Cosmology from Group Field Theory

Daniele Oriti

Albert Einstein Institute

“Quantum Gravity in Paris 2013”
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Plan:

- intro to GFT formalism
- relation to LQG, spin foams and tensor models
- GFT states $\longleftrightarrow$ (approximate) continuum geometries
- examples of GFT condensates
- effective dynamics for GFT condensates (general)
- special case and approximate Friedmann equations
- conclusions and outlook

GFT basics

recent general introductions and reviews:

D. Oriti, arXiv: gr-qc/0607032

D. Oriti, arXiv: 0912.2441 [hep-th]

R. Gurau, J. Ryan, arXiv: 1109.4812 [hep-th]

D. Oriti, arXiv: 1111.5606 [hep-th]

V. Rivasseau, arXiv:1112.5104 [hep-th]

work by:

Baratin, Ben Geloun, Bonzom, Carrozza, De Pietri, Fairbairn, Freidel, Gielen, Girelli, Gurau, Livine, Louapre, Krajewski, Krasnov, Magnen, Noui, Oriti, Perez, Raasakka, Reisenberger, Rivasseau, Rovelli, Ryan, Sindoni, Smerlak, Tanasa, Vitale, ...

GFT basics (4d case): kinematics

Quantum field theory over group manifold $SL(2,\mathbb{C})^{\times 4}$ Lorentzian signature

(or corresponding Lie algebra) $Spin(4)^{\times 4}$ Riemannian signature

$$\varphi(g_1, g_2, g_3, g_4) \leftrightarrow \varphi(B_1, B_2, B_3, B_4) \to \mathbb{C}$$

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with ("simplicity") conditions enforced on the field or the dynamics to impose "geometricity" of simplicial structures

$$\varphi(g_1, g_2, g_3, g_4) \hookrightarrow \varphi(x_1, x_2, x_3, x_4) \qquad x_i \in X \subset G$$

classical phase space of reference:

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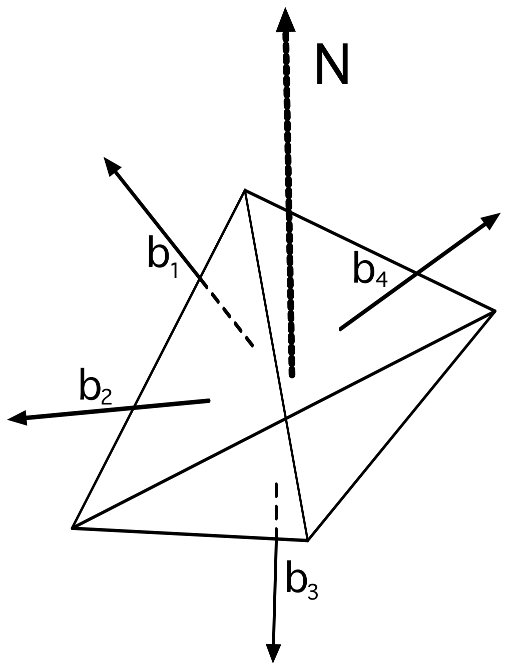
$$B_i^{IJ} \simeq N^I \wedge b_i^J$$



group ~ elementary holonomy Lie algebra ~ "discretized triad"

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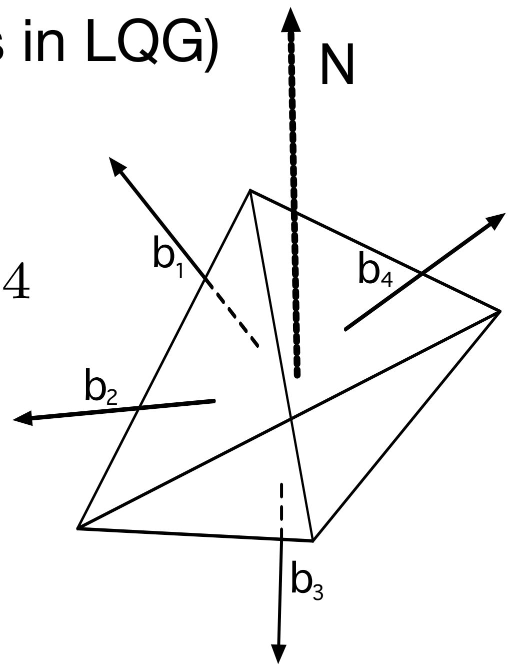

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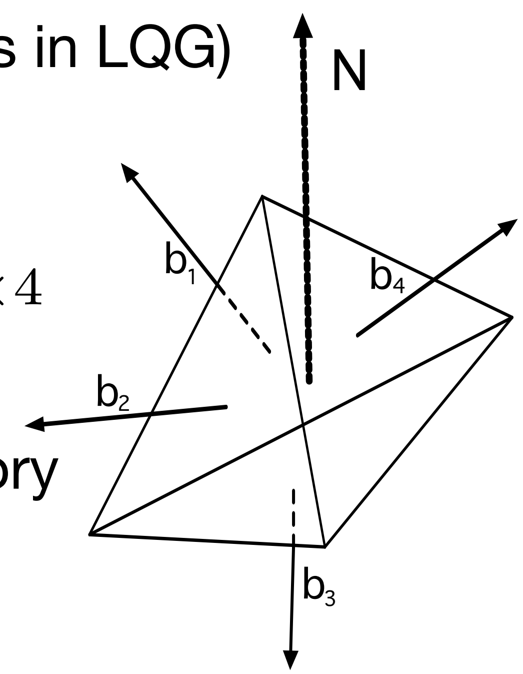

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$B_i^{IJ} \simeq N^I \wedge b_i^J$ also obtained from discretization of continuum theory (gravity = BF theory + constraints)

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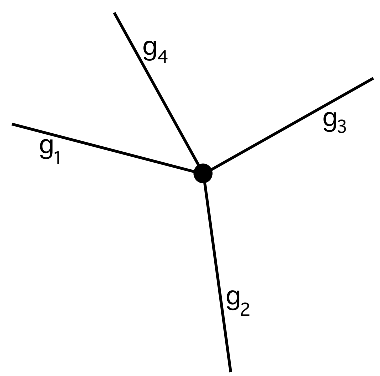

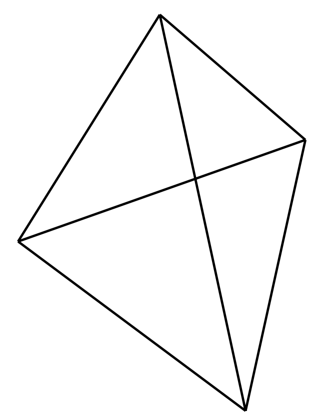

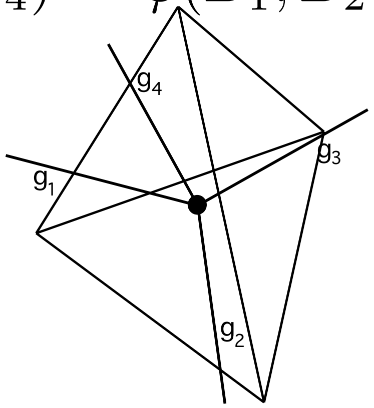

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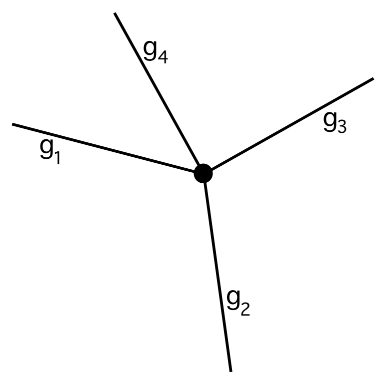

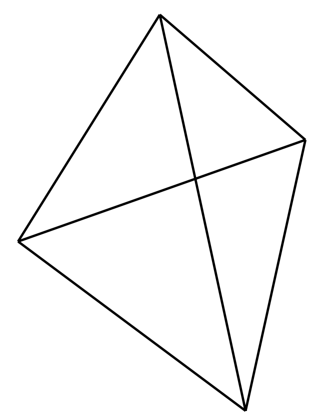

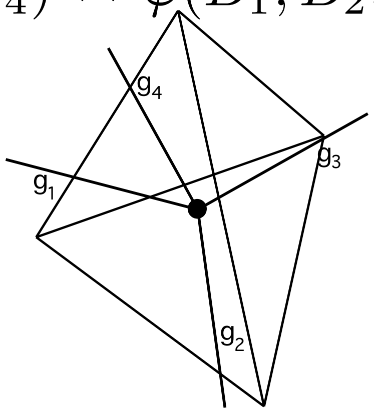

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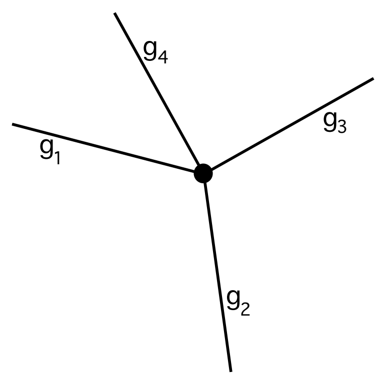

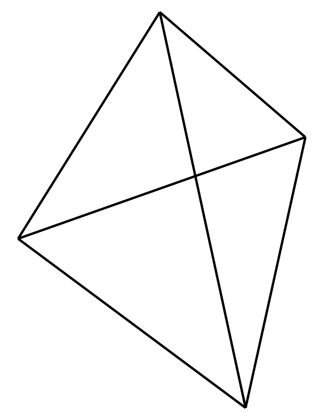

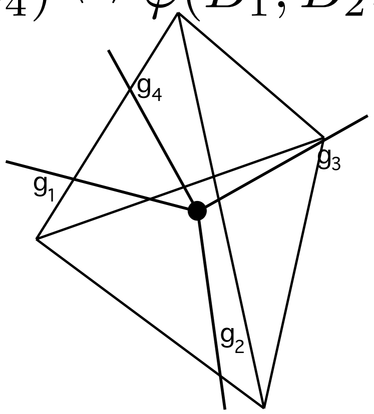

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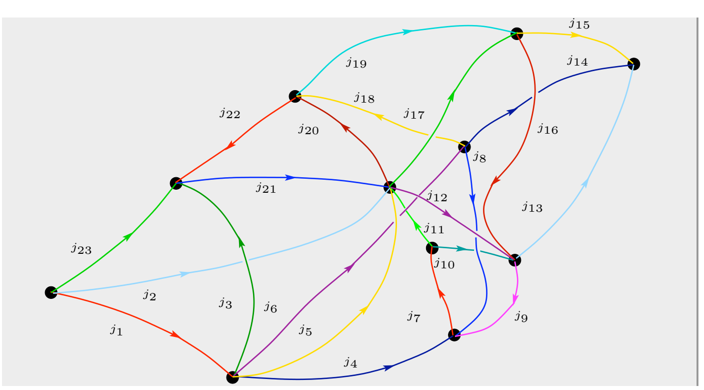

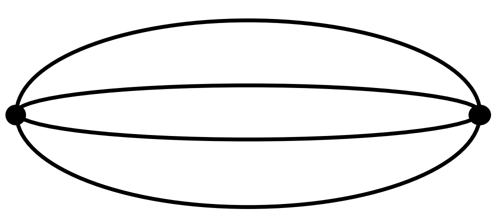

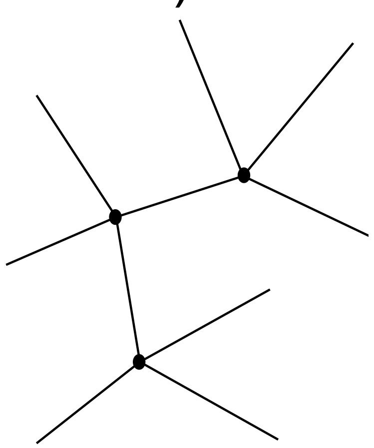

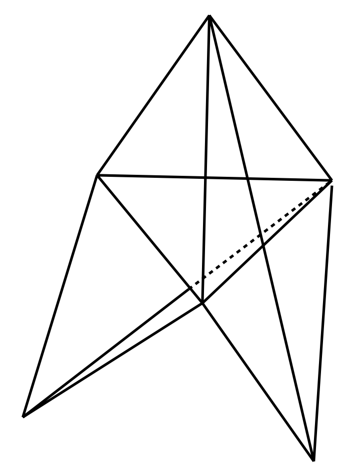

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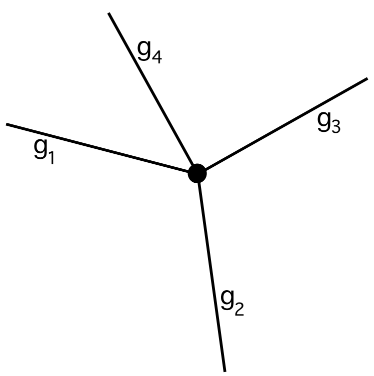

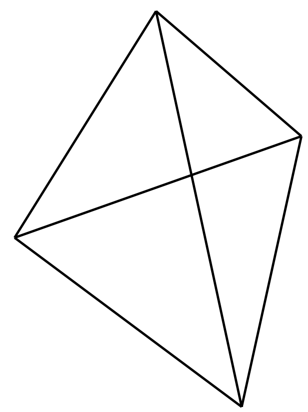

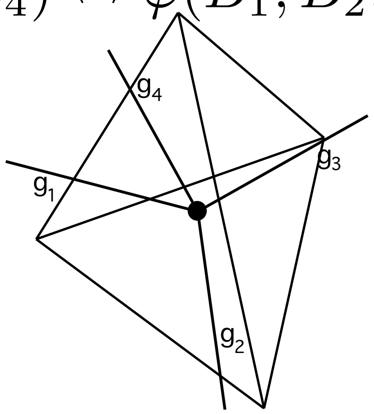

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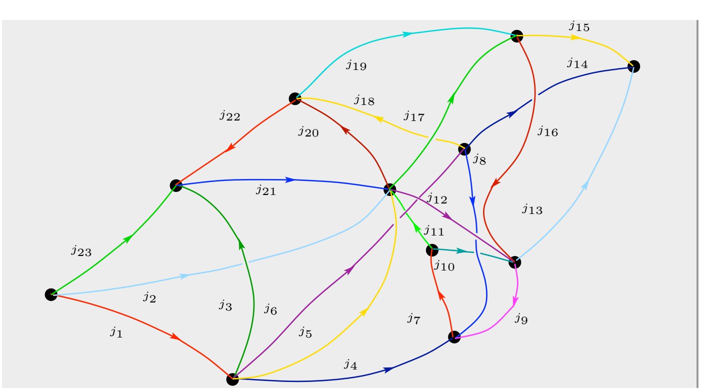

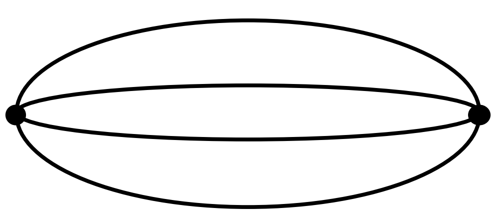

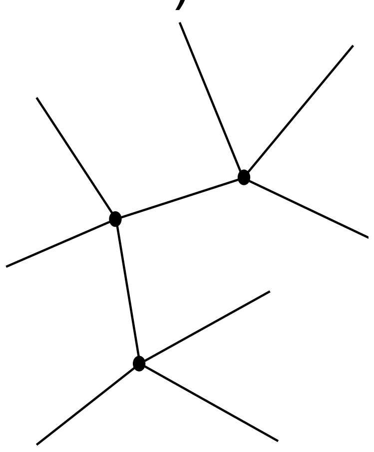

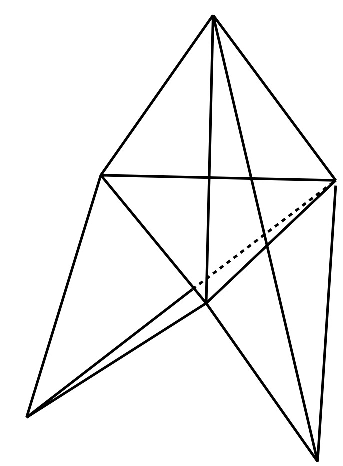

second quantized version of (generalized) LQG (adapted to simplicial context), but dynamics not derived from canonical quantization of GR

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classical action: kinetic (quadratic) term + (higher order) interaction (convolution of GFT fields)

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

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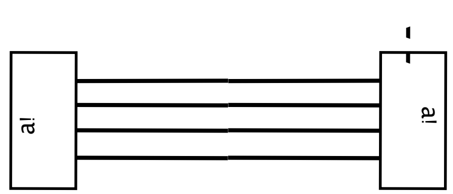

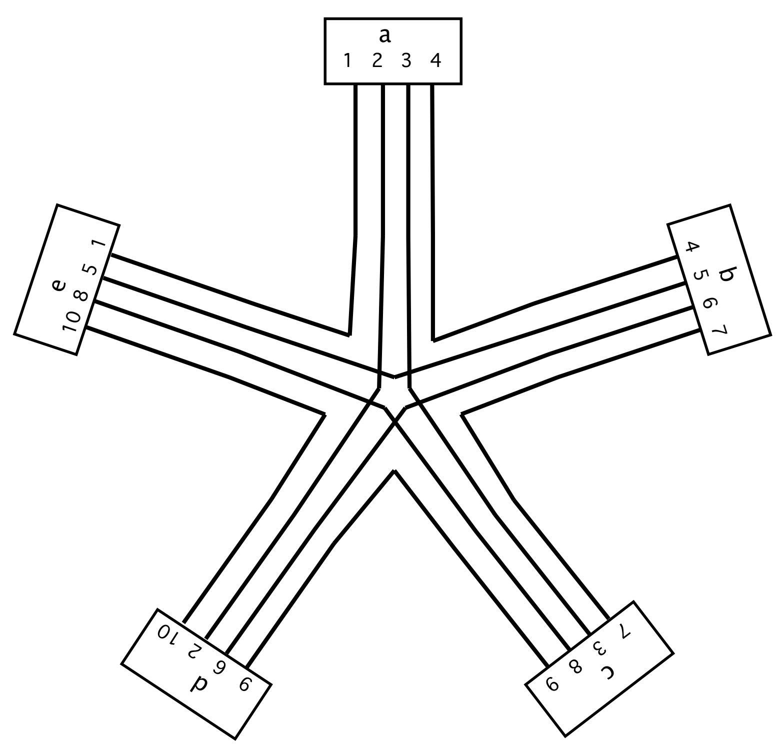

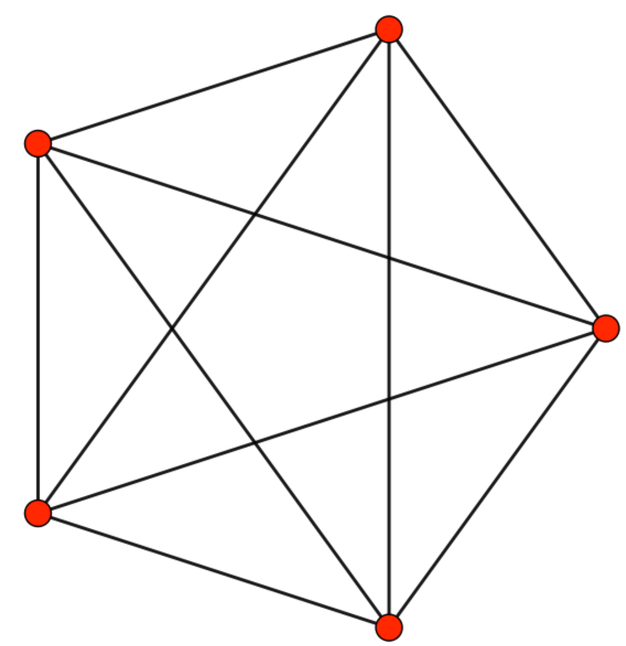

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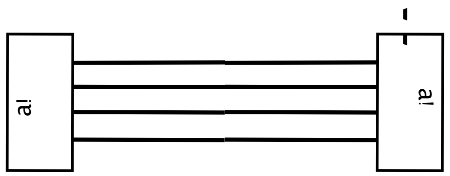

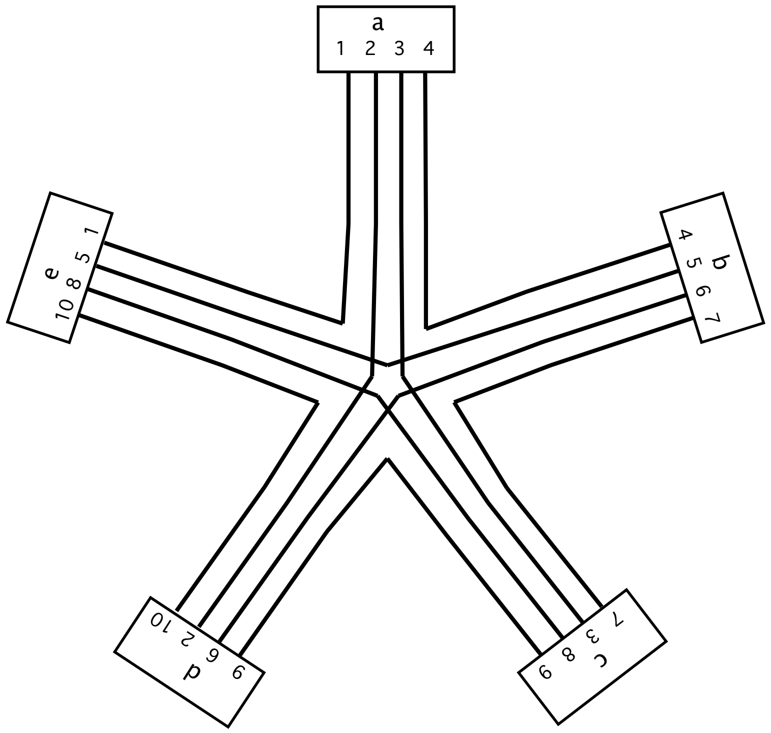

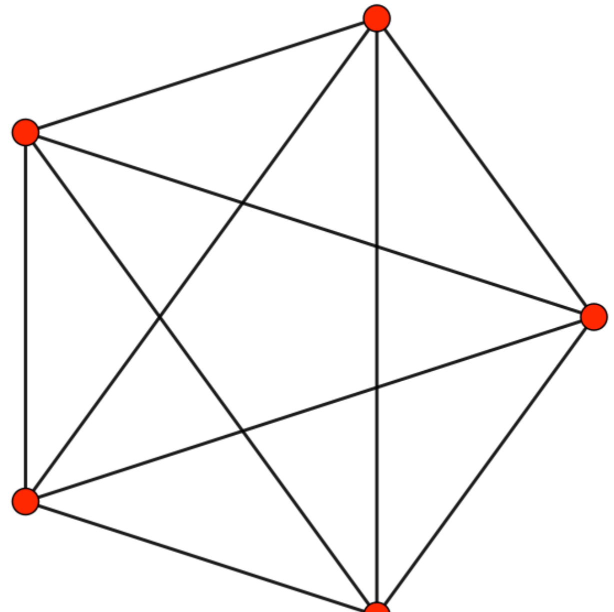

with fields constrained to satisfy "geometricity" conditions

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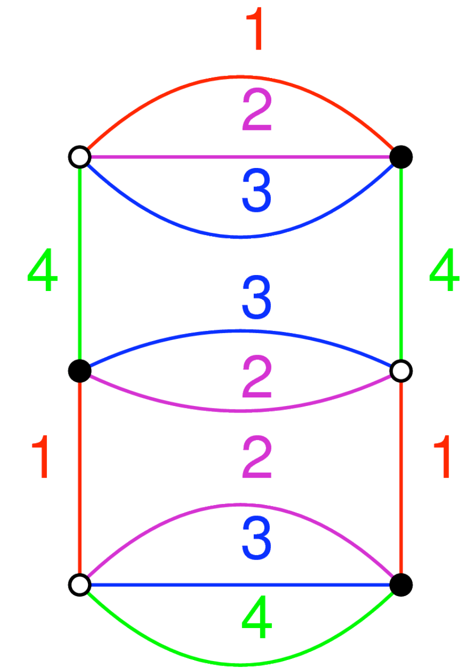

$$S(\varphi, \overline{\varphi}) = \sum_{b \in \mathcal{B}} t_b I_b(\varphi, \overline{\varphi})$$

indexed by d-colored “bubbles”

example: $$\int [\mathrm{d}g_i]^{12} \varphi(g_1, g_2, g_3, g_4) \overline{\varphi}(g_1, g_2, g_3, g_5) \varphi(g_8, g_7, g_6, g_5) \overline{\varphi}(g_8, g_9, g_{10}, g_{11}) \varphi(g_{12}, g_9, g_{10}, g_{11}) \overline{\varphi}(g_{12}, g_7, g_6, g_4)$$

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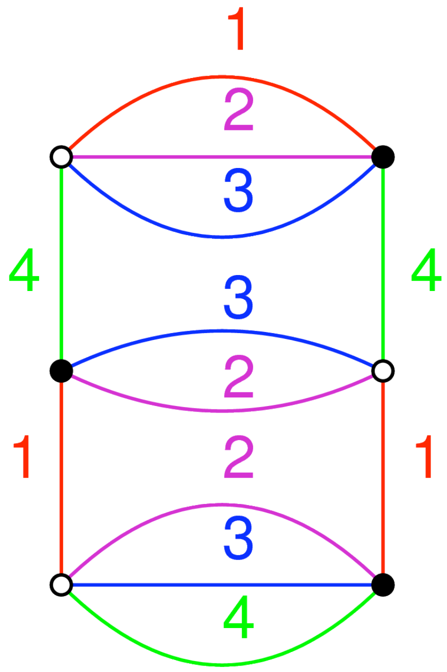

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with non-trivial propagator: $$\left(m^2 - \sum_{\ell=1}^{d} \Delta_\ell\right)^{-1}$$ Laplace-Beltrami on group manifold

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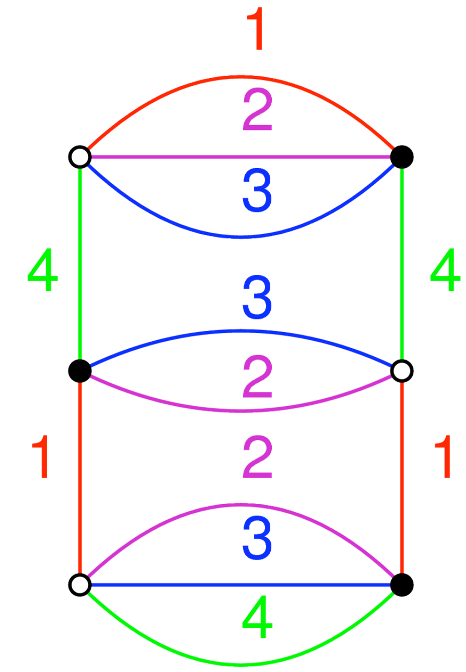

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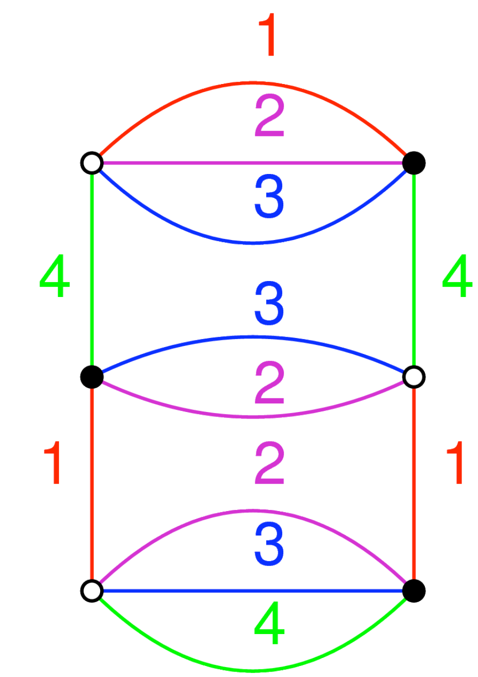
with non-trivial propagator: $$\left(m^2 - \sum_{\ell=1}^{d}\Delta_\ell\right)^{-1}$$ Laplace-Beltrami on group manifold

- interesting models exist with: nice simplicial geometry, direct links with discrete GR and simplicial path integrals, LQG-like Hilbert space, ...

GFT basics (4d case): dynamics

other possibility (motivated by tensor models and renormalization):
(tensor) invariant interactions



$$S(\varphi, \overline{\varphi}) = \sum_{b \in \mathcal{B}} t_b I_b(\varphi, \overline{\varphi})$$

indexed by d-colored "bubbles"

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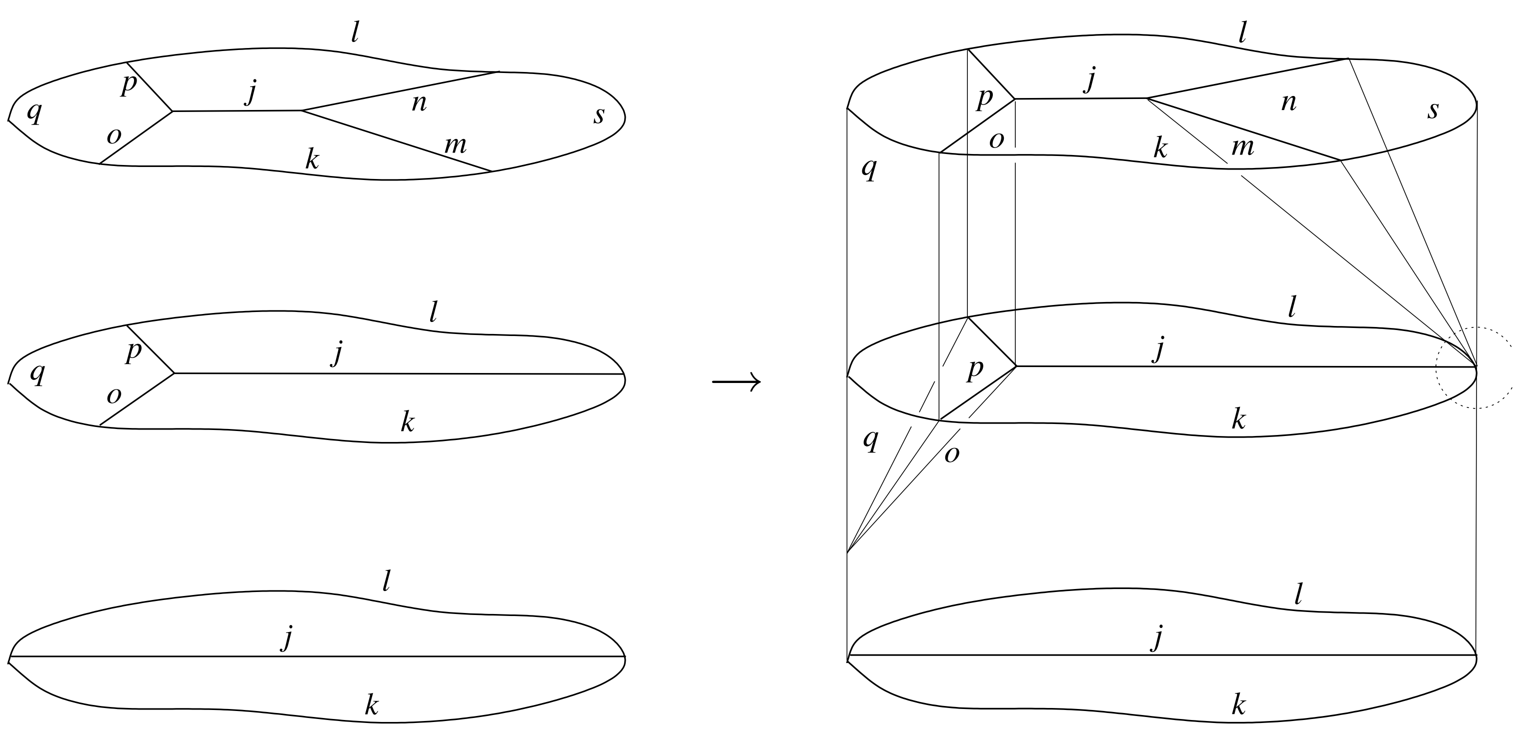

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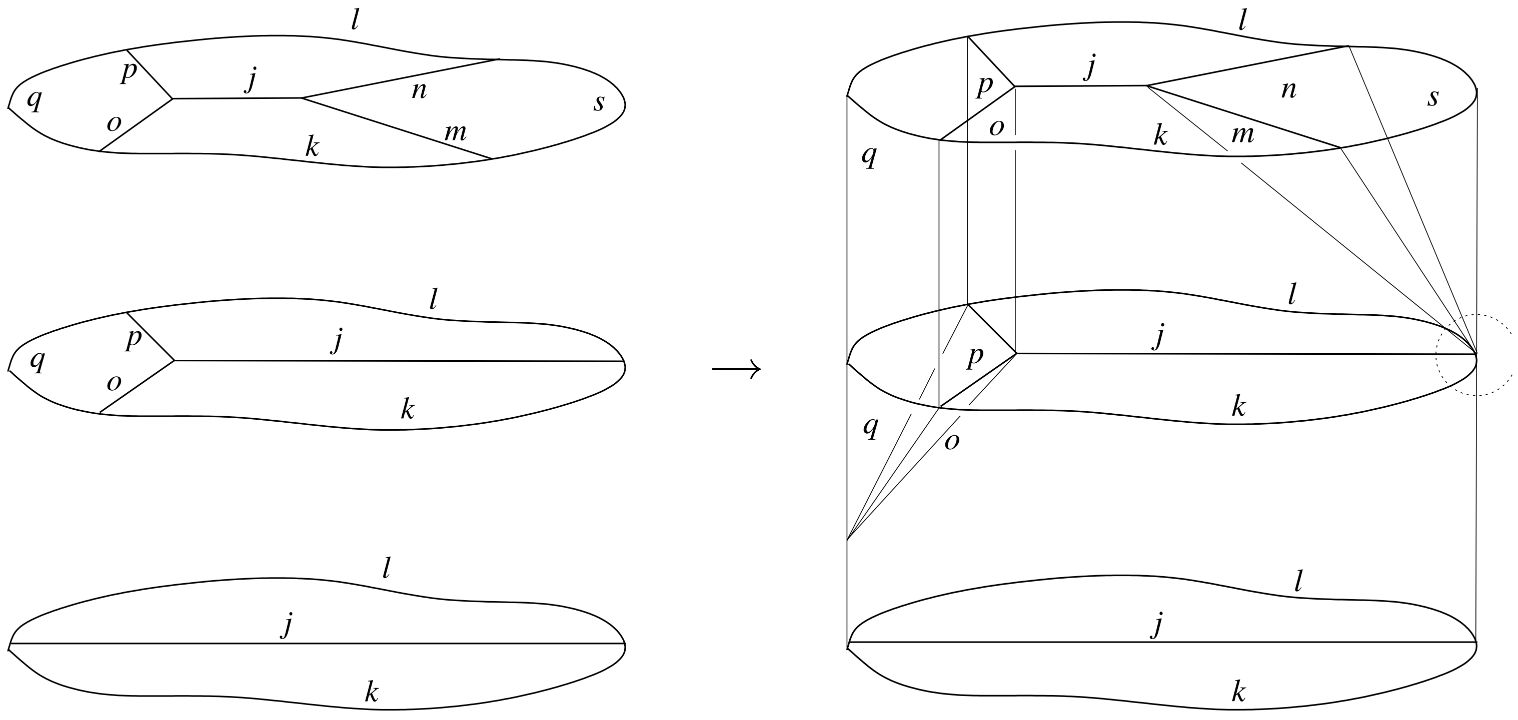

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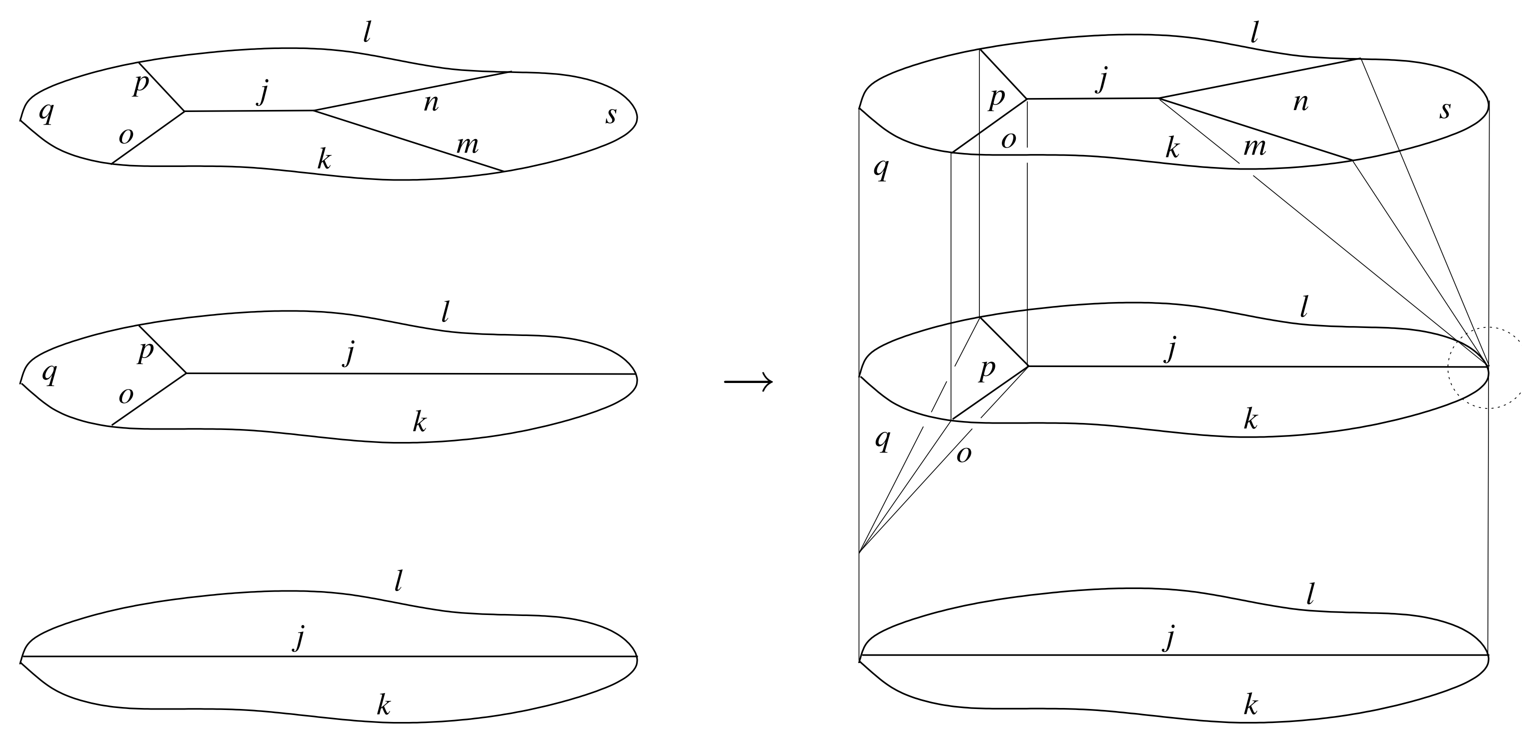

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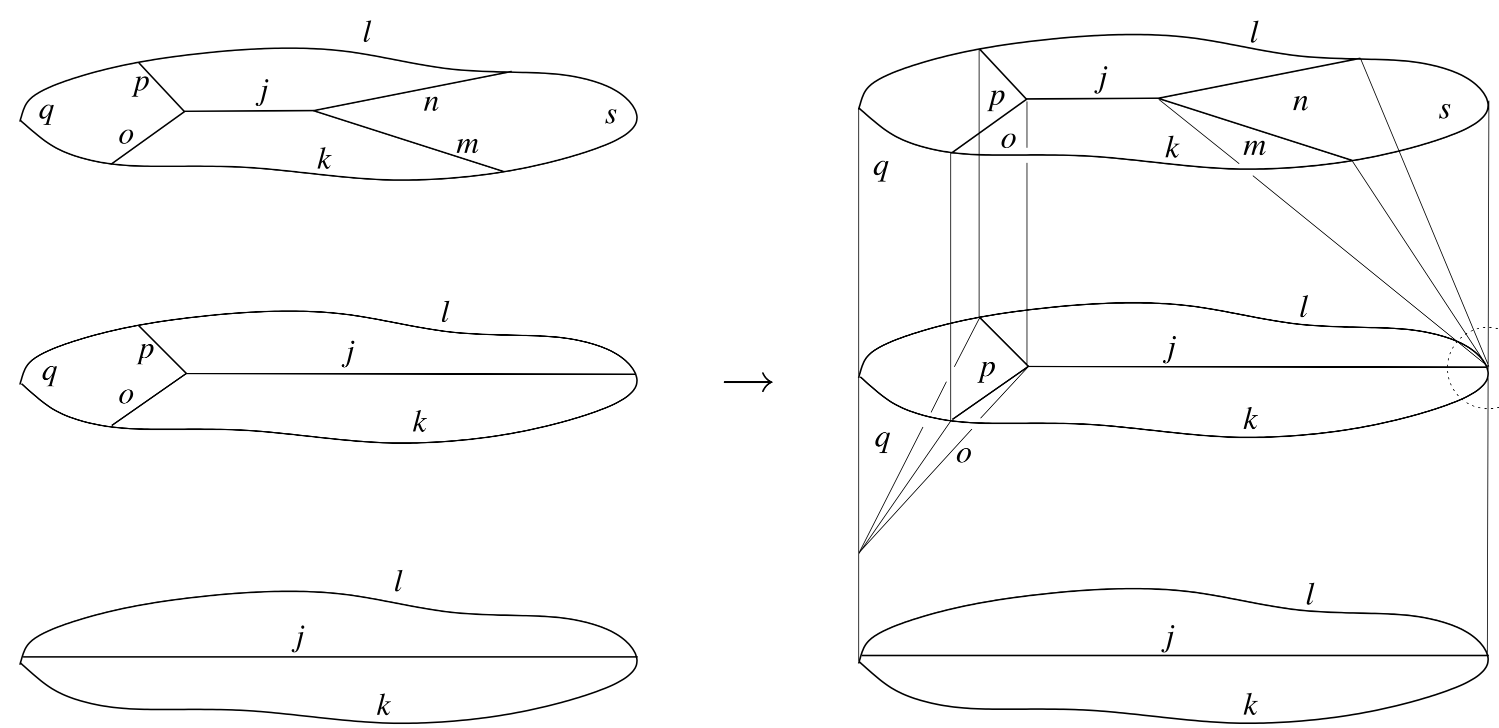

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Group Field Theory and Tensor Models

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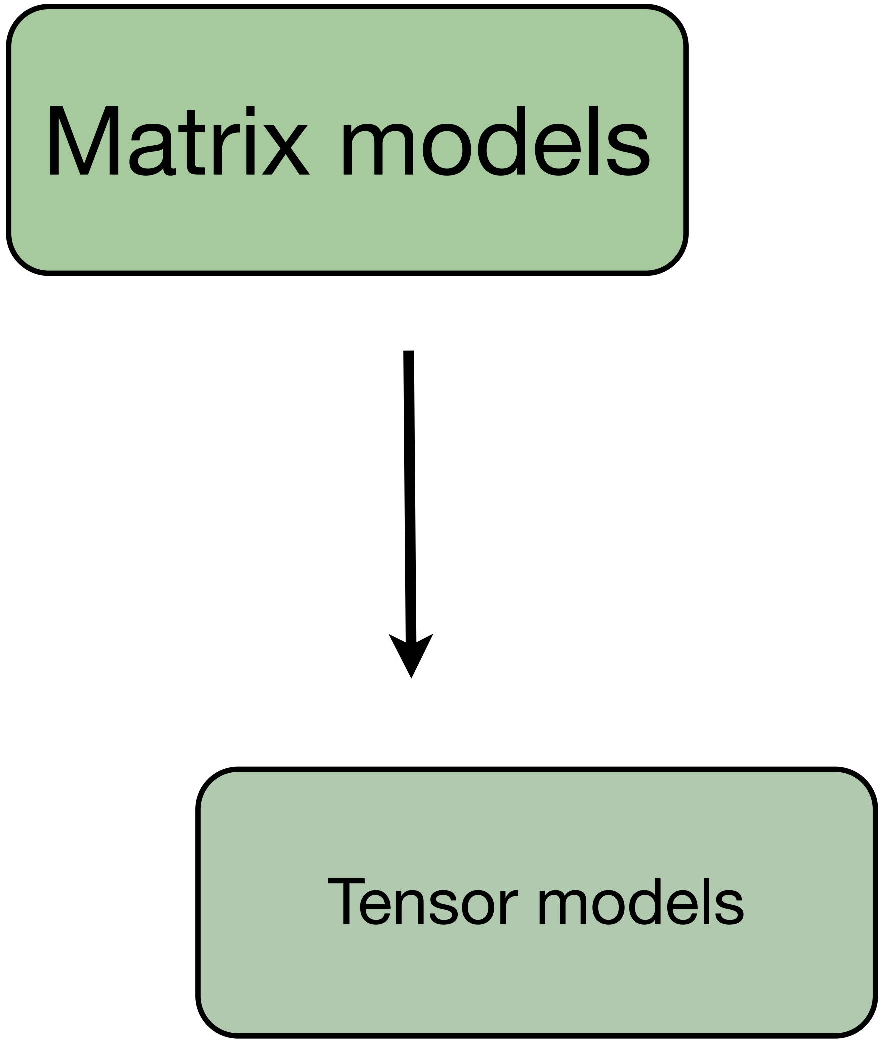
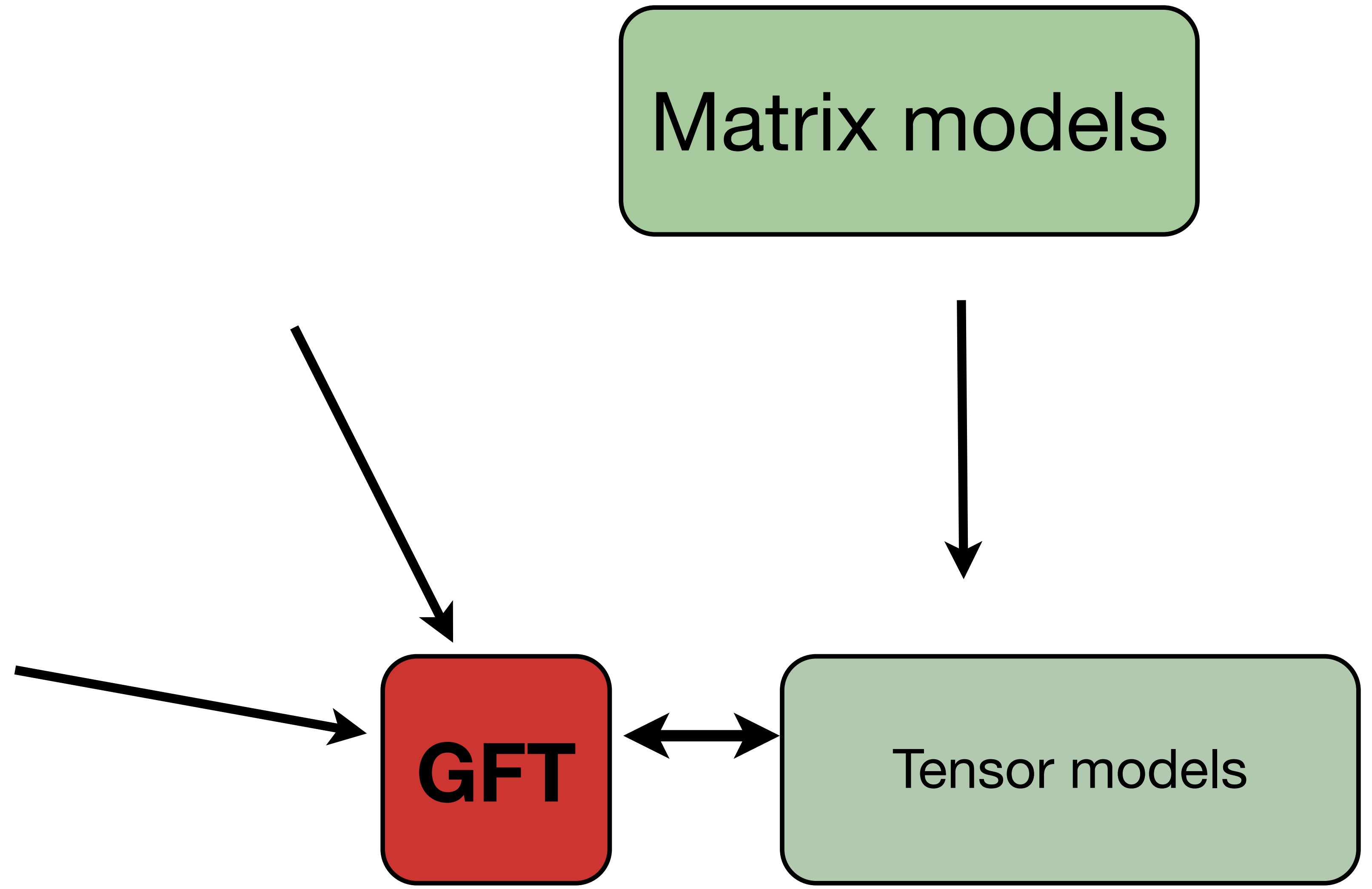
Matrix models

Group Field Theory and Tensor Models



Group Field Theory and Tensor Models



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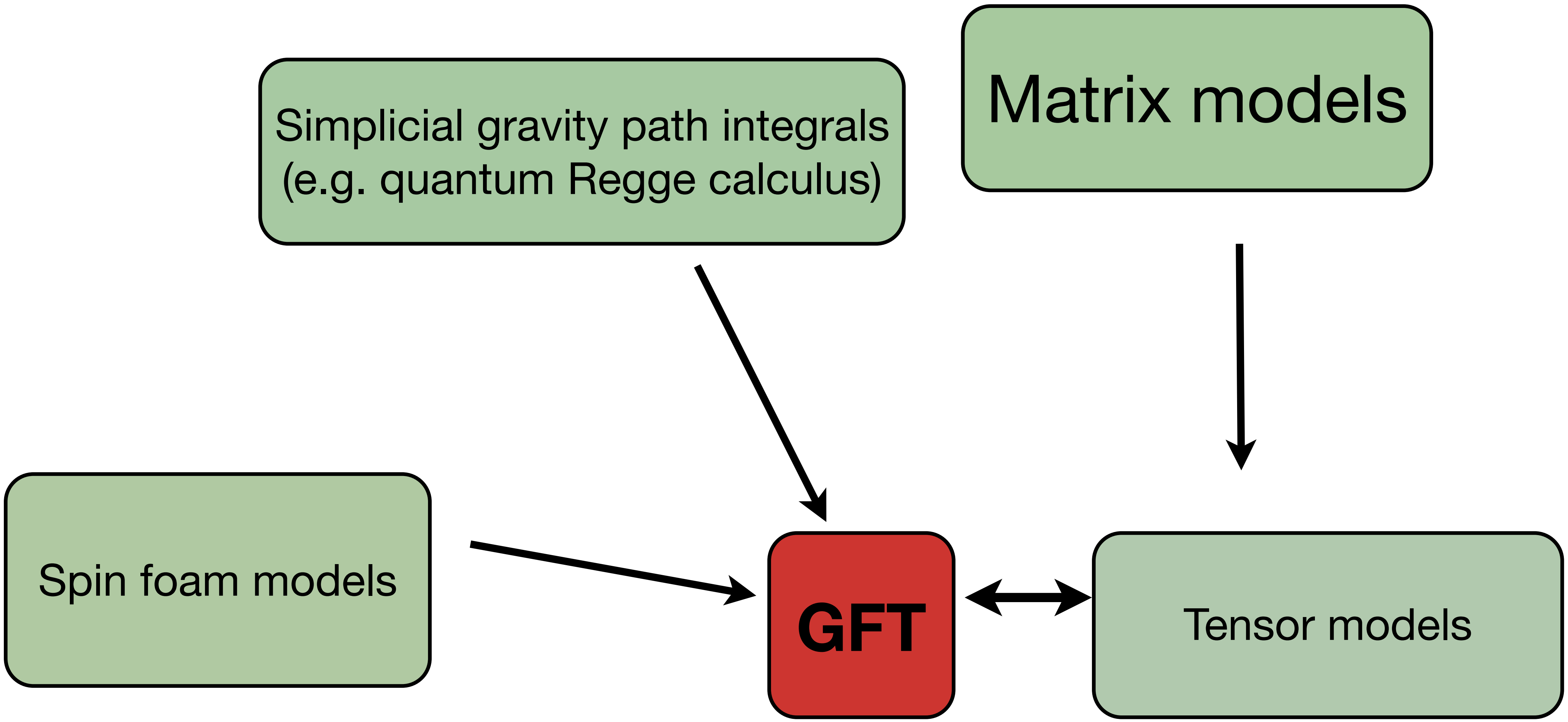

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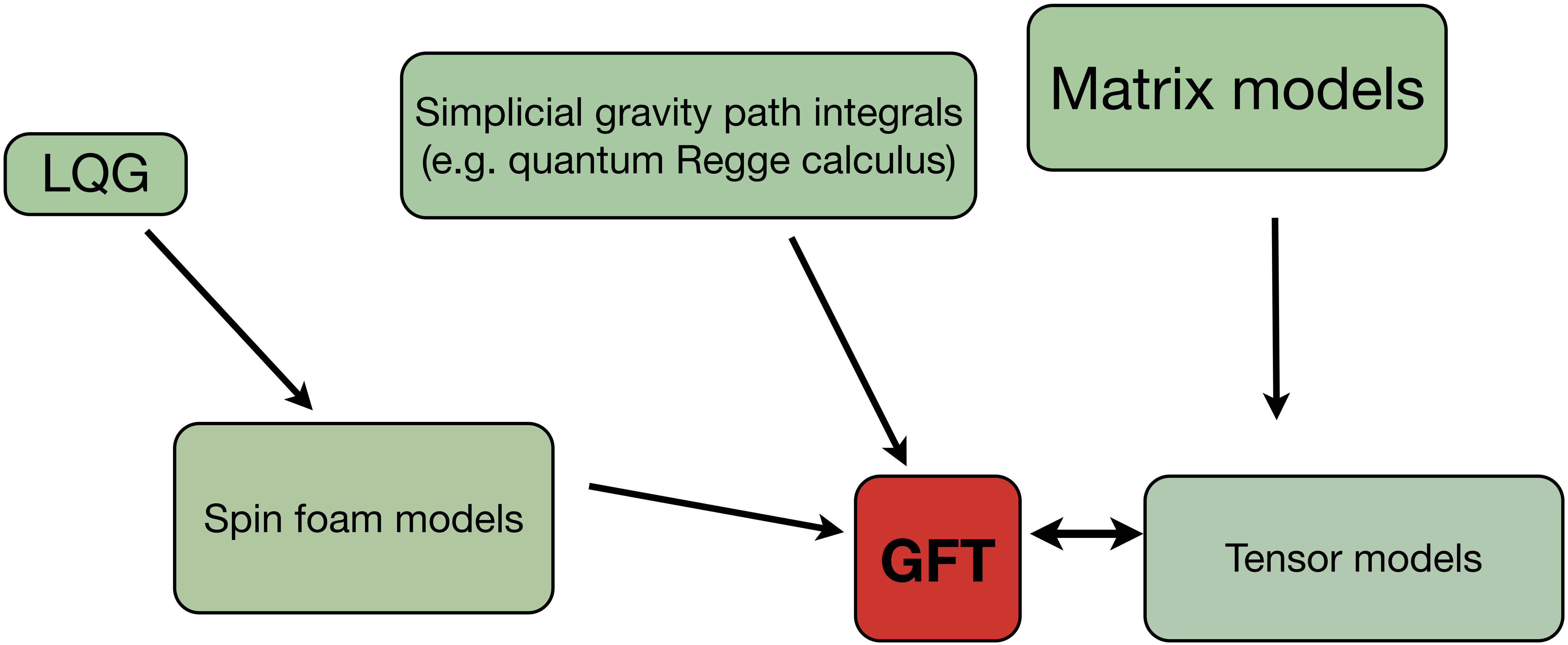

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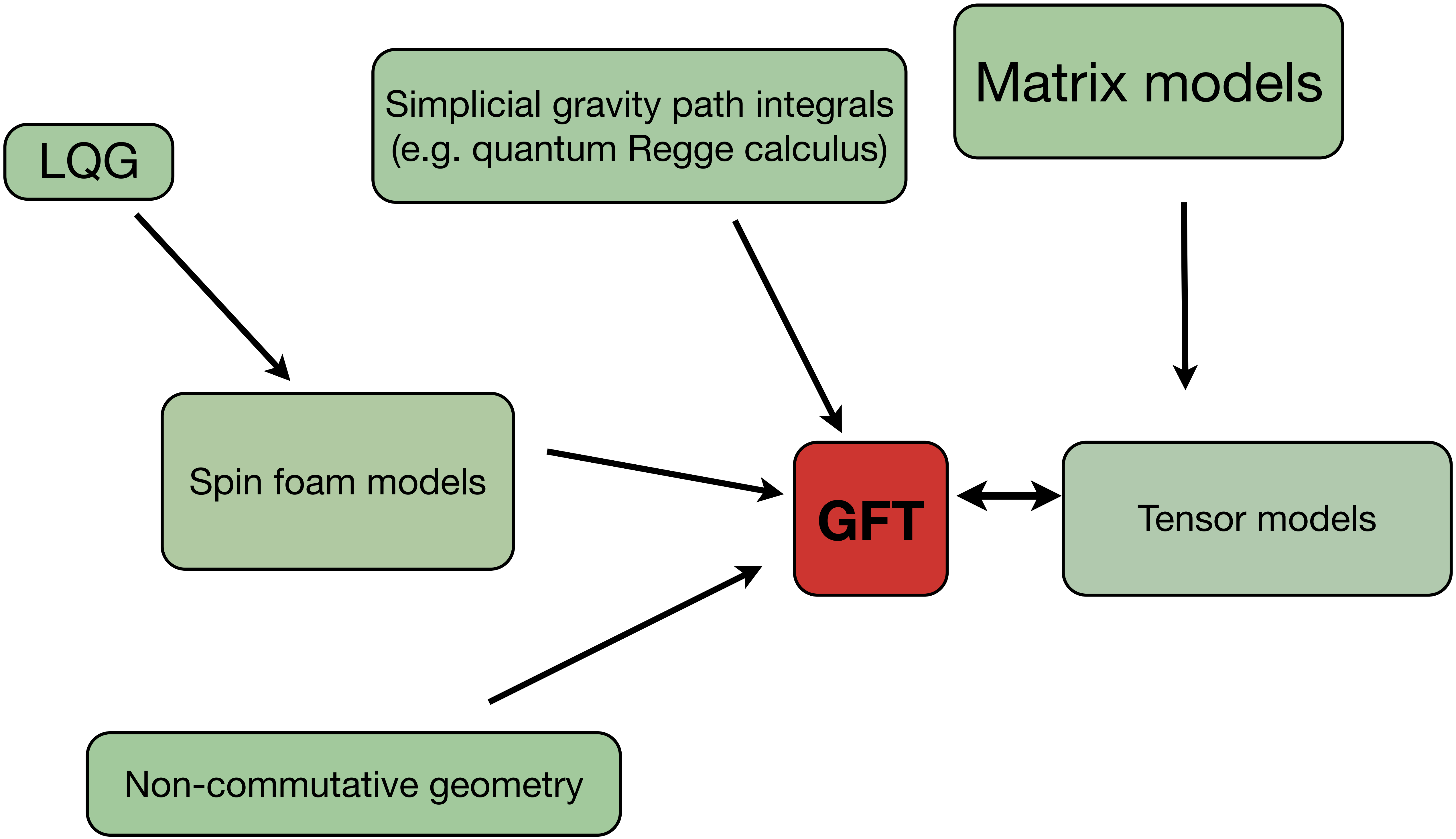

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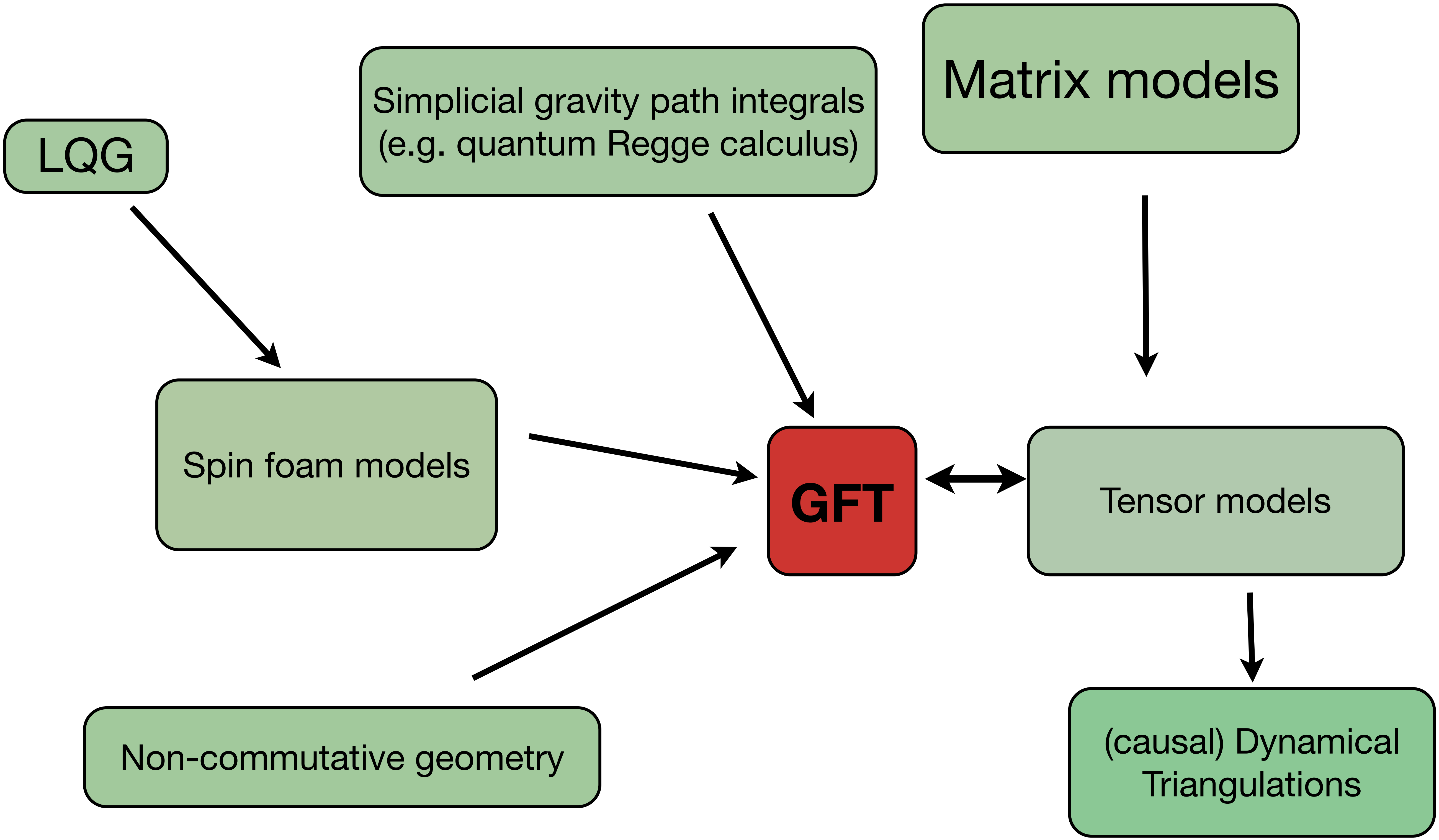

GFTs, spin foams, tensor models: many recent results

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- encoding of simplicial geometry
- field theory symmetries
- understanding of combinatorial structures (GFT Feynman diagrams)
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- GFT renormalization (various renormalizable models)
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(physics?)

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general vision and specific hypotheses

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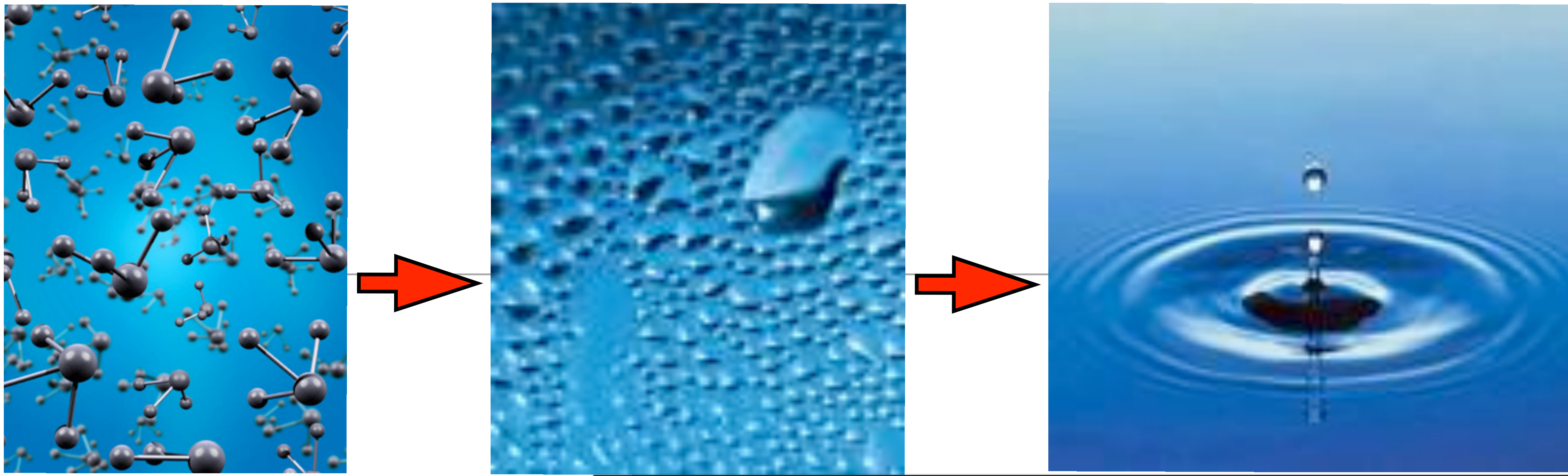
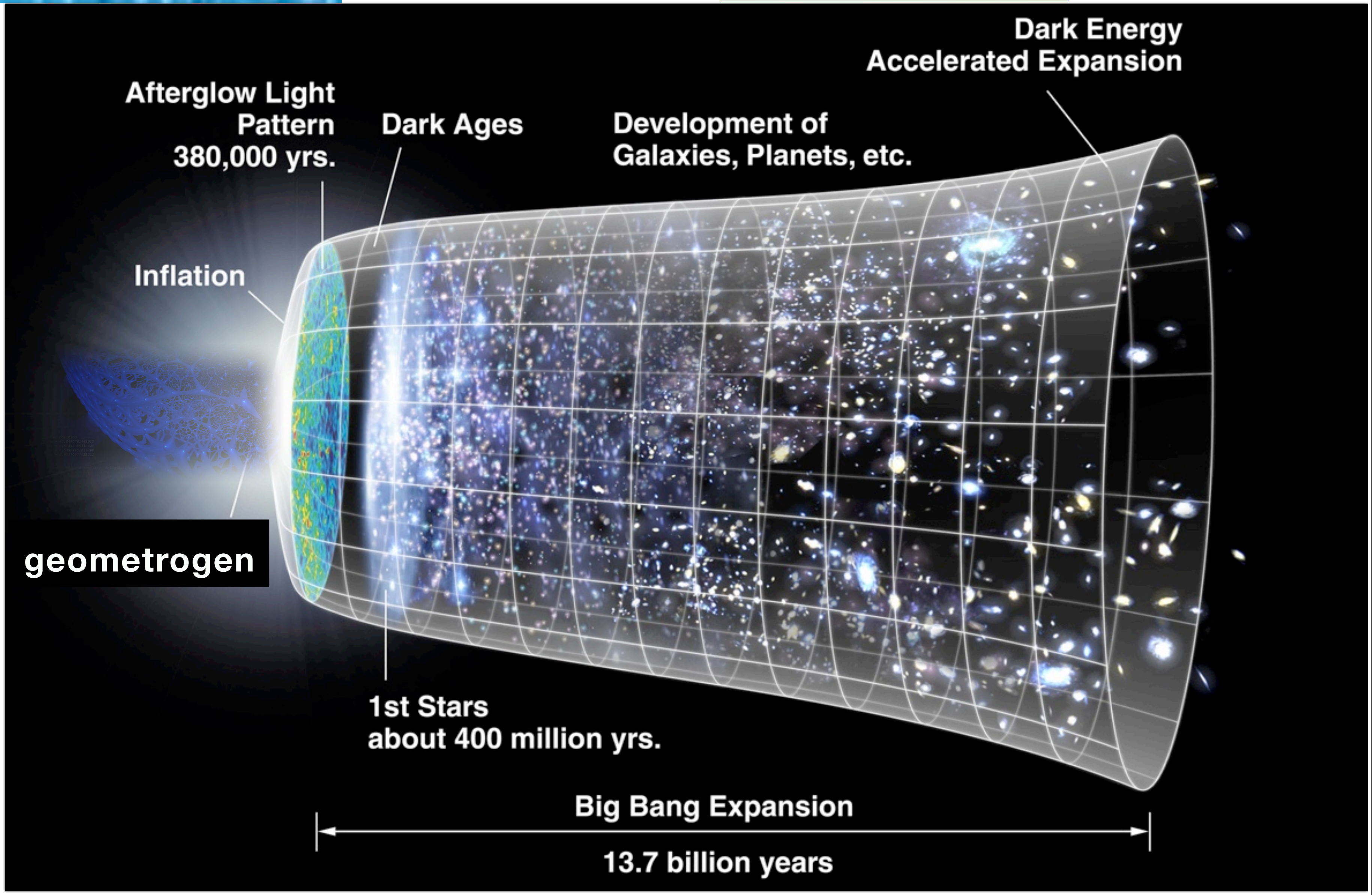
(Oriti ’07, ’11, ’13, Rivasseau ’11, ’12, Sindoni ’11)



spacetime as condensate
of QG building blocks

Big Bang as phase transition
(condensation)



Cosmology from GFT

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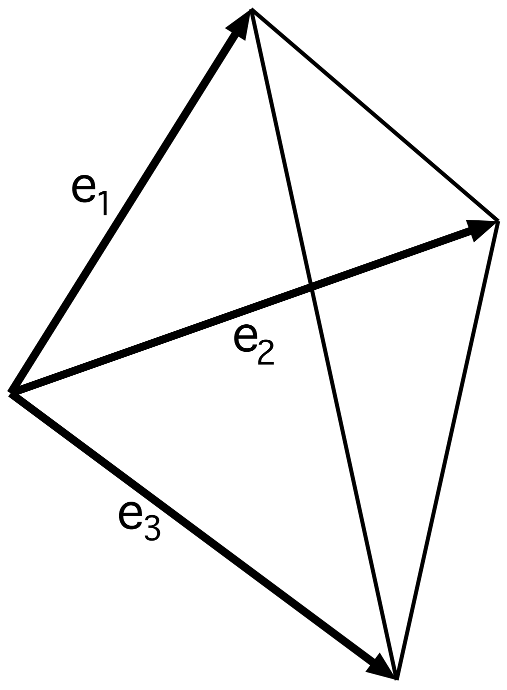
S. Gielen, DO, L. Sindoni,
arXiv:1303.3576 [gr-qc]

GFT states and approximate continuum geometries

- work with GFT with simplicial geometric interpretation (Riemannian SO(4) case for simplicity)

$$\varphi(g_1, g_2, g_3, g_4) \leftrightarrow \varphi(B_1, B_2, B_3, B_4) \to \mathbb{C}$$ describes geometric tetrahedron



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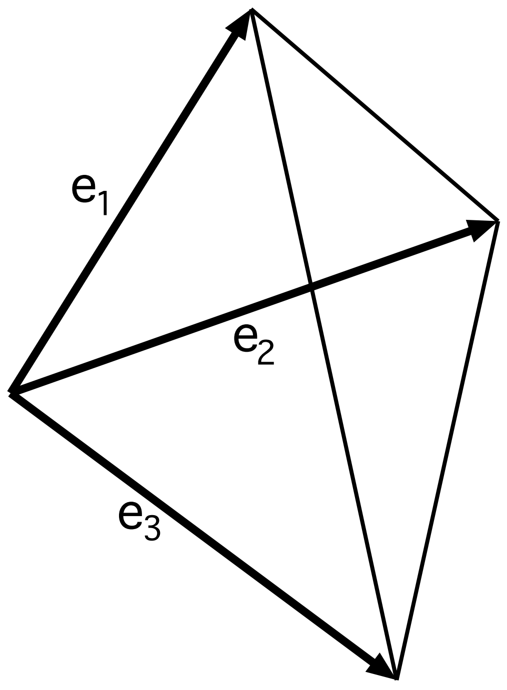

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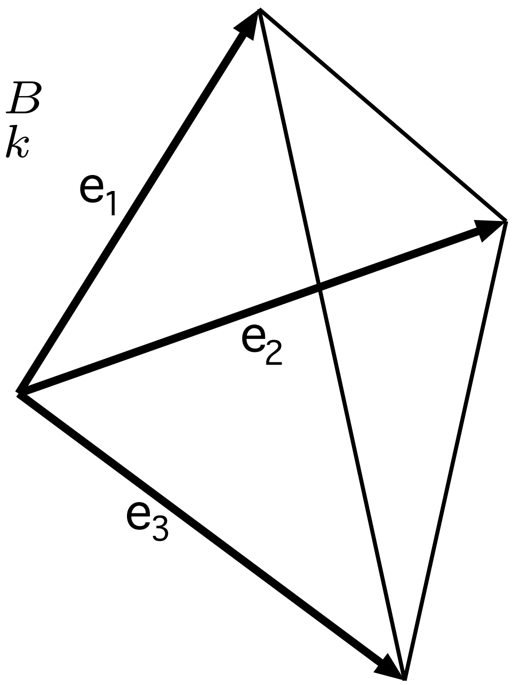

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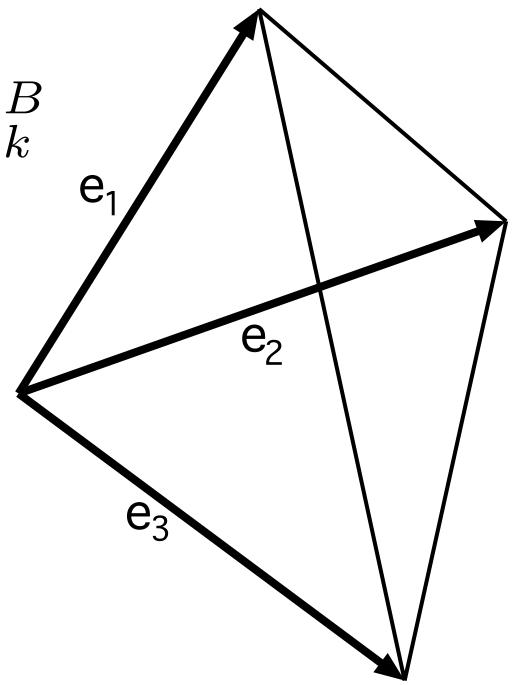

- generic N-particle GFT state (N geometric tetrahedra):

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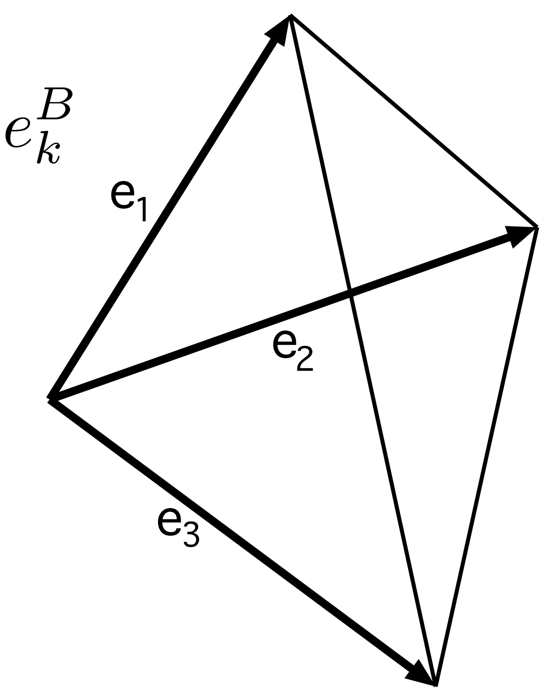

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- think of tetrahedra as embedded in symmetric 3-manifold (wrt group H) - implies choosing embedding point and 3 reference vectors:

$$\triangle_m \mapsto \left\{x_m \in \mathcal{M}, \ \left\{\mathbf{v}_{1(m)}, \mathbf{v}_{2(m)}, \mathbf{v}_{3(m)}\right\} \subset T_{x_m}\mathcal{M}\right\}$$

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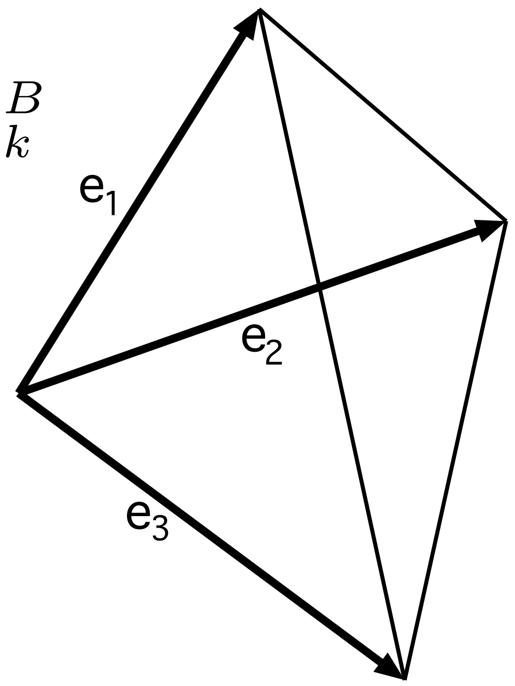
$$\varphi(g_1, g_2, g_3, g_4) \hookrightarrow \varphi(x_1, x_2, x_3, x_4) \qquad x_i \in X \subset G \longleftrightarrow B_i^{AB} = \epsilon_i{}^{jk} e_j^A e_k^B$$



- generic N-particle GFT state (N geometric tetrahedra):

$$|B_{I(m)}\rangle := \prod_{m=1}^{N} \hat{\varphi}^\dagger(B_{1(m)}, \ldots, B_{4(m)})|0\rangle$$

- think of tetrahedra as embedded in symmetric 3-manifold (wrt group H) - implies choosing embedding point and 3 reference vectors:

$$\triangle_m \mapsto \left\{x_m \in \mathcal{M}, \left\{\mathbf{v}_{1(m)}, \mathbf{v}_{2(m)}, \mathbf{v}_{3(m)}\right\} \subset T_{x_m}\mathcal{M}\right\}$$

- choose embedding vectors to be aligned with left-invariant vector fields of H

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- if GFT state satisfy additional gauge invariance condition under SO(4) at every "point", then it can be put in 1-1 correspondence with such approximate continuum metric

$$B_{i(m)} \mapsto \left(h_{(m)}\right)^{-1} B_{i(m)} h_{(m)}\,, \quad e_{i(m)} \mapsto e_{i(m)} h_{(m)}$$

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similar constructions in LQG (Alesci, Cianfrani) and LQC (Bojowald, Wilson-Ewing, ...)

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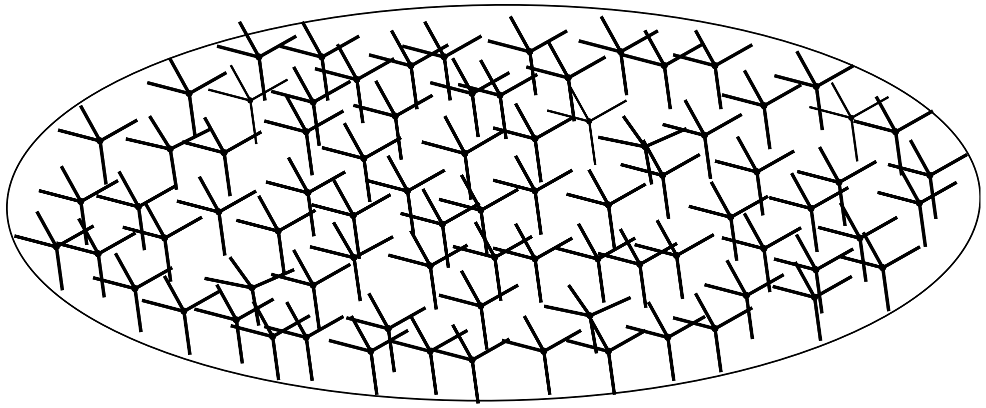

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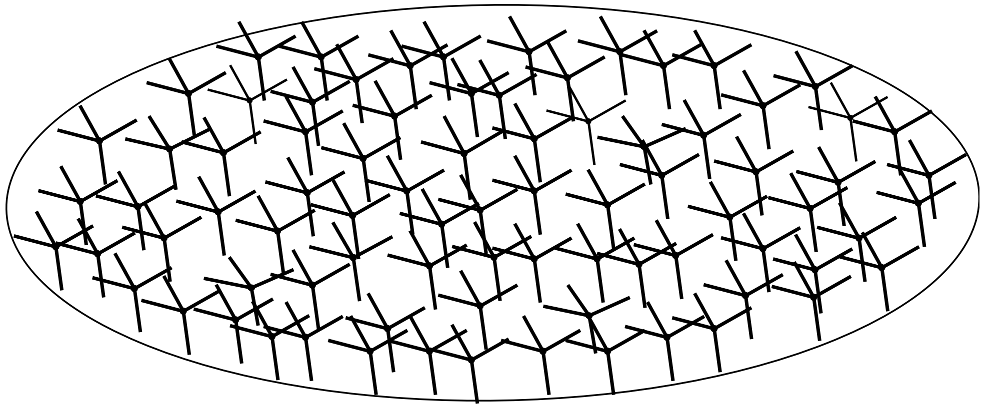

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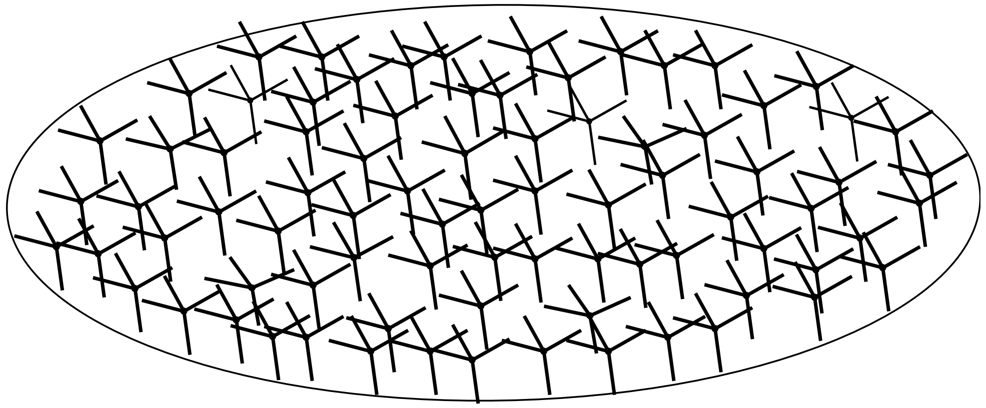

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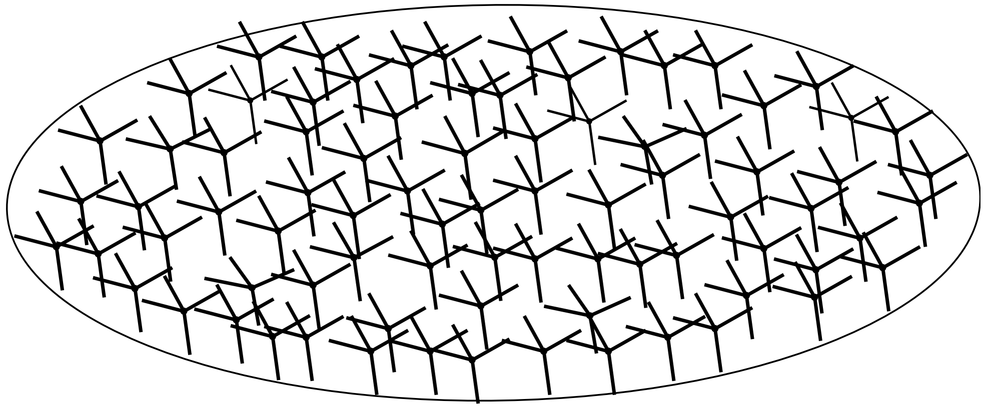

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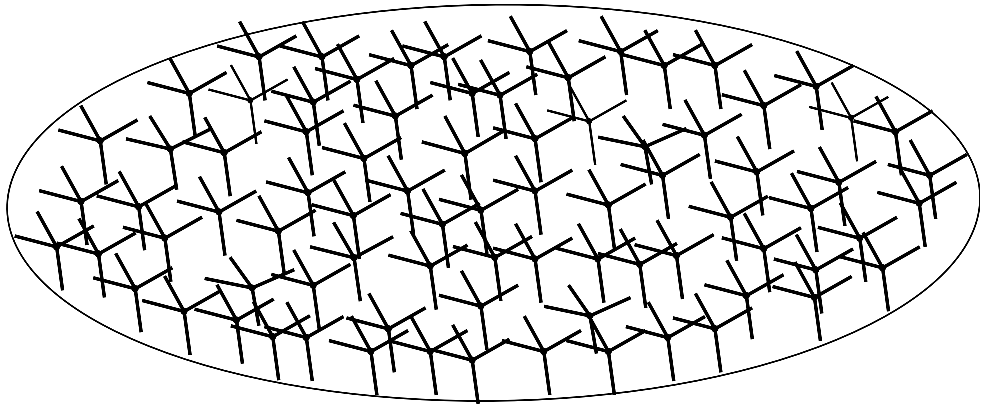

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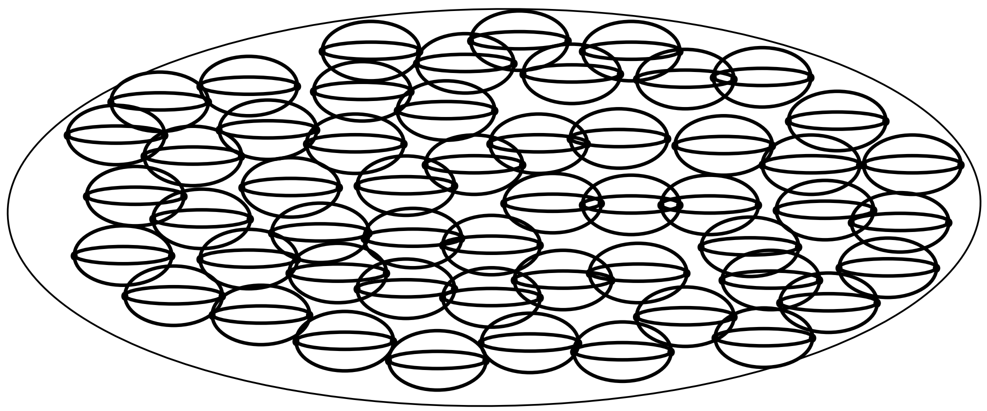

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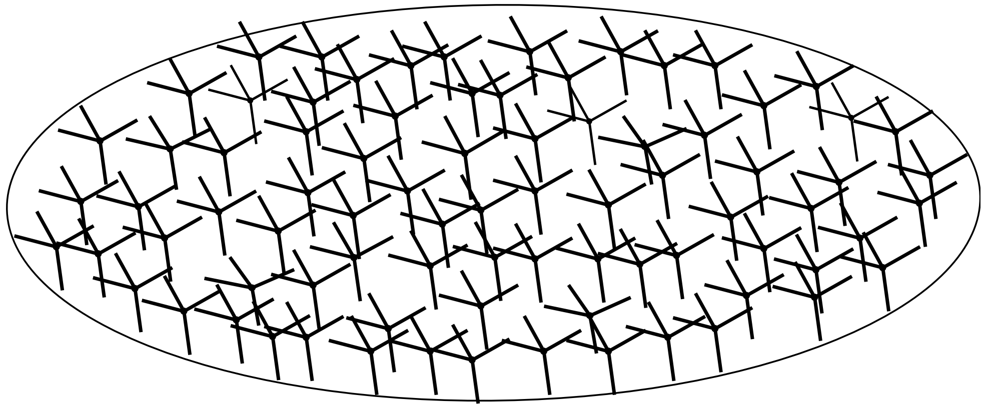

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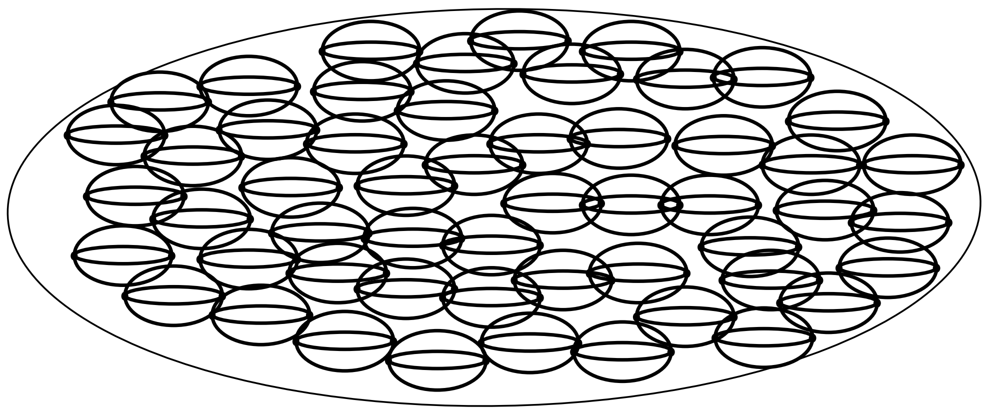

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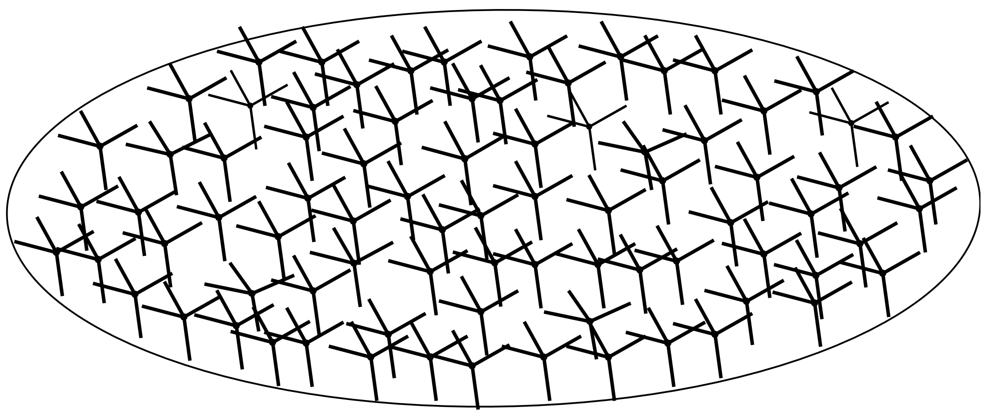

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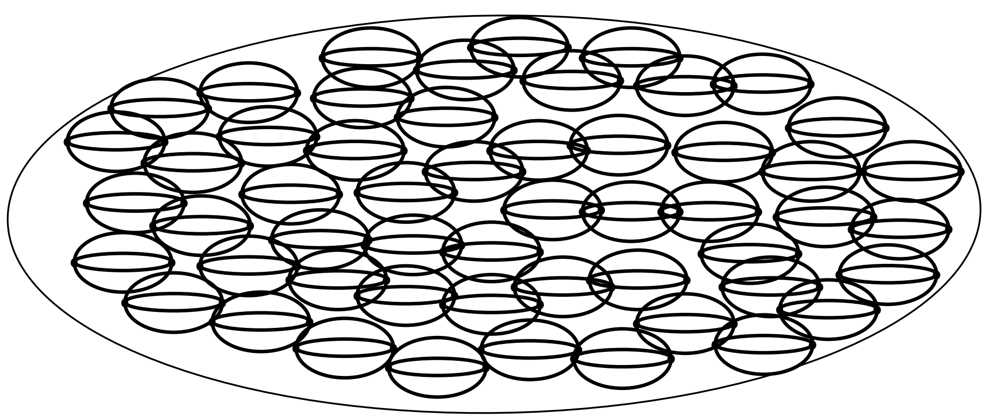

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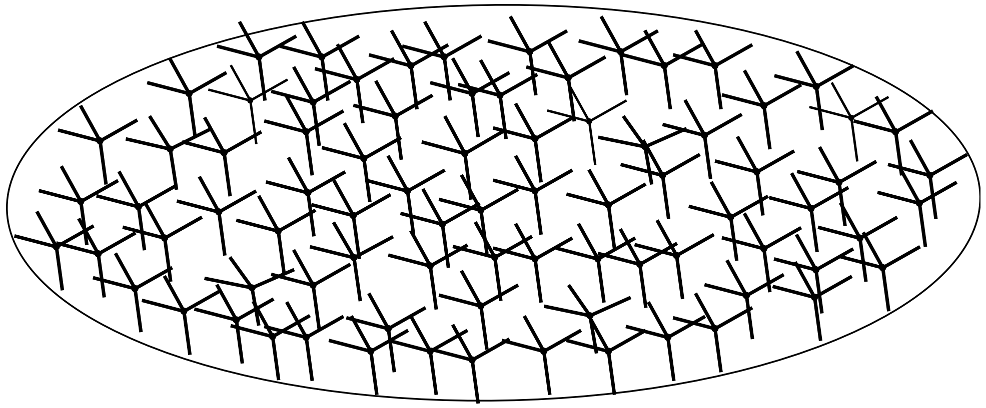

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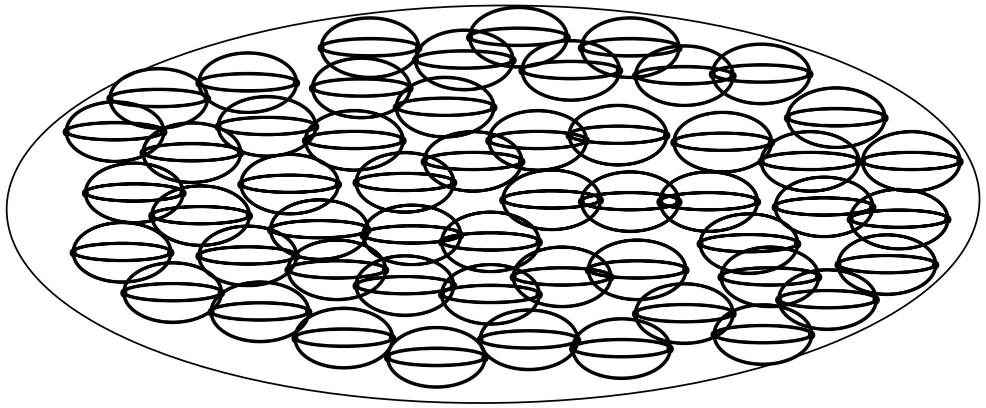

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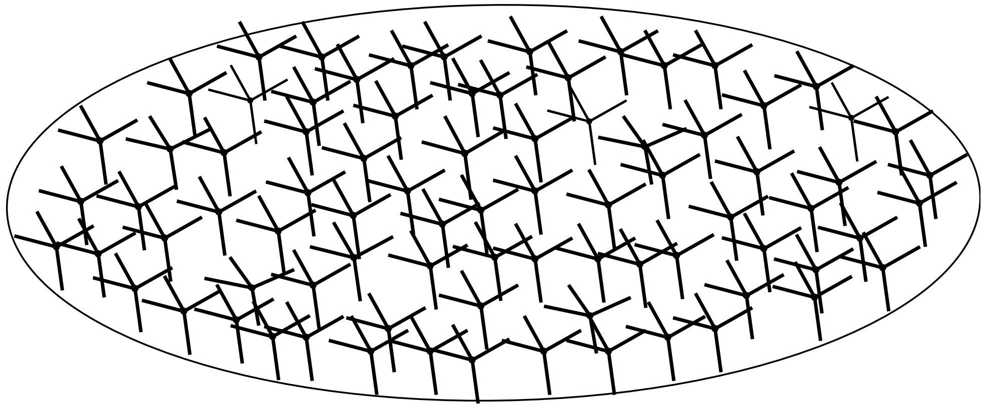

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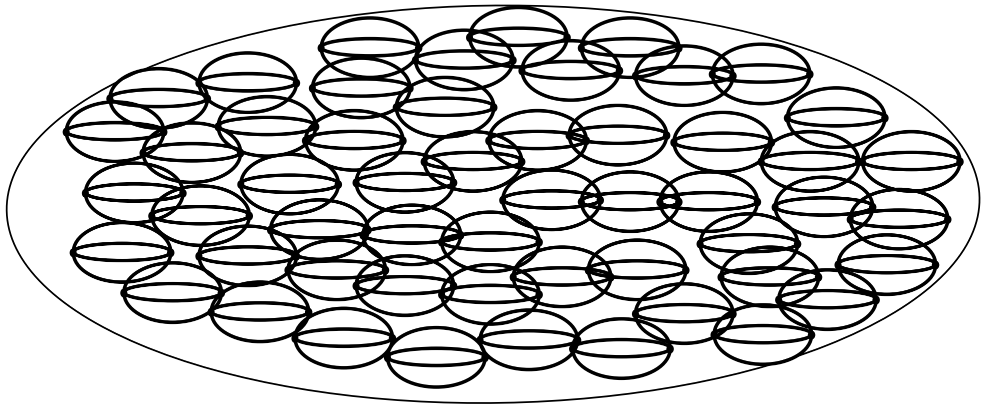

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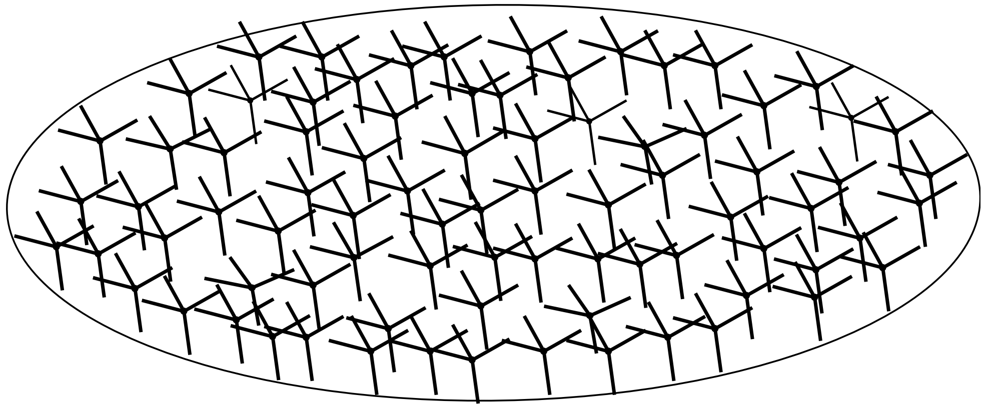

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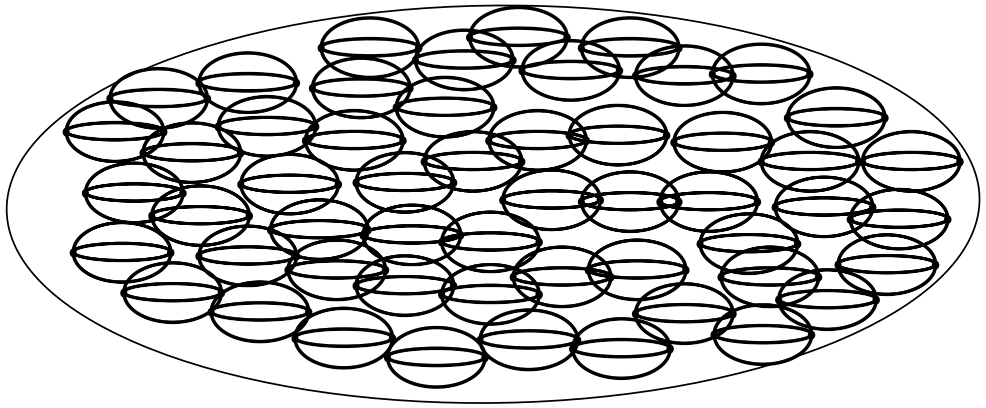

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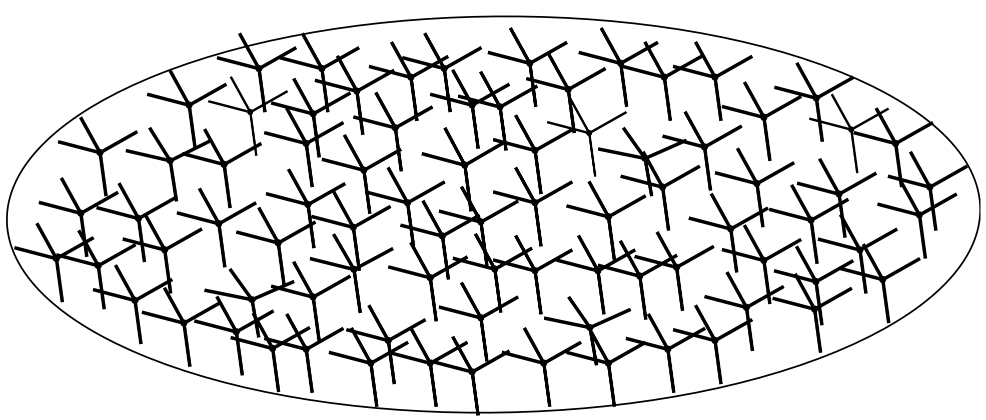

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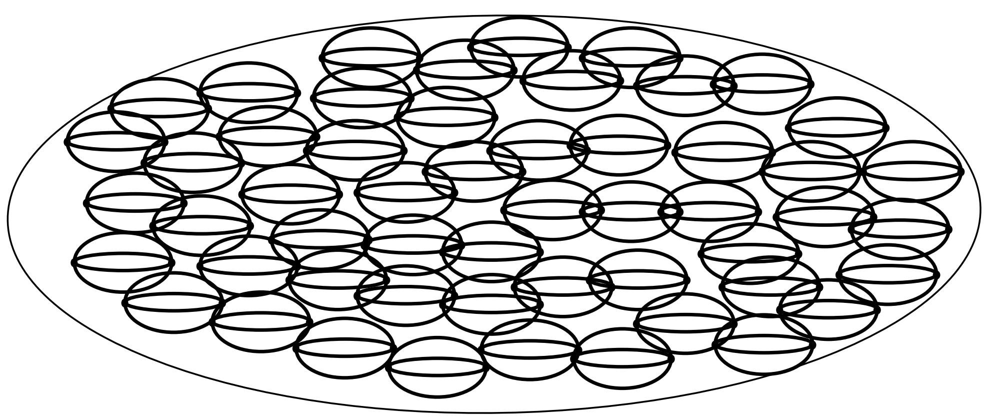

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with extra approximations required for consistent continuum geometric interpretation: GFT quanta “flat enough”:

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non-linear and non-local extension of quantum cosmology-like equation for “collective wave function

QG (GFT) analogue of Gross-Pitaevskii hydrodynamic equation in BECs

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with extra approximations required for consistent continuum geometric interpretation: GFT quanta "flat enough":

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GFT dipole condensation requires effective kinetic term with non-trivial kernel

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similar equations obtained in non-linear extension of LQC (Bojowald et al. '12)

Approximate FRW equations for GFT condensate

$B_I = a_I^2 T_I \qquad \pi_I = p_I V_I$

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special case: (effective) kinetic term = Laplacian on SU(2)^4
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- using geometric interpretation of states and variables, we can identify:

$B_I = a_I^2 T_I \qquad \pi_I = p_I V_I \qquad I = 1, 2, 3$ a 's are scale factors

$B_4 = B_4(B_1, B_2, B_3) \qquad \pi_4 = \pi_4(\pi_1, \pi_2, \pi_3)$

T, V = normalized dimensionless Lie algebra elements (state dependent)

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another way to extract effective classical equations from GFT hydrodynamics: take order parameter to be coherent state for mini-superspace (DO, L. Sindoni, '10)

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derivation of cosmology from full QG formalism!

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Thank you for your attention!

GFT renormalization

interactions given by “tensor invariants” $$S(\varphi,\overline{\varphi}) = \sum_{b\in\mathcal{B}} t_b I_b(\varphi,\overline{\varphi})$$ indexed by d-colored “bubbles”

- abelian renormalizable models in 3d and 4d - without gauge invariance (Ben Geloun, Rivasseau)
- proven to be asymptotically free (Ben Geloun)
- abelian renormalizable model in 4d with gauge invariance (Carrozza, DO, Rivasseau)
- other renormalizable models (Samary, Vignes-Tourneret, Ben Geloun, Livine)

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non-trivial propagator: $$\left(m^2 - \sum_{\ell=1}^{d} \Delta_\ell\right)^{-1}$$ ← Laplace-Beltrami on group manifold

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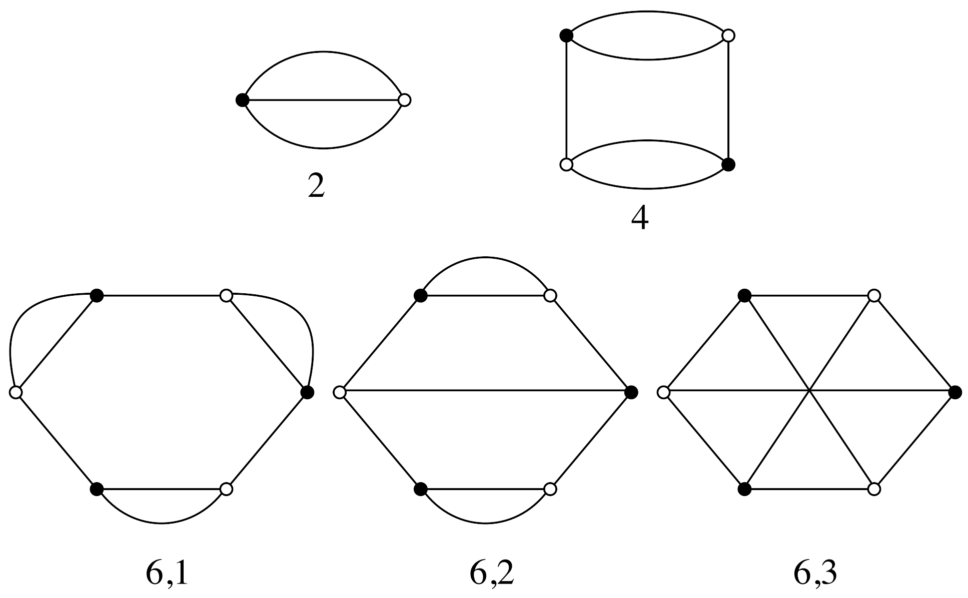
GFT renormalization

latest achievement: renormalizability of SU(2) GFT model in 3 dimensions with gauge invariance

Boulatov-like for 3d quantum gravity

Carrozza, DO, Rivasseau, to appear

$$S_\Lambda = \frac{t_4^\Lambda}{2} S_4 + \frac{t_{6,1}^\Lambda}{3} S_{6,1} + t_{6,2}^\Lambda S_{6,2} + CT_m^\Lambda S_m + CT_\varphi^\Lambda S_\varphi$$



Feynman amplitudes have “lattice gauge theory” structure

$$\mathcal{A}_\mathcal{G} = \left[\prod_{e \in L(\mathcal{G})} \int \mathrm{d}\alpha_e \, e^{-m^2 \alpha_e} \int \mathrm{d}h_e \right] \left(\prod_{f \in F(\mathcal{G})} K_{\alpha(f)} \left(\overrightarrow{\prod_{e \in \partial f}} {h_e}^{\epsilon_{ef}} \right) \right)$$
$$\left(\prod_{f \in F_{ext}(\mathcal{G})} K_{\alpha(f)} \left(g_{s(f)} \left[\overrightarrow{\prod_{e \in \partial f}} {h_e}^{\epsilon_{ef}} \right] g_{t(f)}^{-1} \right) \right) .$$

- renormalizability proven by rigorous multi-scale analysis
- requires adaptation of QFT techniques to GFT combinatorial structures
- crucial: notion of “face-connectedness”
- many results on combinatorics of colored GFT diagrams (in particular, melonic graphs)