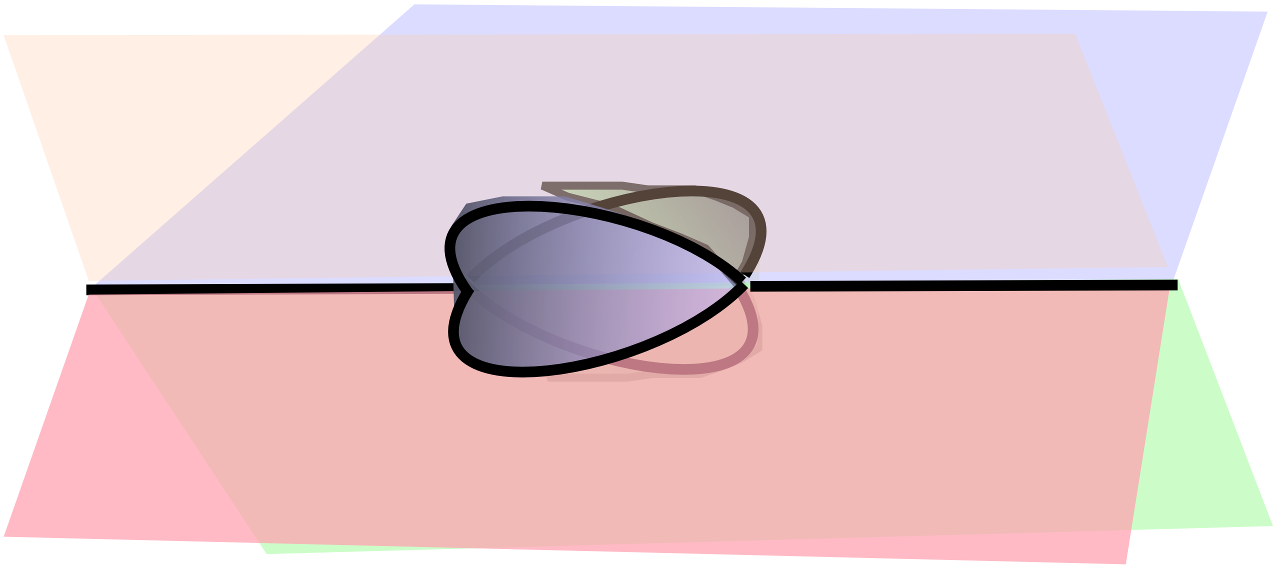
$$S = k\frac{Ac^3}{4\hbar G}$$

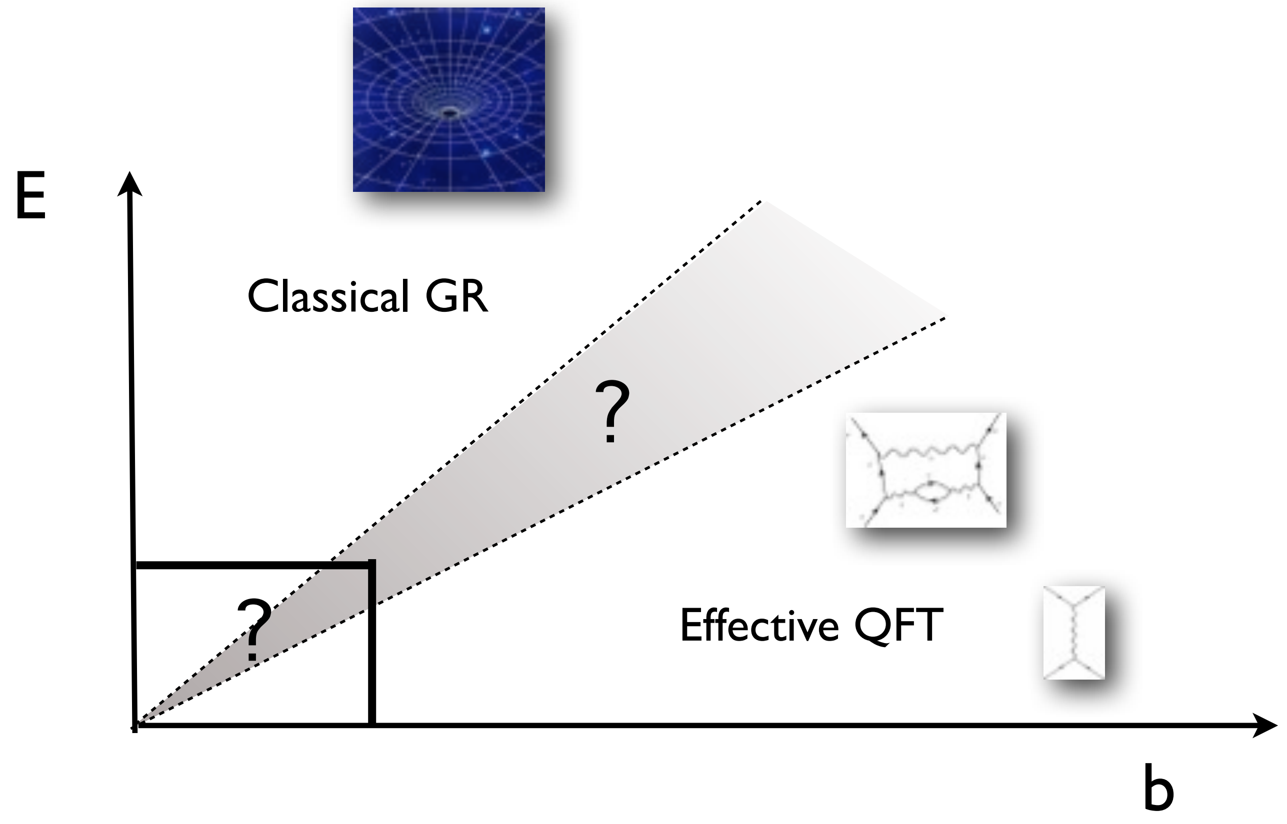
Radiative corrections in covariant LQG

carlo rovelli

Important new result:
“Self-Energy in the Lorentzian ERPL-FK Spinfoam model of Quantum Gravity”
Aldo Riello: ArXives 1302:1781



Is there a consistent quantum theory
whose classical limit is general relativity,
in 4 lorentzian dimensions,
with its standard matter couplings?



Two separate problems:

I. Unification of all interactions

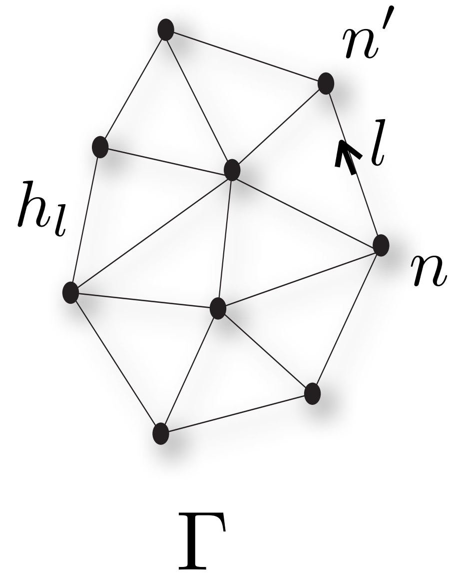
II. Quantum properties of the gravitational field

Covariant loop quantum gravity. Full definition.

Kinematics

State space $$\mathcal{H}_\Gamma = L^2[SU(2)^L/SU(2)^N]_\Gamma \quad \ni \psi(h_l)$$

Operators: $$\vec{L}_l = \{L_l^i\}, i = 1,2,3 \quad \text{where} \quad L^i\psi(h) \equiv \frac{d}{dt}\psi(he^{t\tau_i})\Big|_{t=0}$$



Dynamics

Transition amplitudes $$W_\mathcal{C}(h_l) = N_\mathcal{C}\int_{SU(2)} dh_{vf} \prod_f \delta(h_f) \prod_v A(h_{vf})$$

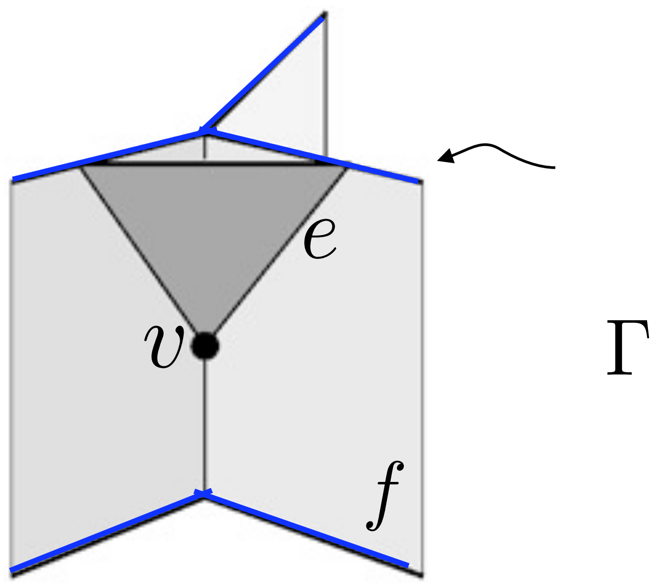
$$h_f = \prod_v h_{vf}$$

Vertex amplitude $$A(h_{vf}) = \int_{SL(2,\mathbb{C})} dg'_e \prod_f \sum_j (2j+1)\ D^j_{mn}(h_{vf}) D^{\gamma(j+1)\,j}_{jmjn}(g_e g_{e'}^{-1})$$

With a cosmological constant λ > 0:
Amplitude: SL(2,C) ➞ SL(2,C)_q network evaluation.

$$q = e^{i\lambda\hbar G}$$

Units: $8\pi\gamma\hbar G = 1$



2-complex $\mathcal{C}$
(vertices, edges, faces)

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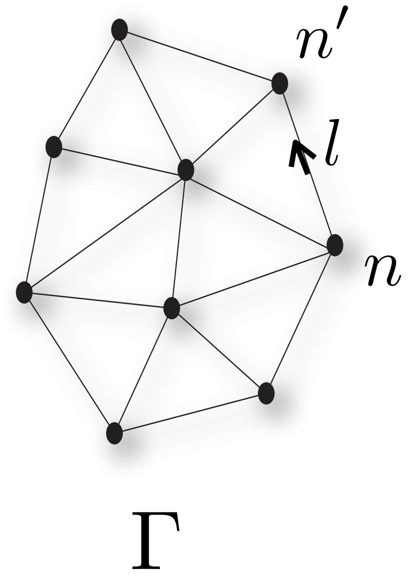

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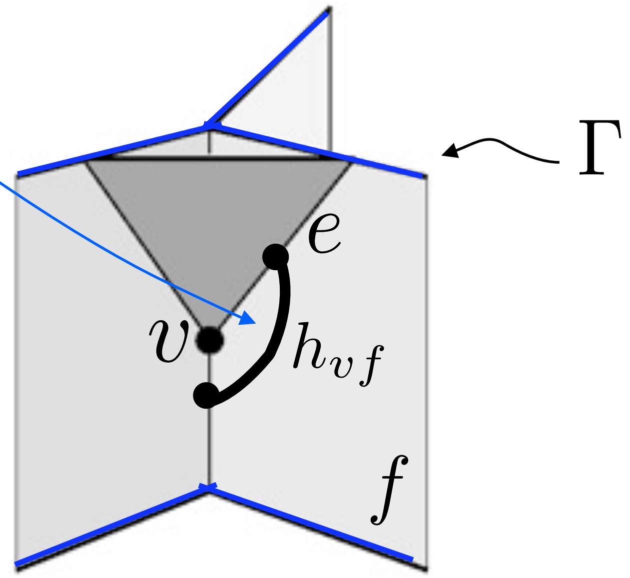

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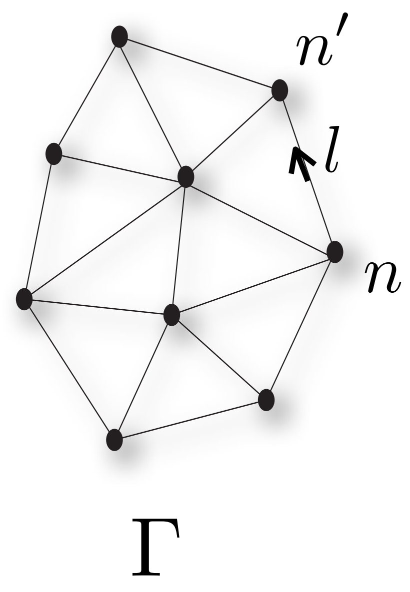

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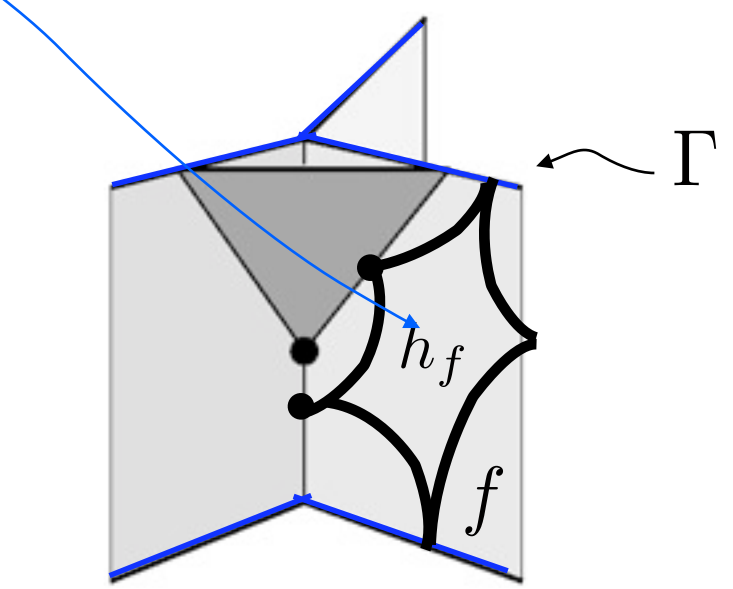

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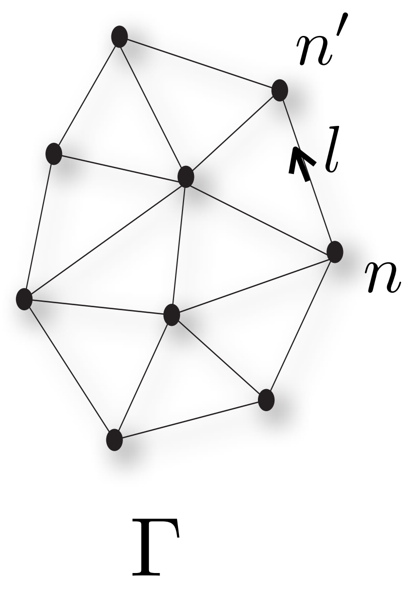

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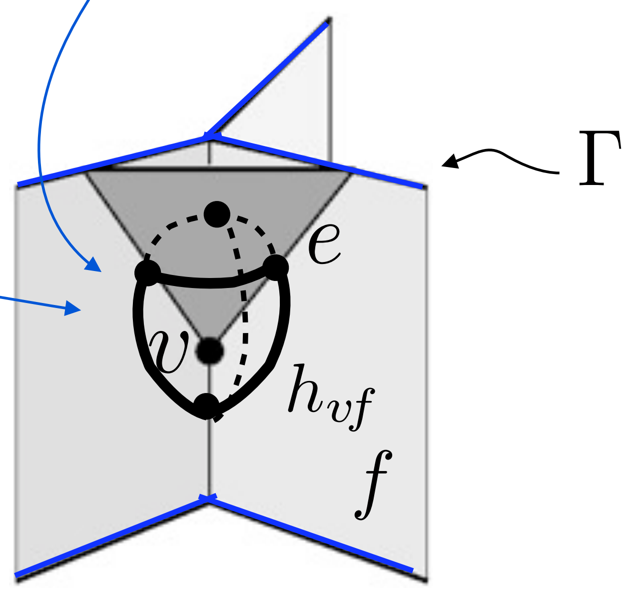

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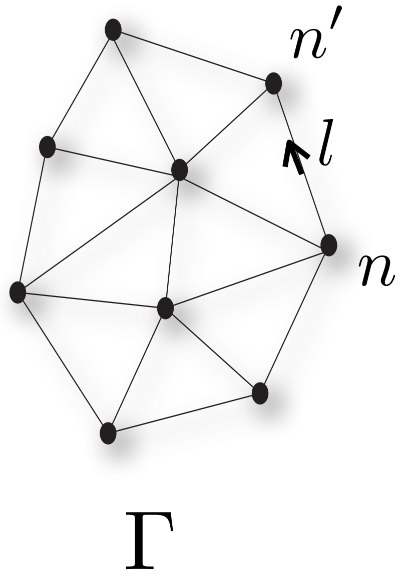

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SU(2)Wigner matrices

SL(2,C) unitary rep matrices

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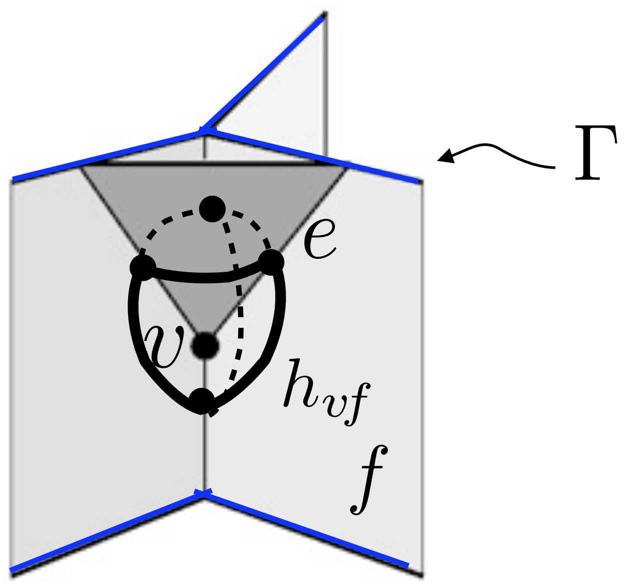


2-complex $\mathcal{C}$
(vertices, edges, faces)

Main Results, old and new

1. Boundary states represent geometries.

   (Canonical LQG 1990', Penrose spin-geometry theorem 1971).

2. Geometry operators have discrete spectra: geometry is discrete at small scale.

   (Canonical LQG main results, 1990').

3. The classical limit of the vertex amplitude is Lorentz invariant and converges to the Regge Hamilton function with λ.

   (Conrady-Freidel, Barrett *et al*, Bianchi-Perini-Magliaro, Engle, Han..., 2009-2012).

4. The amplitudes (with positive cosmological constant) are (UV and IR) finite: $W_{\mathcal{C}}^{q} < \infty$

   (Han, Fairbairn, Moesburger, 2011).

Main features

$$G \sim 6 \cdot 10^{-8} \frac{cm^3}{gr \cdot s}, \qquad \lambda \sim 10^{-56} cm^{-2}$$

1. Two dimensionfull constants: $l_P \sim \hbar G, \qquad \Lambda = 1/\lambda$

2. Two dimensionless constants: $\frac{\Lambda}{l_p} = \frac{1}{\lambda \hbar G} \sim 10^{120}, \qquad \gamma \sim 1,$

3. UV finiteness is given by a finite physical cut-off.

   - Same as in string theory! No infinite renormalization.
   - Semiclassical QG arguments (Bronstein, Wheeler, Isham ...): there is no real physical scale below l_P
   - Amati-Ciafaloni-Veneziano scattering.
   - Finite number of bits on black hole surface.
   - Finite number of bits along a strings.
   - Discreteness of Area and Volume spectra.
   - ...
     - Special relativity: discovery that there is maximal physical velocity c
     - Quantum mechanics: discovery there is minimal physical action $\hbar$
     - Quantum gravity: discovery there is a minimal length l_P .

4. IR finiteness given by cosmological constant.

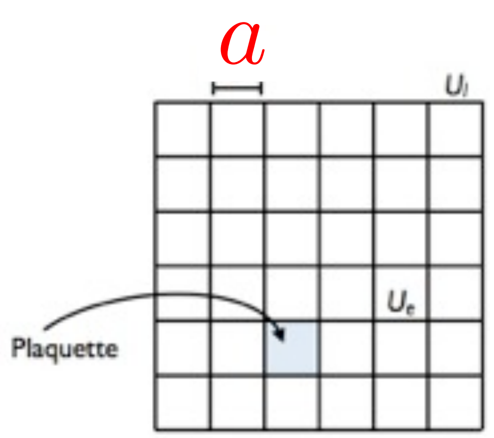
Tassonomy of quantum gravity approaches

	Just gravitational field	New physics
Fundamental scale	**Loops**	**Strings**
Standard qft defined by a critical point	**CDT** **Asymptotic safety** **Tensor models**	**Supergravity**

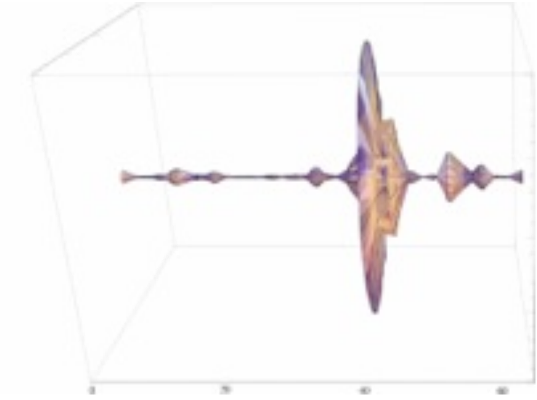
Main features

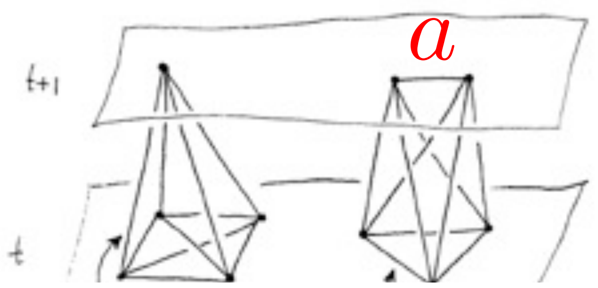
QCD. Large distance: non perturbative (lattice!) Short distance: perturbative.

Gravity. Large distance: perturbative. Short distance: non perturbative (lattice!).

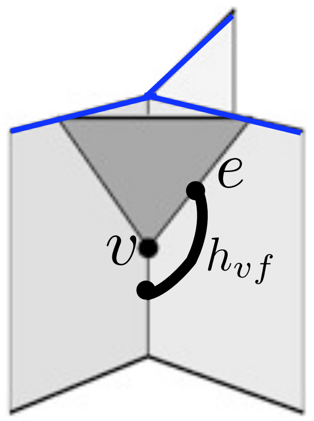


QCD: $$Z_{QCD} = \lim_{\substack{a\to 0\\ N\to\infty}} \int dU_e \; e^{iS_{a,N}(U_e)}$$





CDT: $$Z_{CDT} = \lim_{\substack{a\to 0\\ N\to\infty}} \sum_{\tau} A(\tau)$$

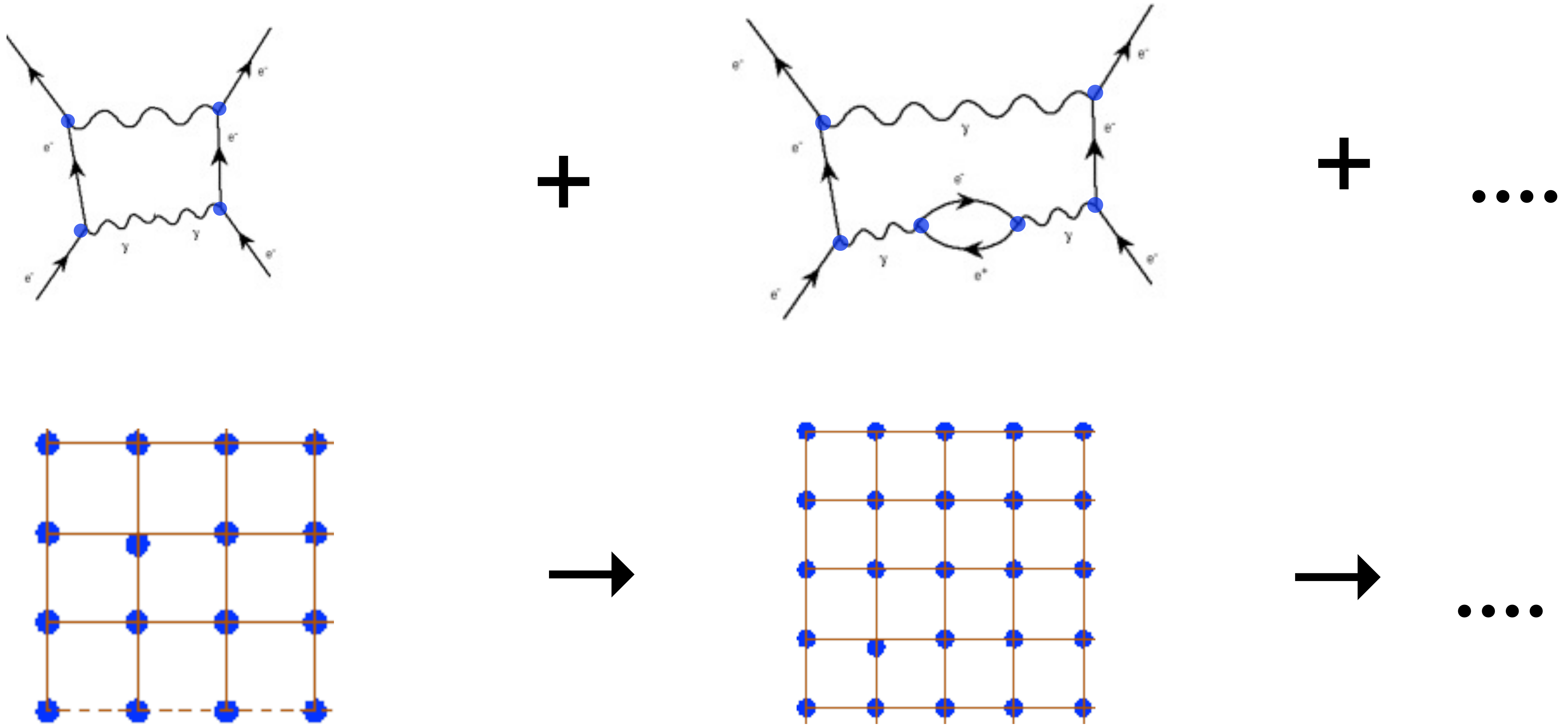


LQG: $$Z_{LQG} = \lim_{N\to\infty} \sum_{\tau,\, j_f} \int dh_e \; A(\tau, h_e, j_f)$$

Length is the variable summed over.
No continuous parameter to tune.
(Like in string theory)
Spins are discrete.

Convergence between the QED and the QCD pictures

- *All* physical QFT are constructed via a truncation of the *d.o.f.* (cfr: QED: particles, QCD Lattice).
- All physical calculation are performed within a truncation.
- The limit in which all *d.o.f.* is then recovered is pretty different in QED qnd QCD:



What about Quantum Gravity?

Lattice site = small region of space = excitations of the gravitational field = quanta of space = quanta of the field

Diff invariance !

General relativity

Tetrads	$g_{ab} \to e^i_a \qquad g_{ab} = e^i_a \; e^i_b \qquad e = e_a dx^a \in R^{(1,3)}$
Spin connection	$\omega = \omega_a dx^a \in sl(2,C) \qquad \omega(e): \quad de + \omega \wedge e = 0$
GR action	$S[e,\omega] = \int e \wedge e \wedge F^*[\omega]$
GR Holst action	$S[e,\omega] = \int e \wedge e \wedge F^*[\omega] + \frac{1}{\gamma} \int e \wedge e \wedge F[\omega]$
Canonical variables	$\omega, \qquad B = (e \wedge e)^* + \frac{1}{\gamma}(e \wedge e)$
On the boundary	$n_i = e^a_i n_a \qquad n_i e^i = 0 \quad SL(2,C) \to SU(2) \qquad n_i = (1,0,0,0)$
	$B \to (K = nB, L = nB^*)$
	$\vec{K} + \gamma \vec{L} = 0$ “Linear simplicity constraint”

Main tool: SL(2,C) ***unitary*** irreducible representations (why so little used in physics?)

SU(2) unitary representations: $2j \in Z$ $|j;m\rangle \in \mathcal{H}_j$

SL(2,C) unitary representations: $2k \in N, \quad \nu \in R$ $|k,\nu;j,m\rangle \in \mathcal{H}_{k,\nu} = \bigoplus_{j=k,\infty} \mathcal{H}^j_{k,\nu},$

γ-simple representations: $\nu = \gamma(k+1)$

SU(2) → SL(2,C) map:

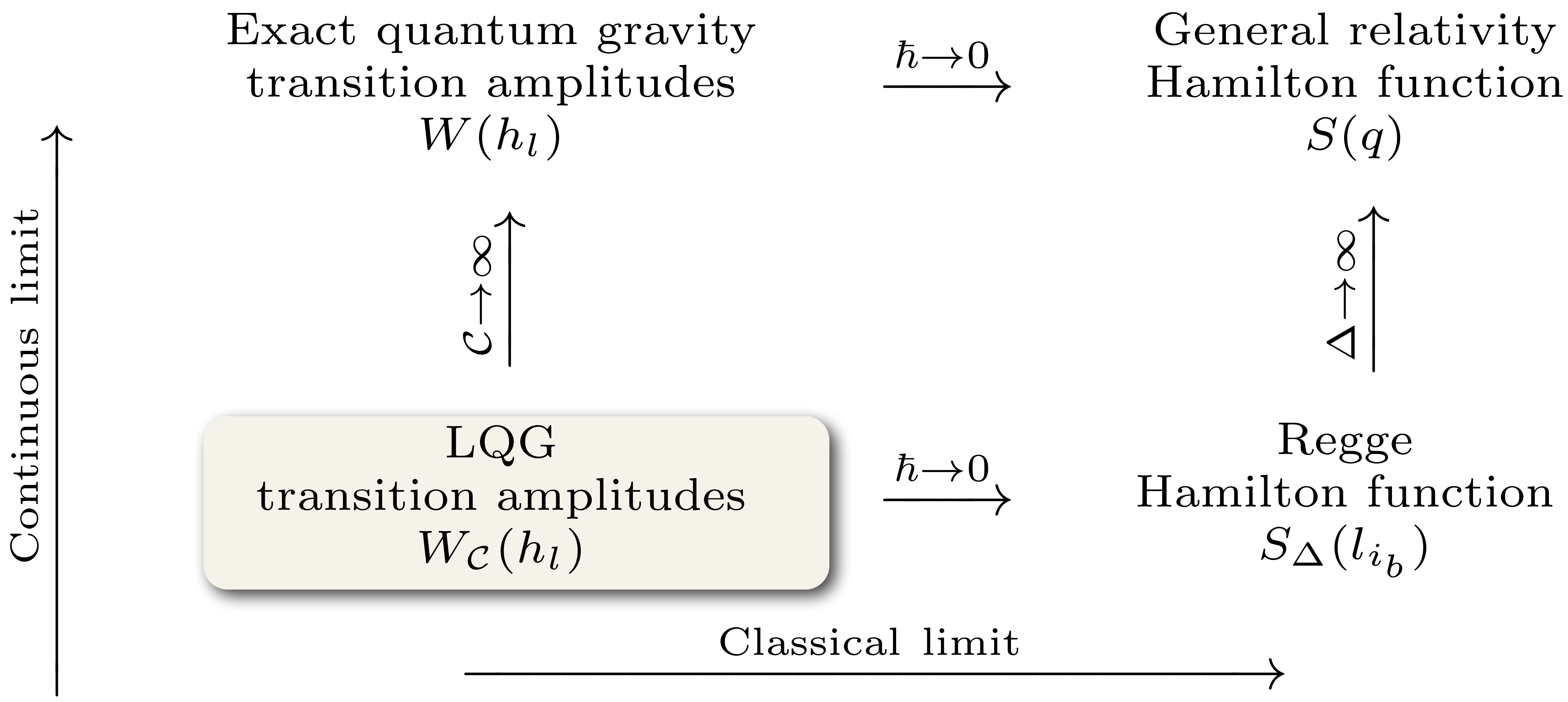
$$\begin{aligned} Y_\gamma : \quad & \mathcal{H}_j \quad \to \mathcal{H}_{j,\gamma j} \\ & |j;m\rangle \mapsto |(j,\gamma(j+1))\,;\; j,m\rangle \end{aligned}$$

Image of Y_γ :
minimal weight subspace $j = k$

Main property: $\vec{K} + \gamma \vec{L} = 0$ weakly on the image of Y_γ

Boost generator Rotation generator

Structure of the theory



$$\begin{tikzcd}
\begin{array}{c}\text{Exact quantum gravity}\\ \text{transition amplitudes}\\ W(h_l)\end{array} \arrow[r, "\hbar\to 0"] & \begin{array}{c}\text{General relativity}\\ \text{Hamilton function}\\ S(q)\end{array} \\
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\end{tikzcd}$$

Continuous limit

Classical limit

Regime of validity of the expansions: $L_{\text{Planck}} \ll L \ll \sqrt{\frac{1}{\text{Curvature}}}$

- No critical point
- No infinite renormalization
- Physical scale: Planck length

Covariant LQG is good

- There is one single known physical spinfoam amplitudes (4d, Lorentzian,correct degrees of freedom.)
- The theory is defined by its transition amplitudes, order by order in the 2-complex.
- The transition amplitudes with cosmological constant λ are finite. [Han, Fairbairn, Moesburger, Zhang.]

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But

- Since $\Lambda=\lambda^{-1}$ is large ($\Lambda\sim10^{120}$), radiative correction might be large, invalidating the expansion!

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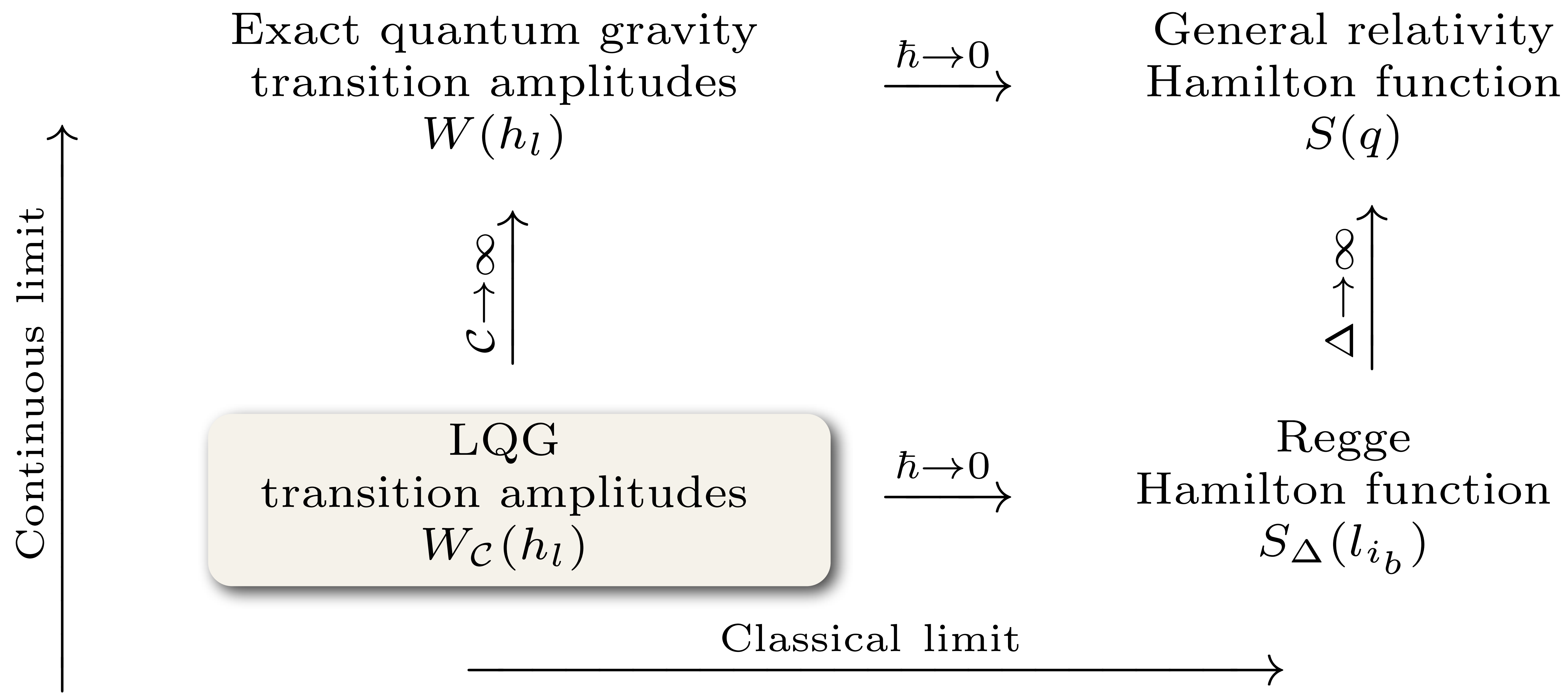

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Problem:

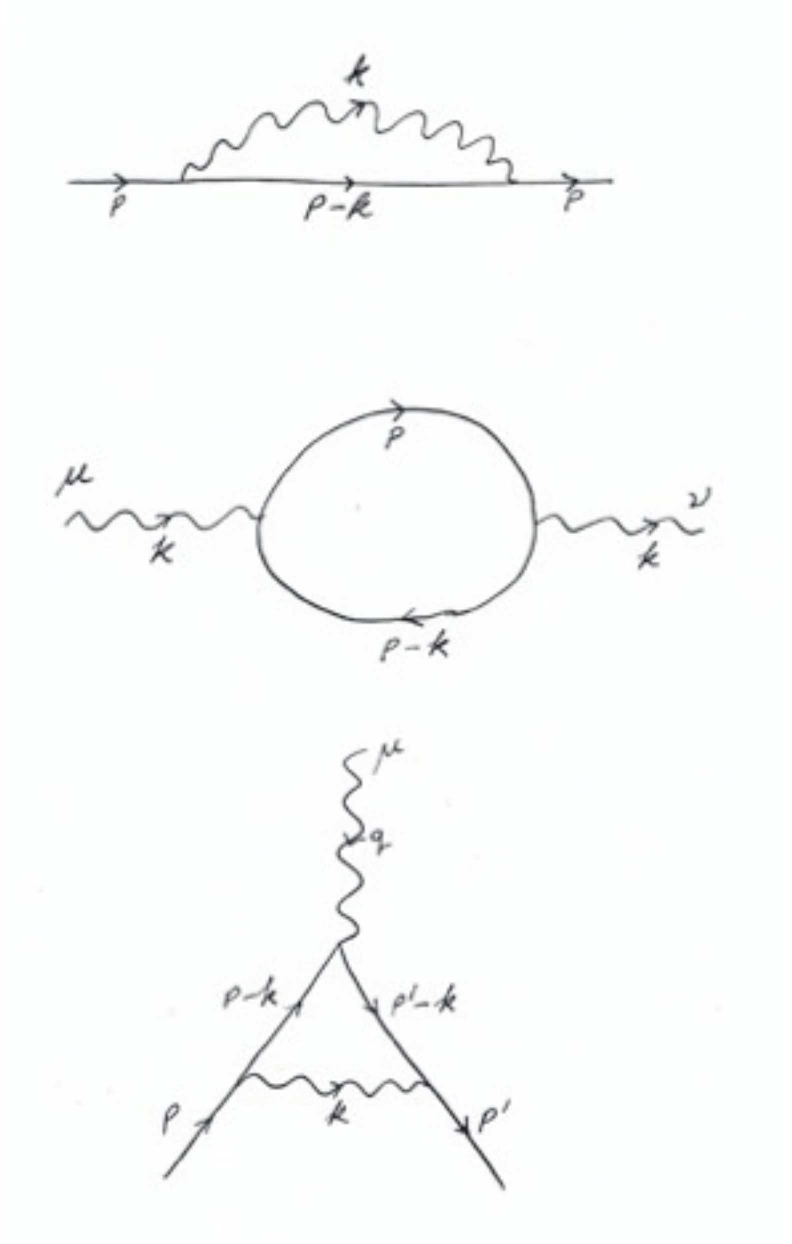
- Since $\Lambda=\lambda^{-1}$ is very large ($\Lambda \sim 10^{120}$), radiative correction might be large, invalidating the expansion.

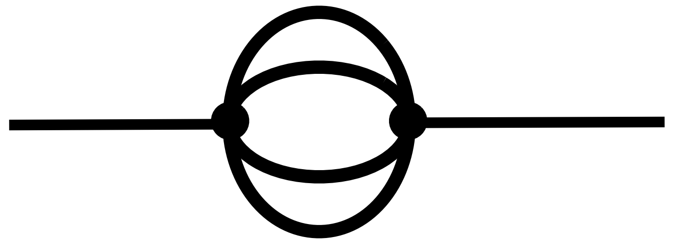
Strategy:

- Large corrections are likely described by the divergences of the $\lambda=0$ theory.
- Study divergences of the $\lambda=0$ theory to understand the viability of the expansion.

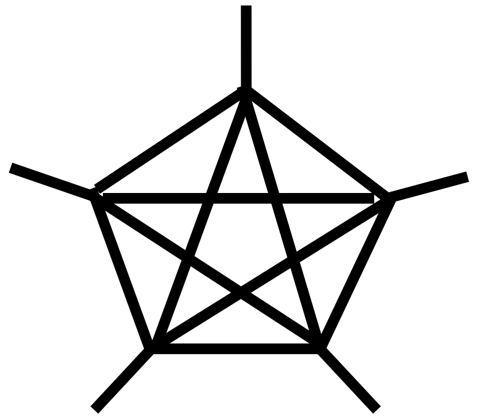
New main message (good news):

cfr:



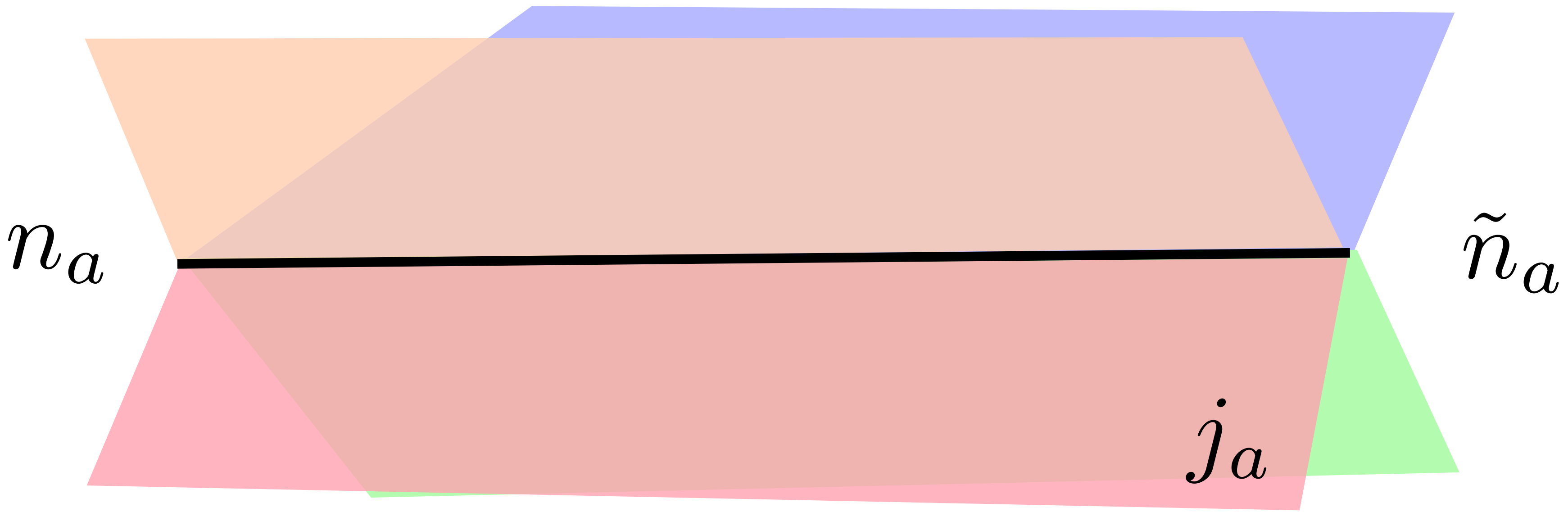


The first radiative correction
to the *edge* amplitude
is logarithmic in λ^{-1}
[Aldo Riello 2013]

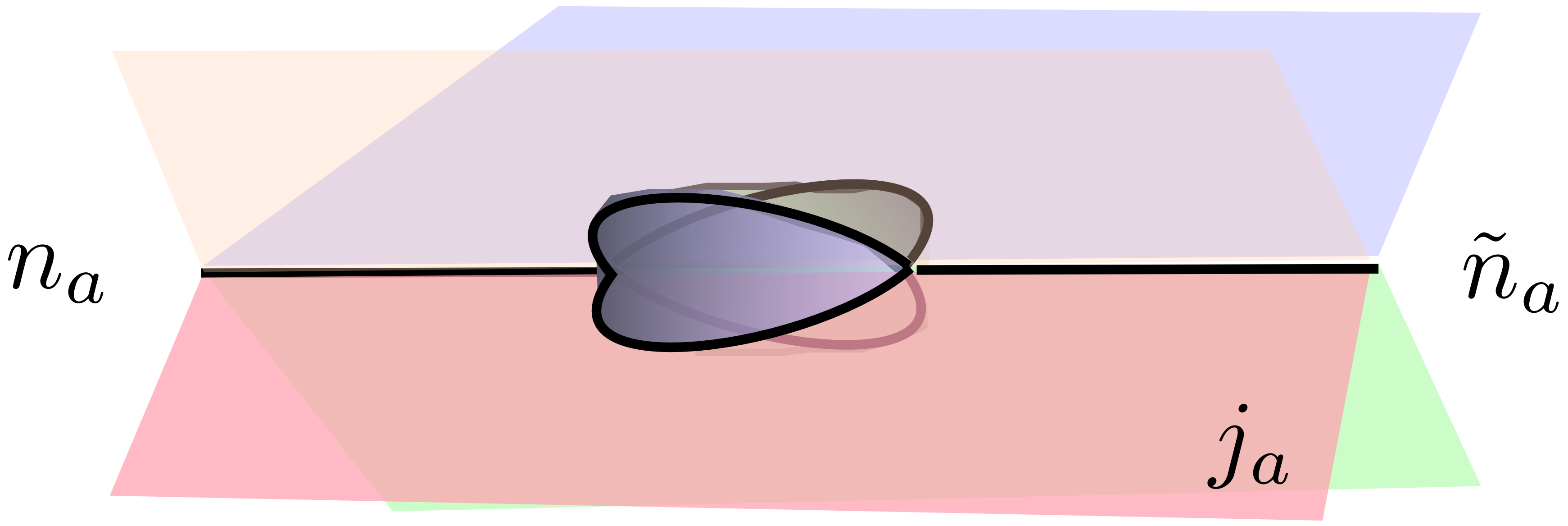


The first radiative correction
to the *vertex* amplitude
is finite.
[Aldo Riello 2013]

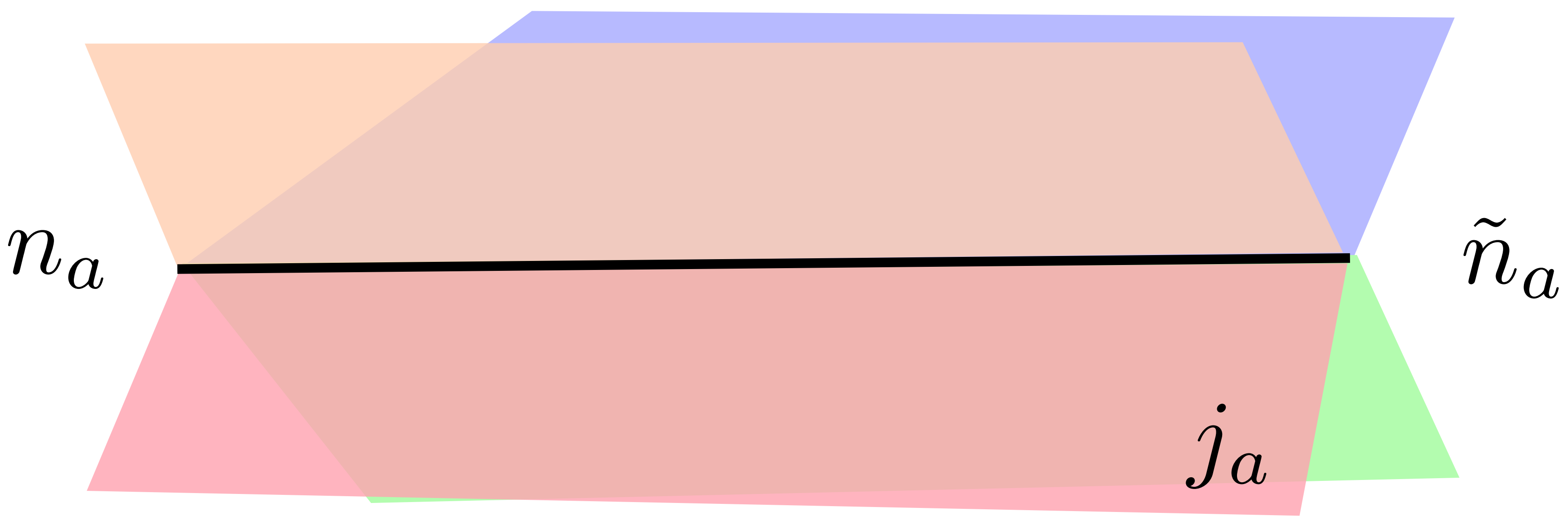
(up to possible technical loopholes, not yet closed)

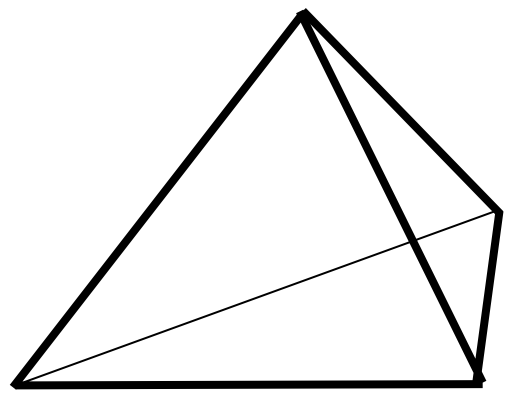


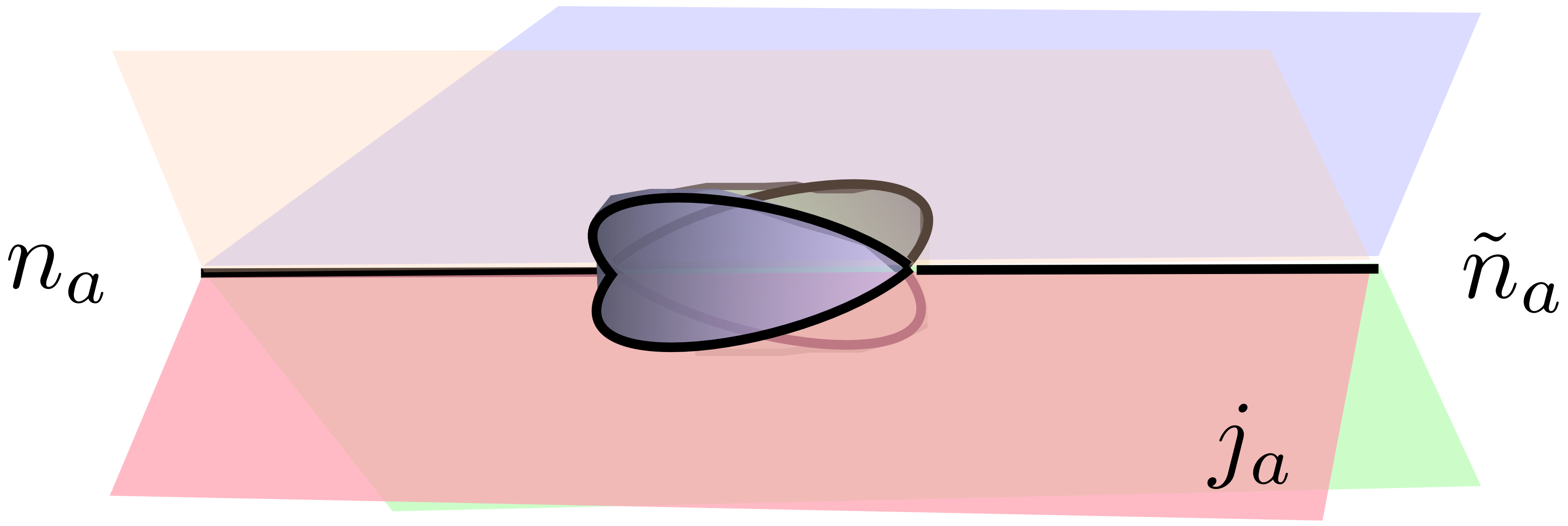
$$\langle j_a n_a | j_a \tilde{n}_a \rangle$$



?


n_a
$\tilde{n}_a$
j_a

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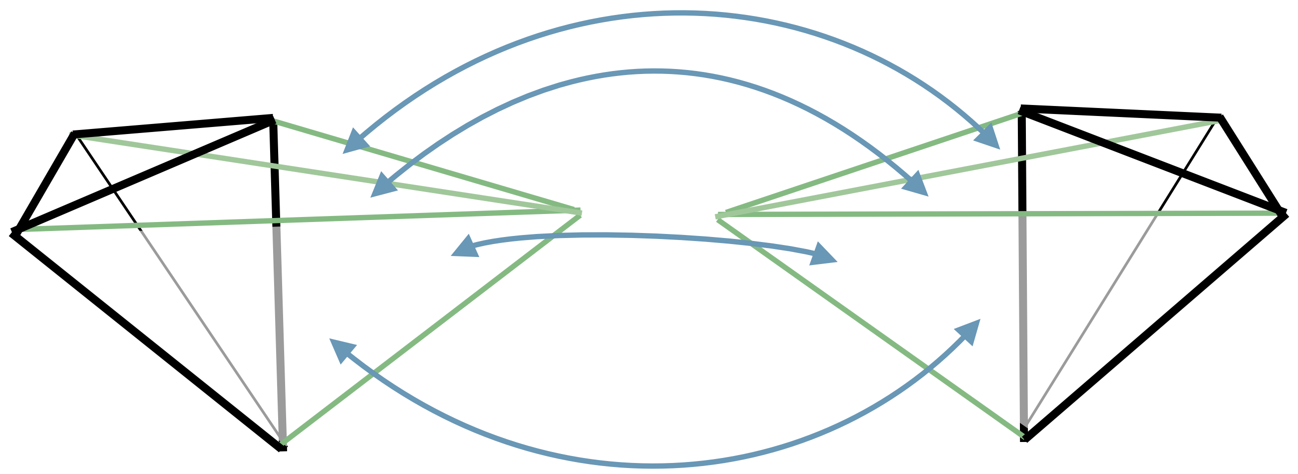
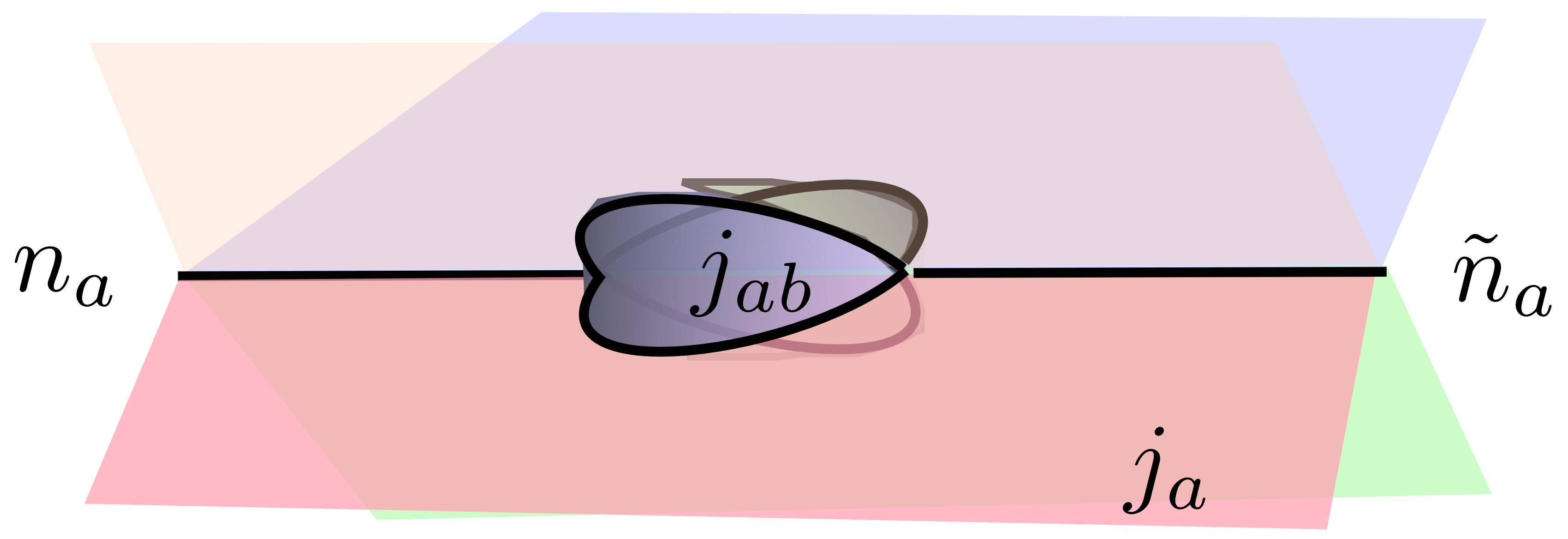




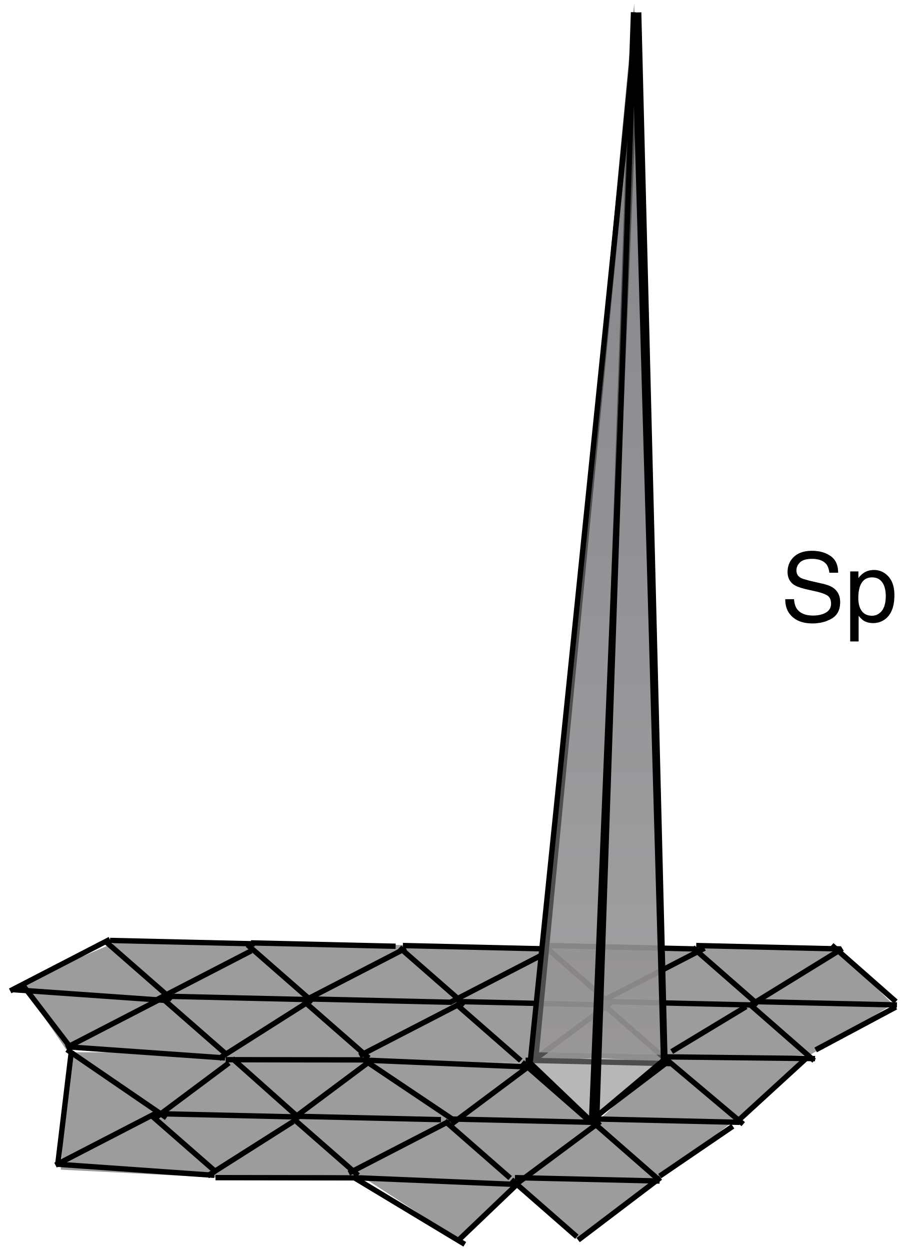
$$W(j_a, n_a, \tilde{n}_a) = \sum_{j_{ab}}^{\color{red}{\Lambda}} \left(\prod_{ab} (2j_{ab}+1)^{\mu} \right) \quad w(j_a, j_{ab}, n_a, \tilde{n}_a)$$

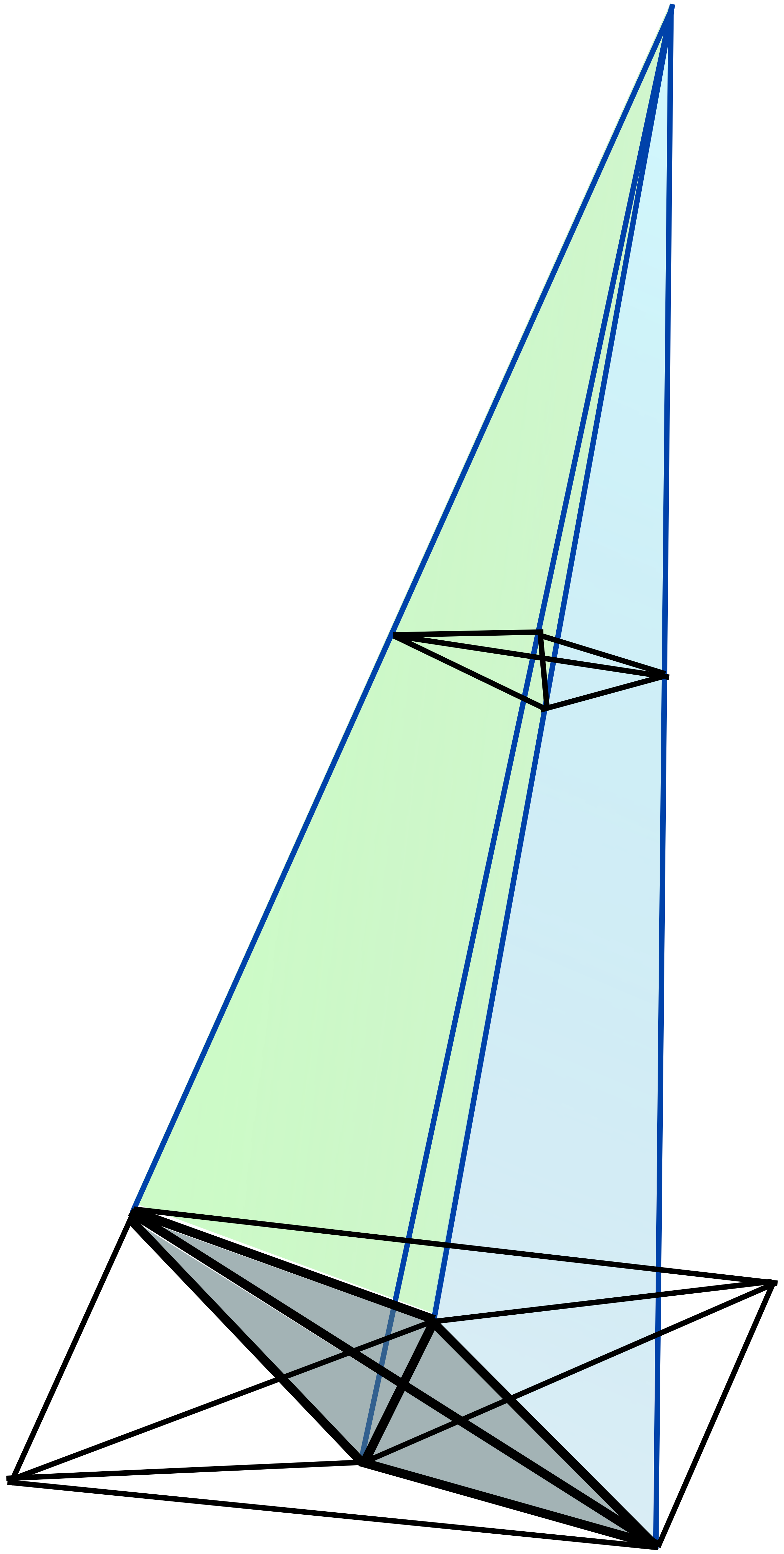
$$w(j_a, j_{ab}, n_a, \tilde{n}_a) = \int_{SL(2,C)} dg_{ab} d\tilde{g}_{ab} \prod_a \langle n_a | Y^\dagger g_{ab}^{-1} Y Y^\dagger \tilde{g}_{ba} Y | \tilde{n}_a \rangle_{j_a} \prod_{ab} \underbrace{Tr_{j_{ab}} [Y^\dagger g_{ba}^{-1} \tilde{g}_{ba} Y Y^\dagger \tilde{g}_{ab}^{-1} g_{ab} Y]}$$

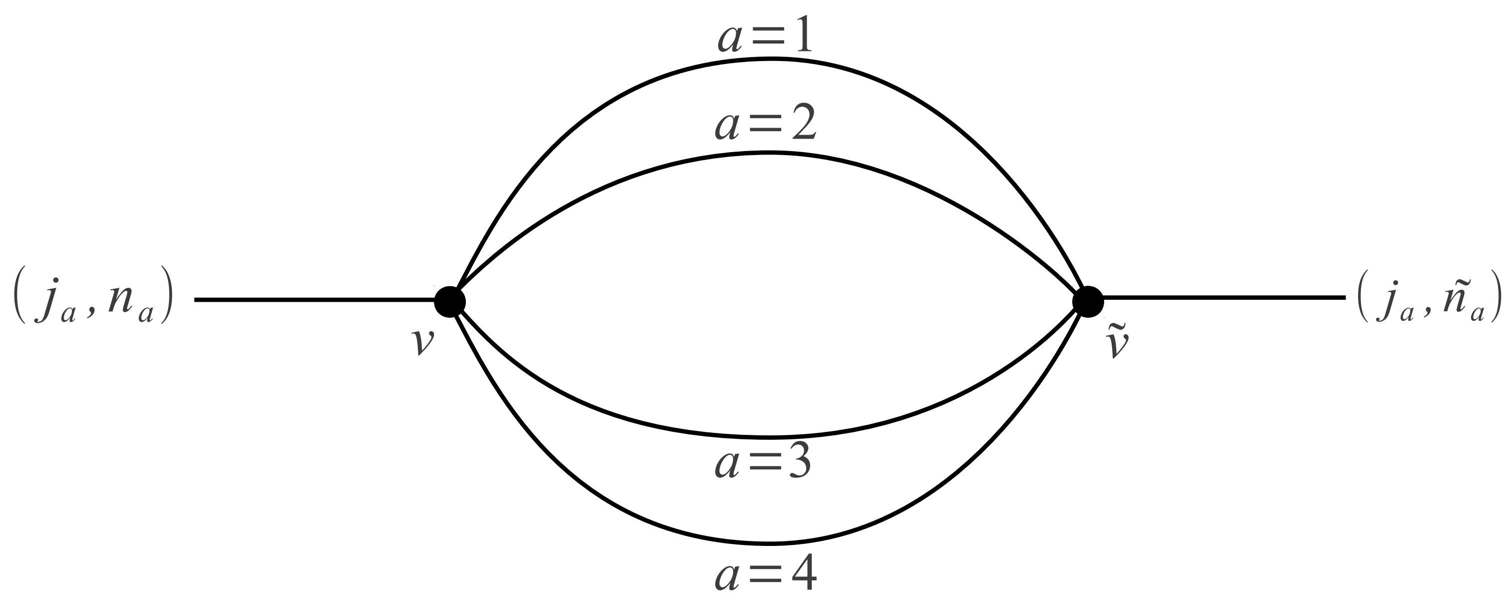
$$Tr_j [Y^\dagger g Y \, Y^\dagger \tilde{g} \, Y] = \int_{S^2} dm \, dm' \; \prod \underbrace{\langle m | Y^\dagger g Y | m' \rangle_j} \langle m' | Y^\dagger \tilde{g} \, Y | m \rangle_j$$

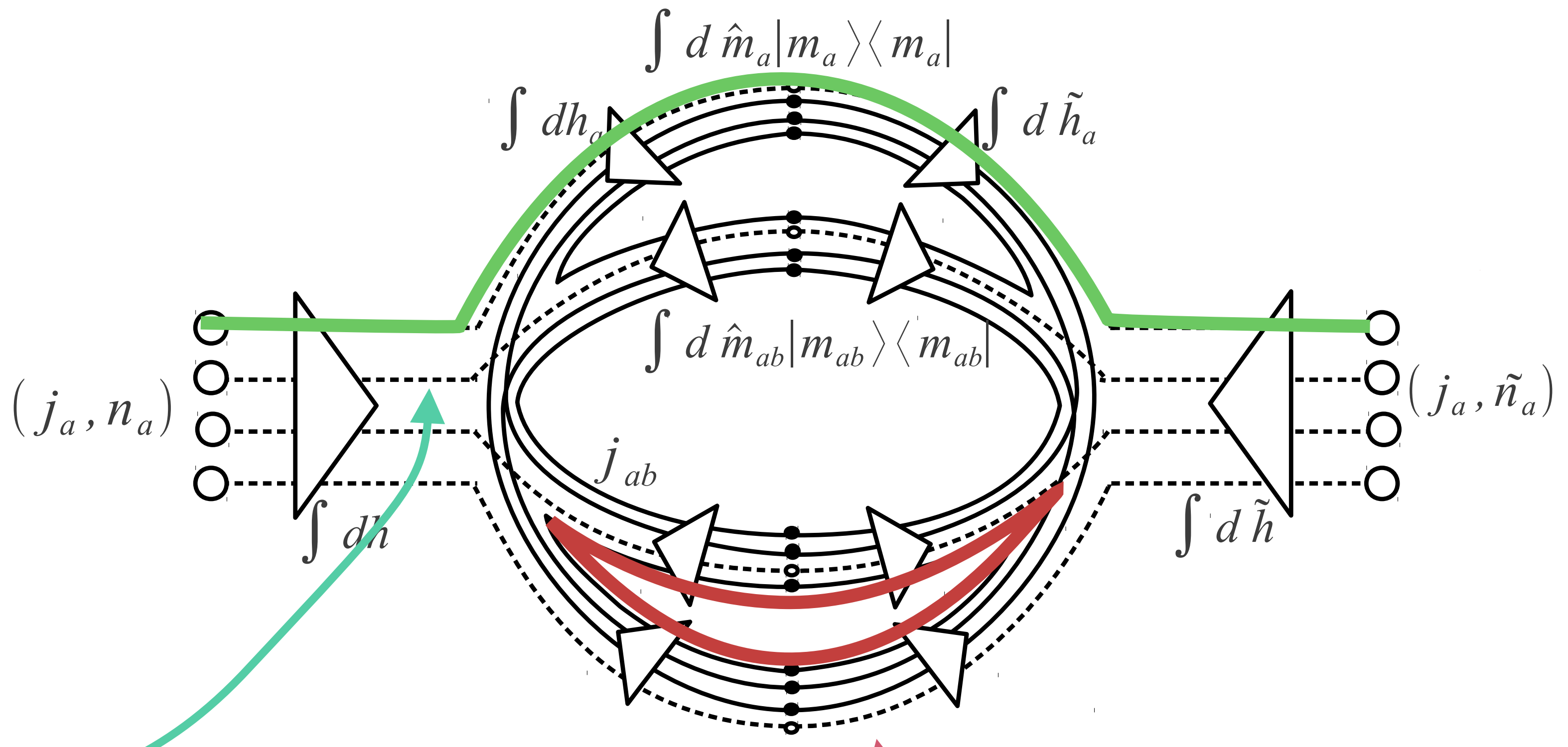
$$\langle m' | Y^\dagger \tilde{g} Y | m \rangle_j = \int_{CP_2} Dz \; F(z, m, m', g, j) = \int Dz \; e^{jS(z,m,m',g)}$$

→ Saddle point approximation


Spike
2d


“Riello tetrahedra”

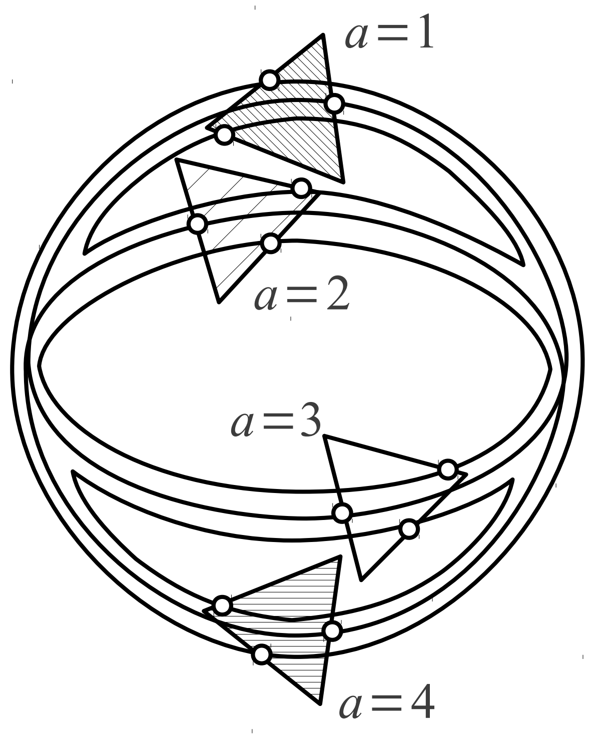


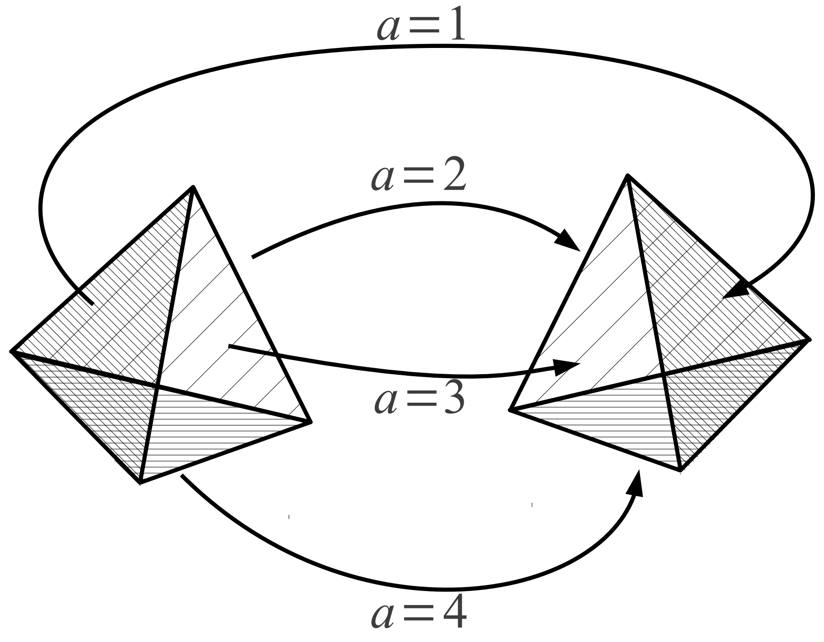


$$w(j_a, j_{ab}, n_a, \tilde{n}_a) = \int dg_{ab} d\tilde{g}_{ab} \prod_a \langle n_a | Y^\dagger g_{ab}^{-1} Y Y^\dagger \tilde{g}_{ba} Y | \tilde{n}_a \rangle_{j_a} \prod_{ab} Tr_{j_{ab}} [Y Y^\dagger g_{ba}^{-1} \tilde{g}_{ba} Y Y^\dagger \tilde{g}_{ab}^{-1} g_{ab}]$$

$$\int dg\ dm\ dz\ \ e^{\sum_{ab}\ j_{ab} S_{ab}(g,m,z)}$$

→ Reduced closure relations for the Riello tetrahedra!



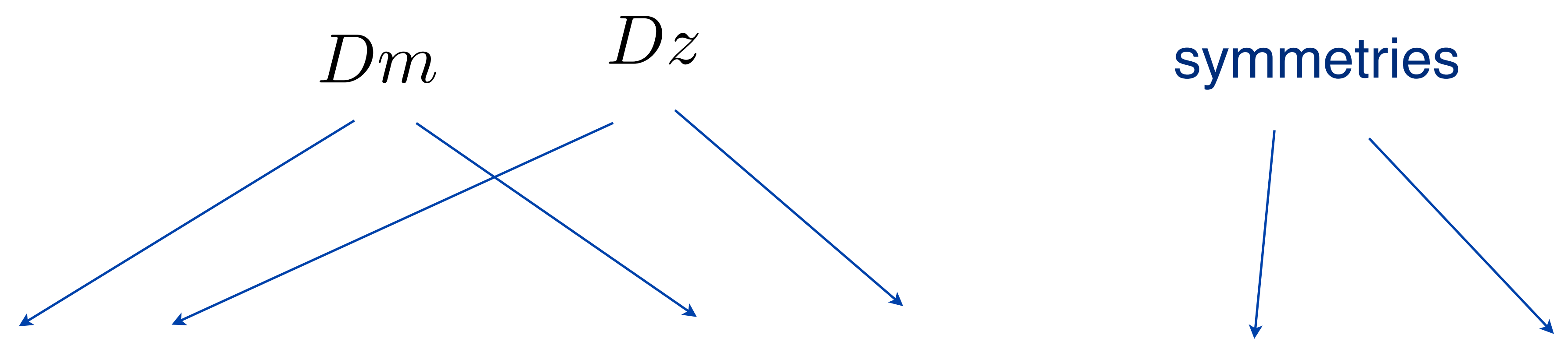


$$\int dg\ dm\ dz\ \ e^{-j_{ab}S(g,m,z)}$$

→ Saddle point $$\int_{R^d} dx^d\ e^{\lambda f(x)} = \left(\frac{2\pi}{\lambda}\right)^{\frac{d}{2}} (detH_2f)^{-\frac{1}{2}}\ \ e^{\lambda f(x_o)}(1+o(\lambda))$$

→ Saddle point equations

→ Compute dimensions of the saddle point



→ Symmetries ! $$w \sim j^{12}\ j^{12}\ j^{-\frac{1}{2}(8\,[SL(2,C)]+12\,[S_2]+12\,[CP^1]-4\,[SU(2)]-2\,[SL(2,C)])}$$

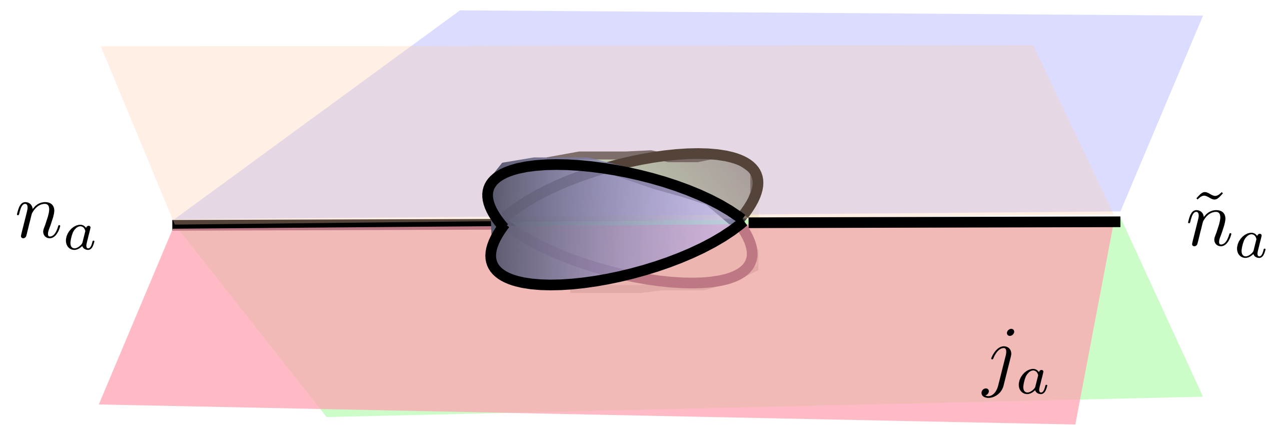
$$w \sim O(j^{-12})$$

face amplitude

→ Summing over spins $$W \sim \sum_{j_{ab}}^{\Lambda} (j_{ab})^{6\mu}\ w(j_{ab}) \sim \left\{ \begin{array}{ll} O(\Lambda^{6(\mu-1)}) & \mu \neq 1 \\ \ln \Lambda & \mu = 1 \end{array} \right.$$

$$w(j_a, j_{ab}, n_a, \tilde{n}_a) = \int dg_{ab} d\tilde{g}_{ab} \prod_a \langle n_a | Y^\dagger g_{ab}^{-1} Y Y^\dagger \tilde{g}_{ba} Y | \tilde{n}_a \rangle_{j_a} \prod_{ab} Tr_{j_{ab}} [Y Y^\dagger g_{ba}^{-1} \tilde{g}_{ba} Y Y^\dagger \tilde{g}_{ab}^{-1} g_{ab}]$$

→ Full amplitude $$W_\lambda(j_a, n_a, \tilde{n}_a) = \left\{ \begin{array}{l} \lambda^{-6(\mu-1)} \\ \ln \lambda^{-1} \end{array} \right\} \int_{SL(2,C)^2} dg d\tilde{g}\ \langle n_a | Y^\dagger g Y^\dagger Y \tilde{g} Y | \tilde{n}_a \rangle$$



$$= W_\lambda(j_a, n_a, \tilde{n}_a) \sim \ln(1/\lambda\hbar G) \int_{SL(2,C)^2} dg d\tilde{g} \; \langle n_a | Y^\dagger g Y^\dagger Y \tilde{g} Y | \tilde{n}_a \rangle$$

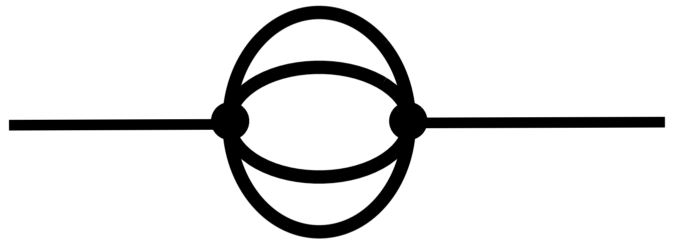
Not a large number !

Proportional to the edge for large j? (Jacek Puchta)

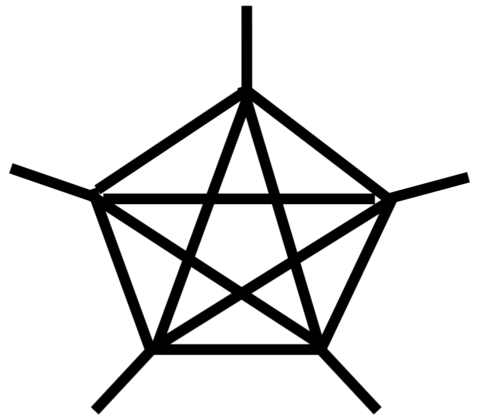
$$\ln(1/\lambda\hbar G) = \ln 10^{120} = 1.7 \; (4\pi)^2$$

New main message (good news):

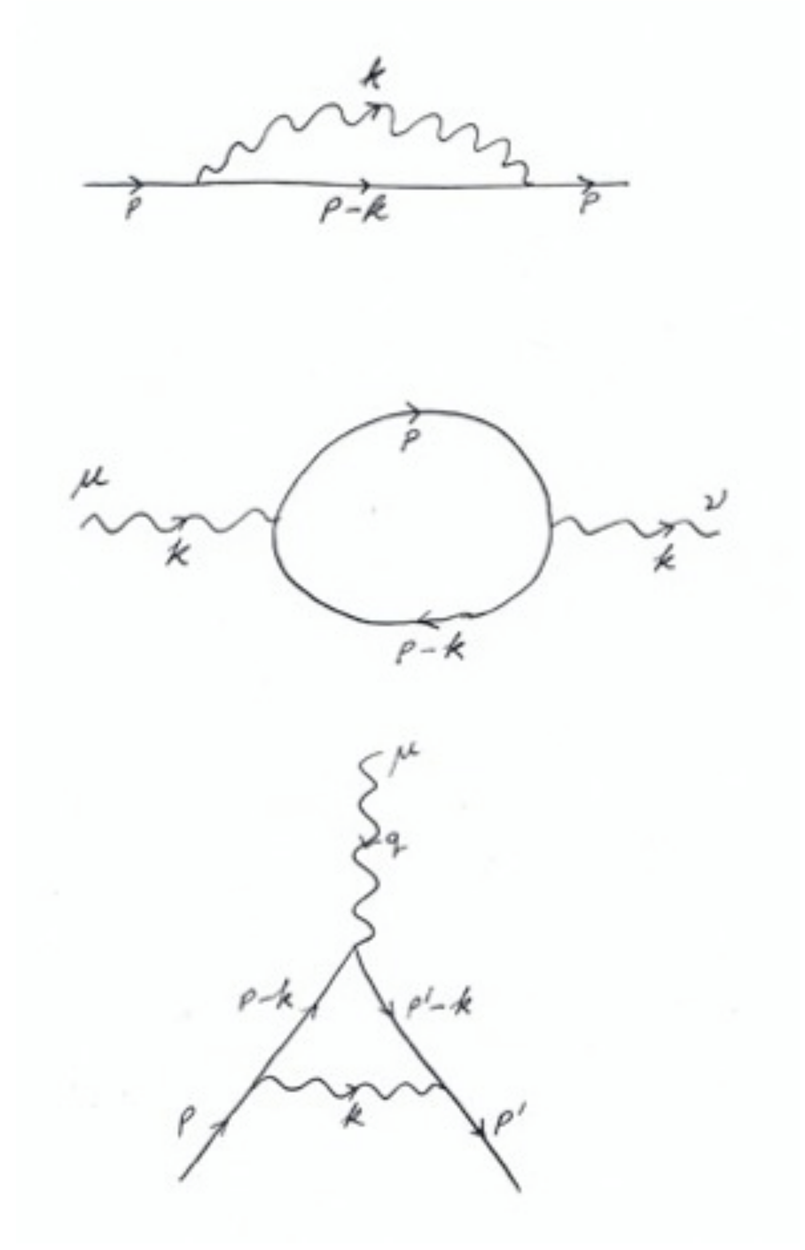
The first radiative correction
to the *edge* amplitude
is logarithmic in Λ^{-1}
[Aldo Riello 2013]



The first radiative correction
to the *vertex* amplitude
is finite.
[Aldo Riello 2013]



cfr:



(up to possible technical loopholes, not yet closed)

General comments:

- Is the large-*j* expansion credible?
    - Yes: it does the correct result in the BF case (large polynomial divergences.)

Additional moral: Gravity is much more convergent than BF!

- Previous results:
    - Euclidean spin-zero external legs [Perini Speziale CR, 09] (using properties of nJ-symbols)
    - Euclidean generic external legs [Krajewski Mangen Rivasseau Tanasa Vitale 10] (using qft techniques).

All consistent.

Additional moral: Euclidean and Lorentzian are rather similar.

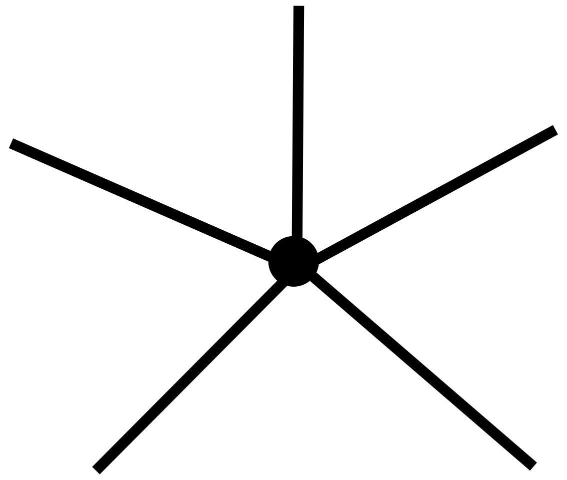
- The edge correction is the “melon” of tensor models: much is known about summing melons !

So:

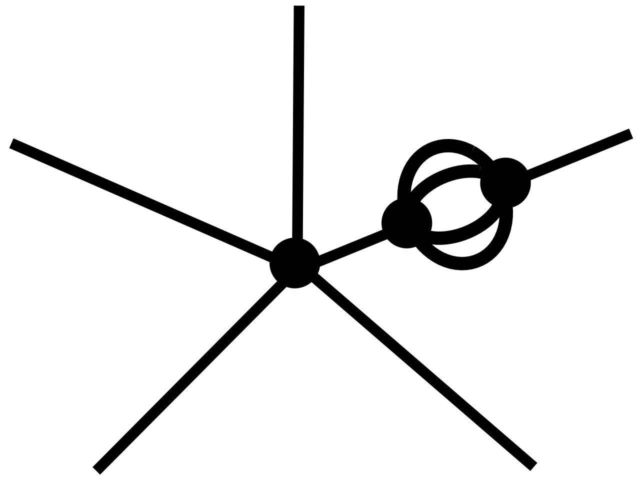
- Can this be used to prove that radiative corrections do not invalidate the expansion?
- Are these the only elementary divergences?
- What about overlapping divergences?

- Can this be used to compute the running of G or λ between the Planck scale and our scale?
- If this is small, there is no naturalness problem for the cosmological constant.

Cartoon calculation



$= e^{\frac{1}{8\pi\hbar G}S}$



$= \ln(\lambda\hbar G_o)e^{\frac{1}{8\pi\hbar G_o}S} = e^{\frac{1}{8\pi\hbar G}S}$

$$\frac{1}{G_o} + \ln\ln(\lambda\hbar G_o) = \frac{1}{G}$$