Statistical interpretation in Quantum Cosmology/Gravity

Diffeomorphism invariance and **emergence of probabilities in QC/QG**

Renaud Parentani[1]

[1]LPT, Paris-Sud Orsay

QUANTUM GRAVITY in Paris, March 2014

Based on:

"Non-adiabatic transitions in QC", NPB 513 (1998) with S. Massar
"Unitary and Non-Unitary Evolution in QC", PRD 59 (1999) with S. Massar

and conversations (2005) with T. Jacobson

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The context

- There is a well-founded **probabilistic interpretation** of the solutions of the Schroedinger eq.,
- but there is **no consensus** on the **interpretation** of the solutions of the WDW eq.

- Several options have been proposed:
  - "Page-Hawking" based on the **norm**: $|\Psi(a, \phi)|^2$ (LQG/LQC)
  - "Vilenkin" based on the **current**: $W = \Psi^* i\overleftrightarrow{\partial}_a \Psi$
  - Third quantization.
  - Square root approach $i\partial_X \Psi = H\Psi$ (Ashtekar (2008))
  - Adding "dust" (Tiemann (2006))

Reviews: *"Conceptual issues in quantum cosmology",* C.Kiefer (1999), also (2014)
"Quantum Cosmology for the XXIst Century", Bojowald, Kiefer, Moniz, (2010).

Plan

- **Compare** (mathematically) the *structure* of the Schr. eq. to that of the WdW eq.
- To perform the **comparison**, compute **twice** the same **transition amplitude** (using molecular physics techniques)
- We shall establish that
  - no **exact** probabilistic interp. could possibly be given: **Diffeo-invar.** + **canon. quantization** $\rightarrow$ **no** proba. interp.
  - the **Schrod. probabilistic interpretation** is an **emergent** property of the solutions of the WDW eq.
  - similar to : Maxwell eqs. $\rightarrow$ no Galilean group, yet, Galilean group re-emerges from the Lorentz group

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Why focusing on Transition Amplitudes ?

- The WdW eq. must *also* describe **matter** transitions, *e.g.* $e^+ e^-$ annihilation.
  $\rightarrow$ How to **compute** their amplitudes ?

- Born statistical interpretation (1926)

$$|n_0\rangle \;\to\; \sum_n c_{n,n_0} \, |n\rangle$$

$$|c_{n,n_0}|^2 \;=\; \text{Proba. to find } |n\rangle \text{ at late time, when starting from } |n_0\rangle. \qquad (1)$$

  followed from his understanding of Transition Amplit.

- Follow here the same approach:
  study the properties of $\mathcal{C}_{n,n_0}(a)$, Trans. Amplit. in QC,
  then consider their *possible* interpretation.

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Why using *molecular physics techniques* ?

- Appropriate for dynamical systems containing **light** and **heavy** (i.e. fast and slow) deg. of freedom.

- **Transition amplitudes** are governed by **frequency ratios** and not by **coupling constants** (*Non-Adiabatic Transition Amplitudes* **NATA**)

- some NATA are **exponentially suppressed** w.r.t. others, thereby introducing a **hierarchy of NATA**.

- Plan: **use** this **hierarchy** to
  - **organize** solutions of the WDW eq.
  - get **algebraic relations** between Schrod's $c_n(t)$ and WDW's $\mathcal{C}_n(a)$.

- *Long tradition*: Born '26, Heisenberg '35, Gottfried '66 + '98 All considering the statistical interpret. of QM.

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The Schroed. equation in cosmology.

- *For definiteness*,
  consider some matter fields governed by an hermitian H_M
  in an expanding **compact** RW sp-time described by $a(t)$.
- The S. eq. is

$$i\partial_t|\psi(t)\rangle = H_M \, |\psi(t)\rangle. \qquad (2)$$

- Because of the expansion $da/dt > 0$,

$$H_M = H_M(a(t))$$

is t-dependent, *but through $a(t)$ only*.

Structure of the S. eq. in Cosmology. 1

- introduce the **instantaneous eigenstates** of $H_m(a)$:

$$\begin{aligned} H_M(a)\,|\chi_n(a)\rangle &= E_n(a)\,|\chi_n(a)\rangle \\ \langle\chi_n(a)|\chi_m(a)\rangle &= \delta_{n,m}, \quad 1 \geq n \geq N. \end{aligned} \tag{3}$$

- decompose $|\psi(t)\rangle$ using this **basis**:

$$|\psi(t)\rangle = \sum_n c_n(t)\, e^{-i\int^t dt' E_n(t')}\, |\chi_n(a(t))\rangle, \tag{4}$$

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Structure of the S. eq. in Cosmology. 2

This gives a $N \times N$ matricial eq.

$$\partial_t c_n = \sum_m \langle \partial_t \chi_m | \chi_n \rangle \; e^{-i \int^t dt' (E_m - E_n)} c_m . \qquad (5)$$

where

$$\langle \chi_m | \partial_t \chi_n \rangle = \frac{\langle \chi_m | \partial_t H_M | \chi_n \rangle}{E_n(a) - E_m(a)} \quad , \quad n \neq m \qquad (6)$$

NB.

$$\partial_t c_n = 0,$$

only when $da/dt \neq 0$ and $dH_M/da \neq 0$.

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Structure of the S. eq. in Cosmology. 2 Remarks.

- The amplitudes $d_n(a)$ defined in the **corresponding contracting** universe, $da/dt \to -da/dt$, obey the **same** equation with $i \to -i$.

- The Schrod. eq. **can** be written in terms of a **only** :

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The cosmic time t drops out since $(dt/da')E_m$ is (diffeo-) invariant under $t \to t' = t'(t)$, since $E = -\partial_t S$.

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NATA: Non-Adiabatic Transition Amplitudes.

- Starting with $c_1(t = -\infty) = 1$, the amplitude to find the system in the state n is $c_n(+\infty)$.
- To first order in NA, it is

$$c_n(+\infty) \simeq \int_{-\infty}^{+\infty} dt \langle \chi_n | \partial_t \chi_1 \rangle \; e^{-i \int_{-\infty}^{t} dt' (E_1(t') - E_n(t'))} \qquad (8)$$

- When $c_n \ll 1$, can be evaluated by a saddle point approx.
- The sp time t^* is complex, and hence c_n is expon. damped:

$$c_n(+\infty) \simeq C \; e^{-i \int_{-\infty}^{t^*} dt' (E_1 - E_n)}$$

NB. $C \to 1$ in the adiabatic limit, see refs.

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WDW Equation

- Replace $[-i\partial_t + H_M]\,|\psi(t)\rangle = 0$ by the WDW eq.

$$[H_G + H_M]\,|\Psi(a)\rangle = 0 \tag{9}$$

where

$$H_G = \frac{-G^2\pi_a^2 - a^2 + \Lambda a^4}{2Ga}. \tag{10}$$

$\hat{\pi}_a = -i\partial_a$ is the momentum conjugated to a.

- Eqs. (9, 10) follow from $S = \int d^4x\sqrt{g}(R + L_M)$ when
  - 3 geometries are compact, and
  - matter distribution is sufficiently homogeneous.
  - N.B. $H_G + H_M = 0$ is the Friedmann eq.
- With (9, 10) we have displaced the *Heisenberg cut* so as to **include** (a, π_a) in the **quantum description.**

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Conserved Wronskian

- The **unique** conserved quantity associated with the Schrod. eq. is the **norm**

$$N = \langle \psi(t) | \psi(t) \rangle \equiv Cst. = \sum_n |c_n(t)|^2. \quad (11)$$

- **Instead** the **unique** one associated with the WDW eq. is the **Wronskian**

$$W = \langle \Psi(a) | \, i\overset{\leftrightarrow}{\partial}_a | \Psi(a) \rangle \equiv Cst. \quad (12)$$

- Question: **Can** W **be written as** $W = \sum_n |\mathcal{C}_n(a)|^2$ **?**

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First attempt

- Decompose

$$|\Psi(a)\rangle = \sum_n \mathcal{C}_n(a)\, \Psi_n(a)\, |\chi_n(a)\rangle, \tag{13}$$

where $|\chi_n(a)\rangle$ is the **same** as for the Schrod. eq. and $\Psi_n(a)$ is the WKB solution of

$$[H_G + E_n(a)]\, \Psi_n(a) = 0. \tag{14}$$

- The unit positive Wronskian WKB solution is

$$\Psi_n(a) = \frac{e^{-i\int^a da\, p_n(a)}}{\sqrt{2p_n(a)}} \tag{15}$$

where the momentum $p_n(a) > 0$ solves $H_G + E_n(a) = 0$.

These solutions correspond to expanding universes.

Their complex conjugated describe **contracting** universes.

Failure of the first attempt

- inserting (13) in the WDW eq. gives a **second** order matricial equation which mixes
  - corrections to WKB corrections with
  - $n \to n'$ matter transitions.
- Moreover the conserved Wronskian W reads

$$W = \sum_n |\mathcal{C}_n(a)|^2 + \sum_n (\mathcal{C}_n^* i\overleftrightarrow{\partial}_a \mathcal{C}_n)\, |\Psi_n(a)|^2. \qquad (16)$$

  The first term is OK, the second unwanted.
- Question: How to sort this out ?

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The correct decomposition

- The "correct" decomposition

$$|\Psi(a)\rangle = \sum_n \left[\mathcal{C}_n(a)\ \Psi_n(a) + \mathcal{D}_n(a)\ \Psi_n^*(a)\right]\ |\chi_n(a)\rangle, \qquad (17)$$

introduces $2N$ arbitrary fonctions $\mathcal{C}_n(a), \mathcal{D}_n(a)$.

- Eq. (17) gives an **under-constrained** system.
- **Exploit** this and impose

$$\langle \chi_n | i\overrightarrow{\partial_a} | \Psi \rangle = p_n \left[\mathcal{C}_n\ \Psi_n - \mathcal{D}_n\ \Psi_n^*\right]. \qquad (18)$$

so that $\mathcal{C}_n$ ($\mathcal{D}_n$) **instantanenously** weigh expanding (contracting) solutions.

- Insert (17) in the WdW eq. *and* use (18) to get

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Structure of the WDW equation, 1

$$\partial_a \mathcal{C}_n = \sum_{m \neq n} \tilde{M}_{nm} \, e^{-i \int^a (p_n - p_m) da} \, \mathcal{C}_m + \sum_m \tilde{N}_{nm} \, e^{-i \int^a (p_n + p_m) da} \, \mathcal{D}_m. \quad (19)$$

+ the same eq. with $\mathcal{C}_n \to \mathcal{D}_n$ and $i \to -i$
where

$$\tilde{M}_{nm} = \langle \partial_a \psi_m | \psi_n \rangle \frac{p_n + p_m}{2\sqrt{p_n p_m}} \quad (20)$$

$$\tilde{N}_{nm} = \langle \partial_a \psi_m | \psi_n \rangle \frac{p_n - p_m}{2\sqrt{p_n p_m}} + \delta_{nm} \frac{\partial_a p_n}{2 p_n} \quad (21)$$

- Eqs. (19,20,21) are **exact** and all coefficients are known
- Eq. (19) **is** the **WDW eq.**

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$$\tilde{N}_{nm} = \langle \partial_a \psi_m | \psi_n \rangle \frac{p_n - p_m}{2\sqrt{p_n p_m}} + \delta_{nm} \frac{\partial_a p_n}{2 p_n} \quad (21)$$

- Eqs. (19,20,21) are **exact** and all coefficients are known
- Eq. (19) is the **WDW eq.**

Structure of the WDW equation, 1

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- When, for a **given** matter Hamiltonian H_M,
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Theorem 2

- **Neglecting only** the $\mathcal{C}_n \to \mathcal{D}_n$ transitions, one gets

$$\partial_a \mathcal{C}_n = \sum_{m \neq n} \tilde{M}_{nm} \, e^{-i \int^a (p_n - p_m) da} \, \mathcal{C}_m \tag{23}$$

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Theorem 3. The recovery of the background sp.t

- **Neglecting** the $\mathcal{C}_n \to \mathcal{D}_n$ transitions, **and** to **first order** in $E_n - E_{\bar{n}}$ around the mean matter state $\bar{n}$,

$$\mathcal{C}_n(a) \equiv c_n(\bar{t}(a)), \tag{24}$$

where

$$\bar{t}(a) = \int^a da' \partial_E p(a')|_{E=E_{\bar{n}}}, \tag{25}$$

is the HJ time to reach a when matter is in the **mean** $\bar{n}$ state.

- The **identity** (24) is Heisenberg-'35 result. It follows from
  - $\tilde{M}_{nm} \equiv M^S_{mn}$
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A remark on NATAmplitudes

- For the Schrod. eq., $c_n(t)$ is given by the **overlap**

$$c_n(t) = \langle \chi_n(t) | e^{+i \int^t dt' E_n} | \psi(t) \rangle. \tag{26}$$

Instead, for the WDW eq., $\mathcal{C}_n(a)$ is given by

$$\mathcal{C}_n(a) = \langle \chi_n(a) | \Psi_n(a)^* \, i\overleftrightarrow{\partial_a} | \Psi(a) \rangle \tag{27}$$

This is the *Vilenkin* **current**,
and not the **Page-Hawking** overlap $\langle \chi_n(a) | \Psi(a) \rangle$.

Theorem 4. The $\mathcal{C}_n \to \mathcal{D}_m$ NATransitions.

- Far from turning points (big bounce),
  the transitions $\mathcal{C}_n \to \mathcal{D}_m$ are **exponentially suppressed**
  w.r.t. to the $\mathcal{C}_n \to \mathcal{C}_m$.

- because, typically

$$\begin{aligned} \mathcal{C}_n \to \mathcal{C}_m &\sim e^{-(p_n - p_m)\, Im\Delta a_C^{sp}} \\ \mathcal{C}_n \to \mathcal{D}_m &\sim e^{-(p_n + p_m)\, Im\Delta a_D^{sp}} \end{aligned} \quad (28)$$

  $p_n + p_m$ scales with the **total matter energy**.

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  The WDW eq. predicts a hierarchy of NAT,
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The $\mathcal{C}_n \to \mathcal{D}_m$ NAT in molecular phys.

- Consider a molecule of mass M and position of CM R
$$H_{\rm molec} = \frac{P^2}{2M} + V(R) + H_e(R),$$
where $H_e(R)$ governs the electronic deg. of freedom.
- There are **two** levels of description:
- The S-LZ way: treat R classically as a given function of time $R(t)$. The electronic d.o.f. then obey the t-dep. Schrod. eq.
$$i\partial_t|\psi_e\rangle = \hat{H}_e(R(t))|\psi_e\rangle$$
- The WdW way: treat **both** R and el. d.o.f on the **same footing**. Then, at fixed total energy E, one has
$$\left[-\frac{\partial_R^2}{2M} + V(R) + H_{el}(R)\right]|\Psi_E(R)\rangle = E\,|\Psi_E(R)\rangle$$

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Decomposing $|\Psi_E(R)\rangle = \sum_n (\mathcal{C}_n \Psi_n + \mathcal{D}_n \Psi_n^*)|\chi_n(R)\rangle$

- **neglecting** $\mathcal{C}_n \to \mathcal{D}_m$ and to **first order in** $E_n^e - E_m^e$, one gets

$$\mathcal{C}_n(R) \equiv c_n(\bar{t}(R)),$$

where c_n are the Landau-Z tr. amplitudes.
Hence the $|\mathcal{C}_n|^2$ give **transition probablities**.

- The $\mathcal{C}_n(R)$ are exactly given by $\mathcal{C}_n \equiv \langle \chi_n | \Psi_n^* \, i\overleftrightarrow{\partial}_R | \Psi_E \rangle.$,
i.e. by the "Vilenkin" current.

- Taking into account the NAT $\mathcal{C}_n \to \mathcal{D}_m$, one gets, as an identity,
$\sum_n |\mathcal{C}_n(R)|^2 - |\mathcal{D}_n(R)|^2 \equiv Const.$
This is a re-expression of **unitarity** in the lab.

- N.B. The H-Page norm $|\langle \chi_n | \Psi_E \rangle|^2 = |\mathcal{C}_n \Psi_n(R) + \mathcal{D}_n \Psi_n^*(R)|^2$
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Conclusions. 1

- The WDW eq. determines the NATA.
  There is NO ambiguity in **computing** these amplitudes.

- It predicts that there is a **hierarchy** of NA-regimes.

Conclusions. 2. The S. regime, the mildest one.

- The first regime, the mildest one, is obtained by
  - neglecting $\mathcal{C}_n \to \mathcal{D}_m$ NAT,
  - the first order in $E_n - E_{\bar{n}}$, but **nothing else**.

- In this regime

$$\mathcal{C}_n(a) \equiv c_n(\bar{t}(a)).$$

Therefore $|\mathcal{C}_n(a)|^2$ is the **proba.** to find the state n in an expand. universe at a (in fact around a).

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- Unitarity,

$$\sum_n |\mathcal{C}_n(a)|^2 \equiv Cst.$$

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Conclusions. 3. The second intermediate regime.

- Neglecting **only** $\mathcal{C}_n \to \mathcal{D}_m$, the WDW gives **two separate 1st order eqs. in $i\partial_a$.**

- **Unitarity**, i.e. $\sum_n |\mathcal{C}_n(a)|^2 = Cst.$ is autom. obtained, from the conservation of W, even though there is **no cosmic time** because **no** backd sp-time **common** to all matter states.

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  The **recovery of bgd sp-t. and cosmic time** requires one extra condition than the **recovery of unitarity**.

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- Taking into account $\mathcal{C}_n \to \mathcal{D}_m$ NAT, one faces the **identity**

$$\sum_n |\mathcal{C}_n(a)|^2 - |\mathcal{D}_n(a)|^2 \equiv Cst.$$

- How to interprete this equation ?
- *Proposal:* as we just did it for cosmic time:
  the recovery of unitarity requires some conditions.
- Hence we conclude/conjecture:

  The **statistical interpretation** of the $\mathcal{C}_n(a)$,
  is **not** as a **fundamental property of QC/QG**, rather
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