UV behavior of half-maximal supergravity theories.

Piotr Tourkine,

Quantum Gravity in Paris 2013,

LPT Orsay

In collaboration with Pierre Vanhove,
based on 1202.3692, 1208.1255

Understand the pertubative structure of supergravity theories.

- Supergravities are theories of gravity with local supersymmetry.

- Those theories naturally arise in the **low energy limit** of superstring theory.

- String theory is then a UV completion for those and thus provides a good framework to study their UV behavior.

  → Maximal and half-maximal supergravities.

Maximal supergravity

- Maximally extended supergravity:
  - Low energy limit of type IIA/B theory,
  - 32 real supercharges, unique (ungauged)
  - N=8 in d=4
- Long standing problem to determine if maximal supergravity can be a consistent theory of quantum gravity in d=4.
- Current consensus on the subject : it is not UV finite, the first divergence could occur at the 7-loop order.

- Impressive progresses made during last 5 years in the field of scattering amplitudes computations.

[Bern, Carrasco, Dixon, Dunbar, Johansson, Kosower, Perelstein, Rozowsky etc.]

Half-maximal supergravity

- Half-maximal supergravity:
  - Heterotic string, but also type II strings on orbifolds
  - 16 real supercharges,
  - N=4 in d=4
- Richer structure, and still a lot of SUSY so explicit computations are still possible.
- There are UV divergences in d=4, [Fischler 1979] at one loop for external matter states
- UV divergence in gravity amplitudes ? As we will see the divergence is expected to arise at the four loop order.

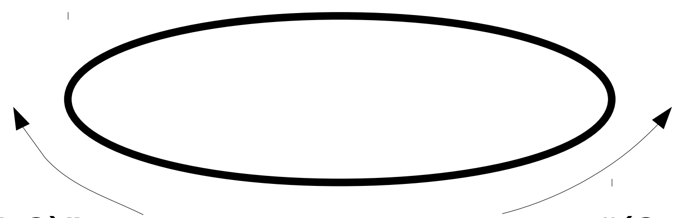
String models that give half-maximal supergravity

- Type IIA/B string = Superstring ⊗ Superstring

Torus compactification : preserves full (4,4) supersymmetry.

Low energy limit: N=8 supergravity

String models that give half-maximal supergravity



- Type IIA/B string = Superstring ⊗ Superstring

Asymmetric orbifold

Symmetric orbifold

Low energy limit: (4,0) supergravity coupled to n_v vector multiplets

Low energy limit: (2,2) supergravity coupled to n_v vector multiplets

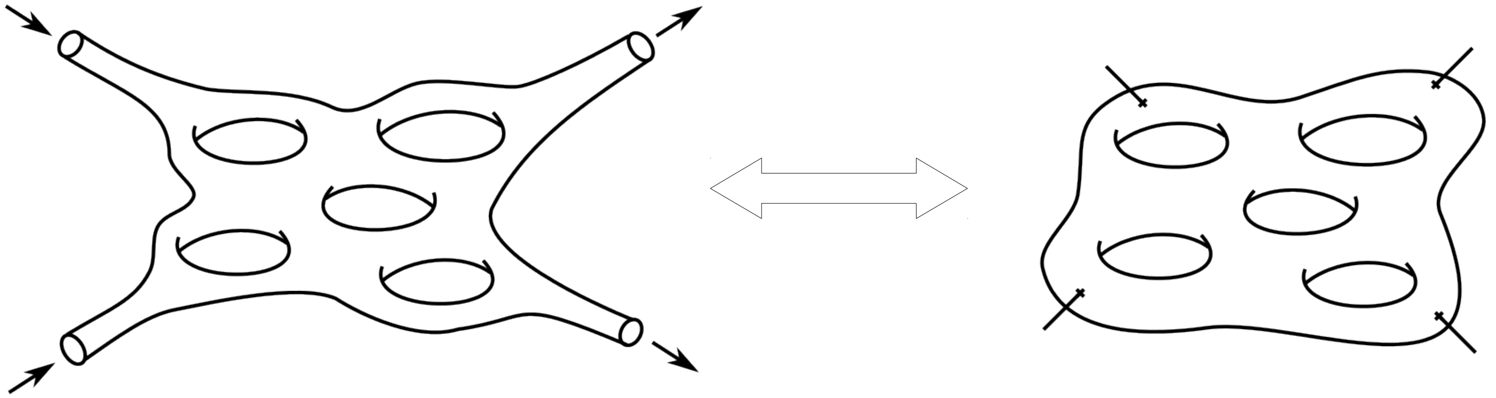
String models that give half-maximal supergravity

- Heterotic string = Superstring ⊗ Bosonic string

Torus

Low energy limit: (4,0) supergravity coupled to n_v vector multiplets

Superstring scattering amplitudes in the R.N.S. formalism



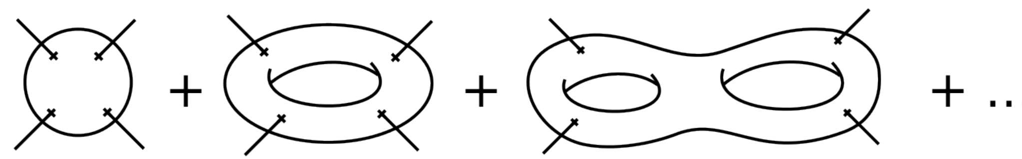
$$\mathcal{A} = \int_{s\mathcal{M}_{g,n}} \langle \prod_i \mathcal{O}_i \rangle$$

Vertex operators
= external states

Supermoduli space of super Riemann surfaces

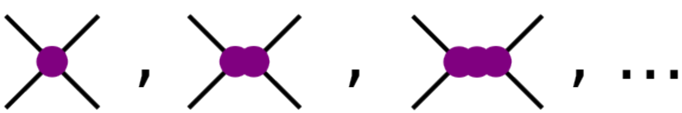
Links between string theory and UV divergences of field theory

String theory amplitude

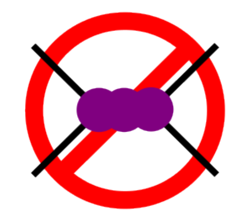


$$\alpha' = \ell_s^2 \to 0$$

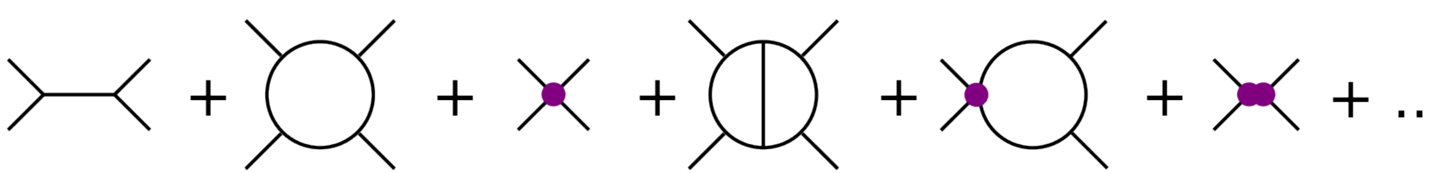
Direct extraction of the UV divergence



Non-renormalisation theorem: direct vanishing of a UV divergence

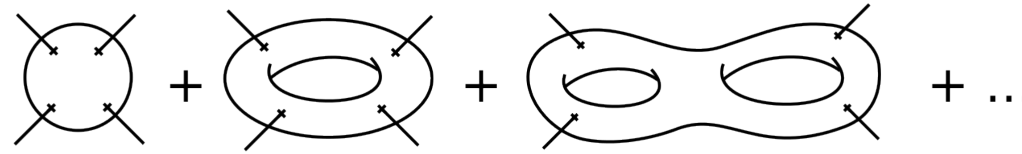


Extraction of the full amplitude with its UV divergences, in the worldline form



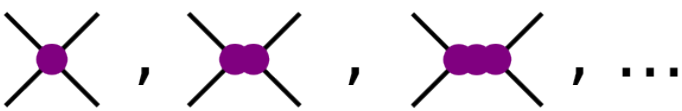
Links between string theory and UV divergences of field theory

String theory amplitude

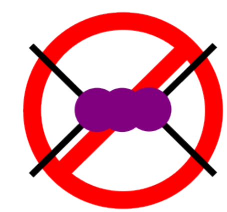


$$\alpha' = \ell_s^2 \to 0$$

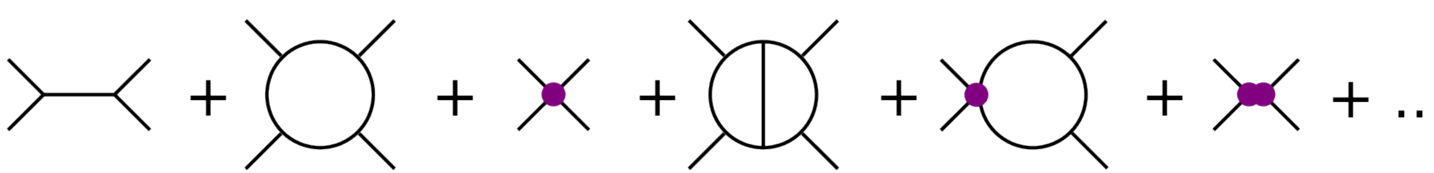
Direct extraction of the UV divergence



Non-renormalisation theorem: direct vanishing of a UV divergence

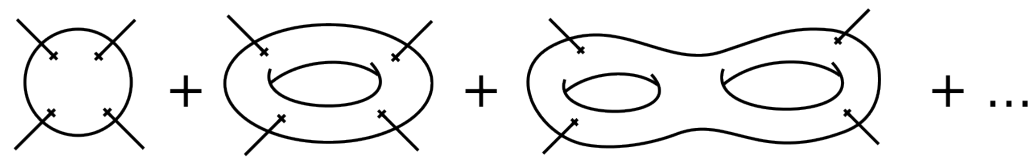


Extraction of the full amplitude with its UV divergences, in the worldline form



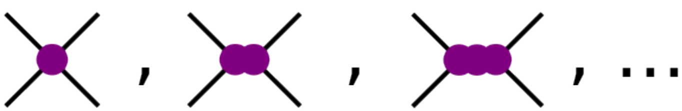
Links between string theory and UV divergences of field theory

String theory amplitude

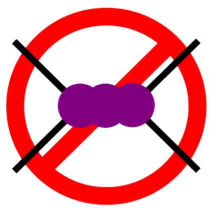


$$\alpha' = \ell_s^2 \to 0$$

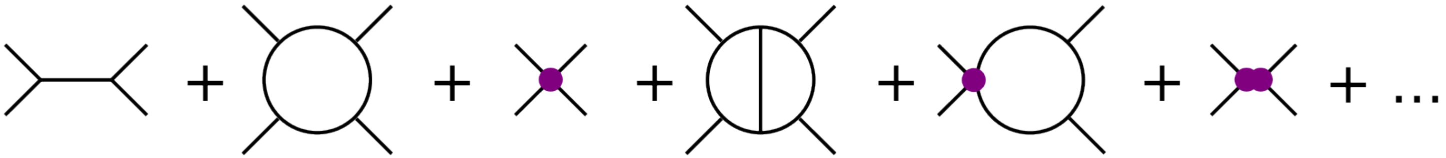
Direct extraction of the UV divergence



Non-renormalisation theorem: direct vanishing of a UV divergence



Extraction of the full amplitude with its UV divergences, in the worldline form



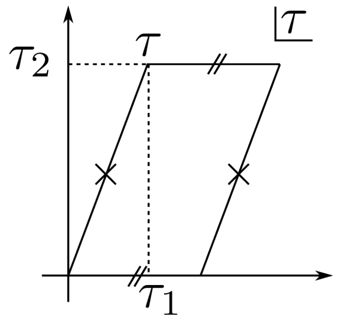
Computation of field theory amplitude : a one loop example.

$$\mathcal{A} = \int_{\mathcal{M}_{g=1,n}} \langle \prod_i \mathcal{O}_i \rangle$$

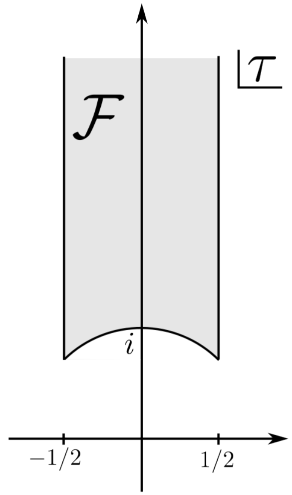
Complex torus:

$\mathbb{C}/(\mathbb{Z}+\tau\mathbb{Z})$

$(\tau = \tau_1 + i\tau_2)$



The moduli space :

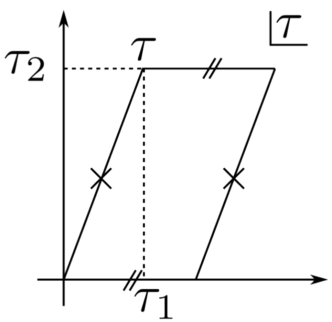


Computation of field theory amplitude : a one loop example.

Complex torus:

$\mathbb{C}/(\mathbb{Z}+\tau\mathbb{Z})$

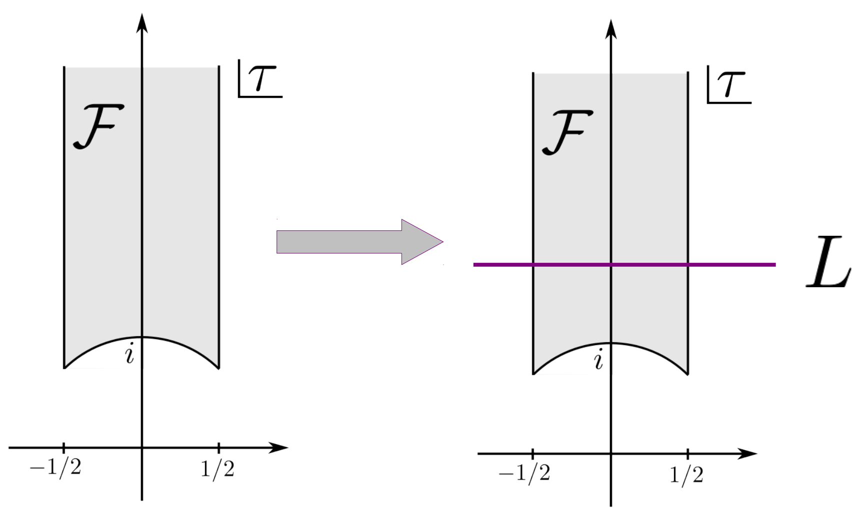
$(\tau = \tau_1 + i\tau_2)$



$$\mathcal{A} = \int_{\mathcal{M}_{g=1,n}} \langle \prod_i \mathcal{O}_i \rangle$$

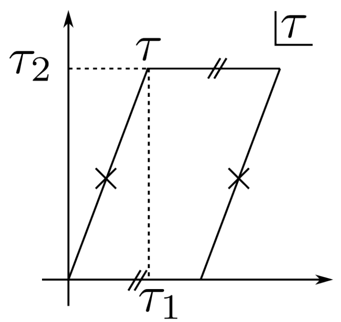
$$\to \left(\int_{\tau_2 \geq L \cap \mathcal{F}} + \int_{\tau_2 < L \cap \mathcal{F}} \right) \langle \prod_i \mathcal{O}_i \rangle$$

The moduli space :



Computation of field theory amplitude : a one loop example.

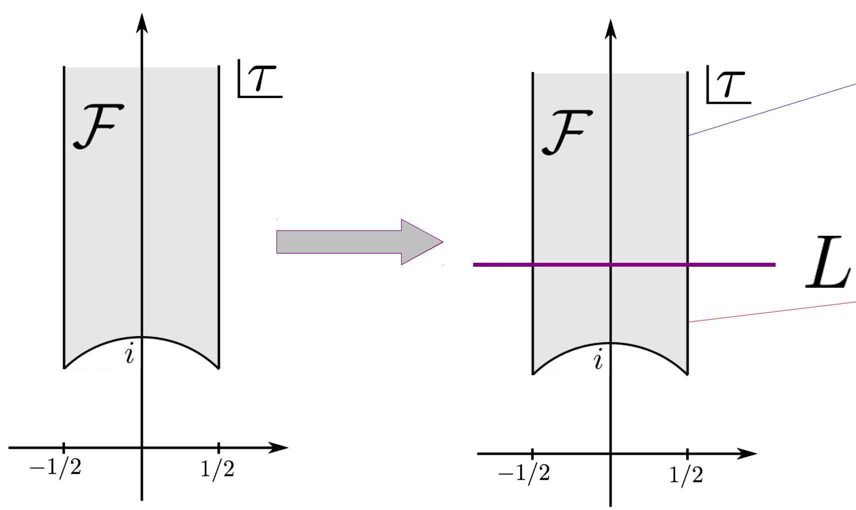
Complex torus:
$\mathbb{C}/(\mathbb{Z}+\tau\mathbb{Z})$
$(\tau = \tau_1 + i\tau_2)$

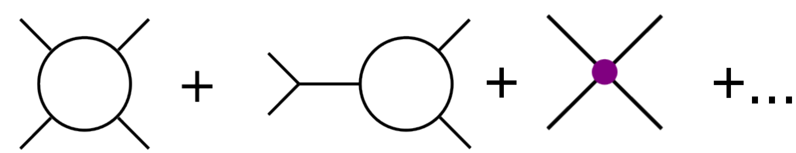


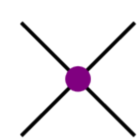
$$\mathcal{A} = \int_{\mathcal{M}_{g=1,n}} \langle \prod_i \mathcal{O}_i \rangle$$

$$\rightarrow \left(\int_{\tau_2 \geq L \cap \mathcal{F}} + \int_{\tau_2 < L \cap \mathcal{F}} \right) \langle \prod_i \mathcal{O}_i \rangle$$

The moduli space :

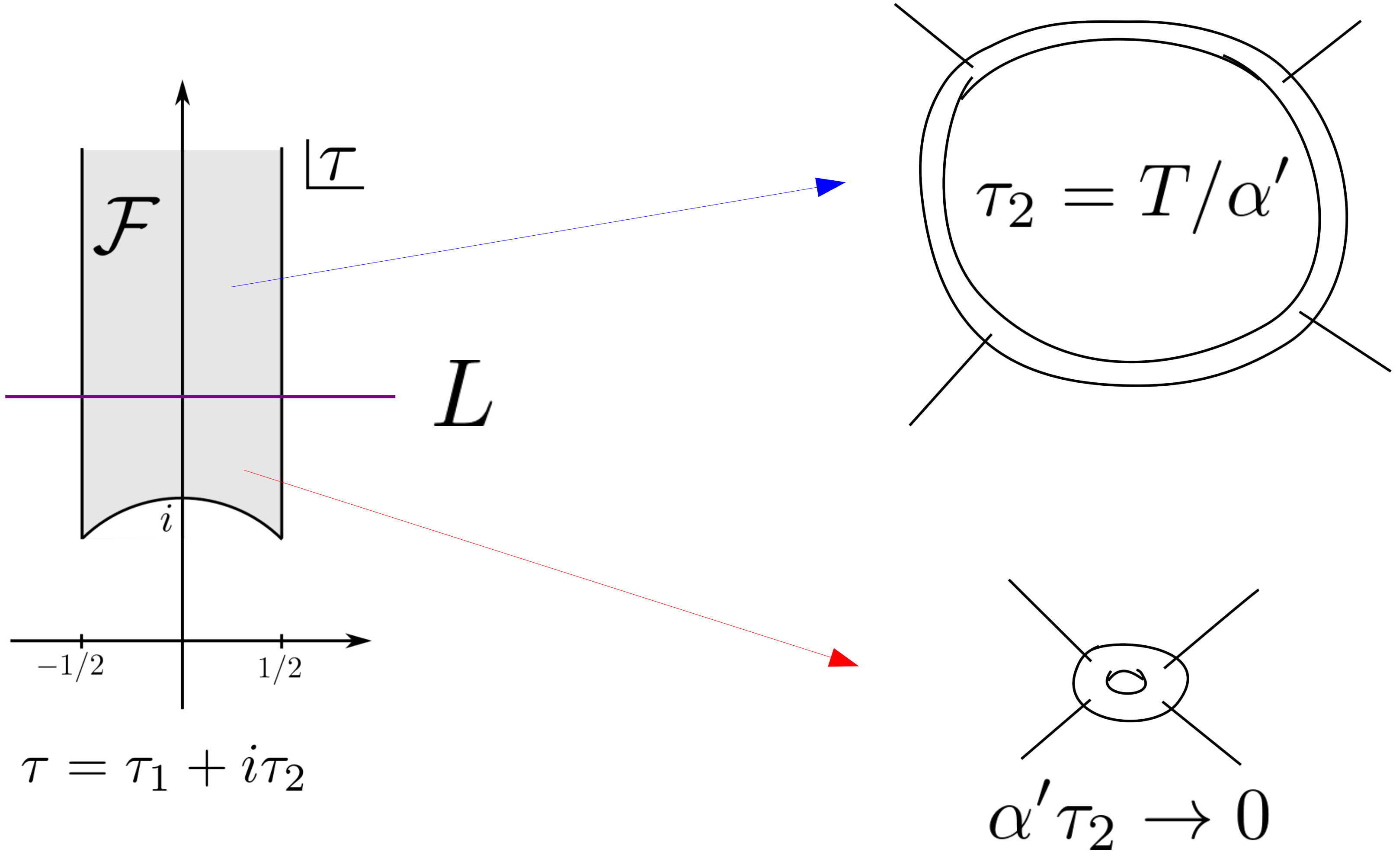




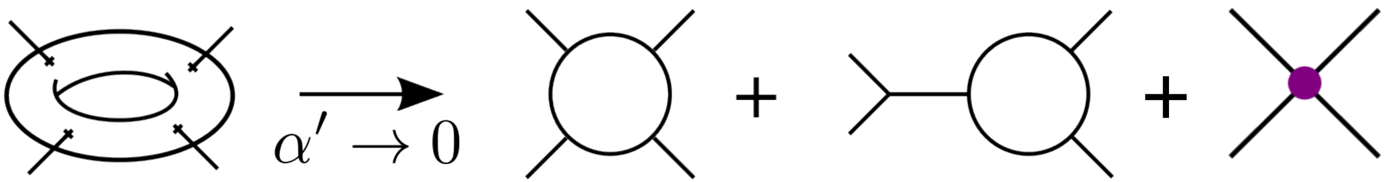
$$\sum_{m,n} c_{m,n} \nabla^m R^n -$$ 

[Green, Miller, Russo, Vanhove] :
Automorphic form program ; determine $c_{m,n}$ and extract directly logarithmic UV divergences.

Computation of field theory amplitude : a one loop example.

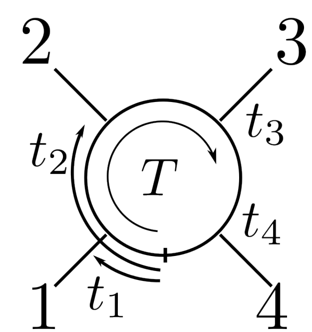


Computation of field theory amplitude : a one loop example.



Worldline graphs =
Schwinger proper time parametrization

$$T > \alpha' L$$



String theory amplitudes give field theory amplitudes computed *within a given renormalization scheme* :
UV cut-off, and infinite tower of massive states.

$$\int_{\mathcal{F}\cap\{\tau_{\leq} L\}} \frac{d^2\tau}{\tau_2^2}\phi_k(\tau)$$

Integrals of weight **k** modular forms over fundamental domains of genus g modular group.

The mathematical aspects.

Very different mathematical methods in the two approaches; give a good way to keep control on the process.

For the computation of the full amplitude :

Algebraic Geometry, *Tropical Geometry, ...*

For the direct extraction of divergences and the computation of higher order couplings in the low energy action :

Number theory, Automorphic forms, Modular forms, ...

Counter-terms.

The counter-terms have to be **local operators** that respect the symmetries of the theory: gauge invariance, full non-linear supersymmetry, duality invariance.

R^n : counter-term to an n-graviton amplitude.

$\longrightarrow \nabla^m R^n, \; m > 0$

$\frac{R^n}{s}$ ~ non local interaction

Example of non-valid counter-term : R^4 in $\mathcal{N} = 8$ in 4-d.

Non linear structure of R^4 terms understood **from string theory** and **field theory.**
This term is actually not invariant under **continuous** $E_{7,7}$ symmetry, and is thus ruled out.

Green, Russo, Vanhove 2010, Green Miller Russo Vanhove 2010,
Beisert Elvang Freedman Kiermaier Morales Stieberger 2010, Bossard, Howe, Stelle 2010

Power counting in supergravity.

Suppose that we have determined that $\nabla^m R^n$ is an admissible counter-term for an amplitude.
The diverging part of this n-graviton amplitude has to be

UV momentum cut-off → $$\Lambda^k \nabla^m R^n$$

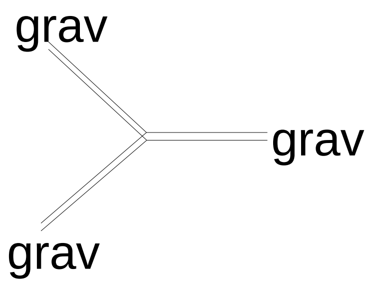
If k=0, log divergence,
If k>0, power divergence,
(if k<0, UV finite)

In other words, if we were to compute the Feynmann diagrams associated to the amplitude, we would find

$$\nabla^m R^n \int \frac{d\ell}{\ell} \ell^k$$

Power counting in supergravity.

In gravity, each graviton vertex carries at least **two derivatives :**



$\sim \partial^2 \sim \ell^2$

Euler relation :

L+V-I=1

V = # vertices

I = # internal lines

L = # loops.

$$A_{gravity} = \int (d^D \ell)^L \frac{\ell^{2V}}{\ell^{2I}} = \int \frac{d\ell}{\ell} \ell^{L(D-2)+2}$$

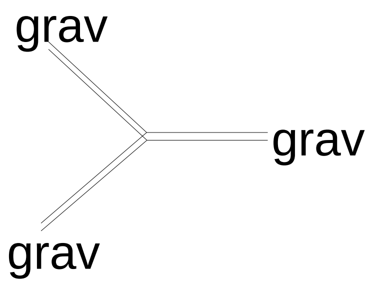
At L loops, superficial degree of divergence is L(D-2)+2

Remark : *this does not depend on the number of external particles, since each external line added to the graph adds an internal propagator, and the UV contribution exactly match.*

This is a specific fact about gravity.

Power counting in supergravity.

In gravity, each graviton vertex carries at least **two derivatives :**



$\sim \partial^2 \sim \ell^2$

Euler relation :

L+V-I=1

V = # vertices

I = # internal lines

L = # loops.

$$A_{gravity} = \int (d^D\ell)^L \frac{\ell^{2V}}{\ell^{2I}} = \int \frac{d\ell}{\ell} \ell^{L(D-2)+2}$$

At L loops, superficial degree of divergence is L(D-2)+2

$$[R] \sim \Lambda^2 \longrightarrow \nabla^m R^n \int \frac{d\ell}{\ell} \ell^{L(D-2)+2-m-2n}$$

Power counting in supergravity.

$$[R] \sim \Lambda^2 \longrightarrow \nabla^m R^n \int \frac{d\ell}{\ell} \ell^{L(D-2)+2-m-2n}$$

Examples for 4 gravitons amplitudes :

R^4 in d=8 →

R^4 in d=4 →

$\nabla^2 R^4$ in d=4 →

$\nabla^8 R^4$ in d=4 →

Power counting in supergravity.

$$[R] \sim \Lambda^2 \longrightarrow \nabla^m R^n \int \frac{d\ell}{\ell} \ell^{L(D-2)+2-m-2n}$$

Examples for 4 gravitons amplitudes :

R^4 in d=8 → L=1

R^4 in d=4 → L=3

$\nabla^2 R^4$ in d=4 → L=4

$\nabla^8 R^4$ in d=4 →

Counter-terms in N=8.

The first authorized counter-term that is

- Fully supersymmetric,
- $E_{7,7}$ invariant,

$$\nabla^8 R^4$$

Would be the CT to the first divergence of the theory at the 7-loop order.

Counter-terms in N=4.

We will see that at two loops, R^4 is actually ruled out by a non-renormalization theorem, [PT, Vanhove 2012], so the first counter-term is $\nabla^2 R^4$

→ 4-loop divergence, **but as well 3-loop finiteness.**

Remark :

N=8 : $$\nabla^2 R^4 \to (s+t+u) R^4 = 0$$ Total derivative.

N=4 : $$\nabla^2 R^4 \qquad \nabla_\mu R^2 \nabla^\mu R^2$$ Bossard, Howe, Stelle, Vanhove, 2011

s,t,u = kinematic Mandelstam invariants

Summary of supergravity four-graviton amplitudes UV behavior in d=4.

- Pure Einstein : UV divergence computed at two loops (R^3),

  Goroff, Sagnotti; van de Ven

- N=4 supergravity : UV divergence expected at 4 loops PT, Vanhove '12
- N=8 supergravity : UV divergence expected at 7 loops [...]

No UV divergence have ever been computed in d=4, despite a lot of explicit results. Bern, Carrasco, Dennen, Dixon, Johansson, Kosower, Roiban, ...

Higher loop orders pose great difficulties.

String models that give half-maximal supergravity.

(4,0) SUSY

- Heterotic CHL models
- Type IIA/B on asymmetric orbifolds

(2,2) SUSY

- Type IIA/B on $K3 \times T^2$ or symmetric orbifolds

Moduli space :

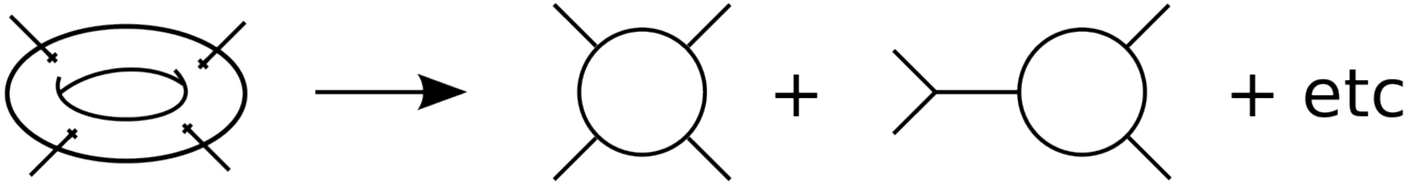
$$\Gamma \backslash SU(1,1)/U(1) \times SO(6,n_v;\mathbb{Z}) \backslash SO(6,n_v)/SO(6) \times SO(n_v)$$

Axion-dilaton

T,U moduli of the T^2

Non perturbatively U dual

One loop computations.



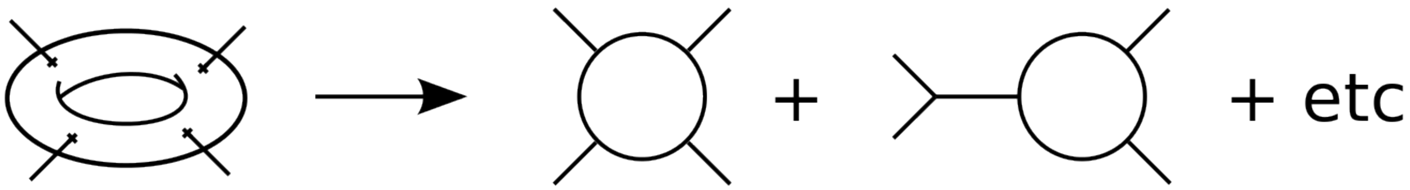
Two step procedure :

- Project onto massless sector of the theory
- Integrate out the real part of the integral

Difference with maximal unbroken SUSY:

- Heterotic models : mixing between tachyon and massive modes,
- Type II models : alteration of supersymmetric projection (GSO projection)

One loop computations.



Two step procedure :

- Project onto massless sector of the theory
- Integrate out the real part of the integral

Difference with maximal unbroken SUSY:

- Heterotic models : mixing between tachyon and massive modes,
- Type II models : alteration of supersymmetric projection (GSO projection)

Result : *four graviton amplitudes are identical in the two types of models at one loop.* [PT, Vanhove 2012]

Asymmetric type of computations in field theory using "Bern-Kosower string inspired rules": [Grisaru, Siegel 1982], [Dunbar Norridge 1995], [Dunbar Julia Seminara Trigante 1999], [Carrasco, Chiodaroli, Gunaydin, Roiban 2012]

Symmetric type of computations in field theory using orbifolds of 10-d maximal supergravity : [Carrasco, Chiodaroli, Gunaydin, Roiban 2012]

Non-renormalisation theorem in heterotic string.

Now, we are going to show that a **non-renormalisation theorem** for the R^4 term might be used to constrain the UV behavior of half-maximal supergravity.

Four-graviton heterotic amplitude.

$$A_{4;g} = \int_{s\mathcal{M}_{g,n}} d\mu \int \prod_{i=1}^{4} d^2\nu_i \, \langle V(\nu_1)V(\nu_2)V(\nu_3)V(\nu_4)\rangle$$

$$\epsilon_{\mu\nu} = \epsilon_\mu \epsilon_\nu$$

Gravitons' polarizations

$$V(z,\bar{z}) = \epsilon_{\mu\nu} : (\partial X^\mu + ik.\psi\psi)\bar{\partial}\bar{X}^\nu e^{ik.X(z,\bar{z})} :$$

Holomorphic : supersymmetric left-moving ⊗ Anti-holomorphic : bosonic right-moving

Holomorphic factorization in the integrand (not trivial for higher genus)

→ Wick's theorem for left / right-movers separately.

Two-loop heterotic amplitude.

Two-loop heterotic amplitude.

$$A_{4;2} = t_8 F^4 \int_{\mathcal{M}_{g,n}} [dM]\, \mathcal{Z}_2^{Het} \int \prod_{i=1}^{4} d^2\nu_i\, \mathcal{W}^{(2)}\, \mathcal{Y}_s\, e^{-\sum_{1\leq i<j\leq 4} 2\alpha' k^i \cdot k^j P(\nu_{ij})}$$

D'Hoker & Phong

7 papers, 396 pages long.

Two-loop heterotic amplitude.

$$A_{4;2} = t_8 F^4 \int_{\mathcal{M}_{g,n}} [dM]\, \mathcal{Z}_2^{Het} \int \prod_{i=1}^{4} d^2\nu_i\, \mathcal{W}^{(2)}\, \mathcal{Y}_s\, e^{-\sum_{1\leq i<j\leq 4} 2\alpha' k^i\cdot k^j P(\nu_{ij})}$$

D'Hoker & Phong

7 papers, 396 pages long.

Two-loop heterotic amplitude.

$$A_{4;2} = t_8 F^4 \int_{\mathcal{M}_{g,n}} [dM]\, \mathcal{Z}_2^{Het} \int \prod_{i=1}^{4} d^2\nu_i\, \mathcal{W}^{(2)}\, \mathcal{Y}_s\, e^{-\sum_{1\leq i<j\leq 4} 2\alpha' k^i\cdot k^j P(\nu_{ij})}$$

$$\langle \prod_{j=1}^{4} e^{ik_j\cdot X(z_j)} \rangle \sim$$ Feynman diagram denominator

Two-loop heterotic amplitude.

$$A_{4;2} = t_8 F^4 \int_{\mathcal{M}_{g,n}} [dM]\, \mathcal{Z}_2^{Het} \int \prod_{i=1}^{4} d^2\nu_i\, \mathcal{W}^{(2)}\, \mathcal{Y}_s\, e^{-\sum_{1\leq i<j\leq 4} 2\alpha' k^i\cdot k^j P(\nu_{ij})}$$

$$\mathcal{W}^{(2)} = \frac{\langle \prod_{j=1}^4 \epsilon^j \cdot \bar\partial \bar X(\bar z_j) e^{ik_j\cdot X(z_j)}\rangle}{\langle \prod_{j=1}^4 e^{ik_j\cdot X(z_j)}\rangle}$$

$$\langle \prod_{j=1}^{4} e^{ik_j\cdot X(z_j)}\rangle \sim$$ Feynman diagram denominator

Two-loop heterotic amplitude.

$$A_{4;2} = t_8 F^4 \int_{\mathcal{M}_{g,n}} [dM]\, \mathcal{Z}_2^{Het} \int \prod_{i=1}^{4} d^2\nu_i\, \mathcal{W}^{(2)}\, \mathcal{Y}_s\, e^{-\sum_{1\le i<j\le 4} 2\alpha' k^i\cdot k^j P(\nu_{ij})}$$

$$\mathcal{W}^{(2)} = \frac{\langle \prod_{j=1}^{4} \epsilon^j \cdot \bar\partial \bar X(\bar z_j) e^{ik_j\cdot X(z_j)}\rangle}{\langle \prod_{j=1}^{4} e^{ik_j\cdot X(z_j)}\rangle}$$

$$\langle \prod_{j=1}^{4} e^{ik_j\cdot X(z_j)}\rangle \sim$$ Feynman diagram denominator

$$3\mathcal{Y}_S = (u-t)\,\Delta_{12}\Delta_{34} + (s-t)\Delta_{13}\Delta_{24} + (s-u)\Delta_{14}\Delta_{23} \;\sim k_i.k_j$$

$$\Delta_{i,j} = \omega_1(\nu_i)\omega_2(\nu_j) - \omega_1(\nu_i)\omega_2(\nu_j) \sim 1$$

Positions of the vertex operators

$$s = -2k_1.k_2$$
$$t = -2k_1.k_4$$
$$u = -2k_1.k_3$$

Two-loop heterotic amplitude.

$$A_{4;2} = t_8 F^4 \int_{\mathcal{M}_{g,n}} [dM]\, \mathcal{Z}_2^{Het} \int \prod_{i=1}^{4} d^2\nu_i\, \mathcal{W}^{(2)}\, \mathcal{Y}_s\, e^{-\sum_{1\leq i<j\leq 4} 2\alpha' k^i\cdot k^j P(\nu_{ij})}$$

$$\mathcal{W}^{(2)} = \frac{\langle \prod_{j=1}^{4} \epsilon^j \cdot \bar\partial \bar X(\bar z_j) e^{ik_j\cdot X(z_j)}\rangle}{\langle \prod_{j=1}^{4} e^{ik_j\cdot X(z_j)}\rangle}$$

$$\langle \prod_{j=1}^{4} e^{ik_j\cdot X(z_j)}\rangle \sim$$ Feynman diagram denominator

$$3\mathcal{Y}_S = (u-t)\,\Delta_{12}\Delta_{34} + (s-t)\Delta_{13}\Delta_{24} + (s-u)\Delta_{14}\Delta_{23} \;\sim k_i.k_j$$

$$\Delta_{i,j} = \omega_1(\nu_i)\omega_2(\nu_j) - \omega_1(\nu_i)\omega_2(\nu_j) \sim 1$$

Positions of the vertex operators

$$s = -2k_1.k_2$$
$$t = -2k_1.k_4$$
$$u = -2k_1.k_3$$

Two-loop heterotic amplitude.

$$A_{4;2} = t_8 F^4 \int_{\mathcal{M}_{g,n}} [dM] \mathcal{Z}_2^{Het} \int \prod_{i=1}^{4} d^2\nu_i \, \mathcal{W}^{(2)} \, \mathcal{Y}_s \, e^{-\sum_{1\leq i<j\leq 4} 2\alpha' k^i \cdot k^j P(\nu_{ij})}$$

$$\mathcal{W}^{(2)} = \frac{\langle \prod_{j=1}^{4} \epsilon^j \cdot \bar\partial \bar X(\bar z_j) e^{ik_j \cdot X(z_j)} \rangle}{\langle \prod_{j=1}^{4} e^{ik_j \cdot X(z_j)} \rangle}$$

$$\langle \prod_{j=1}^{4} e^{ik_j \cdot X(z_j)} \rangle \sim$$ Feynman diagram denominator

$$3\mathcal{Y}_S = (u-t)\,\Delta_{12}\Delta_{34} + (s-t)\Delta_{13}\Delta_{24} + (s-u)\Delta_{14}\Delta_{23} \;\sim k_i . k_j$$

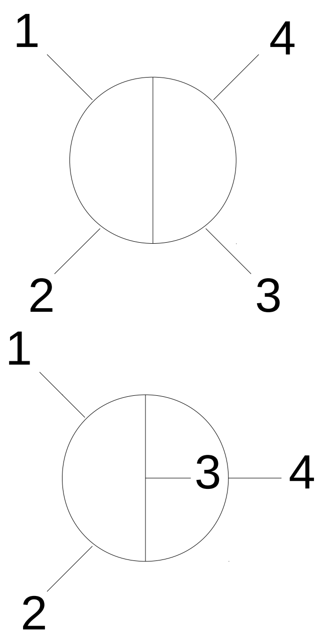
$$\Delta_{i,j} = \omega_1(\nu_i)\omega_2(\nu_j) - \omega_1(\nu_i)\omega_2(\nu_j) \sim 1$$

Positions of the vertex operators

$$s = -2k_1.k_2$$
$$t = -2k_1.k_4$$
$$u = -2k_1.k_3$$

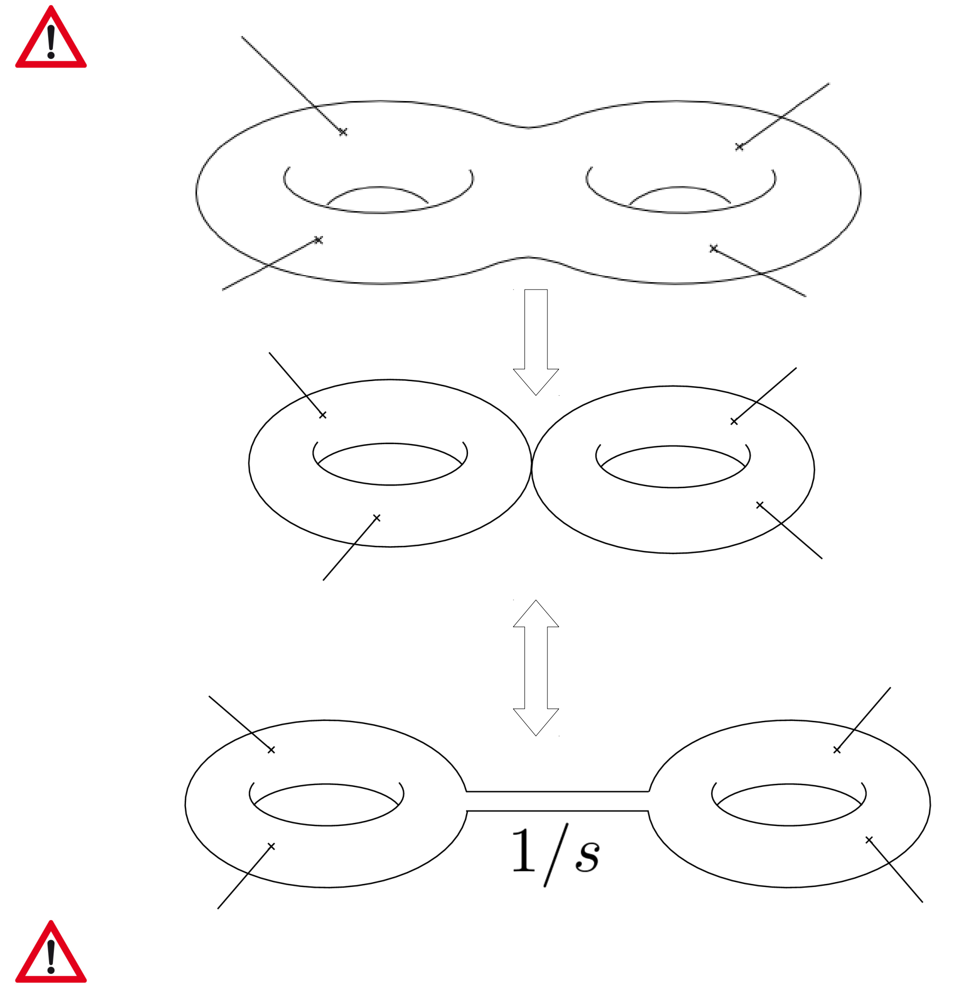
Two-loop N=4 amplitude.

In the field theory limit, $\mathcal{Y}_S \to Y_S$ where $Y_S \propto k_i.k_j$ or $Y_S = 0$ depending on the graph



$$Y_S = s$$

And so on for other orderings.



The only way to use up the two powers of momentum that we have factored out is to see an IR divergence grow and create a 1/s pole.

3-point function :

$$\mathcal{Y}_s \to 0$$

Due to SUSY

Two-loop N=4 amplitude.

$$A_{4;2} = t_8 F^4 \int_{\mathcal{M}_{g,n}} [dM]\, \mathcal{Z}_2^{Het} \int \prod_{i=1}^{4} d^2\nu_i\, \mathcal{W}^{(2)}\, \mathcal{Y}_s\, e^{-\sum_{1\leq i<j\leq 4} 2\alpha' k^i\cdot k^j P(\nu_{ij})}$$

$$k_i.k_j \neq \ell^2$$ Expected

This $k_i.k_j$ indicates that two powers of mass are taken out of the momentum integral.
For counter-terms, it means that we have a $\nabla^2 R^4$ instead of a R^4.

From two loops on, R^4 does not receive perturbative quantum corrections.

Three-loop N=4 amplitude.

As we said, R^4 corresponds to a three-loop logarithmic UV divergence

However, in the four graviton amplitude, **two derivatives** are factored out of the integral.

$$A_{3-loop} \sim \int (d^D \ell)^L \frac{\ell^{2V}}{\ell^{2I}} \sim \nabla^2 R^4 \int \frac{d\ell}{\ell^3}$$

UV finite !

General predictions.

$$A_{L-loop} \sim \int (d^D\ell)^L \frac{\ell^{2V}}{\ell^{2I}} \sim \nabla^2 R^4 \int \frac{d\ell}{\ell} \ell^{L(D-2)-8}$$

General predictions.

$$A_{L-loop} \sim \int (d^D\ell)^L \frac{\ell^{2V}}{\ell^{2I}} \sim \nabla^2 R^4 \int \frac{d\ell}{\ell} \ell^{L(D-2)-8}$$

$L(D-2)-8 \geq 0 \leftrightarrow$ UV divergent, e.g. L=4, D=4
L=2, D=6

$L(D-2)-8 < 0 \leftrightarrow$ UV finite, e.g. L=3, D=4
L=2, D=5

General predictions.

$$A_{L-loop} \sim \int (d^D\ell)^L \frac{\ell^{2V}}{\ell^{2I}} \sim \nabla^2 R^4 \int \frac{d\ell}{\ell} \ell^{L(D-2)-8}$$

$L(D-2)-8 \geq 0 \leftrightarrow$ UV divergent, e.g.

L=4, D=4 ??
L=2, D=6 OK

$L(D-2)-8 < 0 \leftrightarrow$ UV finite, e.g.

L=3, D=4 OK
L=2, D=5 OK

Bern, Davies, Dennen, Huan 2012

$$\mathcal{M}^{(2)}(1,2,3,4)\Big|_{D=6\,\text{div.}} = \frac{1}{(4\pi)^6}\left(\frac{\kappa}{2}\right)^6 st A^{\text{tree}}_{Q=16}(1,2,3,4)\Big\{\Big(\frac{(D_s-6)(26-D_s)}{576\epsilon^2} + \frac{(19D_s-734)}{864\epsilon}\Big) \times \Big[s\,(F_1F_2)(F_3F_4) + t\,(F_1F_4)(F_2F_3) + u\,(F_1F_3)(F_2F_4)\Big] + \frac{(48D_s-1248)}{864\epsilon}\Big[u\,(F_1F_2F_3F_4) + t\,(F_1F_3F_4F_2) + s\,(F_1F_4F_2F_3)\Big]\Big\}, \quad (4.7)$$

$$\sim \nabla^2 R^4$$

Bern, Davies, Dennen, Huan 2012

Conclusion

- Geometric picture of the UV divergences in the moduli space of string theory, and the way they are renormalized,
- Tested perturbative equality between (4,0) and (2,2) models at one loop,
- Gave reason based on a non-renormalisation theorem in heterotic string for the 3-loop finiteness of N=4 with any number of vector multiplets,
- Postulated a 4 loop divergence.

Outlook

- Computation of the four loop divergence in N=4.
- Is N=8 UV finite ?
- Better understanding of the links between supergravity and string theory ?
- Understand in a top down approach the properties of UV completion(s?) of maximal supergravity,
- First quantized World-line formalism for maximal supergravity using *tropical geometry* ?