Asymptotic safety

Roberto Percacci

[1]SISSA, Trieste

QG in Paris, March 21, 2013

A quantum theory of gravity

Expand

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

Gauge fix using a background gauge fixing condition e.g.

$$S_{GF} = \frac{1}{2}\int dx \sqrt{-\bar{g}}\, \bar{g}^{\mu\nu}\chi_\mu\chi_\nu \ ; \qquad \chi_\mu = \bar{\nabla}^\nu h_{\nu\mu} - \frac{1}{2}\bar{\nabla}_\mu h$$

Add ghost Lagrangian

$$S_{ghost} = \int dx \sqrt{-\bar{g}}\, \bar{c}^\mu(-\delta^\nu_\mu \bar{\nabla}^2 - \bar{R}^\nu{}_\mu)c_\nu$$

Compute $\Gamma(\bar{g}_{\mu\nu}, h_{\mu\nu})$.
Formalism preserves background gauge invariance
$\delta_\epsilon \bar{g}_{\mu\nu} = \bar{\nabla}_\mu\epsilon_\nu + \bar{\nabla}_\nu\epsilon_\mu$, $\delta_\epsilon h_{\mu\nu} = \mathcal{L}_\epsilon h_{\mu\nu}$.

ISSUES

Non-renormalizable (Goroff and Sagnotti 1985)

- interaction strength grows like $\tilde{G} = Gk^2$
- violation of unitarity
- lack of predictivity

General EFT recipe

- fix the level of precision that is required in the calculation
- fix the ratio E/M. This determines the order of the expansion that will be required
- at the given order in the expansion, determine all the terms in the action that can contribute to the given process. By power counting there can only be finitely many.
- if the fundamental theory is known, their coefficients can in principle be calculated
- otherwise, measure them with as many experiments.
- use the EFT Lagrangian to compute the amplitudes to the desired precision

Chiral perturbation theory

Strong interactions at low energy described by chiral model

$$S = \int dx \left[\frac{f_\pi^2}{4} tr(U^{-1}\partial U)^2 + \ell_1 tr((U^{-1}\partial U)^2)^2 + \ell_2 tr((U^{-1}\partial U)^2)^2 + O(\partial^6) \right]$$

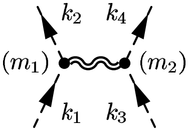
Expansion parameter $E/4\pi f_\pi$.
For low energy meson physics need f_π, ℓ_1, ℓ_2 and perhaps a few others. One loop in f_π, tree level in ℓ_1, ℓ_2.
Successfully describe a rich phenomenology.

Gravity

$$S = \int dx \sqrt{g} \left[2m_P^2 \Lambda - m_P^2 R + \ell_1 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \ell_2 R_{\mu\nu} R^{\mu\nu} + \ell_3 R^2 + O(\partial^6) \right]$$

m_P similar to f_π.

Newtonian potential



$$\sim \; \frac{G m_1 m_2}{\mathrm{q}^2}$$

$$V(r) = \int \frac{d^3 q}{(2\pi)^3} \frac{G m_1 m_2}{\mathrm{q}^2} e^{i\mathrm{qr}} = -\frac{G m_1 m_2}{r}$$

Leading quantum correction

For dimensional reasons the leading correction is

$$V(r) = -\frac{Gm_1m_2}{r}\left[1 + \alpha\frac{G(m_1+m_2)}{rc^2} + \beta\frac{G\hbar}{r^2c^3} + \dots\right]$$

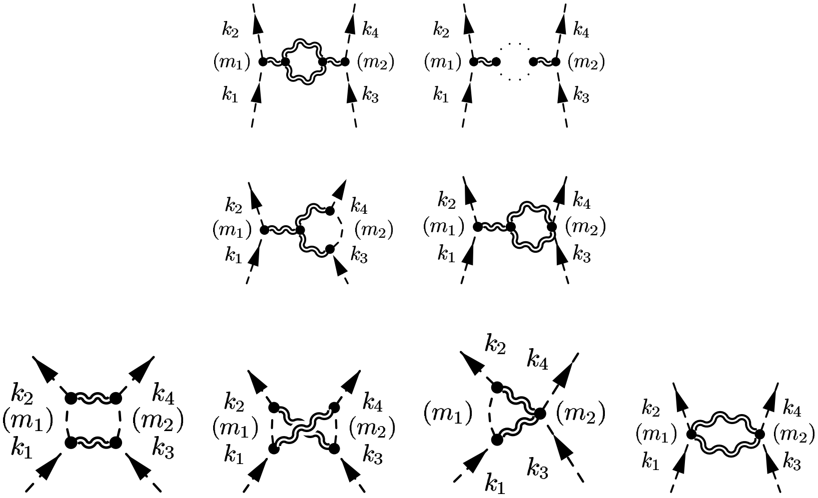
and

$$\int \frac{d^3q}{(2\pi)^3}\log\left(\frac{\mathrm{q}^2}{\mu^2}\right)e^{i\mathrm{qr}} = -\frac{1}{2\pi^2r^3}$$

Clearly distinct from contributions of local counterterms that give analytic corrections to the amplitude. E.g.

$$\int \frac{d^3q}{(2\pi)^3}\ell e^{i\mathrm{qr}} = \ell\delta(r)$$

One loop graphs



(from Bjerrum Bohr, Donoghue, Holstein, 2002)

Evaluation

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A prediction of quantum gravity

$$V(r) = -\frac{Gm_1m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{rc^2} + \frac{41}{10\pi}\frac{G\hbar}{r^2c^3} + \dots\right]$$

No renormalization ambiguities.
Probably the most reliable result in quantum gravity.

$M = M_\odot$	$\frac{GM_\odot}{rc^2}$	$\frac{G\hbar}{r^2c^3}$
$r = R_\odot$	10^{-6}	10^{-88}
$r = r_{S\odot}$	0.5	10^{-76}

Lessons

- no clash between QM and GR
- EFT is consistent and predictive at low enough energy
- experimentally indistinguishable from classical GR
- agrees with all experimental data
- to some extent, background independent
- open issues in the UV, IR, strong field

Outline

The Problems of QG, and the RG solution

Try to extend beyond Planck scale

- interaction strength grows like $\tilde{G} = Gk^2$
- lack of predictivity

first problem solved if $\tilde{G} = G(k)k^2 \to \tilde{G}_*$
more generally, if $\Gamma_k(\bar{g}_{\mu\nu}, h_{\mu\nu}) = \sum_i g_i(k)\mathcal{O}_i(\bar{g}_{\mu\nu}, h_{\mu\nu})$
require $\tilde{g}_i \to \tilde{g}_{i*}$, where $\tilde{g}_i = k^{-d_i} g_i$

THE RG SOLUTION CONT'D

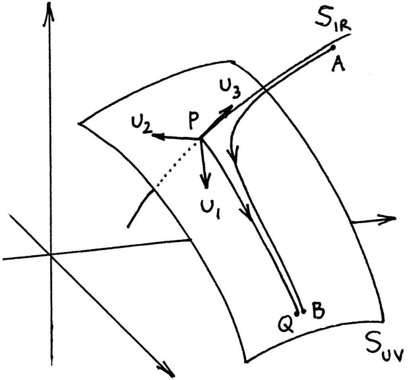
Define a theory space (fields, symmetry, action functionals) parameterized by $\tilde{g}_i$.

- RG trajectory is “renormalizable” or “Asymptotically Safe” if it flows to a FP in the UV
- The RG trajectories that flow into the FP for $k \to \infty$ form the UV critical surface S_{UV}
- Predictivity demands $\dim(S_{UV})$ is finite

Example: QCD

- Gaußian Fixed Point at $\tilde{g}_{i*} = 0$.
- $\tilde{g}_i = k^{-d_i} g_i$
- $\tilde{\beta}_i = \partial_t \tilde{g}_i = -d_i \tilde{g}_i + k^{-d_i} \beta_i$
- $M_{ij}\big|_* = \frac{\partial \tilde{\beta}_i}{\partial \tilde{g}_j}\big|_* = -d_i \delta_{ij}$
- relevant couplings=renormalizable couplings

GENERAL PICTURE



One Loop Corrections in Einstein's Theory

$$k\frac{d}{dk}\frac{1}{16\pi G(k)} = ck^{d-2}$$

$$k\frac{dG}{dk} = -16\pi c G^2 k^{d-2}$$

$$\tilde{G} = Gk^{d-2}$$

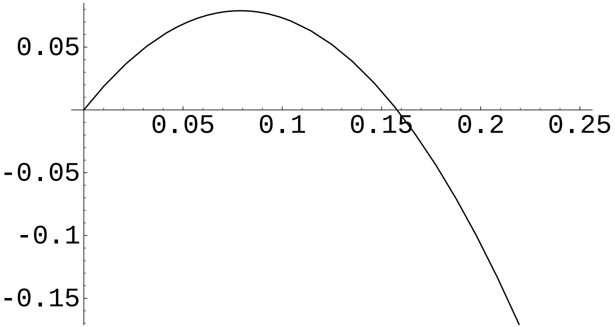
$$k\frac{d\tilde{G}}{dk} = (d-2)\tilde{G} - 16\pi c\tilde{G}^2$$

fixed point at $\tilde{G} = (d-2)/16\pi c$

Gravity in $2+\epsilon$

$d - 2 = \epsilon$

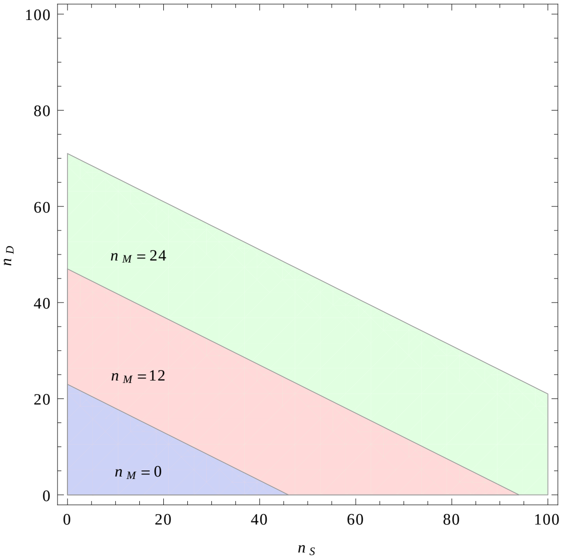
$$\tilde{G} = G k^{\epsilon}$$
$$\beta_{\tilde{G}} = \epsilon \tilde{G} - \frac{38}{3} \tilde{G}^2$$



One loop, $d = 4$

$$c = \frac{11}{3\pi}, \frac{35}{8\pi}, \frac{23}{3\pi}, \ldots$$

One loop, with matter



Higher derivative gravity

$$\Gamma_k = \int d^4x \sqrt{g} \left[2Z\Lambda - ZR + \frac{1}{2\lambda} \left(C^2 - \frac{2\omega}{3} R^2 + 2\theta E \right) \right]$$

$$Z = \frac{1}{16\pi G}$$

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E.S. Fradkin, A.A. Tseytlin,Phys. Lett. **104 B**, 377 (1981).
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G. de Berredo–Peixoto and I. Shapiro, Phys.Rev. **D71** 064005 (2005).
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Beta functions I

$$\beta_\lambda = -\frac{1}{(4\pi)^2}\frac{133}{10}\lambda^2$$

$$\beta_\omega = -\frac{1}{(4\pi)^2}\frac{25+1098\omega+200\omega^2}{60}\lambda$$

$$\beta_\theta = \frac{1}{(4\pi)^2}\frac{7(56-171\theta)}{90}\lambda$$

$$\lambda(k) = \frac{\lambda_0}{1+\lambda_0\frac{1}{(4\pi)^2}\frac{133}{10}\log\left(\frac{k}{k_0}\right)}$$

$$\omega(k) \to \omega_* \approx -0.0228$$

$$\theta(k) \to \theta_* \approx 0.327$$

BETA FUNCTIONS II

$$\beta_{\tilde{\Lambda}} = -2\tilde{\Lambda} + \frac{1}{(4\pi)^2}\left[\frac{1+20\omega^2}{256\pi\tilde{G}\omega^2}\lambda^2 + \frac{1+86\omega+40\omega^2}{12\omega}\lambda\tilde{\Lambda}\right]$$
$$-\frac{1+10\omega^2}{64\pi^2\omega}\lambda + \frac{2\tilde{G}}{\pi} - q(\omega)\tilde{G}\tilde{\Lambda}$$
$$\beta_{\tilde{G}} = 2\tilde{G} - \frac{1}{(4\pi)^2}\frac{3+26\omega-40\omega^2}{12\omega}\lambda\tilde{G} - q(\omega)\tilde{G}^2$$

where $q(\omega) = (83+70\omega+8\omega^2)/18\pi$

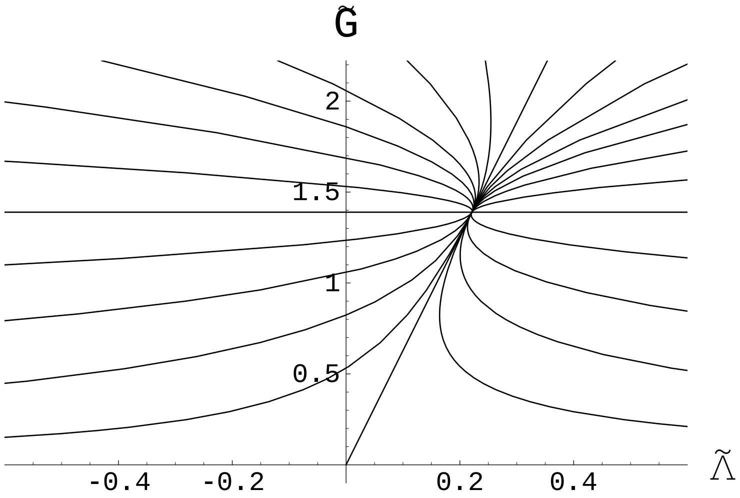
Flow in $\tilde{\Lambda}$–$\tilde{G}$ plane I

$$\beta_{\tilde{\Lambda}} = -2\tilde{\Lambda} + \frac{2\tilde{G}}{\pi} - q_* \tilde{G}\tilde{\Lambda}$$
$$\beta_{\tilde{G}} = 2\tilde{G} - q_* \tilde{G}^2$$

where $q_* = q(\omega_*) \approx 1.440$

$$\tilde{\Lambda}_* = \frac{1}{\pi q_*} \approx 0.221 \,, \qquad \tilde{G}_* = \frac{2}{q_*} \approx 1.389 \,.$$

Flow in $\tilde{\Lambda}$–$\tilde{G}$ plane II



Topologically massive gravity

Action

$$S(g) = Z \int d^3x \sqrt{g} \left(-2\Lambda + R + \frac{1}{2\mu} \varepsilon^{\lambda\mu\nu} \Gamma^{\rho}_{\lambda\sigma} \left(\partial_\mu \Gamma^{\sigma}_{\nu\rho} + \frac{2}{3} \Gamma^{\sigma}_{\mu\tau} \Gamma^{\tau}_{\nu\rho} \right) \right)$$

$$Z = \frac{1}{16\pi G}$$

Dimensionless combinations of couplings

$$\nu = \mu G \; ; \qquad \tau = \Lambda G^2 \; ; \qquad \phi = \mu / \sqrt{|\Lambda|}$$

Beta functions of

$$\begin{aligned} \beta_\nu &= 0\ , \\ \beta_{\tilde{G}} &= \tilde{G} + B(\tilde{\mu})\tilde{G}^2\ , \\ \beta_{\tilde{\Lambda}} &= -2\tilde{\Lambda} + \frac{1}{2}\tilde{G}\left(A(\tilde{\mu},\tilde{\Lambda}) + 2B(\tilde{\mu})\tilde{\Lambda}\right) \end{aligned}$$

Since $\nu = \mu G = \tilde{\mu}\tilde{G}$ is constant

can replace $\tilde{\mu}$ by $\nu/\tilde{G}$

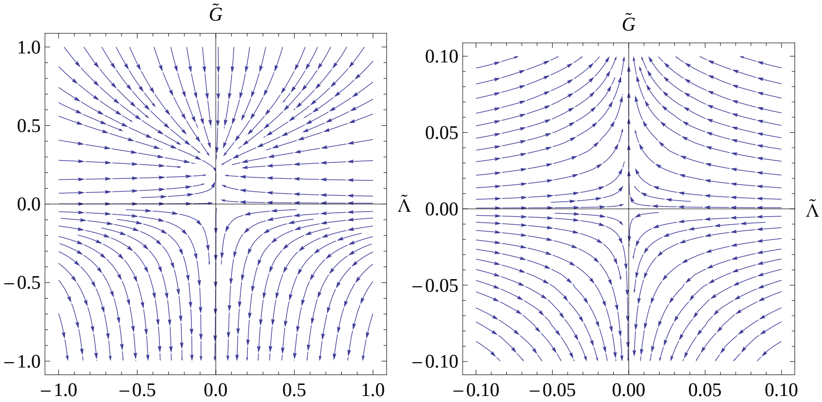


Figure : The flow in the $\tilde{\Lambda}$-$\tilde{G}$ plane for $\alpha = 0$, $\nu = 5$. Right: enlargement of the region around the origin, showing the Gaussian FP. The beta functions become singular at $|\tilde{G}| = 1.9245$.

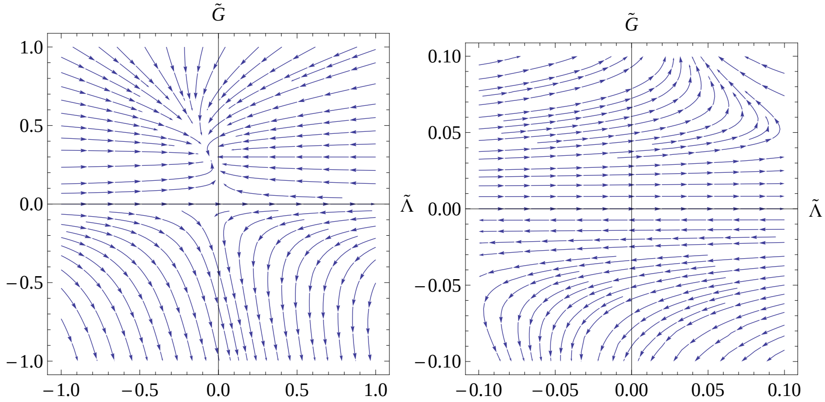


Figure : The flow in the $\tilde{\Lambda}$-$\tilde{G}$ plane for $\alpha = 0$, $\nu = 0.1$. Right: enlargement of the region around the origin, showing that there is no Gaussian FP. The beta functions diverge on the $\tilde{\Lambda}$ axis.

Recently extended to TM SUGRA

[R.P., C. Pope, M. Perry, E. Sezgin,
arXiv 1302.0868]

FRGE

Define $\Gamma_k(\bar{g}_{\mu\nu}, h_{\mu\nu})$. It satisfies a simple functional differential equation

$$k\frac{d\Gamma_k}{dk} = \beta(\bar{g}_{\mu\nu}, h_{\mu\nu})$$

The quantity β is UV and IR finite.
Since $\lim_{k\to 0}\Gamma_k = \Gamma$, can use FRGE to calculate the effective action.

Single-field truncations:

$$\Gamma_k(g_{\mu\nu}) = \Gamma_k(g_{\mu\nu}, 0)$$

Einstein–Hilbert truncation I

$$\Gamma_k(g) = \frac{1}{16\pi G_k}\int dx\sqrt{g}(2\Lambda_k - R) + S_{GF} + S_{ghost}$$

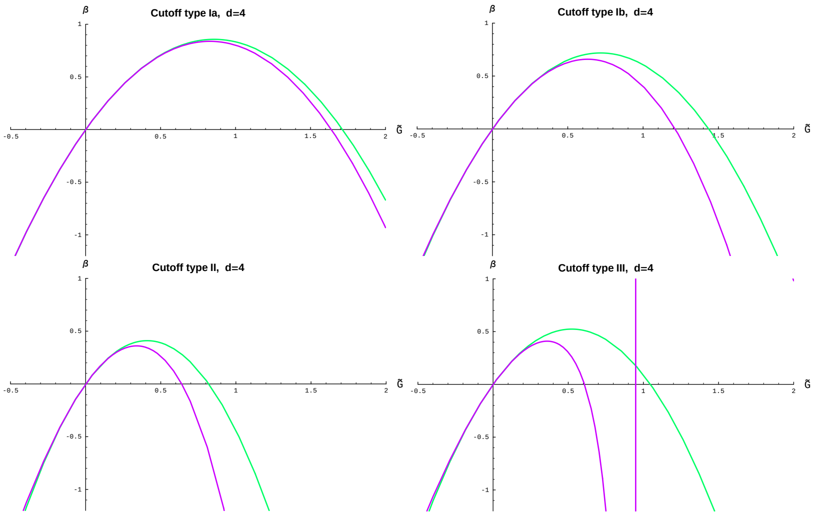
with $h_{\mu\nu} \to Z_h^{1/2} h_{\mu\nu}$, $c^\mu \to Z_c^{1/2} c^\mu$ and $c_\nu \to Z_c^{1/2} c_\nu$

where $\eta_h = -\frac{\partial_t Z_h}{Z_h}$, $\eta_c = -\frac{\partial_t Z_c}{Z_c}$
Assume $\eta_h = -\frac{\partial_t Z}{Z}$, $\eta_c = 0$.

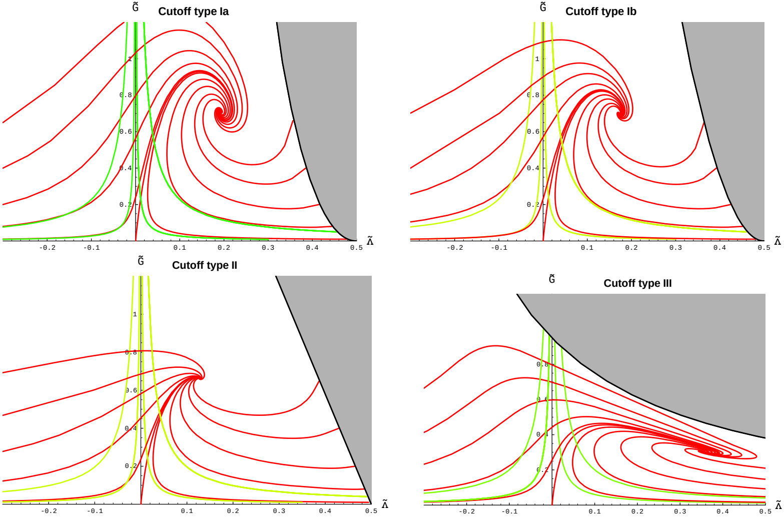
$$\beta_{\tilde\Lambda} = \frac{-2(1-2\tilde\Lambda)^2\tilde\Lambda + \frac{36-41\tilde\Lambda+42\tilde\Lambda^2-600\tilde\Lambda^3}{72\pi}\tilde G + \frac{467-572\tilde\Lambda}{288\pi^2}\tilde G^2}{(1-2\tilde\Lambda)^2 - \frac{29-9\tilde\Lambda}{72\pi}\tilde G}$$

$$\beta_{\tilde G} = \frac{2(1-2\tilde\Lambda)^2\tilde G - \frac{373-654\tilde\Lambda+600\tilde\Lambda^2}{72\pi}\tilde G^2}{(1-2\tilde\Lambda)^2 - \frac{29-9\tilde\Lambda}{72\pi}\tilde G}$$

Einstein–Hilbert truncation II



Einstein–Hilbert truncation III



Fourth order gravity

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- M. Niedermaier, Nucl. Phys. B833, 226-270 (2010)

$f(R)$

talk by Dario Benedetti

BI-FIELD TRUNCATION I

$$\Gamma_k(\bar{g}_{\mu\nu}, h_{\mu\nu}) = \frac{1}{16\pi G_k} \int dx \sqrt{g}(2\Lambda_k - R) + S_{GF} + S_{ghost}$$

flow G_k, Λ_k, Z_h, Z_c.

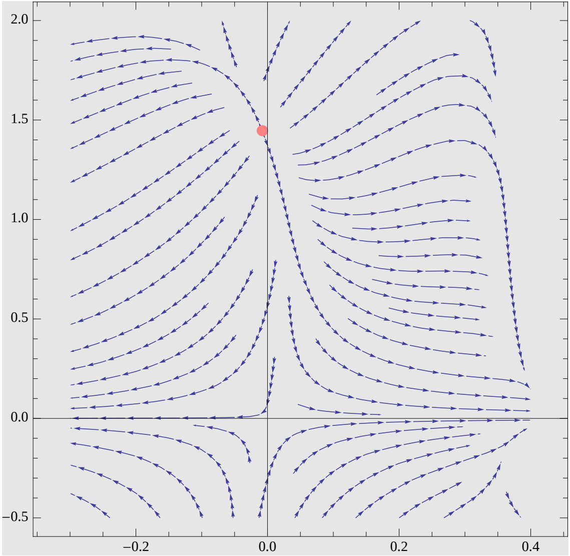
FP at $\tilde{\Lambda} = -0.00812786$, $\tilde{G} = 1.44627$

scaling exponents -3.32286, -1.95403

anomalous dimensions $\eta_h = 0.0720708$, $\eta_c = -1.50325$.

[A. Codello, G. d'Odorico, G. Pagani, in preparation]

Bi-field truncation II



Summary

- AS of gravity by now quite plausible
- use continuum covariant QFT
- bottom up approach, use EFT framework
- almost guaranteed to give correct low energy limit
- hints of agreement with CDT

Outlook

- more complicated/complete truncations
- matter coupled to gravity
- observables
- describe as CFT