in Gromov-Hausdorff sense) to a universal object as $n \to \infty$, namely the CRT.

The Continuous Random Tree

Plane trees are quite universal. The number of plane trees with n edges is the Catalan number $C_n = \frac{1}{n+1}\binom{2n}{n}$.



The 5 plane trees at $n = 3$

The equidistributed measure on plane trees converges (in Gromov-Hausdorff sense) to a universal object as $n \to \infty$, namely the CRT.

Main Properties of the Continuous Random Tree

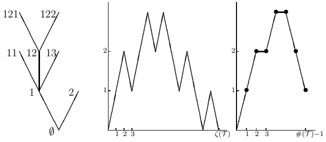
The Harris (or Dyck) Walk of a CRT is exactly a Brownian excursion quotiented by an equivalence relation.

In physics, the CRT is also called a branched polymer.

$$d_{Hausdorff} = 2, \; d_{spectral=4/3}.$$

Main Properties of the Continuous Random Tree

The Harris (or Dyck) Walk of a CRT is exactly a Brownian excursion quotiented by an equivalence relation.



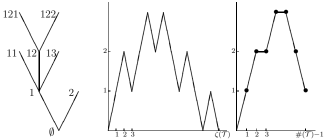
tree **t** contour function height function

In physics, the CRT is also called a branched polymer.

$$d_{Hausdorff} = 2, \; d_{spectral=4/3}.$$

Main Properties of the Continuous Random Tree

The Harris (or Dyck) Walk of a CRT is exactly a Brownian excursion quotiented by an equivalence relation.



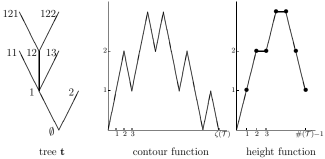
tree **t** contour function height function

In physics, the CRT is also called a branched polymer.

$d_{Hausdorff} = 2, \; d_{spectral=4/3}.$

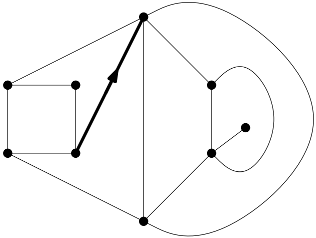
Main Properties of the Continuous Random Tree

The Harris (or Dyck) Walk of a CRT is exactly a Brownian excursion quotiented by an equivalence relation.

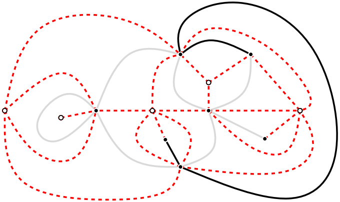


In physics, the CRT is also called a branched polymer.

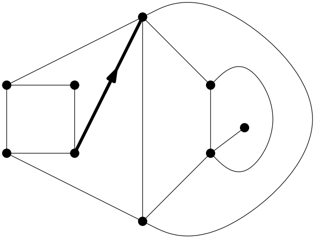
$$d_{Hausdorff} = 2,\ d_{spectral=4/3}.$$



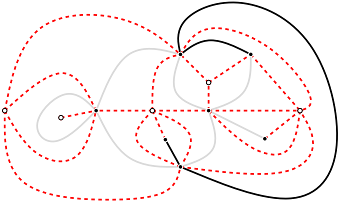
Rooted planar quadrangulations are simple objects



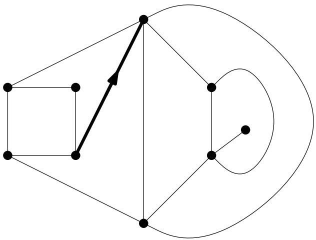
They are quite universal...



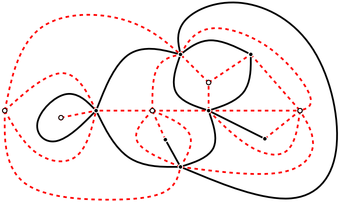
Rooted planar quadrangulations are simple objects



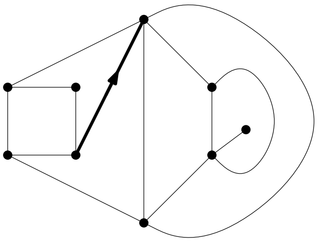
They are quite universal...



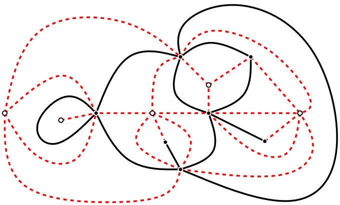
Rooted planar quadrangulations are simple objects



They are quite universal...



Rooted planar quadrangulations are simple objects

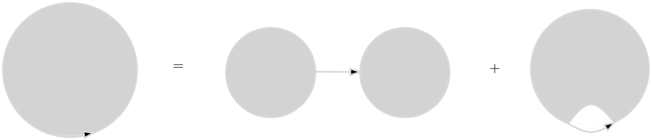


They are quite universal...

Counting Planar Graphs à la Tutte (1963)

Q_n= number of rooted planar quadrangulations with n faces

Adding boundaries Tutte found in 1963 a quadratic recursive equation (à la Polchinski),



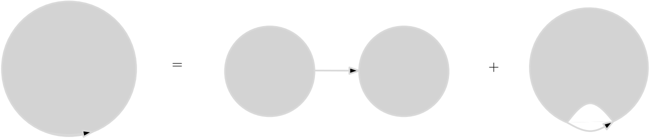
and solved it, getting:

$$Q_n = 3^n \frac{2}{n+2} \frac{1}{n+1} \binom{2n}{n}.$$

Counting Planar Graphs à la Tutte (1963)

Q_n= number of rooted planar quadrangulations with n faces

Adding boundaries Tutte found in 1963 a quadratic recursive equation (à la Polchinski),



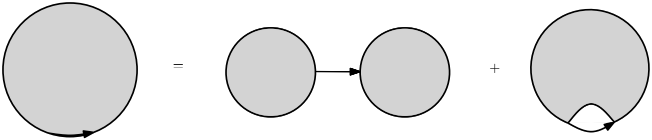
and solved it, getting:

$$Q_n = 3^n \frac{2}{n+2} \frac{1}{n+1} \binom{2n}{n} .$$

Counting Planar Graphs à la Tutte (1963)

Q_n= number of rooted planar quadrangulations with n faces

Adding boundaries Tutte found in 1963 a quadratic recursive equation (à la Polchinski),



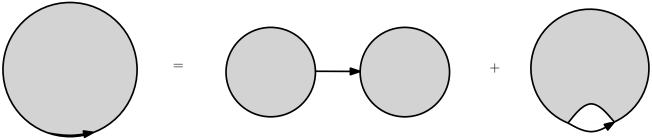
and solved it, getting:

$$Q_n = 3^n \frac{2}{n+2} \frac{1}{n+1} \binom{2n}{n}.$$

Counting Planar Graphs à la Tutte (1963)

Q_n= number of rooted planar quadrangulations with n faces

Adding boundaries Tutte found in 1963 a quadratic recursive equation (à la Polchinski),



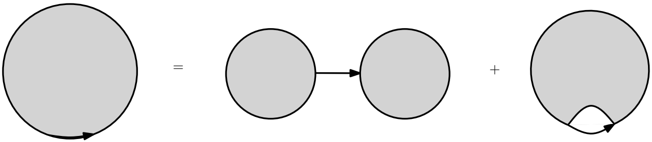
and solved it, getting:

$$Q_n = 3^n \frac{2}{n+2} \frac{1}{n+1} \binom{2n}{n}.$$

Counting Planar Graphs à la Tutte (1963)

Q_n= number of rooted planar quadrangulations with n faces

Adding boundaries Tutte found in 1963 a quadratic recursive equation (à la Polchinski),



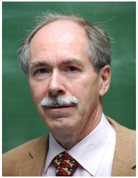
and solved it, getting:

$$Q_n = 3^n \frac{2}{n+2} \frac{1}{n+1} \binom{2n}{n} .$$

Counting Planar Graphs à la 'tHooft and Brezin-Itzykson-Parisi-Zuber (1978)

Why planar quadrangulations?

QFT answer: because they are dual to the Feynman graphs which dominate the $1/N$ expansion of a matrix model...

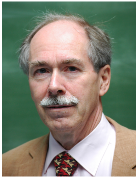


$$
\begin{aligned}
Z &= \int dM \, \exp(-\frac{1}{2}\mathrm{Tr} M^2 + \frac{\lambda}{N}\mathrm{Tr} M^4) \\
&= \sum_{n,g} a_{n,g} \lambda^n N^{2-2g}
\end{aligned}
$$

Counting Planar Graphs à la 'tHooft and Brezin-Itzykson-Parisi-Zuber (1978)

Why planar quadrangulations?

QFT answer: because they are dual to the Feynman graphs which dominate the $1/N$ expansion of a matrix model...

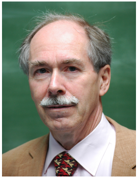


$$Z = \int dM \exp(-\frac{1}{2}\mathrm{Tr} M^2 + \frac{\lambda}{N}\mathrm{Tr} M^4)$$
$$= \sum_{n,g} a_{n,g} \lambda^n N^{2-2g}$$

Counting Planar Graphs à la 'tHooft and Brezin-Itzykson-Parisi-Zuber (1978)

Why planar quadrangulations?

QFT answer: because they are dual to the Feynman graphs which dominate the $1/N$ expansion of a matrix model...

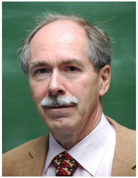


$$Z = \int dM \exp(-\frac{1}{2}\mathrm{Tr} M^2 + \frac{\lambda}{N}\mathrm{Tr} M^4)$$
$$= \sum_{n,g} a_{n,g}\lambda^n N^{2-2g}$$

Counting Planar Graphs à la 'tHooft and Brezin-Itzykson-Parisi-Zuber (1978)

Why planar quadrangulations?

QFT answer: because they are dual to the Feynman graphs which dominate the $1/N$ expansion of a matrix model...

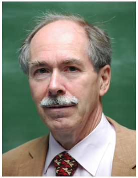


$$
\begin{aligned}
Z &= \int dM \exp(-\frac{1}{2}\mathrm{Tr} M^2 + \frac{\lambda}{N}\mathrm{Tr} M^4) \\
&= \sum_{n,g} a_{n,g}\lambda^n N^{2-2g}
\end{aligned}
$$

Counting Planar Graphs à la 'tHooft and Brezin-Itzykson-Parisi-Zuber (1978)

Why planar quadrangulations?

QFT answer: because they are dual to the Feynman graphs which dominate the $1/N$ expansion of a matrix model...

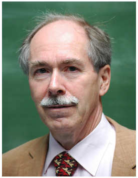


$$Z = \int dM \exp(-\frac{1}{2}\mathrm{Tr} M^2 + \frac{\lambda}{N}\mathrm{Tr} M^4)$$
$$= \sum_{n,g} a_{n,g}\lambda^n N^{2-2g}$$

Counting Planar Graphs à la 'tHooft and Brezin-Itzykson-Parisi-Zuber (1978)

Why planar quadrangulations?

QFT answer: because they are dual to the Feynman graphs which dominate the $1/N$ expansion of a matrix model...



$$
\begin{aligned}
Z &= \int dM \exp\left(-\frac{1}{2}\mathrm{Tr} M^2 + \frac{\lambda}{N}\mathrm{Tr} M^4\right) \\
&= \sum_{n,g} a_{n,g} \lambda^n N^{2-2g}
\end{aligned}
$$

The Cori-Vauquelin-Schaeffer Map

The connection with random metrics and their (random) geodesics remained obscure. Recent progress came from better combinatoric counting.

$$(n+2)Q_n = 2 \cdot 3^n C_n, \ .$$

There exists a two-to-one map between rooted pointed planar quadrangulations with n faces and well-labeled plane trees with n edges.

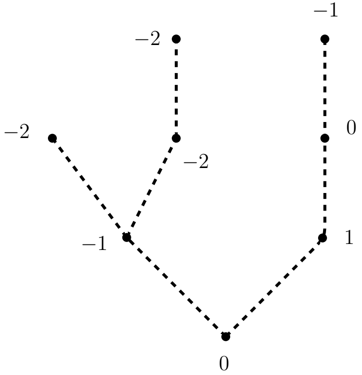
The Cori-Vauquelin-Schaeffer Map

The connection with random metrics and their (random) geodesics remained obscure. Recent progress came from better combinatoric counting.

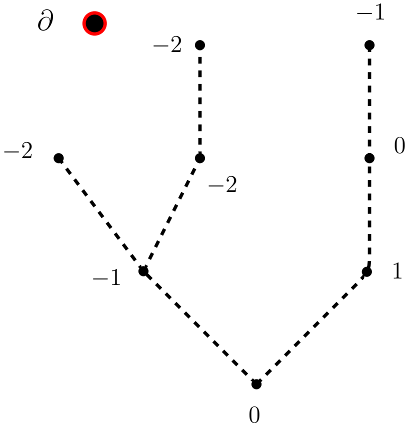
$$(n+2)Q_n = 2 \cdot 3^n C_n, \ .$$

There exists a two-to-one map between rooted pointed planar quadrangulations with n faces and well-labeled plane trees with n edges.

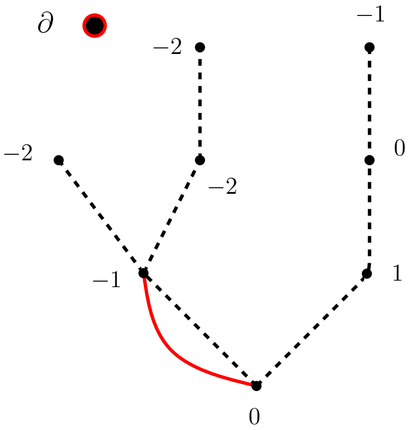
The Cori-Vauquelin-Schaeffer Map



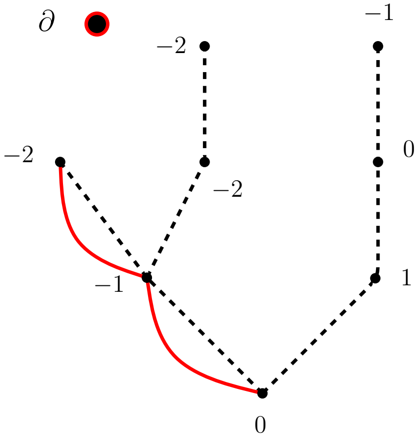
The Cori-Vauquelin-Schaeffer Map



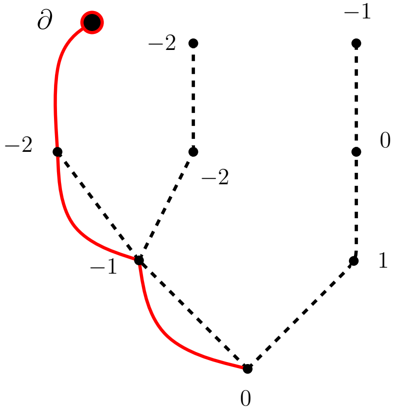
The Cori-Vauquelin-Schaeffer Map



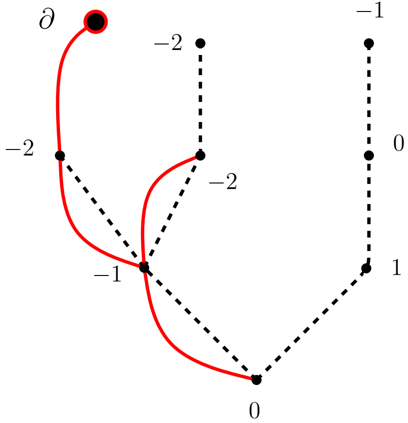
The Cori-Vauquelin-Schaeffer Map



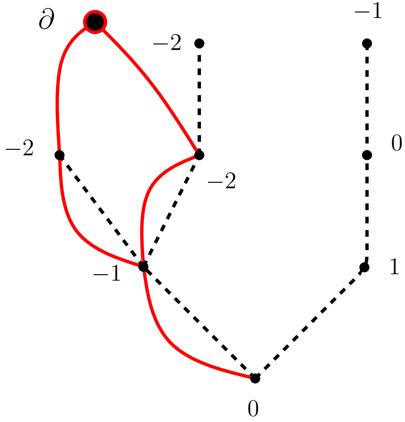
The Cori-Vauquelin-Schaeffer Map



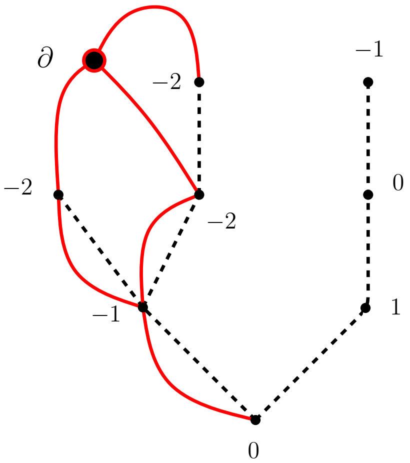
The Cori-Vauquelin-Schaeffer Map



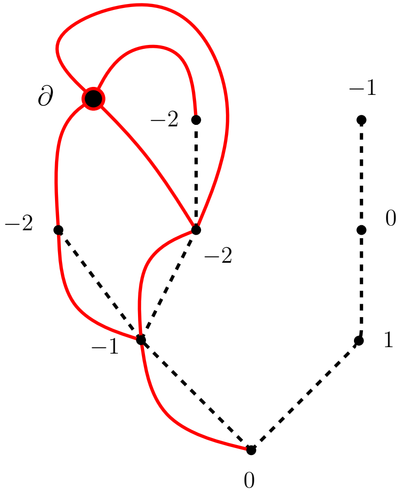
Large Quadrangulations



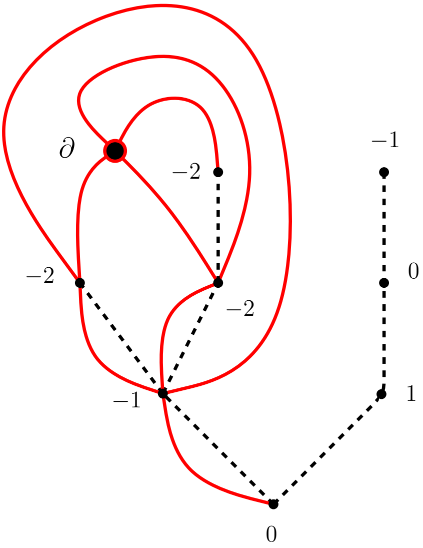
Large Quadrangulations



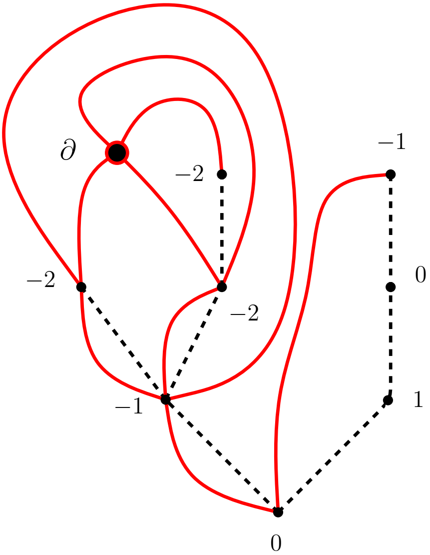
Large Quadrangulations



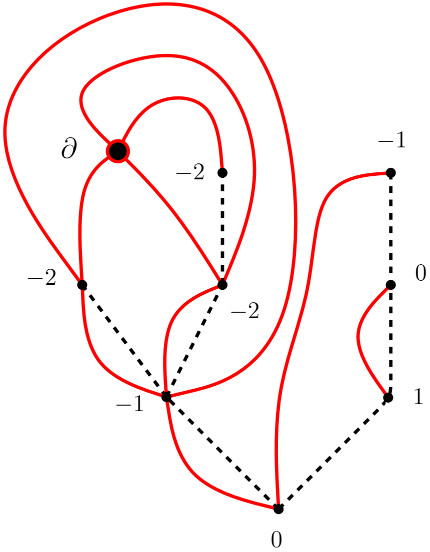
Large Quadrangulations



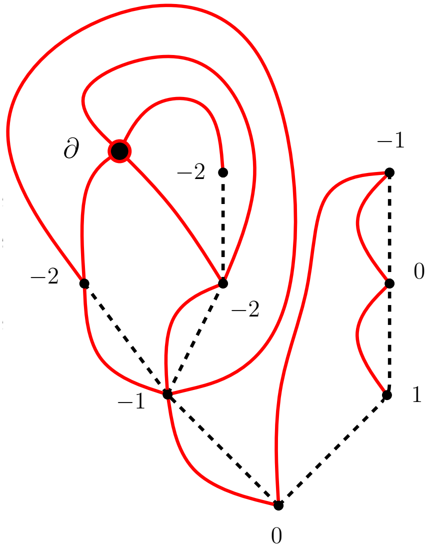
Large Quadrangulations



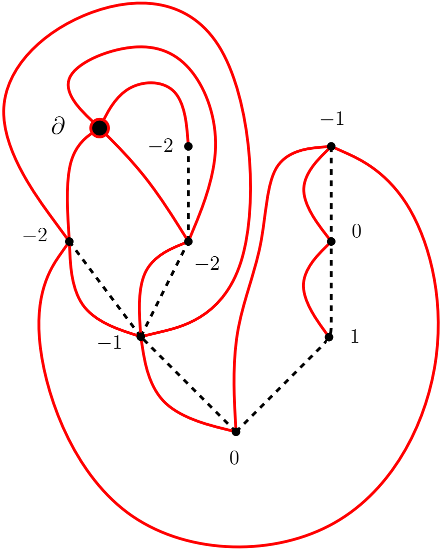
Large Quadrangulations



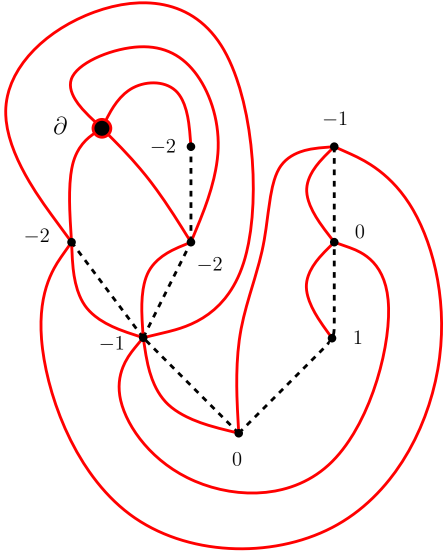
Large Quadrangulations



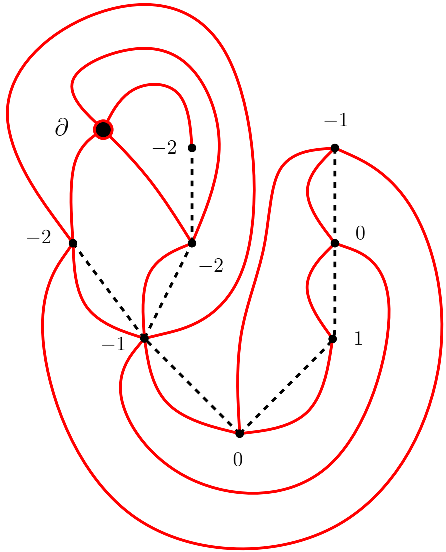
Large Quadrangulations



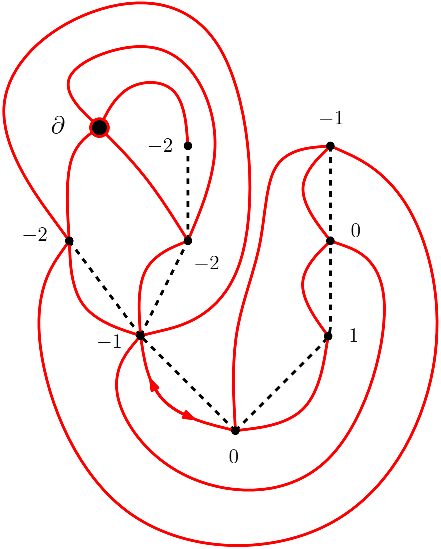
Large Quadrangulations



Large Quadrangulations



Large Quadrangulations



2D Random Geometry à la Le-Gall-Miermont

Theorem (Le Gall, Miermont (2007-2011)

Equidistributed planar quadrangulations of order n converge after rescaling the graph distance by $n^{-1/4}$ (in the Gromov-Hausdorff sense), towards a universal random compact space, called the brownian 2-sphere.

This space has Hausdorff dimension 4 and is almost surely homeomorphic to the two-dimensional sphere. It is expected to have spectral dimension 2.

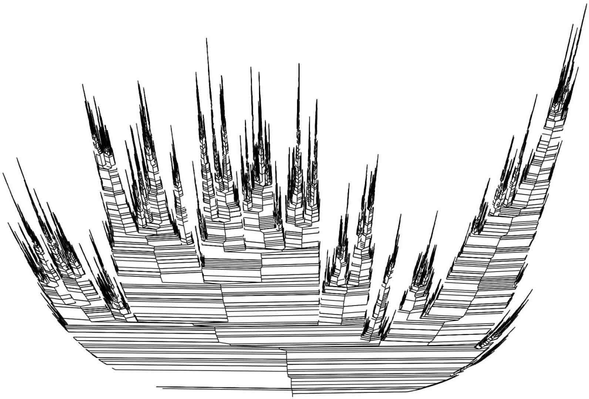
2D Random Geometry à la Le-Gall-Miermont

Theorem (Le Gall, Miermont (2007-2011)

Equidistributed planar quadrangulations of order n converge after rescaling the graph distance by $n^{-1/4}$ (in the Gromov-Hausdorff sense), towards a universal random compact space, called the brownian 2-sphere.

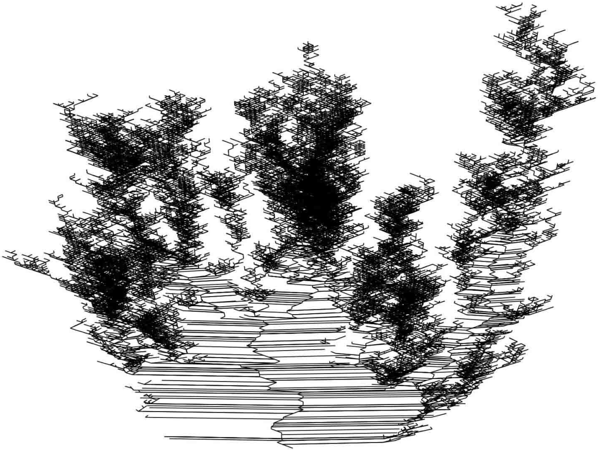
This space has Hausdorff dimension 4 and is almost surely homeomorphic to the two-dimensional sphere. It is expected to have spectral dimension 2.

A Look at Large Random Quadrangulations



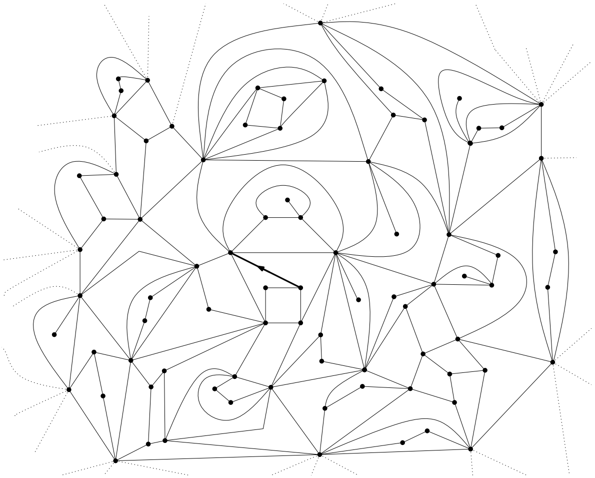
The Probabilist's View: The Brownian Snake, Head on

A Look at Large Random Quadrangulations



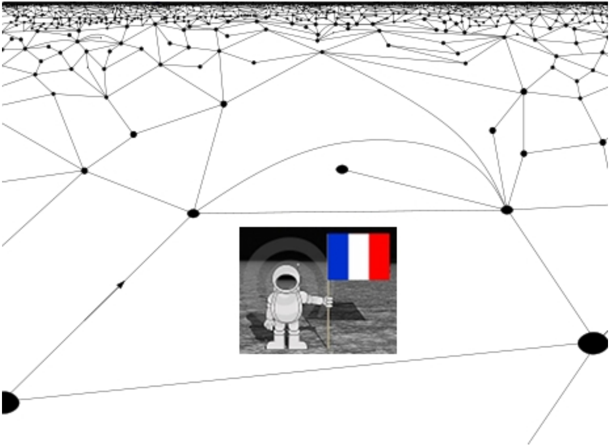
The Probabilist's View: The Brownian Snake, Part Profile

A Look at Large Random Quadrangulations



The Topological View

A Look at Large Random Quadrangulations

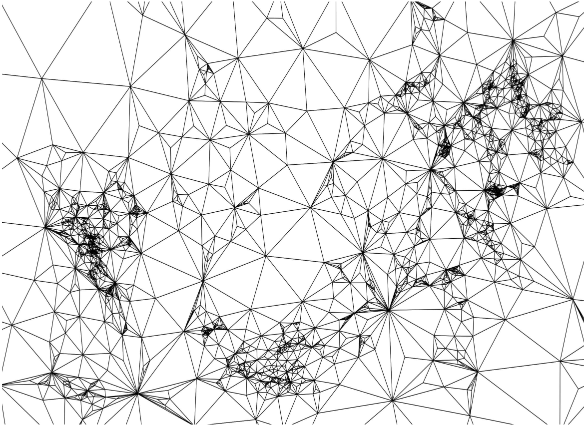


Landing on the Brownian sphere

A Look at Large Random Quadrangulations

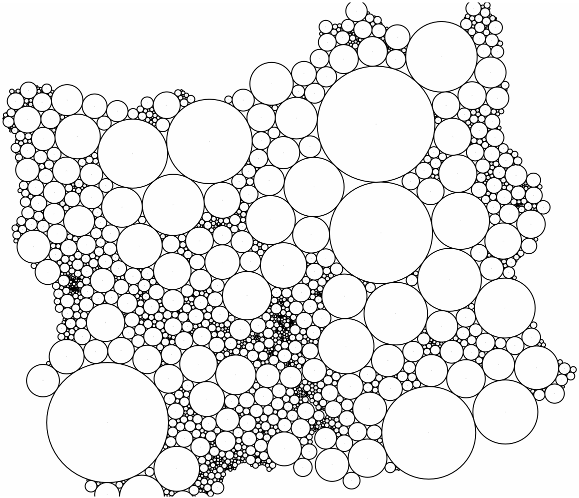
Artist's view in 3D (Courtesy: Marckert)

A Look at Large Random Quadrangulations



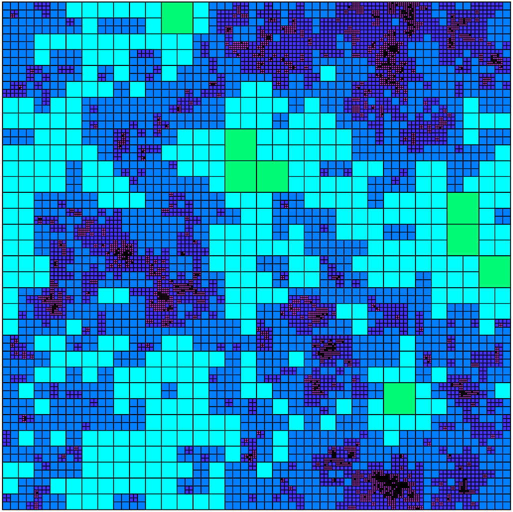
Uniformized Through Riemann Mapping Theorem

A Look at Large Random Quadrangulations



Using the Circle Packing Theorem (Courtesy: Krikun)

A Look at Large Random Quadrangulations



The Liouville Theory (Courtesy: Duplantier)

2D Random Geometry à la KPZ-DDK-DS (1984-2011)

There exists a relationship between critical exponents x and Δ of matter on a fixed (x) and on a random (Δ) geometry.

$$x = \frac{\gamma^2}{4}\Delta^2 + (1 - \frac{\gamma^2}{4})\Delta$$

The matter type is characterized by a number $\gamma \in [0, 2[$, related to the Schramm-Loewner evolution parameter κ through $\gamma = \sqrt{\min(\kappa, 16/\kappa)}$, and to the central charge $c = \frac{(8-3\kappa)(\kappa-6)}{2\kappa}$ (for Ising, $c = 1/2, \kappa = 3, \gamma = \sqrt{3}$).

2D Random Geometry à la KPZ-DDK-DS (1984-2011)

There exists a relationship between critical exponents x and Δ of matter on a fixed (x) and on a random (Δ) geometry.

$$x = \frac{\gamma^2}{4}\Delta^2 + (1 - \frac{\gamma^2}{4})\Delta$$

The matter type is characterized by a number $\gamma \in [0, 2[$, related to the Schramm-Loewner evolution parameter κ through $\gamma = \sqrt{\min(\kappa, 16/\kappa)}$, and to the central charge $c = \frac{(8-3\kappa)(\kappa-6)}{2\kappa}$ (for Ising, $c = 1/2, \kappa = 3, \gamma = \sqrt{3}$).

2D Random Geometry à la KPZ-DDK-DS (1984-2011)

There exists a relationship between critical exponents x and Δ of matter on a fixed (x) and on a random (Δ) geometry.

$$x = \frac{\gamma^2}{4}\Delta^2 + \left(1 - \frac{\gamma^2}{4}\right)\Delta$$

The matter type is characterized by a number $\gamma \in [0, 2[$, related to the Schramm-Loewner evolution parameter κ through $\gamma = \sqrt{\min(\kappa, 16/\kappa)}$, and to the central charge $c = \frac{(8-3\kappa)(\kappa-6)}{2\kappa}$ (for Ising, $c = 1/2, \kappa = 3, \gamma = \sqrt{3}$).

2D Random Geometry à la KPZ-DDK-DS (1984-2011)

There exists a relationship between critical exponents x and Δ of matter on a fixed (x) and on a random (Δ) geometry.

$$x = \frac{\gamma^2}{4}\Delta^2 + (1 - \frac{\gamma^2}{4})\Delta$$

The matter type is characterized by a number $\gamma \in [0, 2[$, related to the Schramm-Loewner evolution parameter κ through $\gamma = \sqrt{\min(\kappa, 16/\kappa)}$, and to the central charge $c = \frac{(8-3\kappa)(\kappa-6)}{2\kappa}$ (for Ising, $c = 1/2, \kappa = 3, \gamma = \sqrt{3}$).

Lessons to draw

2d Random geometry can be based on the careful counting of large triangulations or on the continuum (Liouville) picture. The two pictures should be equivalent, but the first one is particularly convincing from a conceptual point of view.

Random 2d planar geometry can be interpreted as trees or branched polymers equipped with fluctuation fields (the labels). These fields generate space-time shortcuts which change the Hausdorff dimension from 2 to 4.

Yes, pure 2d quantum gravity is topological, but there is much more in it.

Lessons to draw

2d Random geometry can be based on the careful counting of large triangulations or on the continuum (Liouville) picture. The two pictures should be equivalent, but the first one is particularly convincing from a conceptual point of view.

Random 2d planar geometry can be interpreted as trees or branched polymers equipped with fluctuation fields (the labels). These fields generate space-time shortcuts which change the Hausdorff dimension from 2 to 4.

Yes, pure 2d quantum gravity is topological, but there is much more in it.

Lessons to draw

2d Random geometry can be based on the careful counting of large triangulations or on the continuum (Liouville) picture. The two pictures should be equivalent, but the first one is particularly convincing from a conceptual point of view.

Random 2d planar geometry can be interpreted as trees or branched polymers equipped with fluctuation fields (the labels). These fields generate space-time shortcuts which change the Hausdorff dimension from 2 to 4.

Yes, pure 2d quantum gravity is topological, but there is much more in it.

Lessons to draw

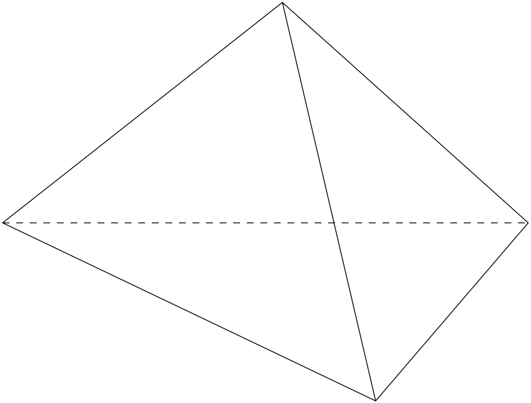
2d Random geometry can be based on the careful counting of large triangulations or on the continuum (Liouville) picture. The two pictures should be equivalent, but the first one is particularly convincing from a conceptual point of view.

Random 2d planar geometry can be interpreted as trees or branched polymers equipped with fluctuation fields (the labels). These fields generate space-time shortcuts which change the Hausdorff dimension from 2 to 4.

Yes, pure 2d quantum gravity is topological, but there is much more in it.

Colored Triangulations

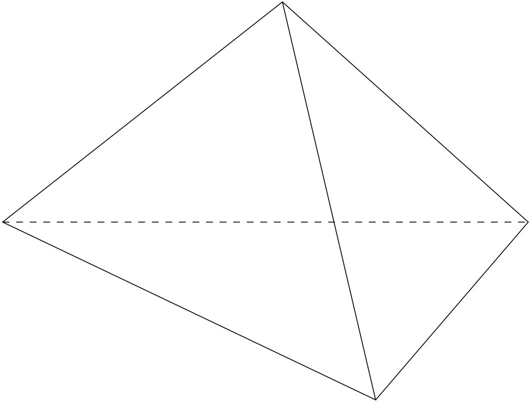
Any d-dimensional triangulation uniquely defines a $(d+1)$ vertex-colored triangulation, its barycentric subdivision.



The dual graph is an edge colored graph (Lins, Crystallization theory).

Colored Triangulations

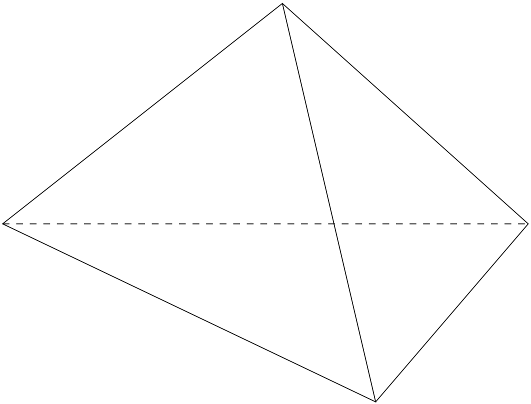
Any d-dimensional triangulation uniquely defines a $(d+1)$ vertex-colored triangulation, its barycentric subdivision.



The dual graph is an edge colored graph (Lins, Crystallization theory).

Colored Triangulations

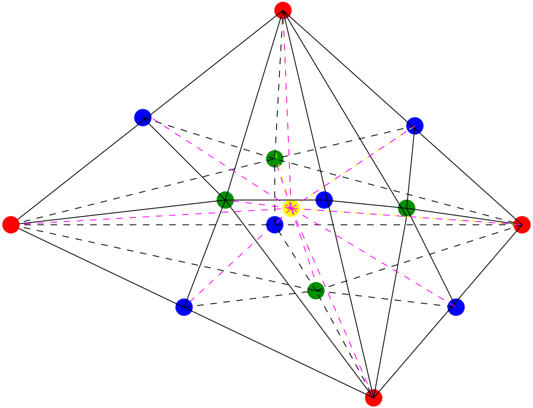
Any d-dimensional triangulation uniquely defines a $(d+1)$ vertex-colored triangulation, its barycentric subdivision.



The dual graph is an edge colored graph (Lins, Crystallization theory).

Colored Triangulations

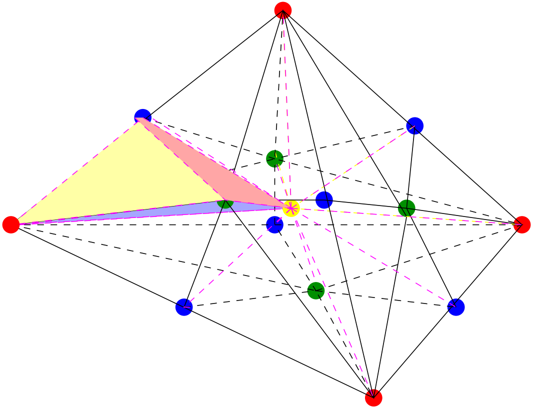
Any d-dimensional triangulation uniquely defines a $(d+1)$ vertex-colored triangulation, its barycentric subdivision.



The dual graph is an edge colored graph (Lins, Crystallization theory..)

Colored Triangulations

Any d-dimensional triangulation uniquely defines a $(d+1)$ vertex-colored triangulation, its barycentric subdivision.



The dual graph is an edge colored graph (Lins, Crystallization theory..)

Random Tensors

Vector Models are probability measures for random vectors of size N.

Matrix models are probability measures for N by N random matrices M.

Tensor models are probability measures for tensors of higher rank $D > 2$, with eg N^D coefficients.

Universal properties when N gets large stem from the existence of a $1/N$ expansion.

There is an algebraic link between random (unsymmetrized) tensors of rank D and $D+1$ colored triangulations, namely classical invariant theory.

Random Tensors

Vector Models are probability measures for random vectors of size N.

Matrix models are probability measures for N by N random matrices M.

Tensor models are probability measures for tensors of higher rank $D > 2$, with eg N^D coefficients.

Universal properties when N gets large stem from the existence of a $1/N$ expansion.

There is an algebraic link between random (unsymmetrized) tensors of rank D and $D + 1$ colored triangulations, namely classical invariant theory.

Random Tensors

Vector Models are probability measures for random vectors of size N.

Matrix models are probability measures for N by N random matrices M.

Tensor models are probability measures for tensors of higher rank $D > 2$, with eg N^D coefficients.

Universal properties when N gets large stem from the existence of a $1/N$ expansion.

There is an algebraic link between random (unsymmetrized) tensors of rank D and $D+1$ colored triangulations, namely classical invariant theory.

Random Tensors

Vector Models are probability measures for random vectors of size N.

Matrix models are probability measures for N by N random matrices M.

Tensor models are probability measures for tensors of higher rank $D > 2$, with eg N^D coefficients.

Universal properties when N gets large stem from the existence of a $1/N$ expansion.

There is an algebraic link between random (unsymmetrized) tensors of rank D and $D+1$ colored triangulations, namely classical invariant theory.

Random Tensors

Vector Models are probability measures for random vectors of size N.

Matrix models are probability measures for N by N random matrices M.

Tensor models are probability measures for tensors of higher rank $D > 2$, with eg N^D coefficients.

Universal properties when N gets large stem from the existence of a $1/N$ expansion.

There is an algebraic link between random (unsymmetrized) tensors of rank D and $D+1$ colored triangulations, namely classical invariant theory.

Random Tensors

Vector Models are probability measures for random vectors of size N.

Matrix models are probability measures for N by N random matrices M.

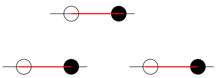
Tensor models are probability measures for tensors of higher rank $D > 2$, with eg N^D coefficients.

Universal properties when N gets large stem from the existence of a $1/N$ expansion.

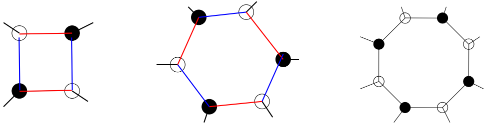
There is an algebraic link between random (unsymmetrized) tensors of rank D and $D + 1$ colored triangulations, namely classical invariant theory.

Classical Invariants

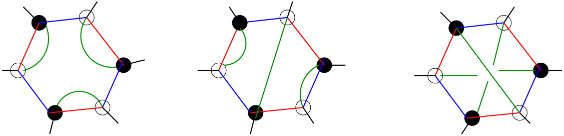
Polynomial $U(N)^{\otimes D}$ invariants for pairs of rank D (unsymmetrized) complex-conjugate tensors are linear combinations of amplitudes associated to D-regular bipartite colored graphs.



Vector Invariants



Matrix Invariants

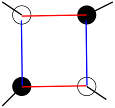


Tensor Invariants

Invariants, II

The algebraic invariants associated to the vector and matrix drawings are

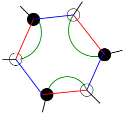
$$= \quad \sum_i \bar{\phi}_i \phi^i$$



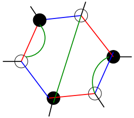
$$= \sum_{i,j,k,l} \bar{M}_{ij} M^{ik} \bar{M}_{lk} M^{lj} = Tr\,[\, M^\dagger M M^\dagger M \,]$$

Invariants, III

The algebraic invariants associated to the tensorial drawings are



$$= \sum_{i,j,k,l,m,n,p,q,r} \bar{T}_{ijp} T^{ikq} \bar{T}_{lkq} T^{lmr} \bar{T}_{nmr} T^{njp}$$



$$= \sum_{i,j,k,l,m,n,p,q,r} \bar{T}_{ijp} T^{ikq} \bar{T}_{lkq} T^{lmp} \bar{T}_{nmr} T^{njr}$$

and so on...

Random Vectors and Matrices

- iid random vectors have a Gaussian limit as $N \to \infty$ (eg in the sense of the central limit theorem);
- invariant random vector models (eg Gaussian plus invariant interactions) have a $1/N$ expansion, dominated by bubble chains;
- iid (centered) random matrices such as GUE have a Gaussian limit as $N \to \infty$, and the invariant observables such as eigenvalues converge to the Wigner-Dyson distribution;
- invariant random matrix models (eg Gaussian plus invariant interactions) have a $1/N$ expansion, dominated by planar graphs;
- until recently there was no corresponding expansion for tensors of rank ≥ 3.

Random Vectors and Matrices

- iid random vectors have a Gaussian limit as $N \to \infty$ (eg in the sense of the central limit theorem);
- invariant random vector models (eg Gaussian plus invariant interactions) have a $1/N$ expansion, dominated by bubble chains;
- iid (centered) random matrices such as GUE have a Gaussian limit as $N \to \infty$, and the invariant observables such as eigenvalues converge to the Wigner-Dyson distribution;
- invariant random matrix models (eg Gaussian plus invariant interactions) have a $1/N$ expansion, dominated by planar graphs;
- until recently there was no corresponding expansion for tensors of rank ≥ 3.

Random Vectors and Matrices

- iid random vectors have a Gaussian limit as $N \to \infty$ (eg in the sense of the central limit theorem);
- invariant random vector models (eg Gaussian plus invariant interactions) have a $1/N$ expansion, dominated by bubble chains;
- iid (centered) random matrices such as GUE have a Gaussian limit as $N \to \infty$, and the invariant observables such as eigenvalues converge to the Wigner-Dyson distribution;
- invariant random matrix models (eg Gaussian plus invariant interactions) have a $1/N$ expansion, dominated by planar graphs;
- until recently there was no corresponding expansion for tensors of rank ≥ 3.

Random Vectors and Matrices

- iid random vectors have a Gaussian limit as $N \to \infty$ (eg in the sense of the central limit theorem);
- invariant random vector models (eg Gaussian plus invariant interactions) have a $1/N$ expansion, dominated by bubble chains;
- iid (centered) random matrices such as GUE have a Gaussian limit as $N \to \infty$, and the invariant observables such as eigenvalues converge to the Wigner-Dyson distribution;
- invariant random matrix models (eg Gaussian plus invariant interactions) have a $1/N$ expansion, dominated by planar graphs;
- until recently there was no corresponding expansion for tensors of rank ≥ 3.

Random Vectors and Matrices

- iid random vectors have a Gaussian limit as $N \to \infty$ (eg in the sense of the central limit theorem);
- invariant random vector models (eg Gaussian plus invariant interactions) have a $1/N$ expansion, dominated by bubble chains;
- iid (centered) random matrices such as GUE have a Gaussian limit as $N \to \infty$, and the invariant observables such as eigenvalues converge to the Wigner-Dyson distribution;
- invariant random matrix models (eg Gaussian plus invariant interactions) have a $1/N$ expansion, dominated by planar graphs;
- until recently there was no corresponding expansion for tensors of rank ≥ 3.

Random Vectors and Matrices

- iid random vectors have a Gaussian limit as $N \to \infty$ (eg in the sense of the central limit theorem);
- invariant random vector models (eg Gaussian plus invariant interactions) have a $1/N$ expansion, dominated by bubble chains;
- iid (centered) random matrices such as GUE have a Gaussian limit as $N \to \infty$, and the invariant observables such as eigenvalues converge to the Wigner-Dyson distribution;
- invariant random matrix models (eg Gaussian plus invariant interactions) have a $1/N$ expansion, dominated by planar graphs;
- until recently there was no corresponding expansion for tensors of rank ≥ 3.

Random Tensors

Random tensors are best analyzed using unsymmetrized colored models. These models

- triangulate pseudo-manifolds (Gurau, 2010)
- admit a $1/N$ expansion (Gurau 2010), whose leading graphs, called melons triangulate only spheres in any dimension (Gurau, R., 2011)
- have computable phase transitions (Bonzom, Gurau, Riello, R. 2011) to a leading melonic phase of branched polymers (Gurau, Ryan, 2013)
- Matter fields can be included (Bonzom et al)
- In short: the theory of $U(N)^{\otimes D}$-invariant random tensors is universal (Gurau 2011; Bonzom, Gurau, R. 2012), and different from the theory of random matrices.

Random Tensors

Random tensors are best analyzed using unsymmetrized colored models. These models

- triangulate pseudo-manifolds (Gurau, 2010)
- admit a $1/N$ expansion (Gurau 2010), whose leading graphs, called melons triangulate only spheres in any dimension (Gurau, R., 2011)
- have computable phase transitions (Bonzom, Gurau, Riello, R. 2011) to a leading melonic phase of branched polymers (Gurau, Ryan, 2013)
- Matter fields can be included (Bonzom et al)
- In short: the theory of $U(N)^{\otimes D}$-invariant random tensors is universal (Gurau 2011; Bonzom, Gurau, R. 2012), and different from the theory of random matrices.

Random Tensors

Random tensors are best analyzed using unsymmetrized colored models. These models

- triangulate pseudo-manifolds (Gurau, 2010)
- admit a $1/N$ expansion (Gurau 2010), whose leading graphs, called melons triangulate only spheres in any dimension (Gurau, R., 2011)
- have computable phase transitions (Bonzom, Gurau, Riello, R. 2011) to a leading melonic phase of branched polymers (Gurau, Ryan, 2013)
- Matter fields can be included (Bonzom et al)
- In short: the theory of $U(N)^{\otimes D}$-invariant random tensors is universal (Gurau 2011; Bonzom, Gurau, R. 2012), and different from the theory of random matrices.

Random Tensors

Random tensors are best analyzed using unsymmetrized colored models. These models

- triangulate pseudo-manifolds (Gurau, 2010)
- admit a $1/N$ expansion (Gurau 2010), whose leading graphs, called melons triangulate only spheres in any dimension (Gurau, R., 2011)
- have computable phase transitions (Bonzom, Gurau, Riello, R. 2011) to a leading melonic phase of branched polymers (Gurau, Ryan, 2013)
- Matter fields can be included (Bonzom et al)
- In short: the theory of $U(N)^{\otimes D}$-invariant random tensors is universal (Gurau 2011; Bonzom, Gurau, R. 2012), and different from the theory of random matrices.

Random Tensors

Random tensors are best analyzed using unsymmetrized colored models. These models

- triangulate pseudo-manifolds (Gurau, 2010)
- admit a $1/N$ expansion (Gurau 2010), whose leading graphs, called melons triangulate only spheres in any dimension (Gurau, R., 2011)
- have computable phase transitions (Bonzom, Gurau, Riello, R. 2011) to a leading melonic phase of branched polymers (Gurau, Ryan, 2013)
- Matter fields can be included (Bonzom et al)
- In short: the theory of $U(N)^{\otimes D}$-invariant random tensors is universal (Gurau 2011; Bonzom, Gurau, R. 2012), and different from the theory of random matrices.

Random Tensors

Random tensors are best analyzed using unsymmetrized colored models. These models

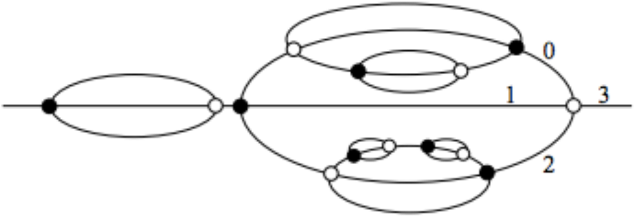
- triangulate pseudo-manifolds (Gurau, 2010)
- admit a $1/N$ expansion (Gurau 2010), whose leading graphs, called melons triangulate only spheres in any dimension (Gurau, R., 2011)
- have computable phase transitions (Bonzom, Gurau, Riello, R. 2011) to a leading melonic phase of branched polymers (Gurau, Ryan, 2013)
- Matter fields can be included (Bonzom et al)
- In short: the theory of $U(N)^{\otimes D}$-invariant random tensors is universal (Gurau 2011; Bonzom, Gurau, R. 2012), and different from the theory of random matrices.

Random Tensors

Random tensors are best analyzed using unsymmetrized colored models. These models

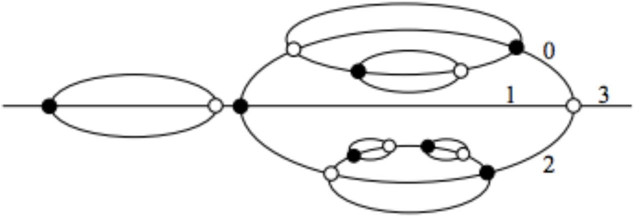
- triangulate pseudo-manifolds (Gurau, 2010)
- admit a $1/N$ expansion (Gurau 2010), whose leading graphs, called melons triangulate only spheres in any dimension (Gurau, R., 2011)
- have computable phase transitions (Bonzom, Gurau, Riello, R. 2011) to a leading melonic phase of branched polymers (Gurau, Ryan, 2013)
- Matter fields can be included (Bonzom et al)
- In short: the theory of $U(N)^{\otimes D}$-invariant random tensors is universal (Gurau 2011; Bonzom, Gurau, R. 2012), and different from the theory of random matrices.

Melonic Graphs



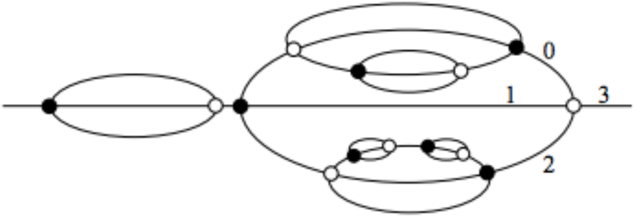
Melonic graphs are trees (branched polymers) without random labels. But we know that a rich structure of labels hides in the sub-melonic contributions, because random tensors in particular include all random matrix models.

Melonic Graphs



Melonic graphs are trees (branched polymers) without random labels. But we know that a rich structure of labels hides in the sub-melonic contributions, because random tensors in particular include all random matrix models.

Melonic Graphs



Melonic graphs are trees (branched polymers) without random labels. But we know that a rich structure of labels hides in the sub-melonic contributions, because random tensors in particular include all random matrix models.

Models/Field Theories

One could distinguish

- Invariant models, with an action fully invariant under $U(N)^{\otimes D}$. These are the analogs of ultralocal quantum field theories.
- Field theories, which have invariant interactions but a propagator which softly break this invariance. This breaking launches their renormalization group flow, just like the soft non-locality of the propagator launches ordinary renormalization group flow in ordinary quantum field theory.

Models/Field Theories

One could distinguish

- Invariant models, with an action fully invariant under $U(N)^{\otimes D}$. These are the analogs of ultralocal quantum field theories.
- Field theories, which have invariant interactions but a propagator which softly break this invariance. This breaking launches their renormalization group flow, just like the soft non-locality of the propagator launches ordinary renormalization group flow in ordinary quantum field theory.

Models/Field Theories

One could distinguish

- Invariant models, with an action fully invariant under $U(N)^{\otimes D}$. These are the analogs of ultralocal quantum field theories.
- Field theories, which have invariant interactions but a propagator which softly break this invariance. This breaking launches their renormalization group flow, just like the soft non-locality of the propagator launches ordinary renormalization group flow in ordinary quantum field theory.

Renormalization

- All standard model interactions (except gravity, until now....) are renormalizable
- Renormalizability is approximate scale invariance over many scales
- Renormalizable (marginal) interactions are the natural ones for physics because they survive long RG flows
- There exist renormalizable tensorial field theories (Ben Geloun, R, 2011).
- More recently we have found how to renormalize Boulatov-type tensorial group field theories (Carrozza, Oriti, R. 2012), building a link between tensorial theory and spin foam amplitudes.

Renormalization

- All standard model interactions (except gravity, until now....) are renormalizable
- Renormalizability is approximate scale invariance over many scales
- Renormalizable (marginal) interactions are the natural ones for physics because they survive long RG flows
- There exist renormalizable tensorial field theories (Ben Geloun, R, 2011).
- More recently we have found how to renormalize Boulatov-type tensorial group field theories (Carrozza, Oriti, R. 2012), building a link between tensorial theory and spin foam amplitudes.

Renormalization

- All standard model interactions (except gravity, until now....) are renormalizable
- Renormalizability is approximate scale invariance over many scales
- Renormalizable (marginal) interactions are the natural ones for physics because they survive long RG flows
- There exist renormalizable tensorial field theories (Ben Geloun, R, 2011).
- More recently we have found how to renormalize Boulatov-type tensorial group field theories (Carrozza, Oriti, R. 2012), building a link between tensorial theory and spin foam amplitudes.

Renormalization

- All standard model interactions (except gravity, until now....) are renormalizable
- Renormalizability is approximate scale invariance over many scales
- Renormalizable (marginal) interactions are the natural ones for physics because they survive long RG flows
- There exist renormalizable tensorial field theories (Ben Geloun, R, 2011).
- More recently we have found how to renormalize Boulatov-type tensorial group field theories (Carrozza, Oriti, R. 2012), building a link between tensorial theory and spin foam amplitudes.

Renormalization

- All standard model interactions (except gravity, until now....) are renormalizable
- Renormalizability is approximate scale invariance over many scales
- Renormalizable (marginal) interactions are the natural ones for physics because they survive long RG flows
- There exist renormalizable tensorial field theories (Ben Geloun, R, 2011).
- More recently we have found how to renormalize Boulatov-type tensorial group field theories (Carrozza, Oriti, R. 2012), building a link between tensorial theory and spin foam amplitudes.

Renormalization

- All standard model interactions (except gravity, until now....) are renormalizable
- Renormalizability is approximate scale invariance over many scales
- Renormalizable (marginal) interactions are the natural ones for physics because they survive long RG flows
- There exist renormalizable tensorial field theories (Ben Geloun, R, 2011).
- More recently we have found how to renormalize Boulatov-type tensorial group field theories (Carrozza, Oriti, R. 2012), building a link between tensorial theory and spin foam amplitudes.

Building Rules

What could be the natural building rules for general field theories of tensors?

- Replace rotation and translation invariance by color permutation symmetry
- Replace locality by tensor invariance (use invariant interactions, but not invariant propagator)
- Natural propagator in group space: Laplacian, possibly with GFT projector
- Replace clustering by decay of correlation functions in the number and type of the boundary components (external legs)

Building Rules

What could be the natural building rules for general field theories of tensors?

- Replace rotation and translation invariance by color permutation symmetry
- Replace locality by tensor invariance (use invariant interactions, but not invariant propagator)
- Natural propagator in group space: Laplacian, possibly with GFT projector
- Replace clustering by decay of correlation functions in the number and type of the boundary components (external legs)

Building Rules

What could be the natural building rules for general field theories of tensors?

- Replace rotation and translation invariance by color permutation symmetry
- Replace locality by tensor invariance (use invariant interactions, but not invariant propagator)
- Natural propagator in group space: Laplacian, possibly with GFT projector
- Replace clustering by decay of correlation functions in the number and type of the boundary components (external legs)

Building Rules

What could be the natural building rules for general field theories of tensors?

- Replace rotation and translation invariance by color permutation symmetry
- Replace locality by tensor invariance (use invariant interactions, but not invariant propagator)
- Natural propagator in group space: Laplacian, possibly with GFT projector
- Replace clustering by decay of correlation functions in the number and type of the boundary components (external legs)

Building Rules

What could be the natural building rules for general field theories of tensors?

- Replace rotation and translation invariance by color permutation symmetry
- Replace locality by tensor invariance (use invariant interactions, but not invariant propagator)
- Natural propagator in group space: Laplacian, possibly with GFT projector
- Replace clustering by decay of correlation functions in the number and type of the boundary components (external legs)

Building Rules

What could be the natural building rules for general field theories of tensors?

- Replace rotation and translation invariance by color permutation symmetry
- Replace locality by tensor invariance (use invariant interactions, but not invariant propagator)
- Natural propagator in group space: Laplacian, possibly with GFT projector
- Replace clustering by decay of correlation functions in the number and type of the boundary components (external legs)

SU(2) Boulatov-type Model

- The 3 dimensional theory with $SU(2)$ group and Boulatov-projector is renormalizable with ϕ^6 type interactions (Carrozza, Oriti, V. R.)
- It is expected to be asymptotically free (work in preparation)
- It corresponds to a quantization of the $B-F$ action
- In the deep ultravilet limit its amplitudes are asymptotic to those of the Boulatov model.

SU(2) Boulatov-type Model

- The 3 dimensional theory with $SU(2)$ group and Boulatov-projector is renormalizable with ϕ^6 type interactions (Carrozza, Oriti, V. R.)
- It is expected to be asymptotically free (work in preparation)
- It corresponds to a quantization of the $B-F$ action
- In the deep ultravilet limit its amplitudes are asymptotic to those of the Boulatov model.

SU(2) Boulatov-type Model

- The 3 dimensional theory with $SU(2)$ group and Boulatov-projector is renormalizable with ϕ^6 type interactions (Carrozza, Oriti, V. R.)
- It is expected to be asymptotically free (work in preparation)
- It corresponds to a quantization of the $B - F$ action
- In the deep ultravilet limit its amplitudes are asymptotic to those of the Boulatov model.

SU(2) Boulatov-type Model

- The 3 dimensional theory with $SU(2)$ group and Boulatov-projector is renormalizable with ϕ^6 type interactions (Carrozza, Oriti, V. R.)
- It is expected to be asymptotically free (work in preparation)
- It corresponds to a quantization of the $B - F$ action
- In the deep ultravilet limit its amplitudes are asymptotic to those of the Boulatov model.

SU(2) Boulatov-type Model

- The 3 dimensional theory with $SU(2)$ group and Boulatov-projector is renormalizable with ϕ^6 type interactions (Carrozza, Oriti, V. R.)
- It is expected to be asymptotically free (work in preparation)
- It corresponds to a quantization of the $B - F$ action
- In the deep ultravilet limit its amplitudes are asymptotic to those of the Boulatov model.

Asymptotic Freedom

- Standard model interactions (except gravity, until now....) are essentially asymptotically free
- Asymptotic freedom (AF) seems generic in the tensor world because the wave-function renormalization is stronger and dominates the coupling renormalization [Ben Geloun, Ben Geloun and Dine, 2012]
- AF is a very desirable physical property: it makes the ultraviolet limit fully consistent and leads typically to phase transitions in the infrared, hence gravitational analogs of quark confinement.

Asymptotic Freedom

- Standard model interactions (except gravity, until now....) are essentially asymptotically free
- Asymptotic freedom (AF) seems generic in the tensor world because the wave-function renormalization is stronger and dominates the coupling renormalization [Ben Geloun, Ben Geloun and Dine, 2012]
- AF is a very desirable physical property: it makes the ultraviolet limit fully consistent and leads typically to phase transitions in the infrared, hence gravitational analogs of quark confinement.

Asymptotic Freedom

- Standard model interactions (except gravity, until now....) are essentially asymptotically free
- Asymptotic freedom (AF) seems generic in the tensor world because the wave-function renormalization is stronger and dominates the coupling renormalization [Ben Geloun, Ben Geloun and Dine, 2012]
- AF is a very desirable physical property: it makes the ultraviolet limit fully consistent and leads typically to phase transitions in the infrared, hence gravitational analogs of quark confinement.

Asymptotic Freedom

- Standard model interactions (except gravity, until now....) are essentially asymptotically free
- Asymptotic freedom (AF) seems generic in the tensor world because the wave-function renormalization is stronger and dominates the coupling renormalization [Ben Geloun, Ben Geloun and Dine, 2012]
- AF is a very desirable physical property: it makes the ultraviolet limit fully consistent and leads typically to phase transitions in the infrared, hence gravitational analogs of quark confinement.

Enumerative Combinatorics in 3D and 4D

Until now it seemed difficult to find analogues of CVS map in 3D. Even the Gromov conjecture is unproven today.

Gromov Conjecture: The number ST_n of triangulations of the sphere with n tetrahedra is exponentially bounded in n:

$$ST_n \leq K^n \tag{1}$$

Random tensors could bring new ideas there. Goal: define and study the random 3d and 4d sphere...

Enumerative Combinatorics in 3D and 4D

Until now it seemed difficult to find analogues of CVS map in 3D. Even the Gromov conjecture is unproven today.

Gromov Conjecture: The number ST_n of triangulations of the sphere with n tetrahedra is exponentially bounded in n:

$$ST_n \leq K^n \tag{1}$$

Random tensors could bring new ideas there. Goal: define and study the random 3d and 4d sphere...

Enumerative Combinatorics in 3D and 4D

Until now it seemed difficult to find analogues of CVS map in 3D. Even the Gromov conjecture is unproven today.

Gromov Conjecture: The number ST_n of triangulations of the sphere with n tetrahedra is exponentially bounded in n:

$$ST_n \leq K^n \tag{1}$$

Random tensors could bring new ideas there. Goal: define and study the random 3d and 4d sphere...

Enumerative Combinatorics in 3D and 4D

Until now it seemed difficult to find analogues of CVS map in 3D. Even the Gromov conjecture is unproven today.

Gromov Conjecture: The number ST_n of triangulations of the sphere with n tetrahedra is exponentially bounded in n:

$$ST_n \leq K^n \tag{1}$$

Random tensors could bring new ideas there. Goal: define and study the random 3d and 4d sphere...

Enumerative Combinatorics in 3D and 4D

Until now it seemed difficult to find analogues of CVS map in 3D. Even the Gromov conjecture is unproven today.

Gromov Conjecture: The number ST_n of triangulations of the sphere with n tetrahedra is exponentially bounded in n:

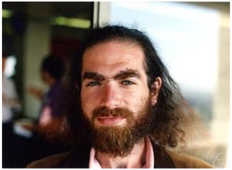
$$ST_n \leq K^n \tag{1}$$

Random tensors could bring new ideas there. Goal: define and study the random 3d and 4d sphere...

Uniformization in 3D

The transfer to a non-diffeo-invariant reference frame (eg a fixed sphere) is an essential step in understanding quantum gravity. It requires some form of uniformization.



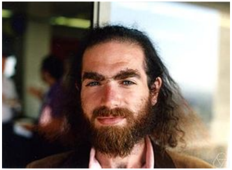


Uniformization in 3D is possible thanks to the works of Thurston, Hamilton and Perelman.

Uniformization in 3D

The transfer to a non-diffeo-invariant reference frame (eg a fixed sphere) is an essential step in understanding quantum gravity. It requires some form of uniformization.

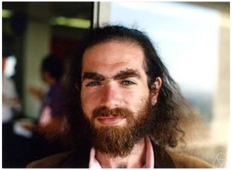




Uniformization in 3D is possible thanks to the works of Thurston, Hamilton and Perelman.

Uniformization in 3D

The transfer to a non-diffeo-invariant reference frame (eg a fixed sphere) is an essential step in understanding quantum gravity. It requires some form of uniformization.

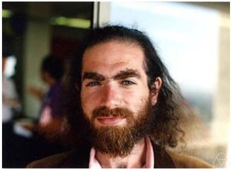
  

Uniformization in 3D is possible thanks to the works of Thurston, Hamilton and Perelman.

Uniformization in 3D

The transfer to a non-diffeo-invariant reference frame (eg a fixed sphere) is an essential step in understanding quantum gravity. It requires some form of uniformization.





Uniformization in 3D is possible thanks to the works of Thurston, Hamilton and Perelman.

Conclusions

• Random Geometry in higher dimensions might be better understood analytically through the use of random tensors and colored triangulations.

• Associated renormalizable tensor field theories exist, are typically asymptotically free hence point towards the generic presence of phase transitions.

Conclusions

• Random Geometry in higher dimensions might be better understood analytically through the use of random tensors and colored triangulations.

• Associated renormalizable tensor field theories exist, are typically asymptotically free hence point towards the generic presence of phase transitions.

Conclusions

• Random Geometry in higher dimensions might be better understood analytically through the use of random tensors and colored triangulations.

• Associated renormalizable tensor field theories exist, are typically asymptotically free hence point towards the generic presence of phase transitions.

Suggestions

To quantize gravity this suggests

- no need for supersymmetry
- no need for Kaluza-Klein dimensions
- no need for any ultraviolet cutoff at Planck scale

Suggestions

To quantize gravity this suggests

- no need for supersymmetry
- no need for Kaluza-Klein dimensions
- no need for any ultraviolet cutoff at Planck scale

Suggestions

To quantize gravity this suggests

- no need for supersymmetry
- no need for Kaluza-Klein dimensions
- no need for any ultraviolet cutoff at Planck scale

Suggestions

To quantize gravity this suggests

- no need for supersymmetry
- no need for Kaluza-Klein dimensions
- no need for any ultraviolet cutoff at Planck scale

Suggestions

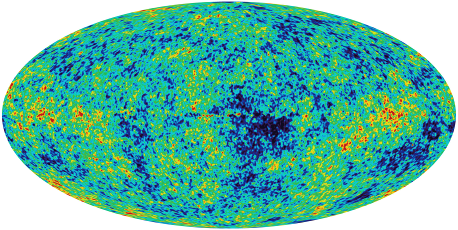
To quantize gravity this suggests

- no need for supersymmetry
- no need for Kaluza-Klein dimensions
- no need for any ultraviolet cutoff at Planck scale

Suggestions

In cosmology, no results yet. However the approach suggests

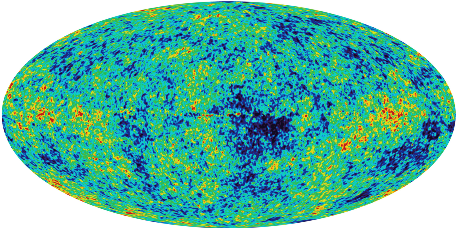
- nothing bounced at the "big bang"
- nothing banged at the "big bang"
- our universe condensed into birth through a cascade of several geometric phase transitions. The first one generated a primitive space-time of the branched polymer kind. This space was rounded later by further RG evolution.
- Quantum Gravity should induce KPZ-like effects on cosmic data $\Lambda \simeq 10^{-122}$ and



Suggestions

In cosmology, no results yet. However the approach suggests

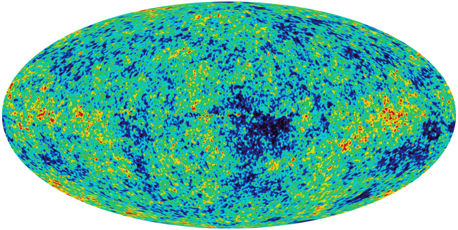
- nothing bounced at the "big bang"
- nothing banged at the "big bang"
- our universe condensed into birth through a cascade of several geometric phase transitions. The first one generated a primitive space-time of the branched polymer kind. This space was rounded later by further RG evolution.
- Quantum Gravity should induce KPZ-like effects on cosmic data $\Lambda \simeq 10^{-122}$ and



Suggestions

In cosmology, no results yet. However the approach suggests

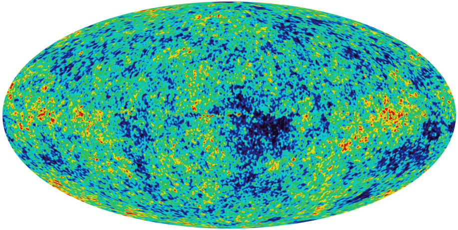
- nothing bounced at the "big bang"
- nothing banged at the "big bang"
- our universe condensed into birth through a cascade of several geometric phase transitions. The first one generated a primitive space-time of the branched polymer kind. This space was rounded later by further RG evolution.
- Quantum Gravity should induce KPZ-like effects on cosmic data $\Lambda \simeq 10^{-122}$ and



Suggestions

In cosmology, no results yet. However the approach suggests

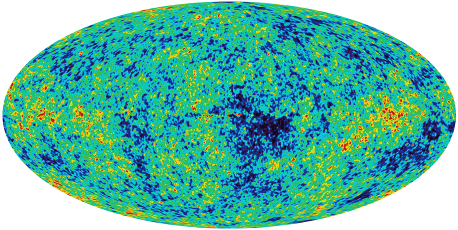
- nothing bounced at the "big bang"
- nothing banged at the "big bang"
- our universe condensed into birth through a cascade of several geometric phase transitions. The first one generated a primitive space-time of the branched polymer kind. This space was rounded later by further RG evolution.
- Quantum Gravity should induce KPZ-like effects on cosmic data $\Lambda \simeq 10^{-122}$ and



Suggestions

In cosmology, no results yet. However the approach suggests

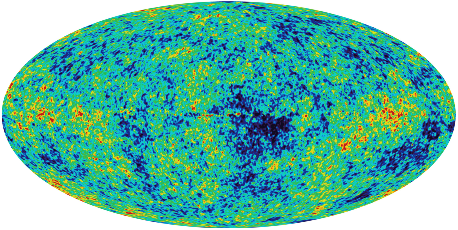
- nothing bounced at the "big bang"
- nothing banged at the "big bang"
- our universe condensed into birth through a cascade of several geometric phase transitions. The first one generated a primitive space-time of the branched polymer kind. This space was rounded later by further RG evolution.
- Quantum Gravity should induce KPZ-like effects on cosmic data $\Lambda \simeq 10^{-122}$ and



Suggestions

In cosmology, no results yet. However the approach suggests

- nothing bounced at the "big bang"
- nothing banged at the "big bang"
- our universe condensed into birth through a cascade of several geometric phase transitions. The first one generated a primitive space-time of the branched polymer kind. This space was rounded later by further RG evolution.
- Quantum Gravity should induce KPZ-like effects on cosmic data $\Lambda \simeq 10^{-122}$ and



Beyond space and time

Quantum gravity is diffeo-invariant but we are not.

There should be measurable consequences of quantum gravity.

Beyond time and space, there could be still physics. A more abstract renormalization group could govern this physics of our universe in its pregeometric phases.

Thank you for your attention!

Beyond space and time

Quantum gravity is diffeo-invariant but we are not.

There should be measurable consequences of quantum gravity.

Beyond time and space, there could be still physics. A more abstract renormalization group could govern this physics of our universe in its pregeometric phases.

Thank you for your attention!

Beyond space and time

Quantum gravity is diffeo-invariant but we are not.

There should be measurable consequences of quantum gravity.

Beyond time and space, there could be still physics. A more abstract renormalization group could govern this physics of our universe in its pregeometric phases.

Thank you for your attention!

Beyond space and time

Quantum gravity is diffeo-invariant but we are not.

There should be measurable consequences of quantum gravity.

Beyond time and space, there could be still physics. A more abstract renormalization group could govern this physics of our universe in its pregeometric phases.

Thank you for your attention!

Beyond space and time

Quantum gravity is diffeo-invariant but we are not.

There should be measurable consequences of quantum gravity.

Beyond time and space, there could be still physics. A more abstract renormalization group could govern this physics of our universe in its pregeometric phases.

Thank you for your attention!

A Citation

The Ricci flow has also been discussed in quantum field theory, as an approximation to the renormalization group (RG) flow (...)
While my background in quantum physics is insufficient to discuss this on a technical level, I would like to speculate on the Wilsonian picture of the RG flow (...) To compute something on a lower energy scale one has to average the contributions of the degrees of freedom corresponding to the higher energy scale. (...)
Note that we have a paradox here: the regions that appear to be far from each other at larger distance scale may become close at smaller distance scale; moreover, if we allow Ricci flow through singularities, the regions that are in different connected components at larger distance scale may become neighboring when viewed through microscope.
Anyway, this connection between the Ricci flow and the RG flow suggests that Ricci flow must be gradient-like; the present work confirms this expectation.

Perelman, arXiv 0211159, 2002