Black hole entropy in LQG; the local perspective

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Quantum Gravity in Paris
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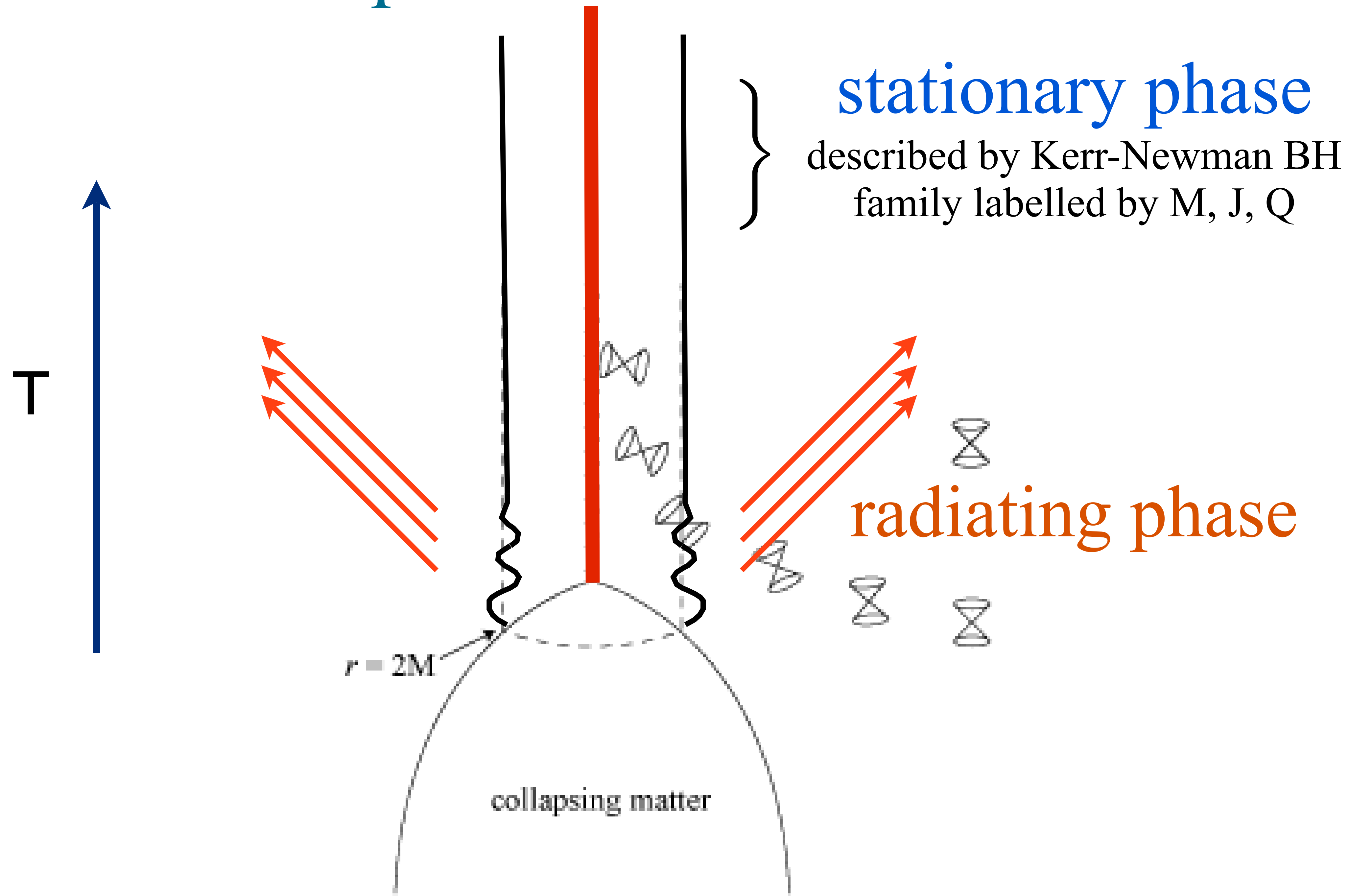
based on resent results obtained in collaboration with
Amit Ghosh and Ernesto Frodden

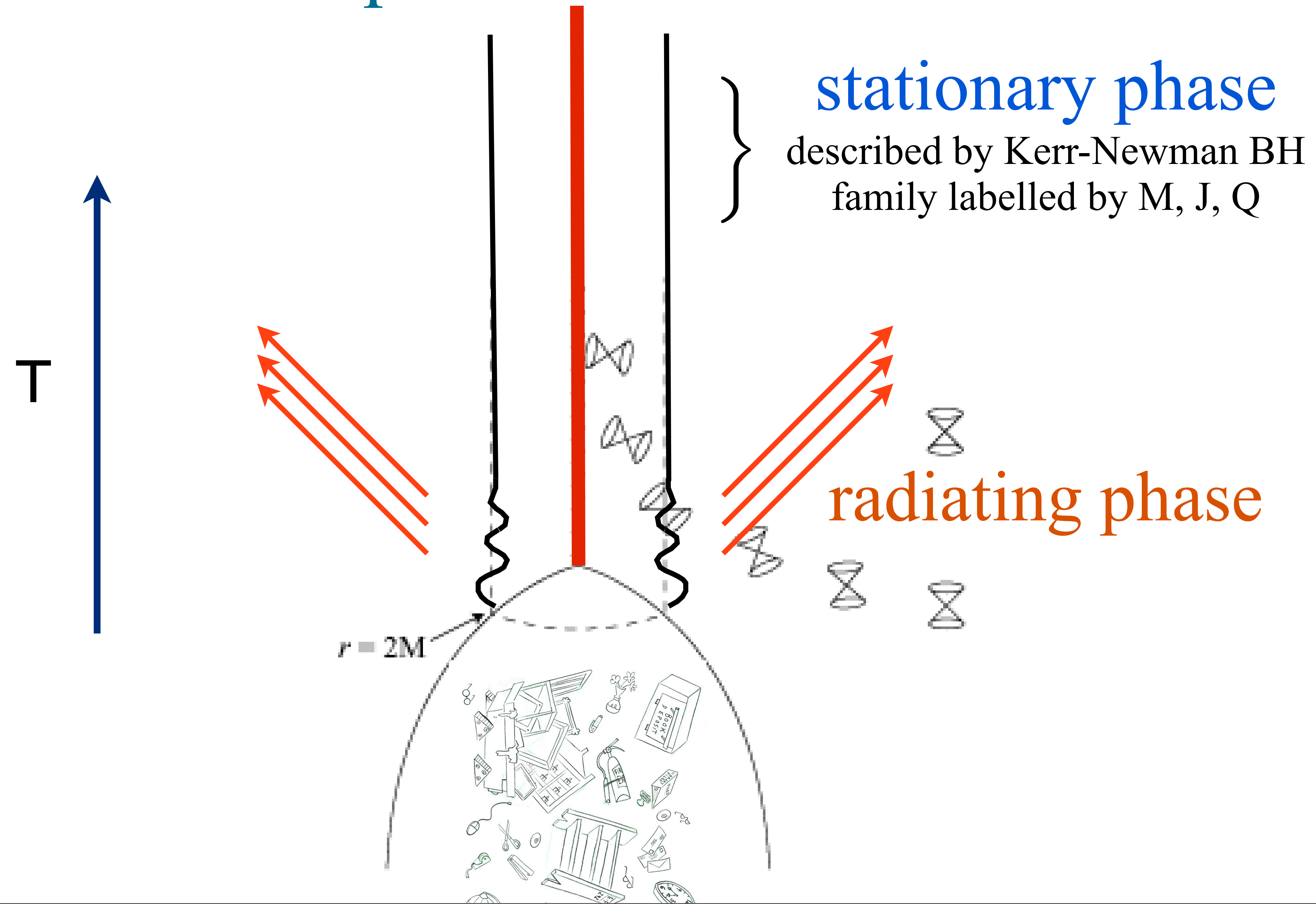
Black Hole Thermodynamics

The Robinson-Carter-Hawking-Israel uniqueness theorems



Black Hole Thermodynamics

The Robinson-Carter-Hawking-Israel uniqueness theorems

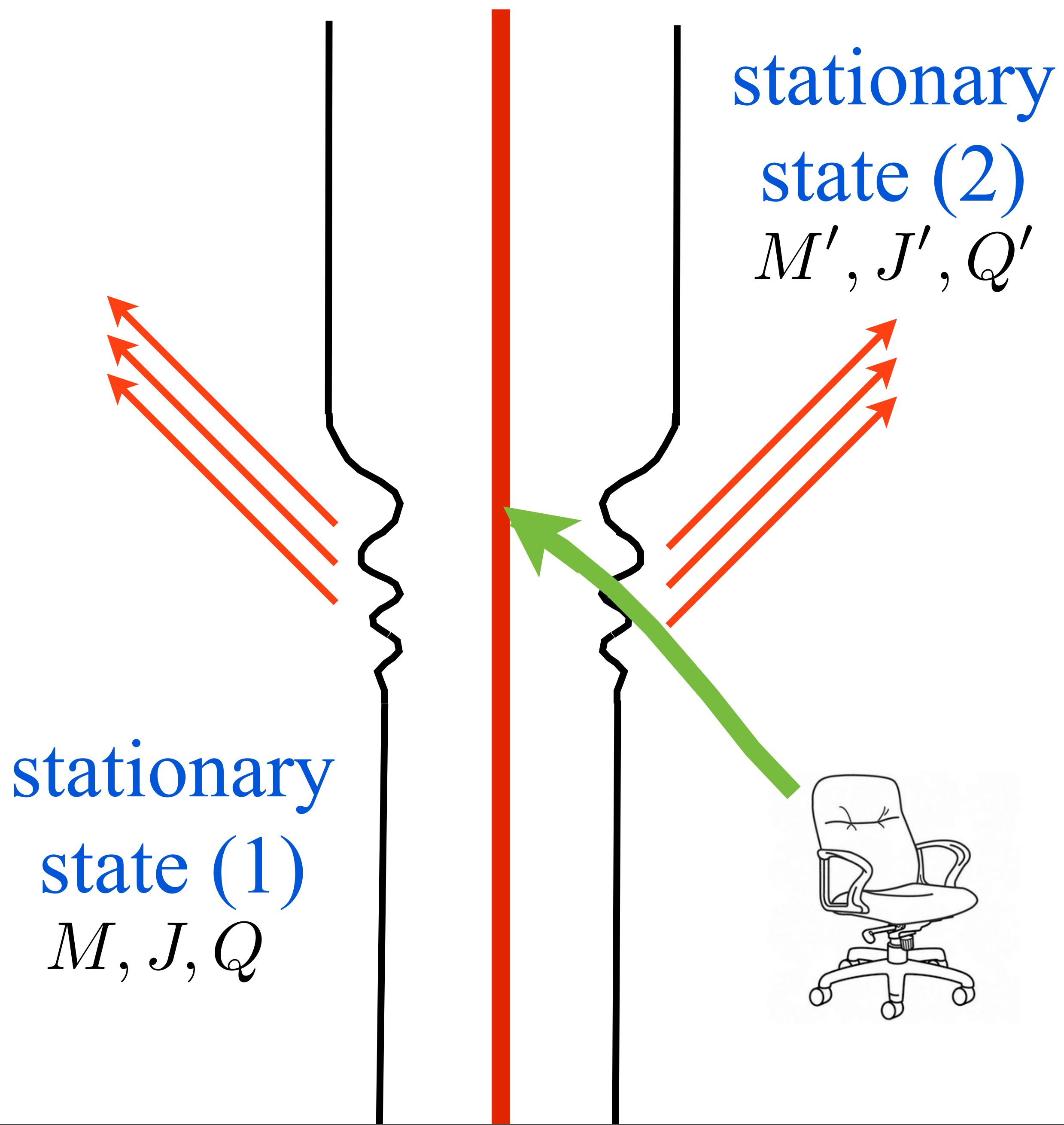


Black Hole Thermodynamics

The 0th, 1st, 2nd and 3rd laws of BH

Some definitions

- $\Omega \equiv$ horizon angular velocity
- $\kappa \equiv$surface gravity ('grav. force' at horizon). If ℓ^a =killing generator, then $\ell^a \nabla_a \ell^b = \kappa \ell^b$.
- $\Phi \equiv$electromagnetic potential.



0th law: the surface gravity κ is constant on the horizon.

1st law:

$$\delta M = \frac{\kappa}{8\pi}\delta A + \underbrace{\Omega \delta J + \Phi \delta Q}_{\text{work terms}}$$

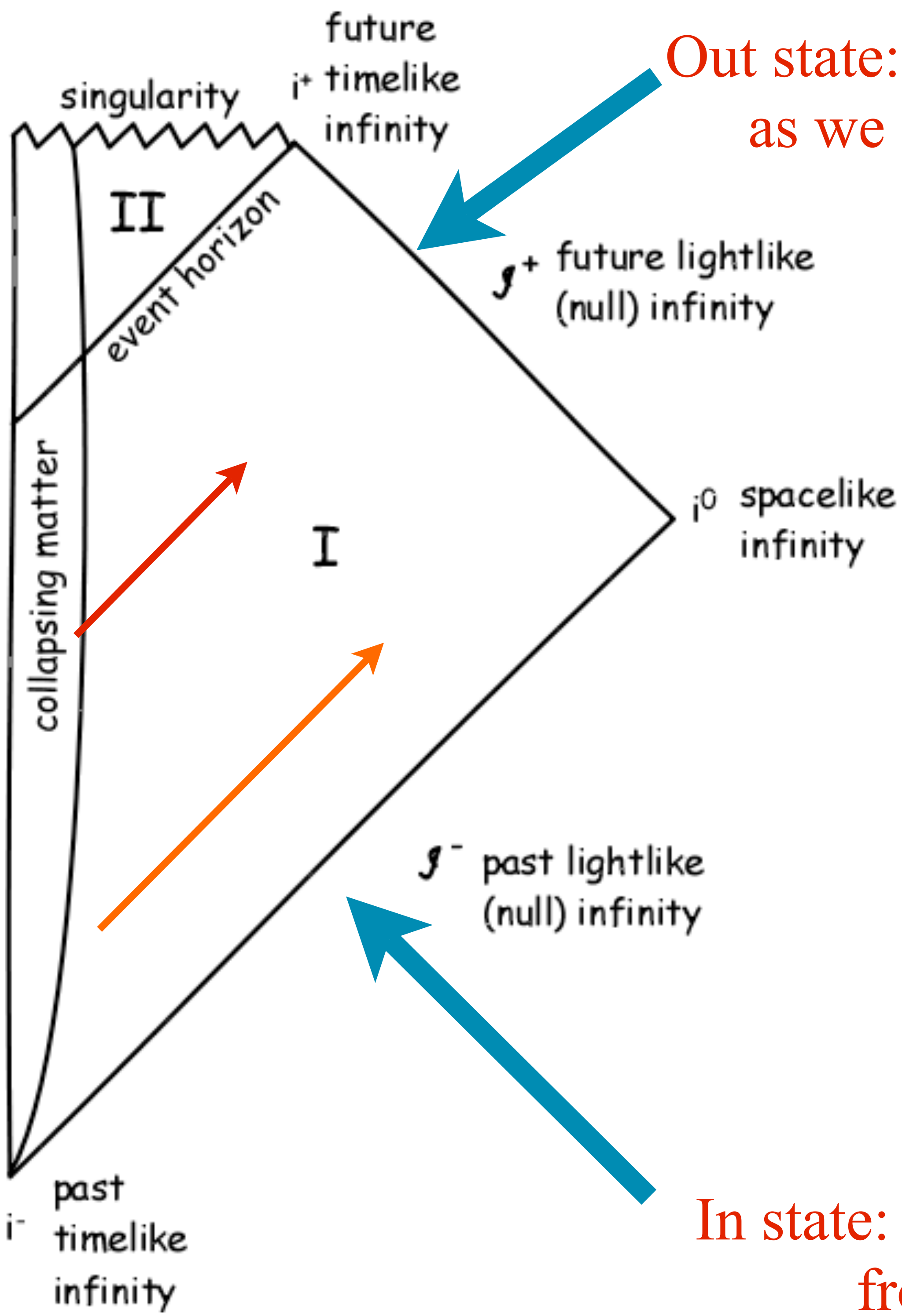
2nd law:

$$\delta A \geq 0$$

3rd law: the surface gravity value $\kappa = 0$ (extremal BH) cannot be reached by any physical process.

Black Hole Thermodynamics

Hawking Radiation: QFT on a BH background



Out state: thermal flux of particles as we approach the point **i**+

In state: vacuum far from **i**-

Temperature at infinity

$$T_\infty = \frac{\kappa}{2\pi}$$

From the first law

$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega\delta J + \Phi\delta Q$$

One gets the ENTROPY

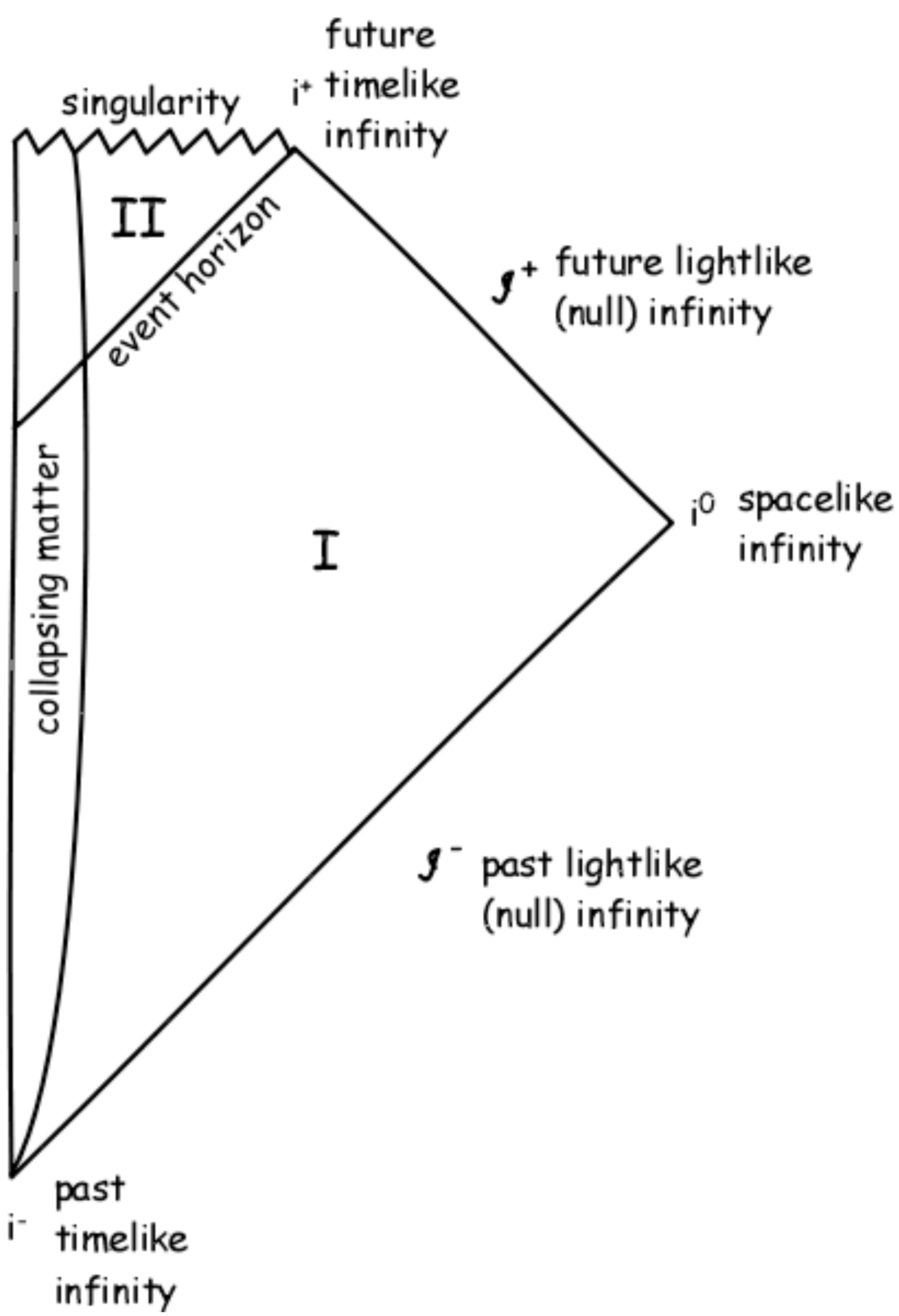
$$S = \frac{A}{4\ell_p^2}$$

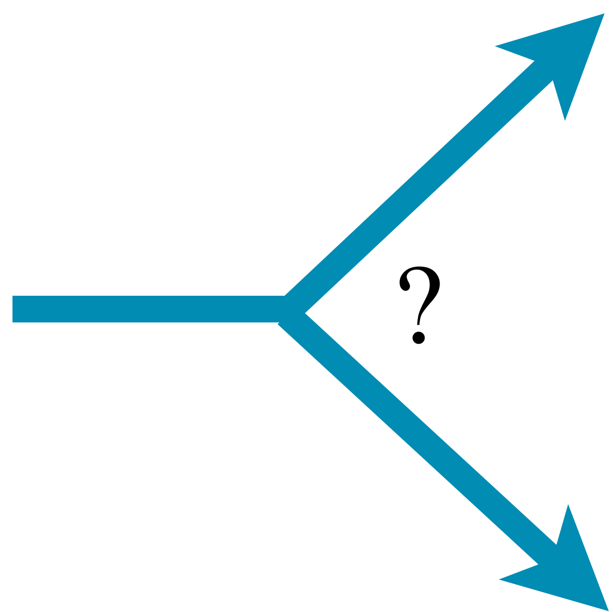
The local laws of BH mechanics

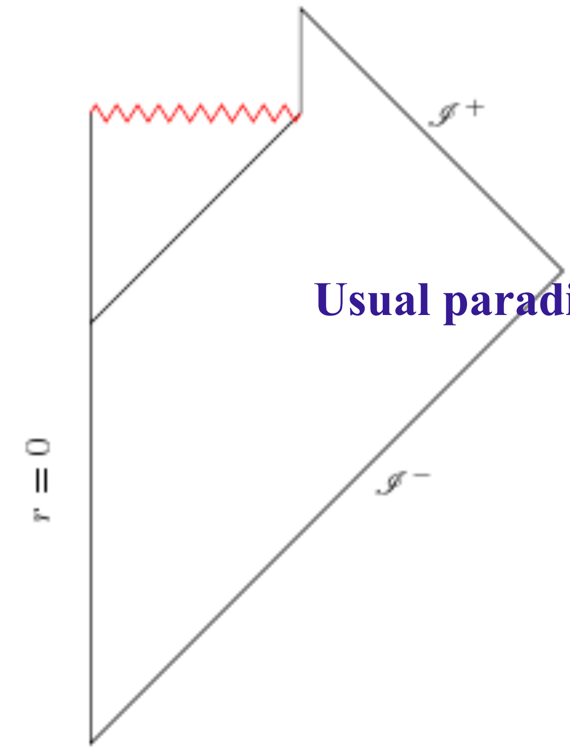
BH thermodynamics from a local perspective

Black Hole entropy in LQG

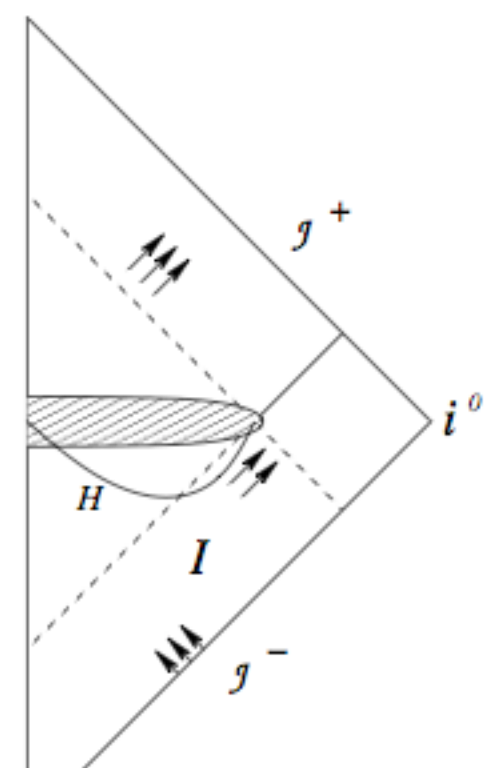
The standard definition of BH is GLOBAL
(need a quasi-local definition)







Usual paradigm



Alternative Paradigm:
Ashtekar-Bojowald (2005),
Ashtekar-Taveras-Varadarajan (2008),
Ashtekar-Pretorius-Ramazanoglu (2011).

Black Hole Thermodynamics

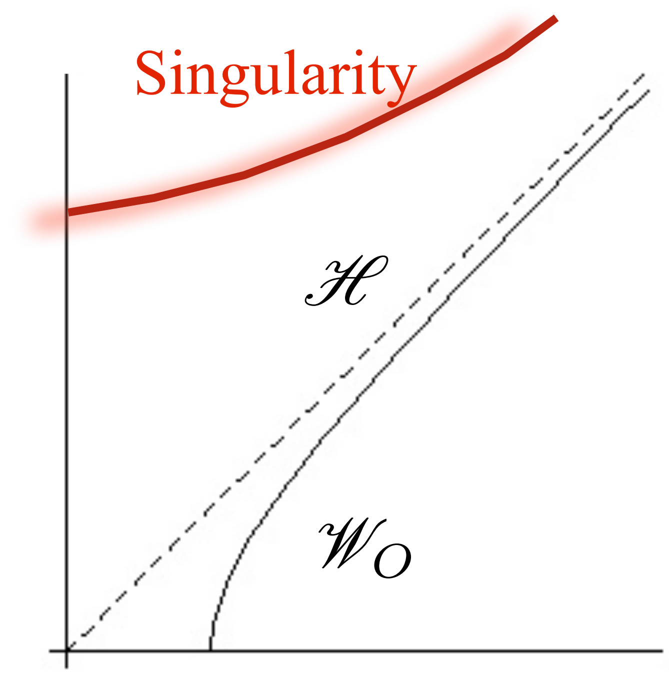
A local perspective

$\ell^2 << A$

Introduce a family of
local stationary observers
~ZAMOS

$$\chi = \xi + \Omega\,\psi = \partial_t + \Omega\,\partial_\phi$$

$$u^a = \frac{\chi^a}{\|\chi\|}$$



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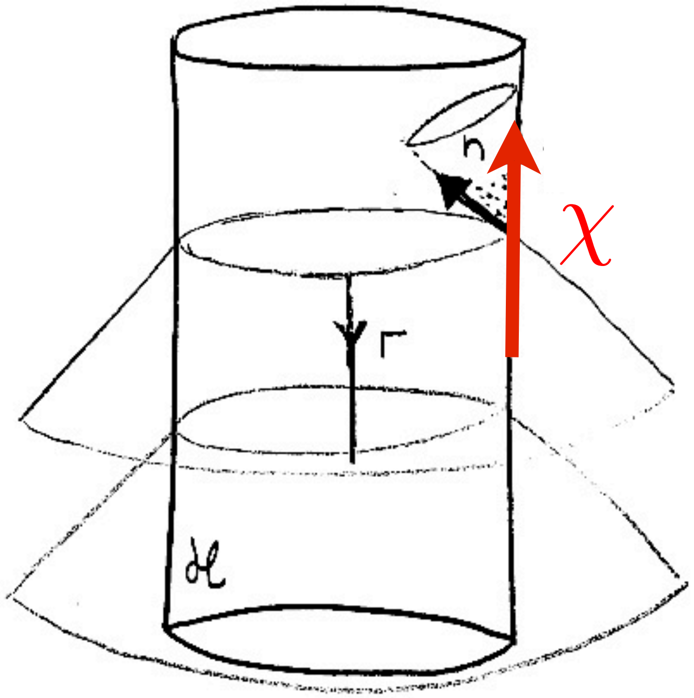
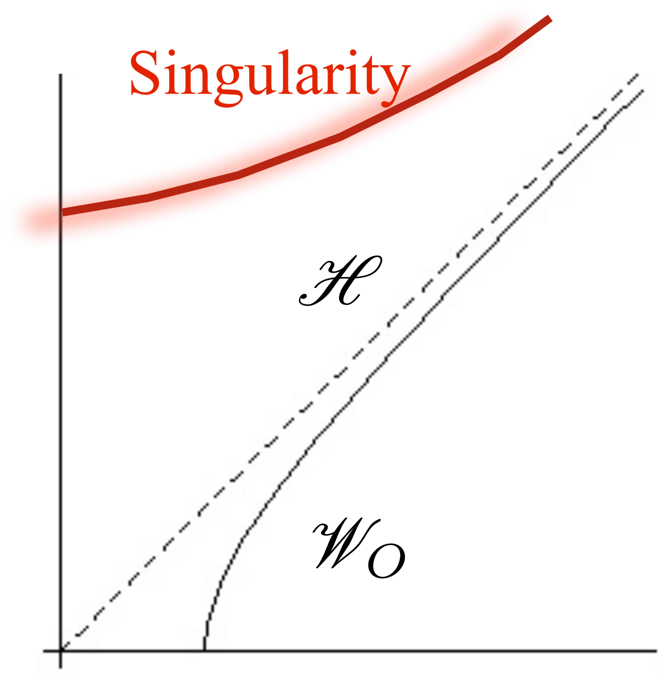


$$\chi = \xi + \Omega\,\psi = \partial_t + \Omega\,\partial_\phi$$

$$u^a = \frac{\chi^a}{\|\chi\|}$$

$$a = ||u^a \nabla_a u^b|| = \frac{1}{\ell}$$

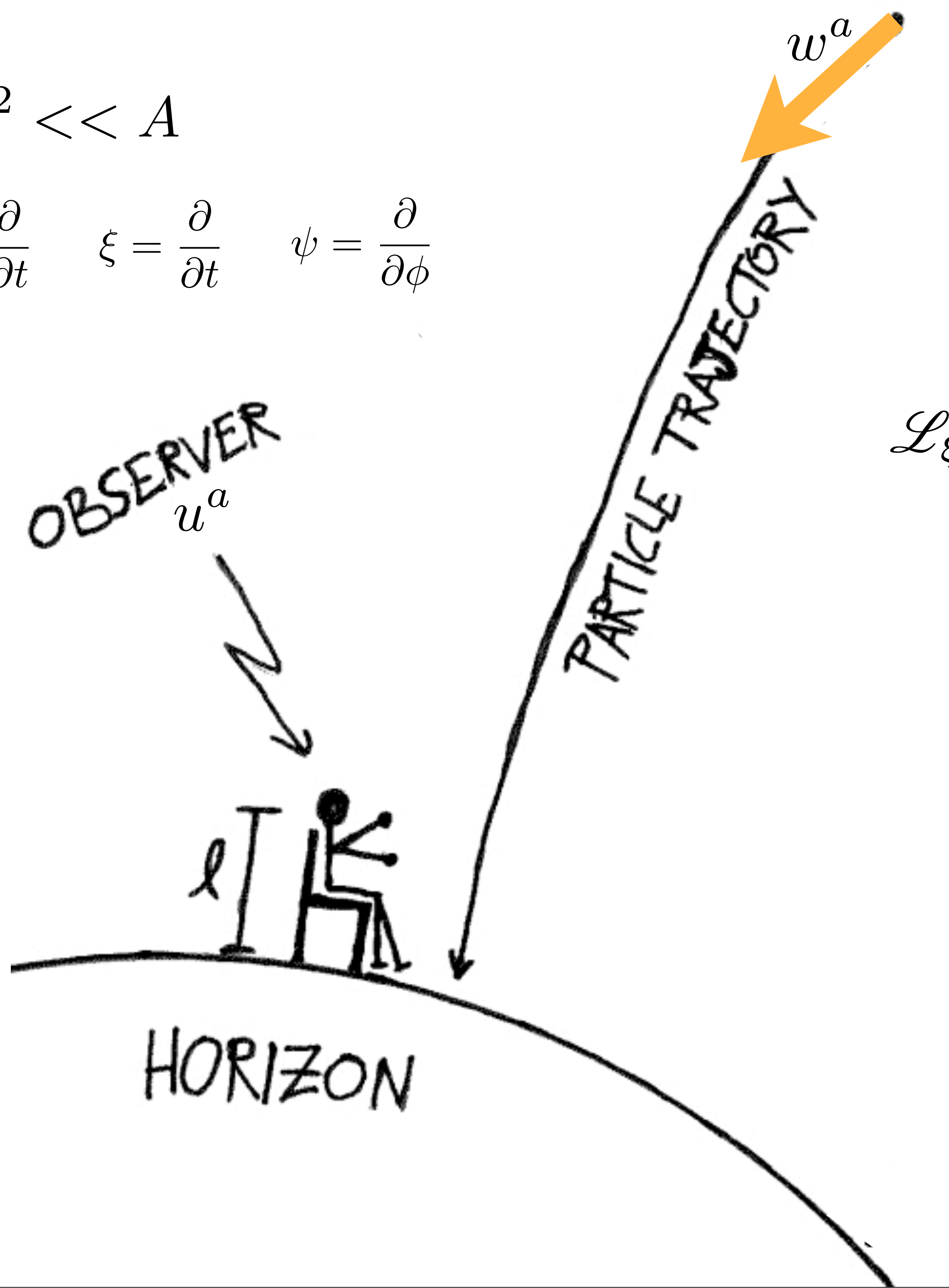


A thought experiment

throwing a test particle from infinity

$$\ell^2 << A$$

$$\chi = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial t} \qquad \xi = \frac{\partial}{\partial t} \qquad \psi = \frac{\partial}{\partial \phi}$$



Particle's equation of motion

$$w^a \nabla_a w_b = q\, F_{bc} w^c$$

Symmetries of the background

$$\mathscr{L}_\xi g_{ab} = \mathscr{L}_\psi g_{ab} = \mathscr{L}_\xi A_a = \mathscr{L}_\psi A_a = 0$$

Conserved quantities

$$\mathcal{E} \equiv -w^a \xi_a - q A^a \xi_a$$

$$L \equiv w^a \psi_a + q A^a \psi_a$$

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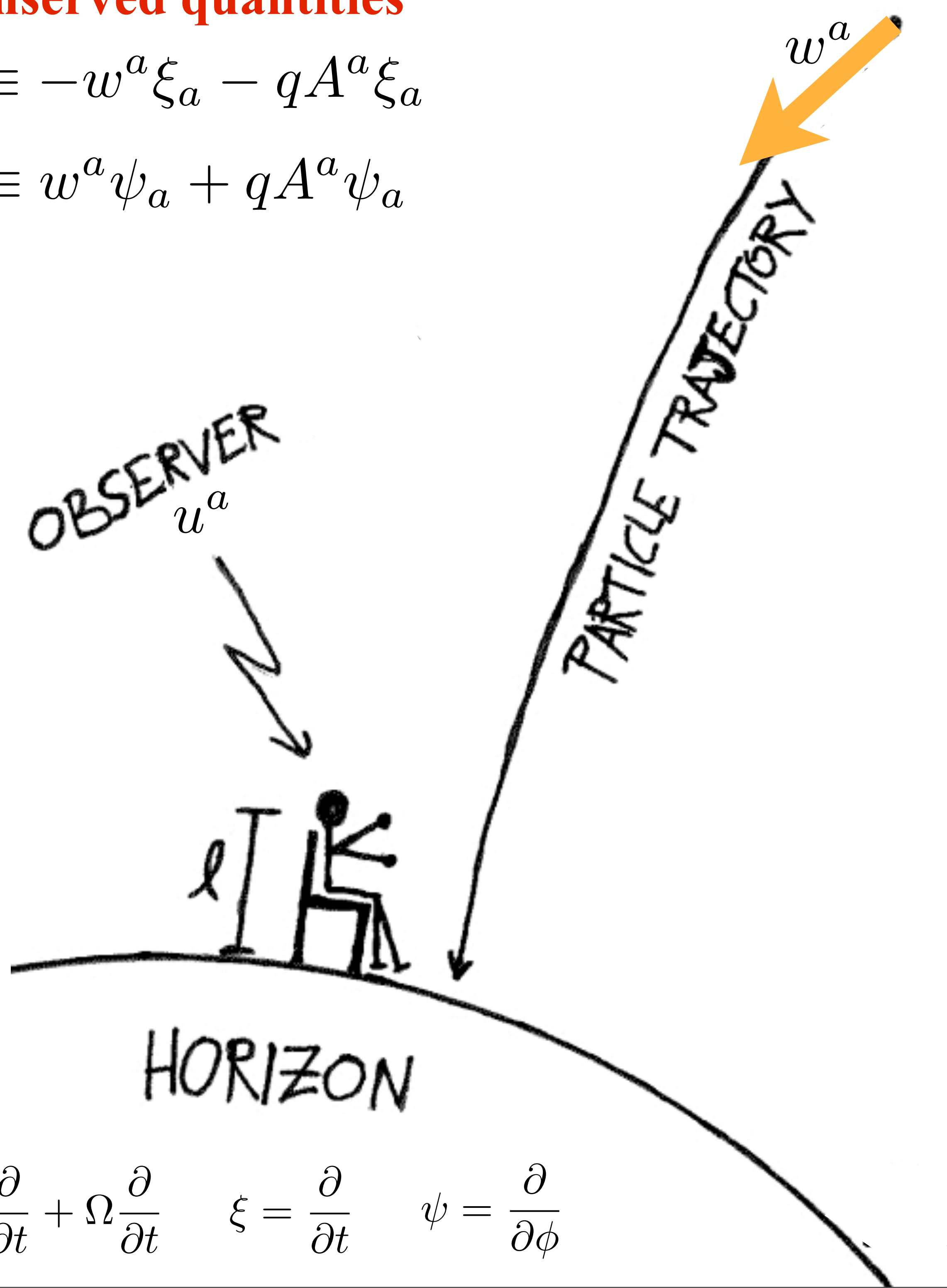

Particle at infinity

$$\mathcal{E} = -w^a \xi_a|_\infty \equiv \text{energy}$$

$$L = w^a \Psi_a|_\infty \equiv \text{angular momentum}$$

$$\chi = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial t} \qquad \xi = \frac{\partial}{\partial t} \qquad \psi = \frac{\partial}{\partial \phi}$$

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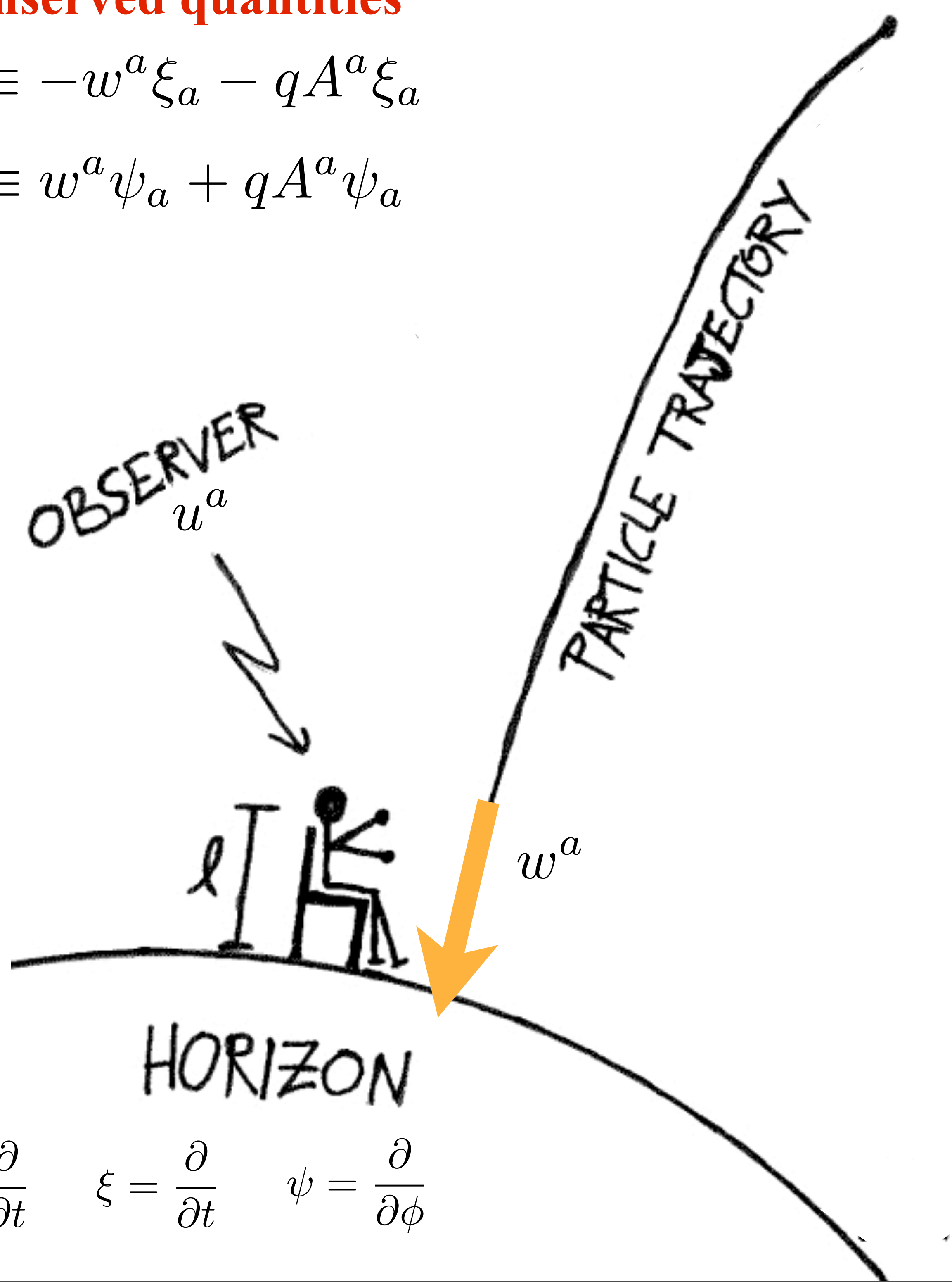

At the local observer

$$\mathcal{E}_{\ell oc} \equiv -w^a u_a \equiv \text{local energy}$$

$$\chi = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial t} \qquad \xi = \frac{\partial}{\partial t} \qquad \psi = \frac{\partial}{\partial \phi}$$

After absorption
seen from infinity

$$\chi = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial t} \qquad \xi = \frac{\partial}{\partial t} \qquad \psi = \frac{\partial}{\partial \phi}$$

$$\mathcal{E} = -w^a \xi_a|_\infty \equiv \text{energy}$$

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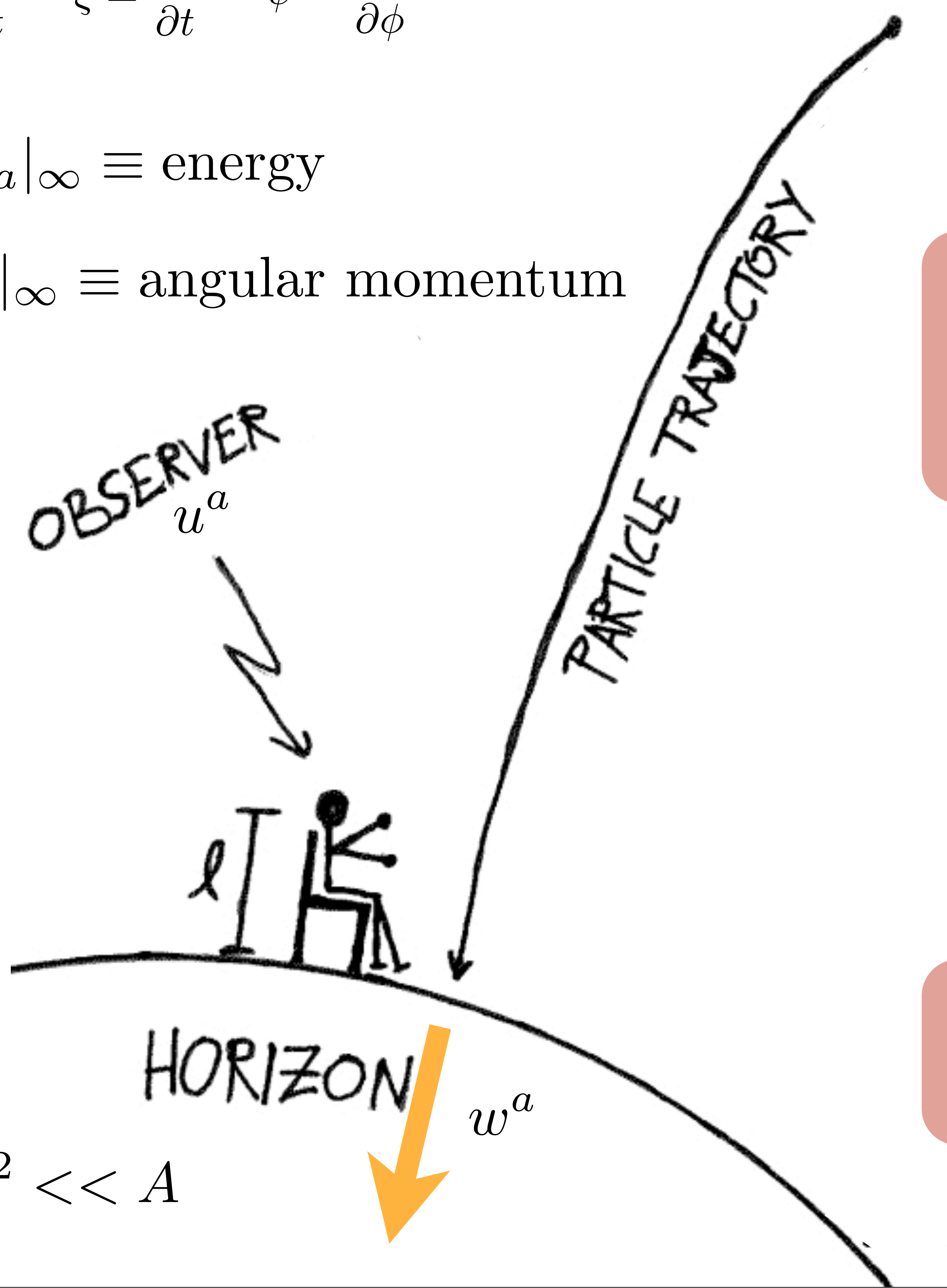


$\ell^2 << A$

The BH readjusts parameters

$$\delta M = \mathcal{E} \qquad \delta J = L$$

$$\delta Q = q$$

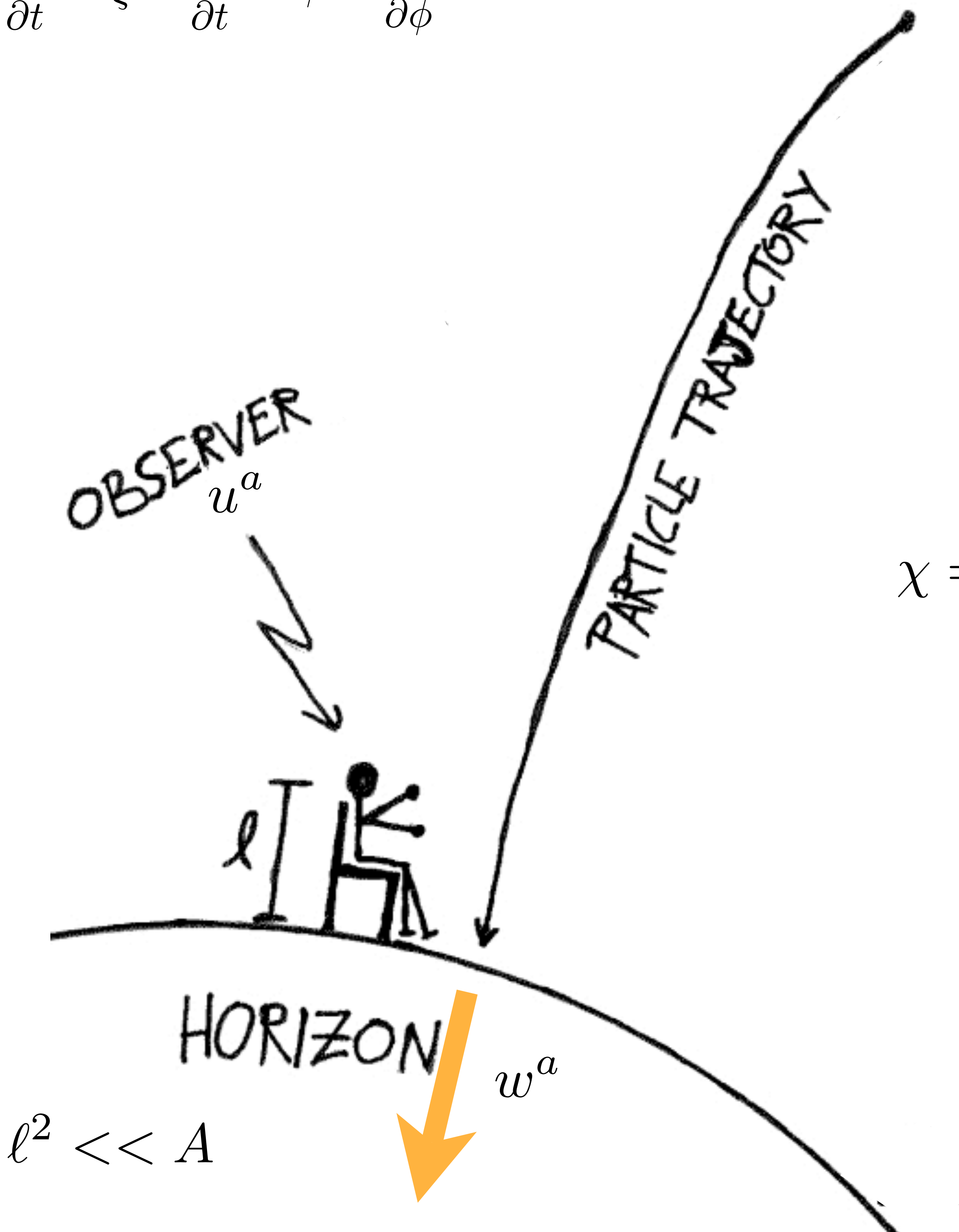
The area change from 1st law

$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega \delta J + \Phi \delta Q$$

$$\frac{\kappa}{8\pi}\delta A = \mathcal{E} - \Omega L - \Phi q$$

After absorption
seen by a local observer

$$\chi = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial t} \qquad \xi = \frac{\partial}{\partial t} \qquad \psi = \frac{\partial}{\partial \phi}$$



$$\boxed{\frac{\kappa}{8\pi}\delta A = \mathcal{E} - \Omega L - \Phi q}$$

At the local observer

$$\mathcal{E}_{\ell oc} \equiv -w^a u_a \equiv \text{local energy}$$

$$\chi = \xi + \Omega\,\psi = \partial_t + \Omega\,\partial_\phi \qquad u^a = \frac{\chi^a}{\|\chi\|}$$

$$\mathcal{E}_{\ell oc} = -\frac{w^a \xi_a + \Omega w^a \psi_a}{\|\chi\|}$$

$$\boxed{\mathcal{E}_{\ell oc} = \frac{\mathcal{E} - \Omega L - q\Phi}{\|\chi\|}}$$

After absorption
seen by a local observer

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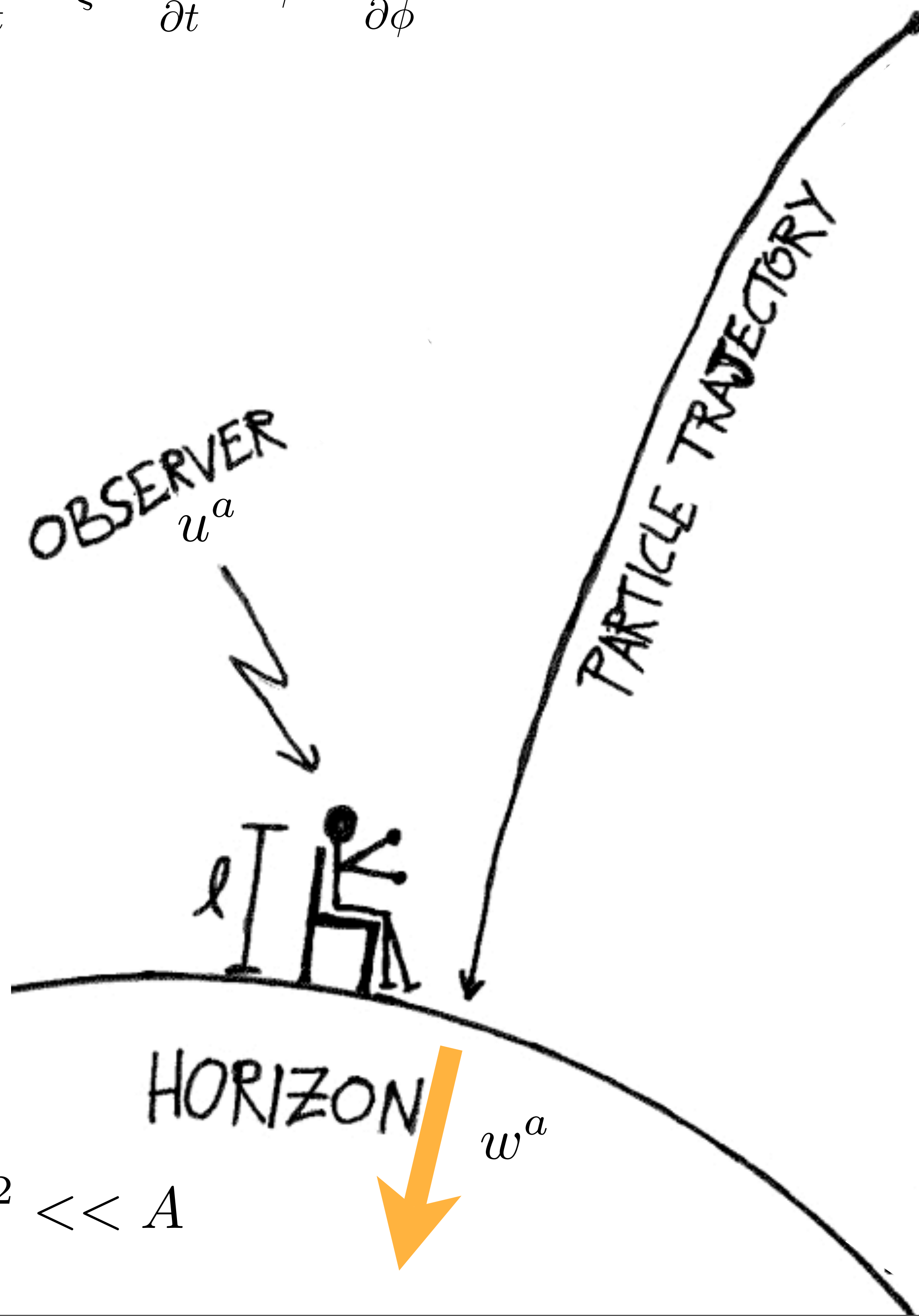


$$\ell^2 << A$$

$$\boxed{\frac{\kappa}{8\pi}\delta A = \mathcal{E} - \Omega L - \Phi q}$$

$$\boxed{\mathcal{E}_{\ell oc} = \frac{\mathcal{E} - \Omega L - q\Phi}{\|\chi\|}}$$

$$\mathcal{E}_{loc} = \frac{\kappa}{8\pi\|\chi\|}\delta A$$

$$\boxed{\mathcal{E}_{loc} = \frac{\overline{\kappa}}{8\pi}\delta A}$$

$$\overline{\kappa} \equiv \frac{\kappa}{\|\chi\|}$$

Local first law

Local BH energy

$$\mathcal{E}_{\ell oc} \equiv -w^a u_a \equiv \text{local energy of the absorbed particle}$$

The appropriate local energy notion must be the one such that:

$$\delta E = \mathcal{E}_{\ell oc}$$

$$\boxed{\delta E = \frac{\overline{\kappa}}{8\pi}\delta A}$$

$$\chi = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial t} \qquad \xi = \frac{\partial}{\partial t} \qquad \psi = \frac{\partial}{\partial \phi}$$

Local first law

A refined argument

$J^a = \delta T^a{}_b \chi^b$ is conserved thus

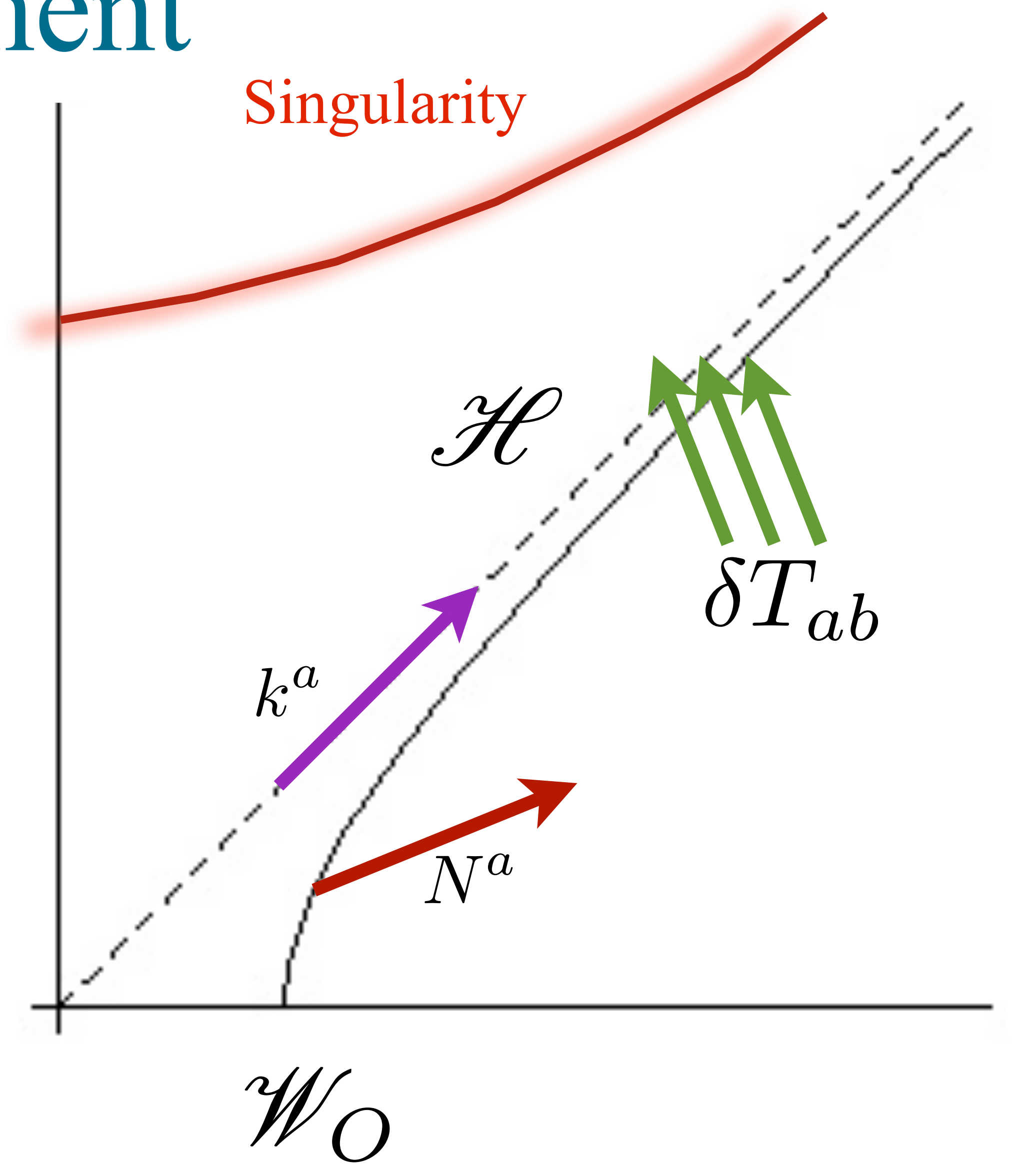
$$\int_{\mathscr{H}} dV dS \; \delta T_{ab} \chi^a k^b = \int_{W_{\mathscr{O}}} J_b N^b$$

$$\int_{\mathscr{H}} dV dS \; \delta T_{ab} \underbrace{\kappa V k^a}_{\chi^a} k^b = \int_{W_{\mathscr{O}}} \| \chi \| \, \delta T_{ab} u^a N^b$$

The Raychaudhuri equation

$$\frac{d\theta}{dV} = -8\pi \delta T_{ab} k^a k^b$$

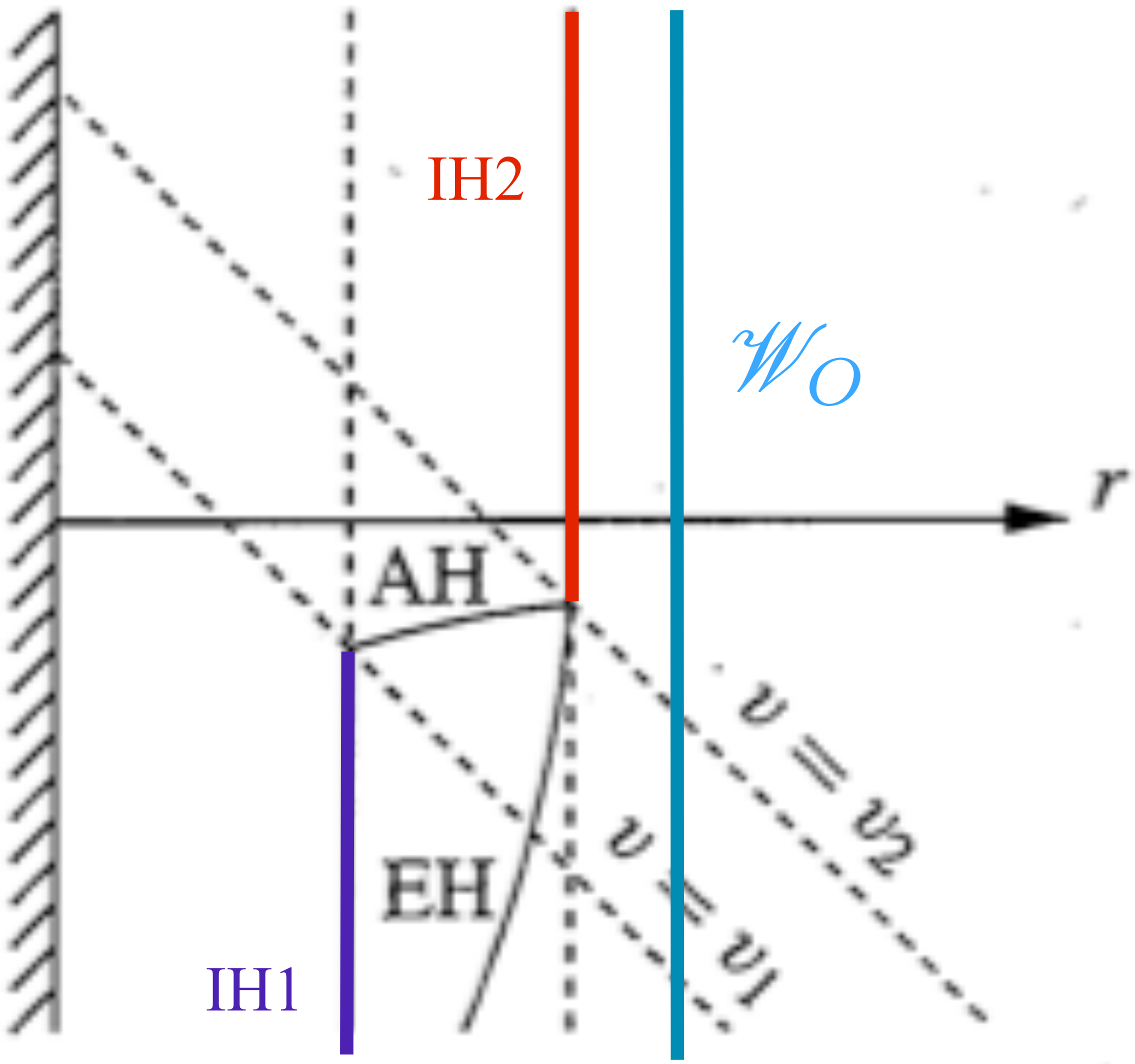
$$\int_{\mathscr{H}} dV dS \; V \frac{d\theta}{dV} = -\frac{8\pi \| \chi \|}{\kappa} \delta E,$$

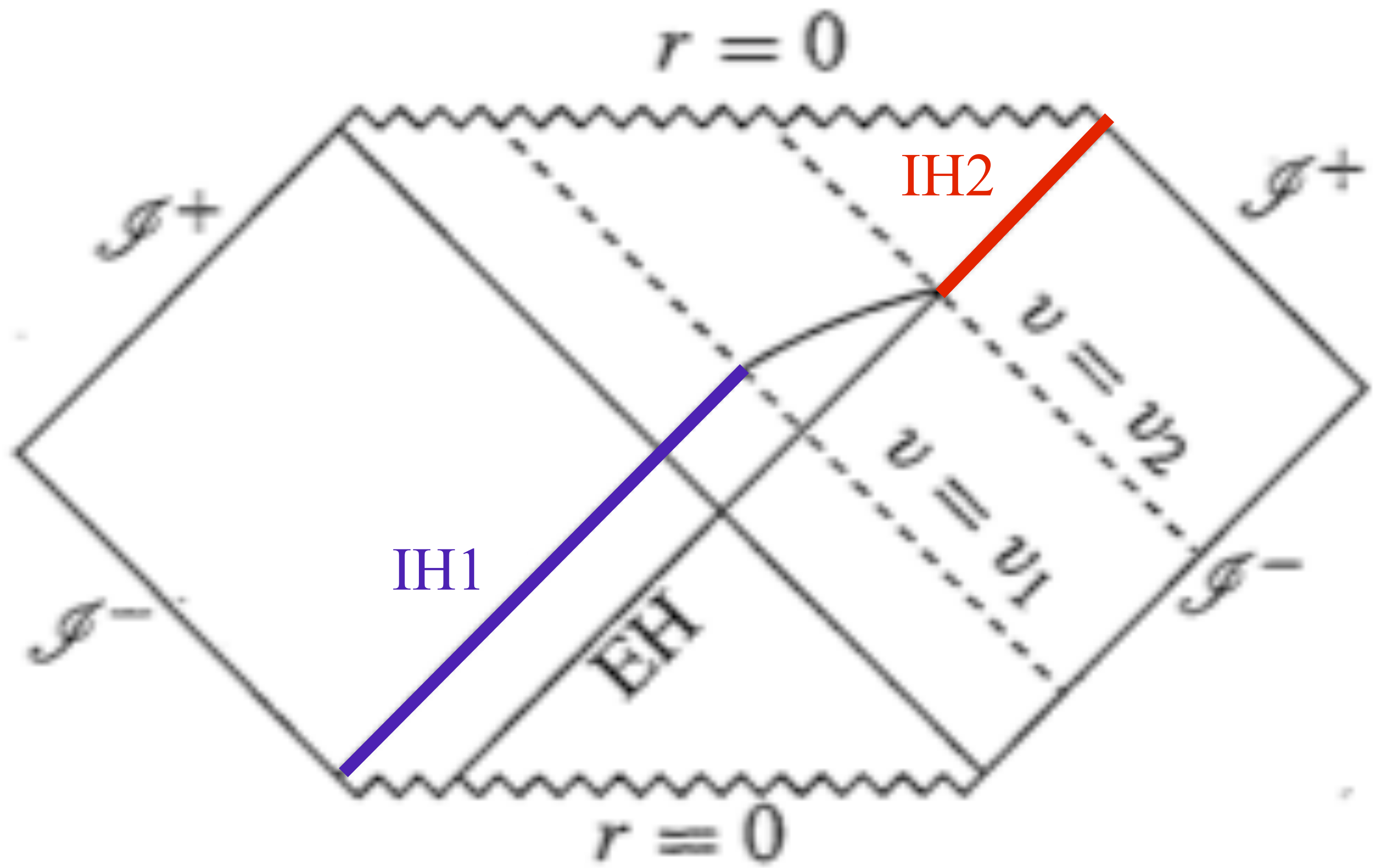


$$\delta E = \frac{\kappa}{8\pi} \delta A$$

The Local first law is dynamical

Simple example: Vaidya spacetime





The same holds in non symmetric situations (detailed proof in progress AP, O. Moreschi, E. Gallo)

$$\delta E = \frac{\overline{\kappa}}{8\pi}\delta A$$

Local first law
Main result

$\ell^2 << A$

$a = ||u^a \nabla_a u^b|| = \frac{1}{\ell}$

$$\delta M = \frac{\kappa}{8\pi} \delta A + \Omega \, \delta J + \Phi \, \delta Q,$$

$$\delta E = \frac{\overline{\kappa}}{8\pi} \delta A$$

$$\overline{\kappa} \equiv \frac{\kappa}{||\chi||} = \frac{1}{\ell} + o(\ell)$$

$$E = \frac{A}{8\pi\ell}$$

Quantization

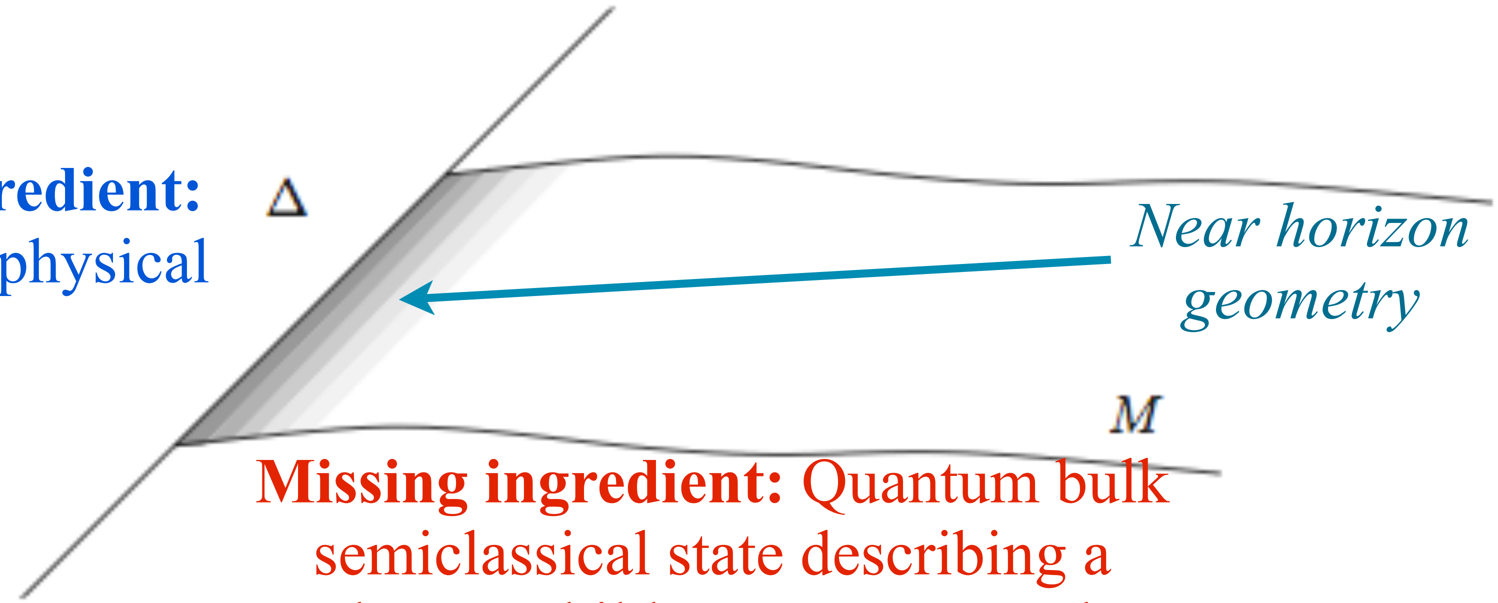
Chern-Simons theory with spin-network punctures

Alesci, Agullo, Ashtekar, Baez, Barbero, Bianchi, Borja, Corichi, Diaz-Polo, Domagala, Engle, Frodden, Ghosh, Krasnov, Kaul, Lewandowski, Livine, Majumdar, Meissner, Mitra, AP, Pranzetti, Rovelli, Sahlmann, Terno, Thiemann, Villasenor, etc.

An important ingredient

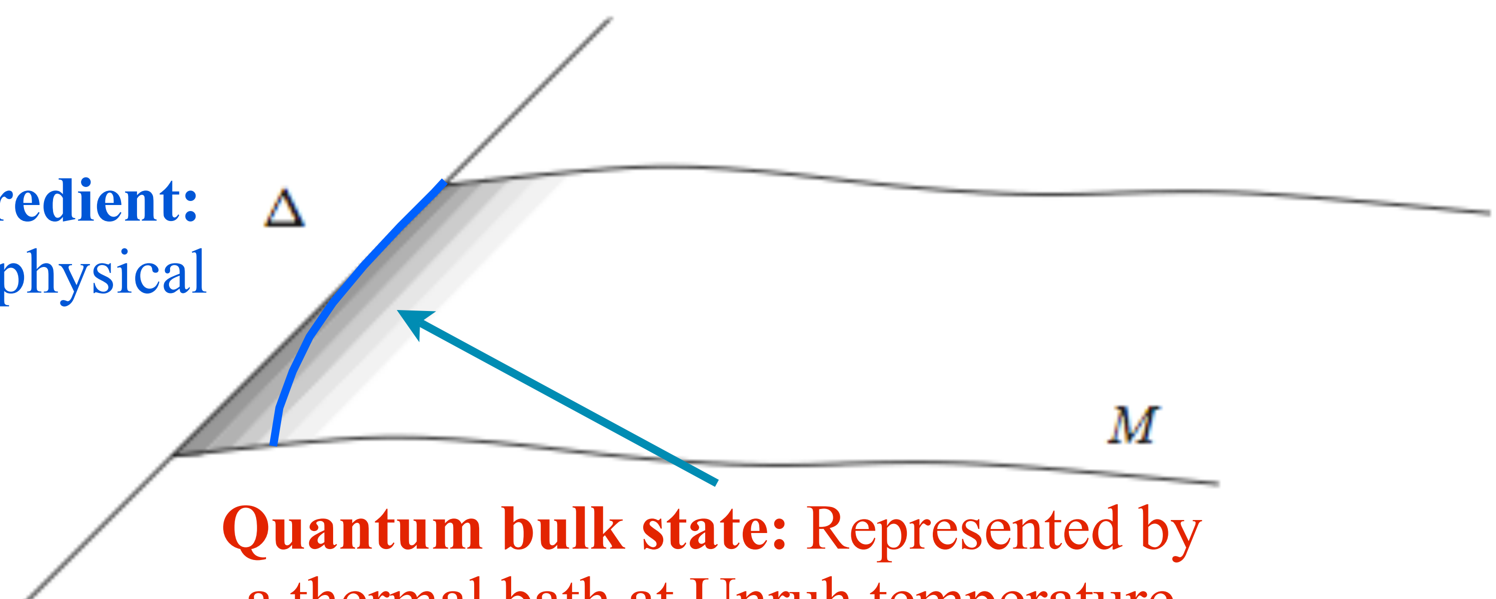
$$T_U = \frac{1}{2\pi\ell}$$

Present ingredient: Quantum IH physical state



Missing ingredient: Quantum bulk semiclassical state describing a Schwarzschild geometry near the horizon

Present ingredient: Quantum IH physical state



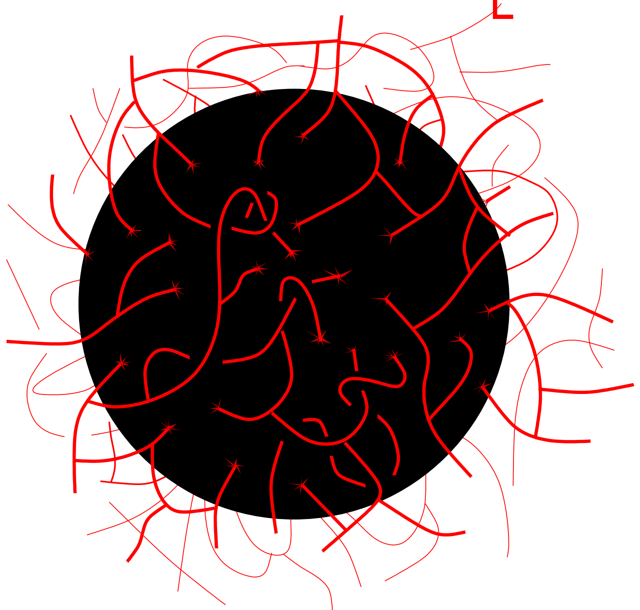
Quantum bulk state: Represented by a thermal bath at Unruh temperature as seen by stationary observers.

$\ell \equiv$ arbitrary fixed proper distance to the horizon

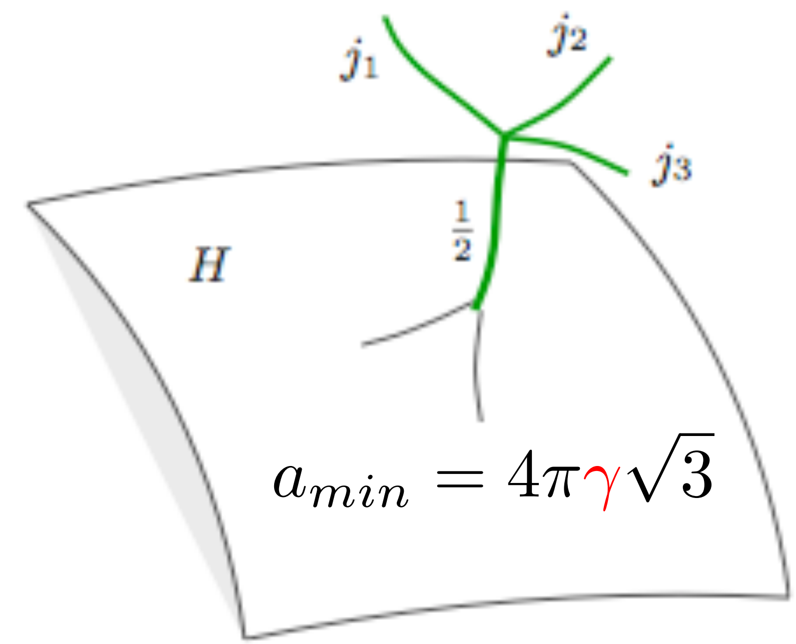
Is the number of punctures an important observable?

The area gap

$$\widehat{A}_S\,|j_1, j_2\cdots\rangle = \left[8\pi\gamma\ell_p^2 \sum_p \sqrt{j_p(j_p+1)}\right] |j_1, j_2\cdots\rangle$$



The AREA gap and BH quantum transitions



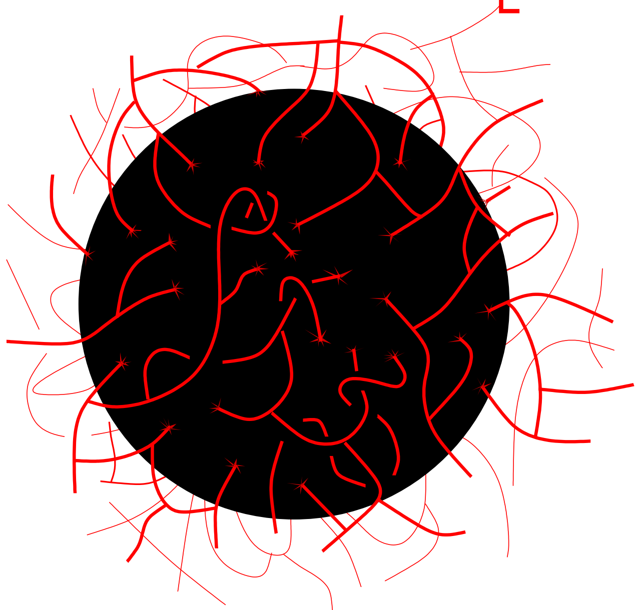
a) By a rearrangement of the spin quantum numbers labelling spin network links ending at punctures on the horizon without changing the number of punctures N (in the large area regime this kind of transitions allows for area jumps as small as one would like as the area spectrum becomes exponentially dense in $\mathbb{R}^+$ [Rovelli 96]

b) By the emission or absorption of punctures with arbitrary spin (such transitions remain discrete at all scales and are responsible for a modification of the first law: a chemical potential arises and encodes the mean value of the area change in the thermal mixture of possible values of spins j).

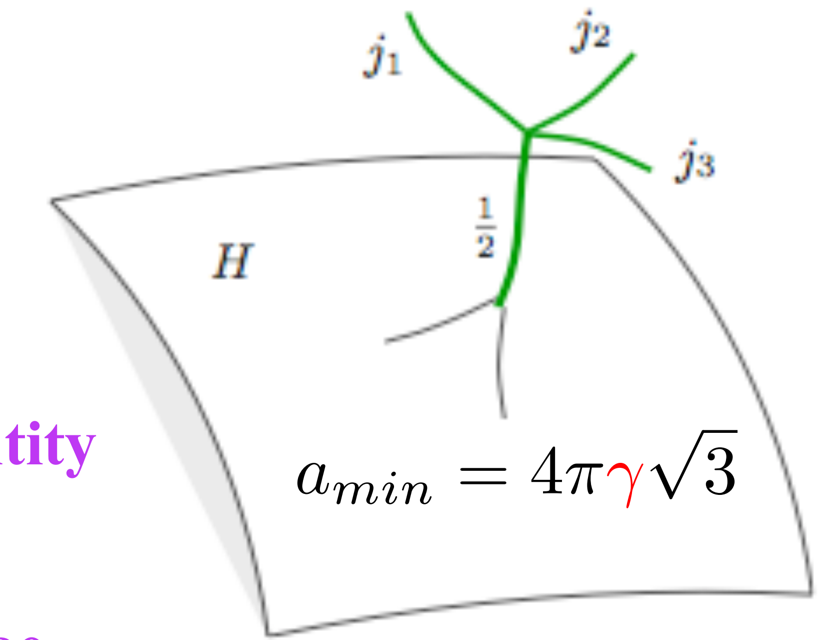
Is the number of punctures an important observable?

The area gap

$$\widehat{H}|j_1, j_2\cdots\rangle = \left[\frac{\gamma\ell_p^2}{\ell}\sum_p \sqrt{j_p(j_p+1)}\right]|j_1, j_2\cdots\rangle$$



The scale \ell is a fiducial quantity (a regulator)
The regulator is natural: York 1983, Hajicek-Israel 1980.



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Is the number of punctures an important observable?

So far people assumed it is not!

The usual LQG calculation was performed in the microcanonical ensemble (with an implicit assumption of a vanishing chemical potential). The number of states $d[\{s_j\}]$ associated with a configuration $\{s_j\}$ is

$$d[\{s_j\}] = \Big(\sum_k s_k\Big)! \prod_j \frac{(2j+1)^{s_j}}{s_j!}.$$

$$\boxed{S = \frac{\gamma_0}{\gamma} \frac{A}{4\ell_p^2}}$$

with γ_0 solution of

$$\log[e^{-\sigma} \sum_j (2j+1) e^{-2\pi\gamma_0\sqrt{j(j+1)}}] = 0 \qquad \gamma_0 = 0.274067....$$

Agullo-Barbero-Borja-DiazPolo-Villasenor, Meissner, Domagala-Lewandowski

but this is inconsistent with

$$\delta M = \frac{\kappa}{2\pi}\,\delta A + \Omega\,\delta J + \Phi\,\delta Q$$

unless $\gamma = \gamma_0$.

Is the number of punctures an important observable?

N is observable! (Ghosh-AP 2011)

The canonical partition function is given by

K. Krasnov (1999), S. Major (2001), F. Barbero E. Villasenor (2011)

$$Z(N,\beta) = \sum_{\{s_j\}} \prod_j \frac{N!}{s_j!}\,[(2j+1)]^{s_j} e^{-\beta s_j E_j} \qquad \Longrightarrow \qquad \log Z = N \log[\sum_j [(2j+1)] e^{-\beta E_j}]$$

where $E_j = \ell_g^2 \sqrt{j(j+1)}/\ell$. A simple calculation gives For the entropy we get

$$S = -\,\beta^2 \frac{\partial}{\partial\beta}\left(\frac{1}{\beta}\log Z\right)\Big|_{\beta=2\pi\ell} = \frac{A}{4\ell_p^2} + \log Z$$

more precisely

$$\boxed{S = \frac{A}{4\ell_p^2} + \sigma(\gamma)N}$$

where

$$\boxed{\sigma(\gamma) \equiv \log[\sum_j (2j+1) e^{-2\pi\gamma\sqrt{j(j+1)}}].}$$

The (thermodynamical) local first law versus the (geometric) local first law

$$\delta M = \frac{\kappa}{2\pi}\,\delta S + \Omega\,\delta J + \Phi\,\delta Q + \mu\,\delta N \qquad \Longleftrightarrow \qquad \delta M = \frac{\kappa}{2\pi}\,\delta A + \Omega\,\delta J + \Phi\,\delta Q$$

$$\mu = -T \frac{\partial S}{\partial N}|_A = -\frac{\kappa}{2\pi}\sigma(\gamma)$$

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more precisely

$$\boxed{S = \frac{A}{4\ell_p^2}\left(1 - \frac{\sigma(\gamma)}{\gamma \frac{d\sigma}{d\gamma}}\right) \qquad \text{from EOS} \qquad \langle E \rangle = -\frac{\partial}{\partial \beta} \log Z\Big|_{\beta=2\pi\ell} \qquad \Longleftrightarrow \qquad N = \frac{-A}{4\ell_p^2 \gamma \frac{d\sigma}{d\gamma}}}$$

The (thermodynamical) local first law versus the (geometric) local first law

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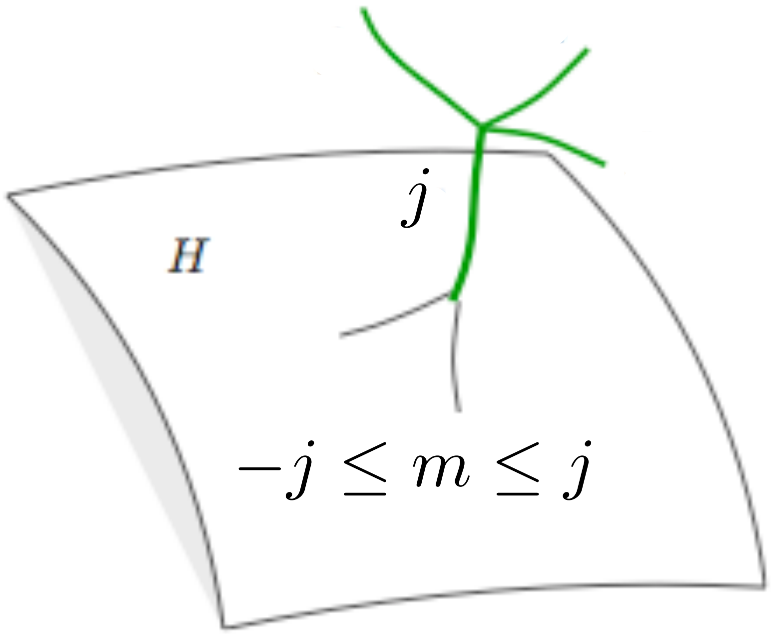
$$\mu = -T \frac{\partial S}{\partial N}|_A = -\frac{\kappa}{2\pi} \sigma(\gamma)$$

Future...

What about matter?

matter d.o.f. do not contribute to energy!

$$\boxed{\widehat{H}|j_1, j_2 \cdots\rangle = \left[\frac{\gamma \ell_p^2}{\ell} \sum_p \sqrt{j_p(j_p+1)}\right] |j_1, j_2 \cdots\rangle}$$



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$$\log Z = N \log[\sum_j [(2j+1)] e^{-\beta E_j}]$$

$$S = -\,\beta^2 \frac{\partial}{\partial \beta}(\frac{1}{\beta}\log Z)\Big|_{\beta=2\pi\ell} \quad \Longleftrightarrow \quad S = \frac{A}{4\ell_p^2} + \sigma[\gamma] N$$

where

$$\sigma[\gamma] \equiv \log[\sum_j (2j+1) e^{-2\pi\gamma\sqrt{j(j+1)}}].$$

What about matter?

matter behaves as if at *infinite temperature*!

$$\boxed{\widehat{H}(\ell)|j_1, j_2 \cdots \rangle = [\frac{\gamma_* G_*}{\ell G(\ell)} \sum_p \sqrt{j_p(j_p+1)}]|j_1, j_2 \cdots \rangle}$$

where γ_*, G_* are the UV values. The canonical partition function is given by

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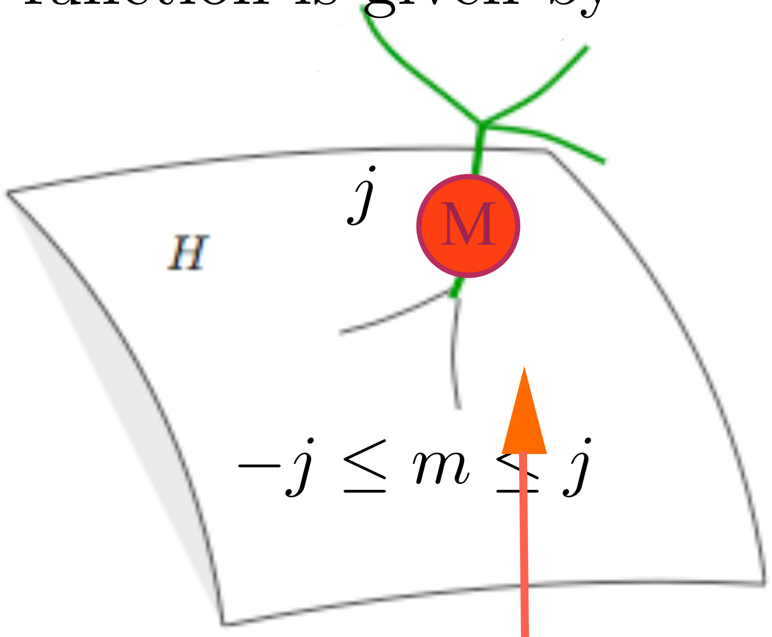

where $g_M = g_M[j, \gamma, \cdots; \ell]$ matter degeneracy.

$$S = -\beta^2 \frac{\partial}{\partial \beta}(\frac{1}{\beta} \log Z)\Big|_{\beta = 2\pi\ell} \qquad \Longleftrightarrow \qquad S = \frac{A}{4G(\ell)} + \Sigma[\gamma, \cdots; \ell] N$$

where

$$\Sigma[\gamma, \cdots; \ell] = \log[\sum_j (2j+1) g_M e^{-2\pi \frac{\gamma_* G_*}{G(\ell)} \sqrt{j(j+1)}}].$$

The puncture is *dressed by matter d.o.f.*

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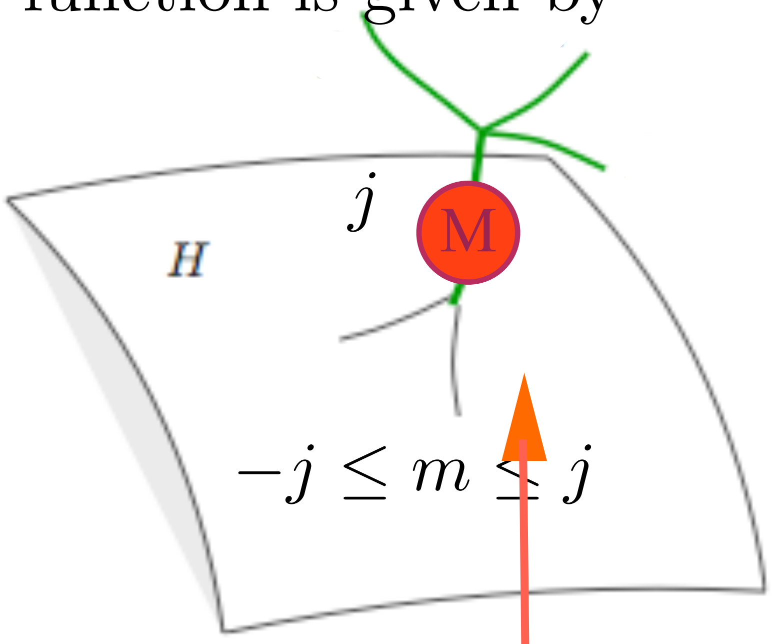

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Fixed dynamically: *Hamiltonian constraint!!*

(see Pranzetti)

What about matter?

we can get the general form of BH entropy

At thermal equilibrium the average energy $\langle E \rangle = -\frac{\partial}{\partial \beta} \log Z$ One can obtain a thermal equilibrium relation between the number of punctures to the area

$$N = \frac{A}{4G(\ell) a(\gamma, \cdots; \ell)}.$$

where we have defined

$$a = \frac{2\pi \frac{\gamma_* G_*}{G(\ell)} \sum_j (2j+1) g_M \sqrt{j(j+1)} e^{-2\pi \frac{\gamma_* G_*}{G(\ell)} \sqrt{j(j+1)}}}{\sum_j (2j+1) g_M e^{-2\pi \frac{\gamma_* G_*}{G(\ell)} \sqrt{j(j+1)}}}.$$

We rewrite the entropy in the Sackur-Tetrode form

$$\boxed{S = \frac{A}{4G(\ell)} \left(1 + \frac{\Sigma[\gamma, \cdots; \ell]}{a(\gamma, \cdots; \ell)} \right).}$$

The scaling of BH entropy

Holography and renormalization condition

$$\log(Z) \equiv N \log[\sum_j (2j+1) g_{M} \exp(-\frac{a_j}{4\ell_p^2})]$$

Previous quantities are well defined as long as

$$\boxed{g_{M} < \exp\left(\frac{a_j}{4G(\ell)}\right).}$$

Therefore, in view of the previous physical argument we assume that

$$g_M(j,...,\gamma,\ell) = \exp\left(\frac{a_j}{4G(\ell)}\right) \times \left[g_m^0(j,...,\gamma) + g_m^1(j,...,\gamma)\frac{G(\ell)}{\ell^2} + \cdots\right],$$

$$S = \frac{A}{4G(\ell)}\left[1 + \frac{G(\ell)}{\gamma_* G_* a_0}\left(\log z_0 + \frac{G(\ell)}{\ell^2}K + \cdots\right)\right]$$

$$\bar{\mu} = -\frac{\log z_0}{2\pi\ell} - \frac{G(\ell)}{\ell^3}\frac{z_1}{2\pi} + \cdots$$

The scaling of BH entropy
Holography and renormalization condition

There are now three ways to get Hawking entropy in the low energy limit

1. First, $a_0\gamma_* G_* \gg G(\ell_{IR})$, and $\ell^2 \gg G(\ell)$. This can happen in more than one ways:
   (a) If gravity in the infrared is much weaker than in the UV, namely $G_* \gg G(\ell_{IR})$ and $a_0\gamma_* \sim o(1)$.
   (b) two $G_* \lesssim G(\ell_{IR})$ but $a_0\gamma_* \gg 1$
2. Second if $z_0 = 1$ or more explicitly

$$1 = \sum_j (2j+1) g_M^0(j, \cdots).$$

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2. Second if $z_0 = 1$ or more explicitly

$$1 = \sum_j (2j+1) g^0_M(j, \cdots).$$

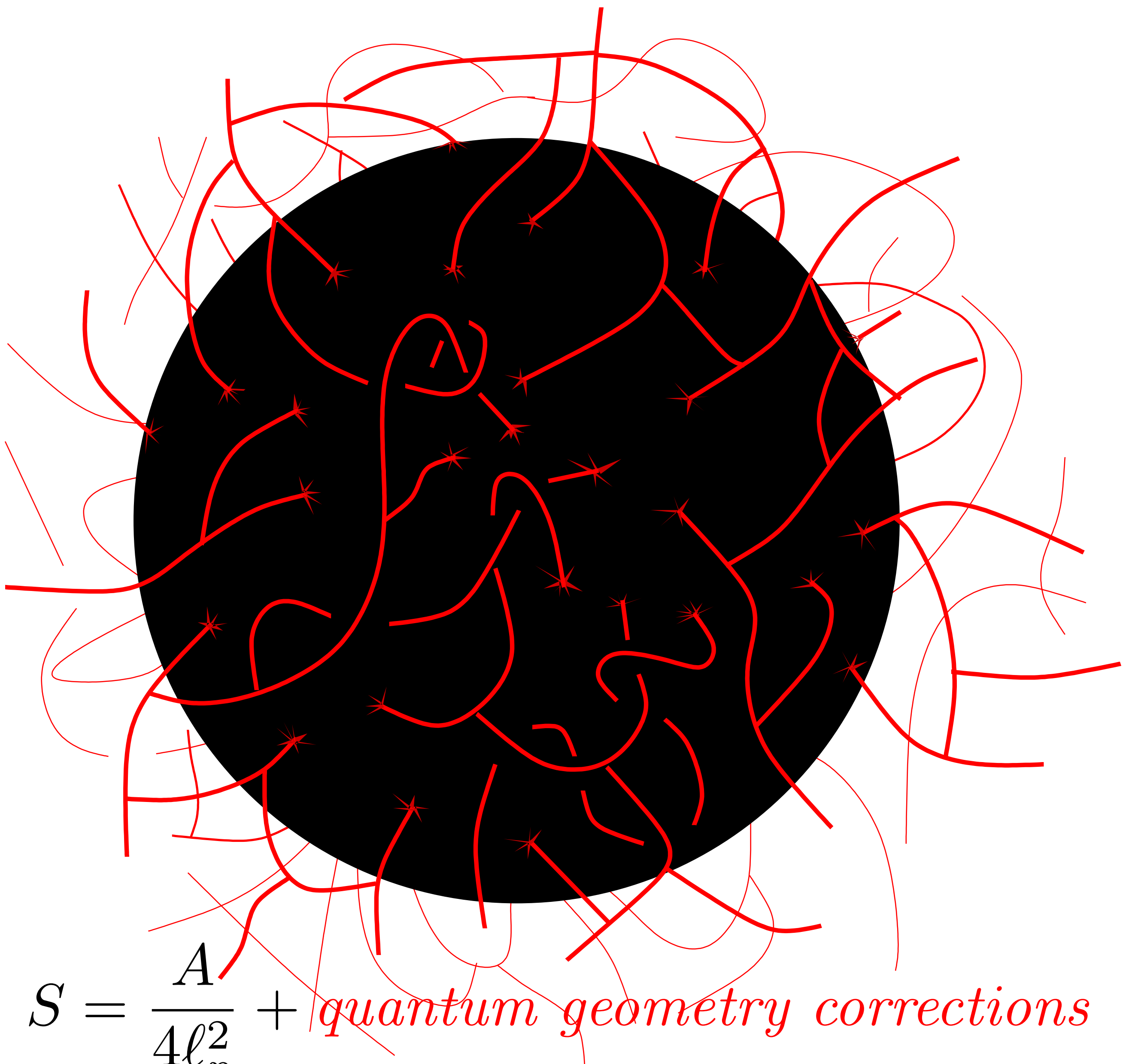
The usual condition on the Immirzi parameter now involves other couplings

The scaling of BH entropy

The puncture chemical potential

In the UV ($\ell \approx \ell_p$) punctures are a relevant.
This is reflected in a nonvanishing chemical potential and
a large deviation from Bekenstein-Hawking entropy.

$\mu \neq 0$



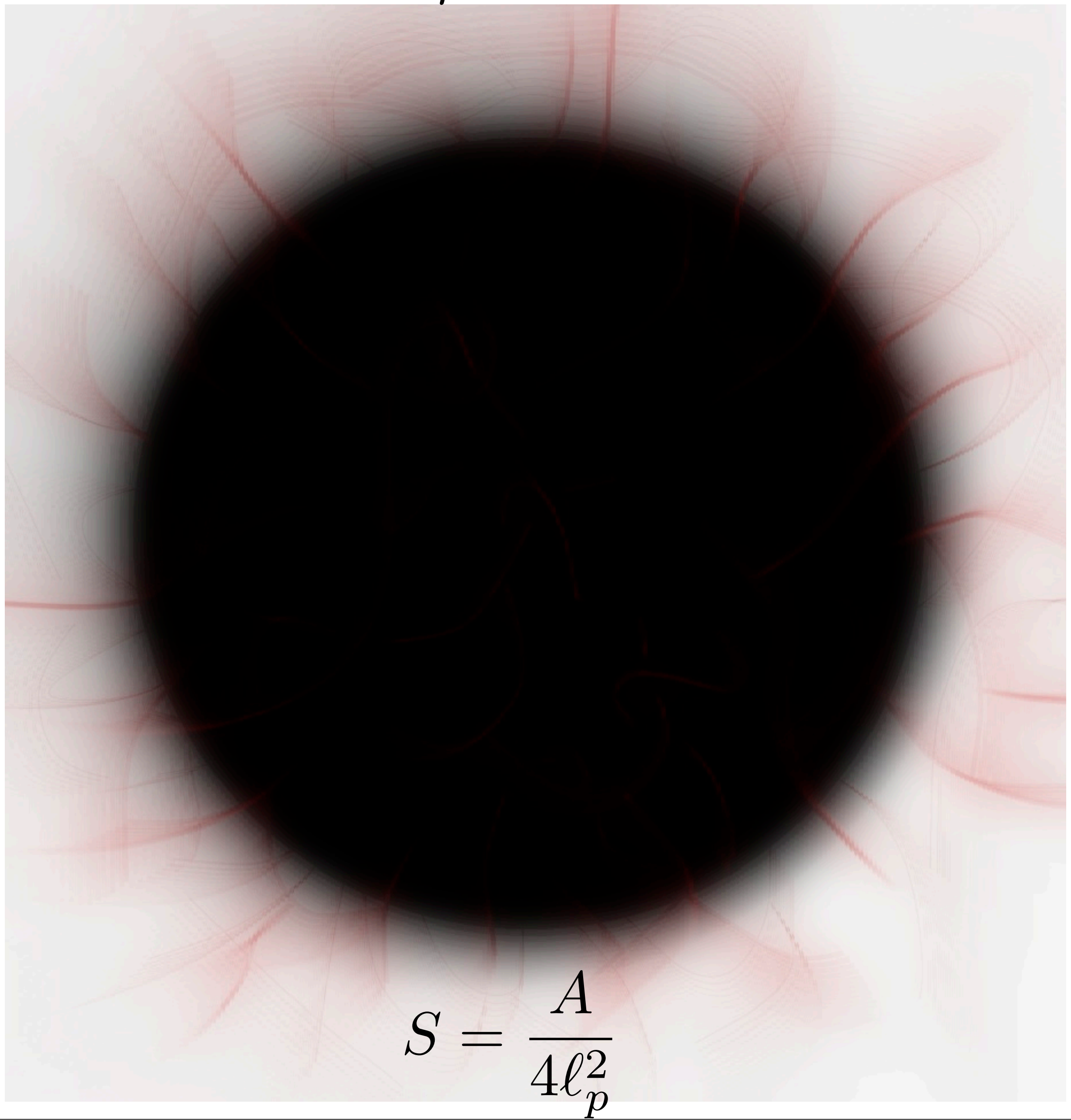
$$S = \frac{A}{4\ell_p^2} + \textit{quantum geometry corrections}$$

The scaling of BH entropy

The puncture chemical potential

In the IR ($\ell >> \ell_p$) punctures are no longer relevant (continuum limit).
This is reflected in a vanishing chemical potential.
BH entropy matches Bekenstein-Hawking entropy.

$\mu = 0$



Conclusions

- A local definition is needed which corresponds to large semiclassical BHs: Isolated horizons [Ashtekar et al.] provides a suitable boundary condition.

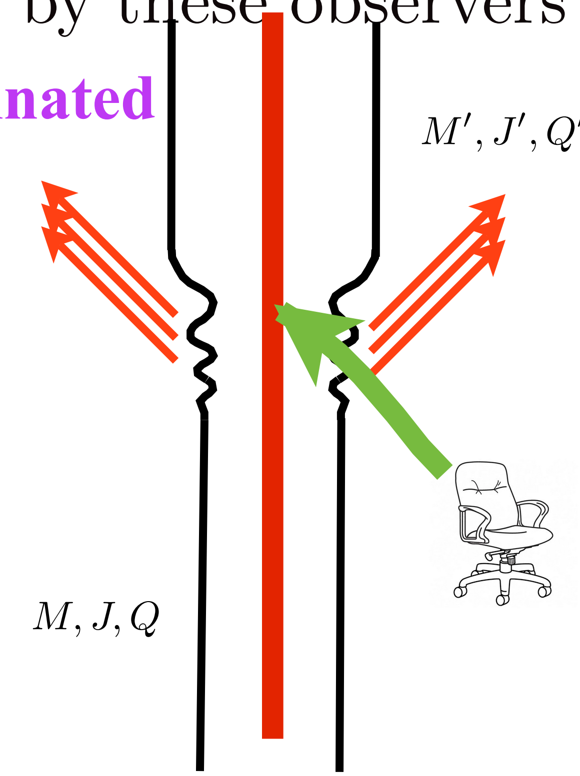
- Yet a little bit more (near horizon geometry) is necessary for dealing with BH thermodynamics.

- New [E. Frodden, A. Ghosh and AP (arXiv:1110.4055)]: A preferred notion of stationary observers can be introduced. These are the suitable observers for local thermodynamical considerations. There is:

  1. a unique notion of energy of the system described by these observers $E = A/(8\pi\ell)$. **Degeneracy of energy notion is eliminated**
  2. a universal surface gravity $\overline{\kappa} = 1/\ell$.
  3. and they are related by a local fist law

$$\delta E = \frac{\overline{\kappa}}{8\pi\ell}\delta A.$$

  4. The first law is of a dynamical nature.



- In progress [E. Wilson-Ewin, AP, D. Forni]: a first law for Rindler Horizons holds

- The area gap of LQG=an energy gap in the local formulation.

- New [A. Ghosh and AP (PRL **107** 2011)]: The entropy computation yields and entropy formula that is consistent with Hawking semiclassical calculations for all values of the Immirzi parameter γ:

$$\delta M = \frac{\kappa}{2\pi}\delta S + \Omega\delta J + \Phi\delta Q + \mu\delta N \qquad \Longleftrightarrow \qquad \delta M = \frac{\kappa}{2\pi}\,\delta A + \Omega\,\delta J + \Phi\,\delta Q$$

- If one ignores matter degrees of freedom

$$S = \frac{A}{4\ell_p^2} + \sigma(\gamma)N \qquad \mu = \frac{\kappa}{2\pi}\sigma(\gamma)$$

where $\sigma(\gamma) = \log[\sum_j (2j+1)\exp\left(-2\pi\gamma\sqrt{j(j+1)}\right)]$. No scaling appart from possible running of γ [Speziale et al.] If one fixes $\gamma = \gamma_0$ then $\mu = 0$!

- If one does not ignore matter degrees of freedom

$$S = \frac{A}{4\ell_p^2} + \Sigma[\gamma, \cdots; \ell]N \qquad \Longleftrightarrow \qquad S = \frac{A}{4\ell_p^2}\left[1 + \frac{\Sigma[\gamma, \cdots; \ell]}{a[\gamma, \cdots; \ell]}\right]$$

where $\Sigma[\gamma, \cdots; \ell] = \log[\sum_j (2j+1) g_{M} \exp\left(-2\pi\gamma\sqrt{j(j+1)}\right)]$.

- Non trivial scaling.
- Convergence requires micro-holography $g_{M} < \exp a_j/(4\ell_p^2)$
- Recovering Bekenstein-Hawking entropy is a RC involving many couplings.

Thank you very much!

Entropy calculation

The old view

The usual LQG calculation was performed in the microcanonical ensemble (with an implicit assumption of a vanishing chemical potential) and gives

$$S = \frac{\gamma_0}{\gamma}\frac{A}{4\ell_p^2} = \frac{\gamma_0}{\gamma}\frac{2\pi\ell}{\ell_p^2}E \qquad (1)$$

while semiclassical considerations (Hawking radiation) imply that

$$T_U^{-1} = \frac{\partial S}{\partial E} = \frac{2\pi}{\overline{\kappa}\ell_p^2} \qquad (2)$$

Thermal equilibrium at Unruh temperature is achieved only if the *Immirzi parameter* is fine tuned according to

$$\gamma = \gamma_0 = 0.274067... \qquad (3)$$

$$\sigma(\gamma) = \log[\sum_{j=1/2}^{\infty}(2j+1)\exp -2\pi\gamma\sqrt{j(j+1)}] \qquad (4)$$