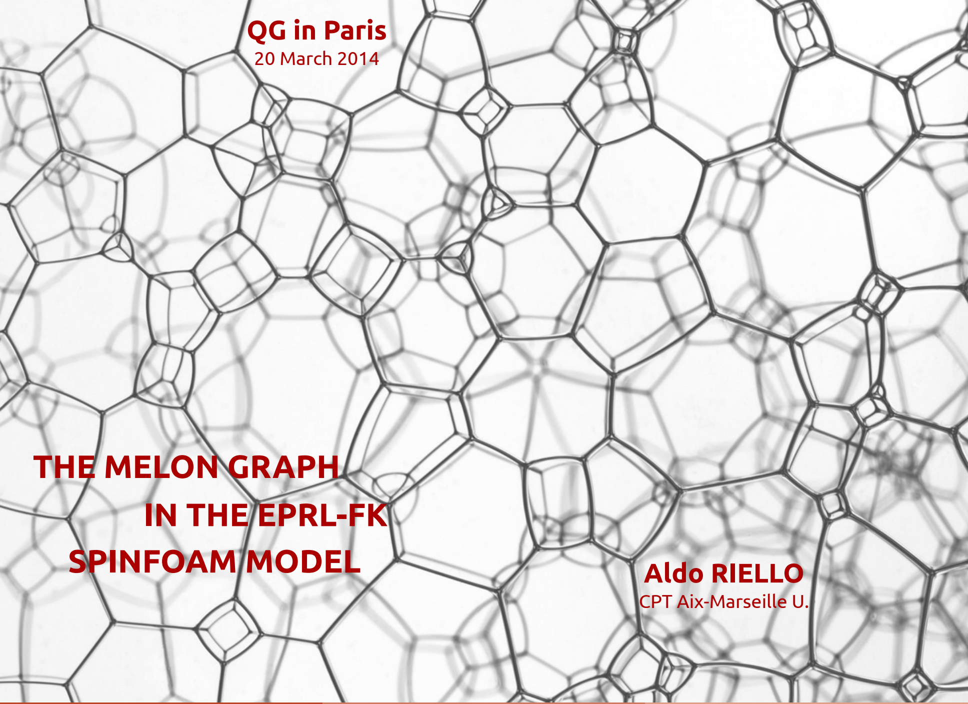

QG in Paris
20 March 2014

THE MELON GRAPH IN THE EPRL-FK SPINFOAM MODEL

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Plan

- *Spinfoam gravity overview* [Part 0]
- *Why studying spinfoam divergences?* [Part I]
- *The melon graph: why, what, and therefore?* [Part II]
  [AR, PRD88 (2013), arXiv:1302.1781]
- *Not just the amplitude: correlations* [Part III]
  [AR, PRD89 (2014), arXiv:1310.2174]

Part 0

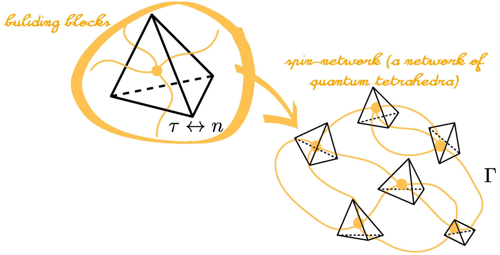
Spinfoam gravity overview

Quantum geometry

Building blocks: quantum states of geometry named **spin networks** $\Psi_{\Gamma j_l \iota_n}$

Labelled by:

- ▷ an abstract graph Γ (say 4-valent)
- ▷ an $SU(2)$ spin j_l at every link
- ▷ an $SU(2)$ (4-valent) invariant tensor ι_n at every node (intertwiner)



Nodes are dual to quantum tetrahedra, whose volume is fixed by the intertwiner ι_n, and whose face areas are

$$A_l = 8\pi G\hbar\gamma\sqrt{j_l(j_l+1)}$$

[The group $SU(2)$ encodes the symmetries of the quantized space]

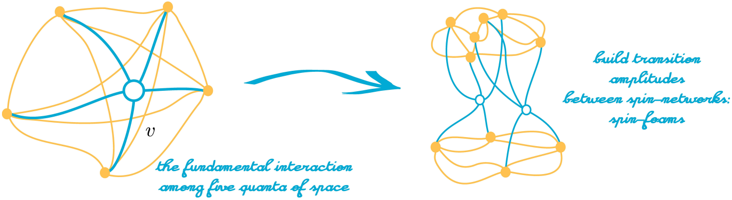
Quantum processes

Spinfoams

Quantum gravitational processes between spin network states

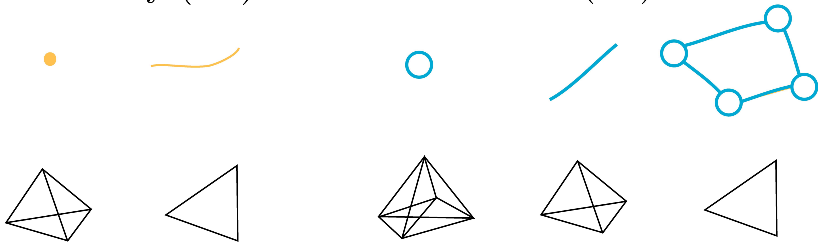
As Feynman graphs, they are built out of fundamental units:

- ▷ interaction vertices among quanta of space
- ▷ gluing of interaction vertices via their boundaries



Geometric interpretation

Boundary (SN) Bulk (SF)



node n link l vertex v edge e face f

Spinfoam amplitude

Spinfoam model

A prescription to assign an amplitude ($\in \mathbb{C}$) to every such process.

Local spinfoam Ansatz:
the amplitude is built out of local amplitudes for faces f, edges e and vertices v depending only on local representations and intertwiners:

$$\mathscr{A}_{SF}[\Psi] = \sum_{\text{colourings}} \prod_f A_f[j_f] \prod_e A_e[\iota_e, j_{f\ni e}] \prod_v A_v[\iota_{e\ni v}, j_{f\ni v}]$$

It is expected to be a regularization of the integral over histories:

$$\mathscr{A}_{SF} \sim \int \mathscr{D} g_{\mu\nu}\, \mathrm{e}^{\mathrm{i}S[g_{\mu\nu}]}$$

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Spinfoams and GFTs

The GFT partition function is a generating function of spinfoam amplitudes

$$Z_{\mathrm{GFT}}[\mu] = \int \mathscr{D}_{\mathrm{Gauss}}[\varphi] \ \exp\left\{ \frac{\mu}{5!} \int_G \mathrm{d}g_i \, V(\{g_i, g_j\}) \varphi^5(\{g_i\}) \right\}$$

Roughly speaking:

- ▷ vertex amplitude: $A_v[\iota, j] \leftrightsquigarrow V(\{g_i, g_j\})$
- ▷ edge and face amplitudes: $A_e[j]$, $A_e[\iota, j] \leftrightsquigarrow \mathscr{D}_{\mathrm{Gauss}}[\varphi]$

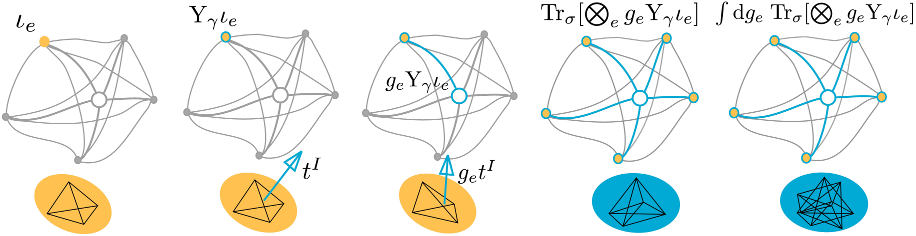
"UV" and "IR"

- ▷ in GFT: UV = high frequency modes on the group ⇒ *large* spins $j \gg \frac{1}{2}$
- ▷ in Spinfoams: UV = small physical geometries ⇒ *small* spins $j \gtrsim \frac{1}{2}$

EPRL-FK

Four-dimensional Lorentzian spinfoam gravity: the **EPRL-FK model**

Vertex amplitude:



Mathematically:

$$\mathrm{Y}_\gamma : \mathscr{V}^{j}_{SU(2)} \to \mathscr{V}^{(\rho=\gamma j, k=j)}_{SL(2,\mathbb{C})} , \quad |j;m\rangle \mapsto |\gamma j, j; j, m\rangle$$

[Engle, Pereira, Rovelli, Livine, Freidel, Krasnov, Speziale]

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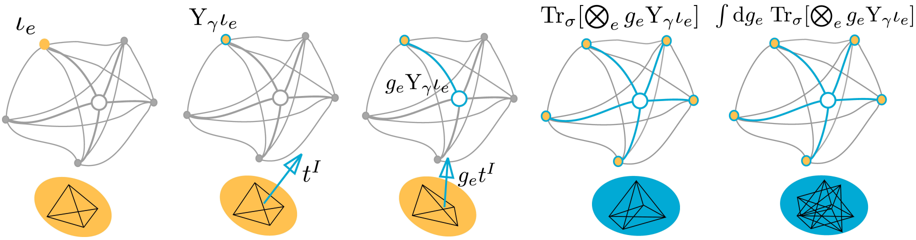


Define the spinfoam model as [see next slide]

$$A_f = (2j+1), \quad A_e = 1, \quad A_v = \int_{SL(2,\mathbb{C})} \mathrm{d}g_e \, \mathrm{Tr}_\sigma[\bigotimes_e g_e \mathrm{Y}_\gamma \iota_e]$$

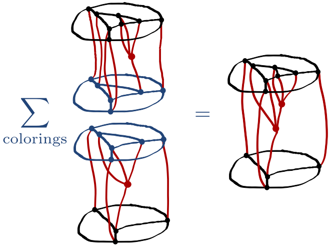
[Engle, Pereira, Rovelli and Livine, Freidel and Krasnov, Speziale]

Choice of face and edge amplitudes

In previous definition of EPRL-FK:

$$A_f = (2j+1) \quad \text{and} \quad A_e = 1$$

▷ this face amplitude is selected by a criterion of composition of spinfoams amplitudes. [Bianchi, Regoli, Rovelli]
However, the following analysis can be straightforwardly applied to any face weight



▷ the edge amplitude is quite arbitrary and receives corrections after renormalization. It could be interesting to study the consequences of its tuning

Indications of viability of EPRL-FK

Most convincing indications of viability come from **semiclassics**:

$$A_v[\text{coherent state}] \quad \xrightarrow{j \gg 1} \quad N \times \left(\mathrm{e}^{\mathrm{i} S_{\text{Regge}}} + \mathrm{e}^{-\mathrm{i} S_{\text{Regge}}} \right),$$

where

$$S_{\text{Regge}} = \sum_{\text{triangles}} A_t \Theta_t$$

[Freidel,Conrady;Barrett,Dowdall,Gomes,Hellmann,Fairbairn,Pereira;Han,Zhang;Kaminski...]

is a well-defined discretization of Einsten-Hilbert action

$\rightsquigarrow$ This leads (in some proper sense) to the expected graviton propagator [Bianchi,Magliaro,Modesto,Perini,Rovelli,Speziale...]

RMK Natural variables are not triangulation bone-lengths (as in Regge calculus), but triangle areas, i.e. spins

Part I

Why studying spinfoam divergences?

Where the wild things are

In Feynman diagrams, divergences are associated to loops. This is because, loops involve integrals on unconstrained variables (momenta)

* * *

Every spinfoam local amplitude A_f, A_e, A_v is finite [Engle and Pereira]

Divergences arise from summations over unconstrained internal colourings

$$\mathscr{A}_{SF}[\Psi] = \sum_{\text{colourings}} \prod_e A_e \prod_f A_f \prod_v A_v$$

Unconstrained colourings are associated to **bubbles**, i.e. to topological structures dual to lower dimensional submanifolds in the triangulation (sides and points) [Perez and Rovelli, Freidel and Louapre, Bonzom and Smerlak, ...]

RMK Even finite models, e.g. q-deformed ones, have a large number embedded in them, i.e. the inverse (bare) cosmological constant $\rightsquigarrow$ **IR cut-off** [Turaev,Viro,Fairbairn,Meusburger,Han,...]

Why studying divergences?
(I) Model consistency

Spinfoam continuous limit $\rightsquigarrow$ triangulation refinement $\rightsquigarrow$ bubbles

Therefore, bubbles are a natural piece of the theory. What is their rôle?

- In a renormalization/corarse graining procedure, divergences "drive" the flow of the theory
  $\rightsquigarrow$ renormalized amplitudes may not have the desired semiclassics
- Divergences may indicate the presence of residual diffeo symmetry (cf. topological 3d gravity) [Freidel,Louapre,...]

Why studying divergences?
(II) New physics

Studying divergent graphs may lead to **new physics**:
[remark that corrections to the classical theory are present even at "tree-level"]

- via associated radiative corrections, as for the Lamb shift in QED
- quantum gravity naturally cuts-off UV divergences, which seem to be traded with IR (long distance) ones. How to interpret this fact? Has it any (physical) consequence?

Plan

- *Spinfoam gravity overview* [Part 0]
- *Why studying spinfoam divergences?* [Part I]
  Naturally present. Related to the study of the continuous limit. Even finite models contain a very large number $\sim \lambda^{-1}$. Crucial for the consistency of the model under renormalization/coarse graining. Possibly related with unfixed non-compact gauge symmetries. Possibly related to new physics.
- *The melon graph: why, what, and therefore?* [Part II]
- *Not just the amplitude: correlations* [Part III]

Part II

The melon graph: why, what, and therefore?

Why melons?

Part (I)

(+) The simplest bubble

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(+) The simplest bubble

(+) Central in (coloured) Tensor Models and GFTs (most diverging building block $\Rightarrow$ role in $1/N$ expansion, renormalization)
[Gurau,Rivasseau,Bonzom,AR,Carrozza,Oriti,BenGeloun,Ryan and many other...]

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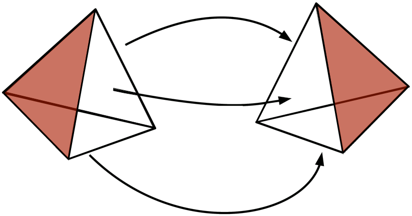
($-$) Topological sphere, but dual to a degenerate triangulation (less natural from a geometric perspective)



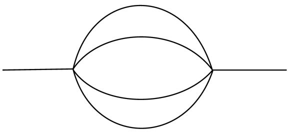
Why melons?

Part (II)

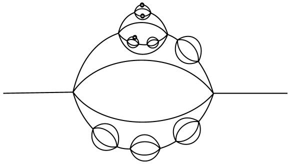
Relevant summations are associated to spinfoam faces (spins)
Therefore, at a given order in vertex expansion:

maximize divergence degree $\rightsquigarrow$ maximize # [unconstrained] faces

The melonic family of graphs is the one satisfying this condition
[Actually, additional hypothesis are needed]



the melon graph



a melonic graph

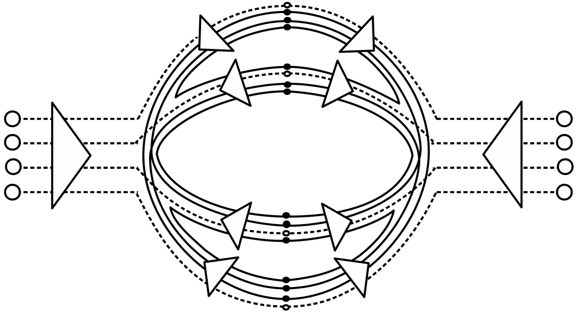
The goal

Caculate the **most divergent** contribution to the melon-graph amplitude in the **Lorentzian EPRL-FK** model

In particular, work out **both**:

- the scaling of the (regularized) amplitude
- and the relation it imposes between *in* and *out* states

The structure of the melon



- ▷ dotted lines represent external faces (spin fixed by boundary data)
- ▷ solid lines represent internal faces (spins summed over)
- ▷ triangles represent integrations over $SL(2, \mathbb{C})$ elements

The strategy

1. Write the amplitude for the internal faces in a path integral form

$$\mathscr{A}_{SF}[\Psi] = \sum_{\{j_f\}} \mu(j_f) \int \mathrm{d}x \; e^{\sum_f j_f S[x]} \prod_{f \; ext} A_f^{ext}[\Psi, x]$$

The strategy

1. Write the amplitude for the internal faces in a path integral form
2. Observe that divergences come from the tail of the sum over spins

$$\mathscr{A}_{SF}[\Psi] \sim \sum_{\{j_f \gg 1\}} \mu(j_f) \int \mathrm{d}x \; \mathrm{e}^{\sum_f j_f S[x]} \prod_{f \; ext} A_f^{ext}[\Psi, x]$$

The strategy

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3. Put a cut-off J on the spins

$$\mathscr{A}_{SF}[\Psi] \sim \sum_{\{j_f \gg 1\}}^{J} \mu(j_f) \int \mathrm{d}x\, \mathrm{e}^{\sum_f j_f S[x]} \prod_{f\ ext} A_f^{ext}[\Psi, x]$$

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4. Simplify this expression using the saddle point approximation $x = x_0[j]$

$$\mathscr{A}_{SF}[\Psi] \sim \sum_{\{j_f \gg 1\}}^{J} \mu(j_f)\, N(j_f)\, e^{\sum_f j_f S[x_0]} \prod_{f\ ext} A_f^{ext}[\Psi, x_0]$$

$\rightsquigarrow$ Semiclassics on the internal faces, which effectively decouple

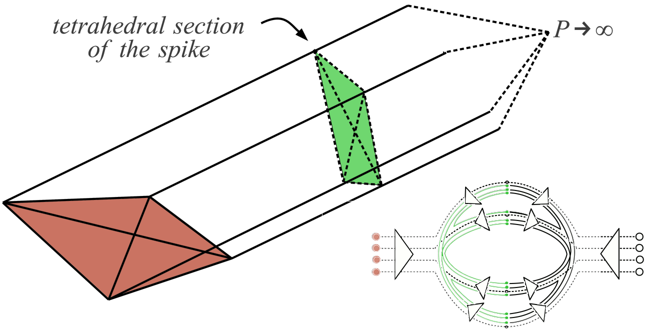
The strategy

1. Write the amplitude for the internal faces in a path integral form
2. Observe that divergences come from the tail of the sum over spins
3. Put a cut-off J on the spins
4. Simplify this expression using the saddle point approximation $x = x_0[j]$
5. Perform an order of magnitude evaluation of the divergence degree, by taking into account all the symmetries of the action

$$\mathscr{A}_{SF}[\Psi] \sim J^k \prod_{f\ ext} A_f^{ext}[\Psi]$$

Keep the sector dominating the sum over spins: $\sum_f j_f S_0 \equiv 0$ and $A_f^{ext}[\Psi, x_0] \equiv A_f^{ext}[\Psi]$

Geometrical interpretation of the calculation



The leading order term comes from a contribution where one spacetime and one “anti”- spacetime interfere, in such a way that their phases (e^{iS_0} and e^{-iS_0}, resp.) cancel. The total action is zero, without flatness

Approximations

Downsides

- All spin are let scale together, a priori to maximize divergence, but maybe other effects enter the game
- No rigorous proof to pick the dominating sector
- Neglected **degenerate sector** of the sum over spins
  [There are indications that this sector could endanger the result. However, there are difficulties in doing a complete analysis]

Upsides

- EPRL-FK with cut-off $\rightsquigarrow$ toy model for cosmological constant

$$J \sim \lambda^{-1} \gg 1$$

The result

$$\mathscr{A}_{SF}[\Psi_{out}, \Psi_{in}] \sim \log\left(\frac{J}{j_{ext}}\right) \langle \Psi_{out} | \mathbb{T}^2 | \Psi_{in} \rangle$$

where the 1-bubble renormalized gluing operator $\mathbb{T}^2$ is the square of

$$\mathbb{T} \triangleright (\bullet) := \int_{SL(2,\mathbb{C})} \mathrm{d}g \, \left[\mathrm{Y}_\gamma{}^\dagger g \mathrm{Y}_\gamma \triangleright (\bullet) \right]$$

RMK $\mathbb{T}^2 \neq \mathbb{T}$, but $\mathbb{T} \xrightarrow{j_{ext} \gg 1} \langle j_{ext} \rangle^{-3/2} \mathbb{P}_{SU(2)}$, i.e. the "bare" gluing [Puchta]

RMK The scaling depends on the choice of face and edge amplitude

Why is this encouraging?

- 1^{st} calculation of a Lorentzian SF radiative process $\rightsquigarrow$ it's **doable**!
- It matches previous calculations in Euclidean QG, a priori not obvious [Perini,Rovelli,Speziale,Krajewski,Magnen,Rivasseau,Tanasa,Vitale]
- The result does **not** spoil the semiclassical limit: $\mathbb{T}^2 \xrightarrow{j \gg 1} \#\mathbb{P}_{SU(2)}$ [Cf. spinfoam graviton propagator melonic corrections]

Other remarks

- If logarithm is relevant, and considering $J \sim \lambda^{-1}$
  - $\rightsquigarrow$ **no** graph is "truly" divergent, since $\log(10^{120}) \approx 300$
  - $\rightsquigarrow$ **any** process is then relevant?
- The logarithm is unlikely to have a geometric origin [cf. symmetries]

Lessons

- ▷ “Anti”-spacetimes play a prominent role
- ▷ IR divergences mean large virtual geometries
  [though not necessarily 4d]
- ▷ If bubble divergences are not due just to symmetries, which physical meaning to such IR divergences?

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- *The melon graph: why, what, and therefore?* [Part II]
  1^{st} calculation of a QG radiative process $A_M = \log(\Lambda/j)\langle out|\mathbb{T}^2|in\rangle$. Only marginally divergent (consequences in the theory with $\lambda \neq 0$?). The gluing gets modified, though consistently with semiclassics. Role of antispacetimes. A glimpse on how renormalization ideas could get modified by QG
- *Not just the amplitude: correlations* [Part III]

Part III

Not just the amplitude: correlations

Question

Since $\mathbb{T}^2 \xrightarrow{j \gg 1} \#\mathbb{P}_{SU(2)}$, then do melonic insertions really have *no consequences* in this regime? [Apart from that of modifying the edge weight]

Indeed, it seems that amplitudes (semiclassically) should not be influenced by melonic insertions.

However, is this the end of the story?

Correlations

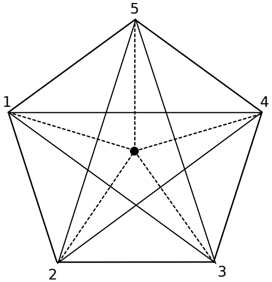
Many interesting physical questions concern **correlations** between fluctuations on the top of some (classical) solution

In this sense, they do not directly concern the amplitude itself

In pure gravity, the best example are metric correlations on a background
$\rightsquigarrow$ **"graviton propagator"**

Graviton propagator

At lowest order, analyse the graviton propagator on a single four simplex
[All faces are external, there is no sum over spins associated to the dynamics]



$$\langle h_{\mathbf{n}}^{ab} h_{\mathbf{m}}^{cd} \rangle_{conn} \overset{j \gg 1}{\approx} \left[\left(S'' \right)^{-1} \right]^{ij} (h_{\mathbf{n}}^{ab})'_i (h_{\mathbf{m}}^{cd})'_j \xrightarrow{\gamma \to 0} G_{\mathbf{nm}}^{abcd}$$

RMK $\gamma \to 0$ is needed to kill correlations carried by variables different from spins, not present in Regge calculus [Indications that this could be needed to select the right semiclassical regime in general, and to avoid flatness]

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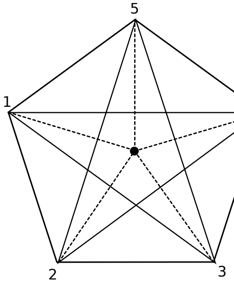

$$\langle h^{ab}_{\mathbf{n}} h^{cd}_{\mathbf{m}} \rangle_{conn} \overset{j\gg1}{\approx} \left[\left(S''\right)^{-1}\right]^{ij} (h^{ab}_{\mathbf{n}})'_i (h^{cd}_{\mathbf{m}})'_j \cancel{\xrightarrow{\gamma\to 0} G^{abcd}_{\mathbf{nm}}}$$

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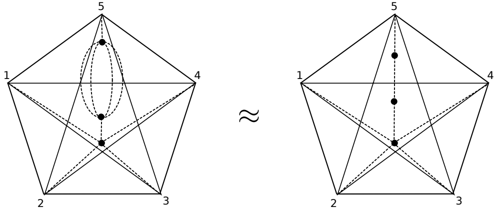
Melonic graviton

Now, let's put a melonic insertion in the previous calculation



[Relation valid at leading order in cut-off, thanks to previous study of the melon graph]

$$\langle h^{ab}_{\mathbf{n}} h^{cd}_{\mathbf{m}} \rangle^{melon}_{conn} \overset{j \gg 1}{\approx} \left[\left(S''_T \right)^{-1} \right]^{ij} (h^{ab}_{\mathbf{n}})'_i (h^{cd}_{\mathbf{m}})'_j$$

This expression is **different** from the previous one:

- the matrix S''_T is larger $\rightsquigarrow$ there are more d.o.f. one can excite
- taking the inverse mixes the entries of S''_T
  - $\rightsquigarrow$ hard to isolate new contributions
  - $\rightsquigarrow$ *a priori* all correlations are modified

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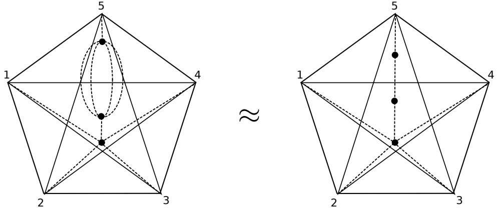

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$$\langle h^{ab}_{\mathbf{n}} h^{cd}_{\mathbf{m}} \rangle^{melon}_{conn} \overset{j \gg 1}{\approx} \left[\left(S''_T \right)^{-1} \right]^{ij} (h^{ab}_{\mathbf{n}})'_i (h^{cd}_{\mathbf{m}})'_j \xrightarrow{\gamma \to 0} G^{abcd}_{\mathbf{nm}}$$

Whence $\gamma \to 0$, correlations related to variables other than spins are killed, hence restoring the usual semiclassical result

Comment

Considering the previous graph, even if the presence of the bubble is essentially irrelevant for its amplitude (in the large spin limit), one cannot say the same for the correlations (fluctuations) on the graph

This is a feature of EPRL-FK, where the dominating contribution to the melon graph is *not* proportional to the bare gluing, and is not a projector

Avoiding this feature would be mostly desirable, if one wanted to identify the divergences with purely topological effects related to residual symmetries

Conclusions

Summary

- ▷ Spin networks, spinfoams, and EPRL-FK model
- ▷ Indications of viability of EPRL-FK: semiclassics

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- The melon graph: simple, crucial in cTGFT, but degenerate
- The EPRL-FK melon: with $SU(2)$ face weights it diverges as $\log(J)$ [vs. J^9 for $SU(2)$ BF on the same graph], it effectively modifies the gluing ($\mathbb{T}^2$) and the edge weights
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- Geometrically it can be interpreted via spikes and "anti"-spacetimes
- At leading order, the failure in reproducing the bare gluing implies a modification in the correlations on a spinfoam with and without a melonic insertion

Outlook

- ▷ Which meaning should we attach to divergences? Should they only arise from residual symmetries? Could there be any other physics attached to them?
- ▷ If there is more physics, what is the meaning of having IR divergences?
- ▷ Adding more and more bubbles: is it an empty process, or does it add details to the physics I am trying to describe? [Cf. correlations]

THANK YOU